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Question01:

step1. we could first label the 20 bottles arbitrary from 1 to 20;
step2. and then take i pills from the ith bottle;
step3. use the scale once to scale the total pills collected from step 2.

step4. if all pills were one gram each, the scale would read 210 (1+2+...+20)
since there is one bottle has pills of weight 1.1 grams
so we could get the bottle by calculate:

(weight - 210) / 0.1

Question02:

P(win game 1) = p;
P(win game 2) = $\binom{3}{1}p^2(1\text{-}p) = 3p^2 - 2p^3$

When P(win game 1) > P(win game 2) =>
(2p – 1)(p – 1) > 0 =>
when 0<p<0.5, we should choose game 1;
when 0.5<p<1, we should choose game 2;
when p = 0, or 0.5 or 1, choose either is ok.

Question03:

Since the chessboard initially has 32 black and 32 white squares, and the opposite corners must be the same color, so once we removing the two corners, we will have either 30 black and 32 white squares left, or 30 white and 32 black squares left.
However, since each domino will always take up one white and one black square, therefore 31 dominos take up 31 white and 31 black squares. That’s why it is impossible to use 31 to cover the entire board.

Question04:

For triangle:
Since the ants move at the same speed, the ants won’t collide only if all of they walk in the same direction (clockwise or counterclockwise).
P(won’t collide) =(½ * ½ * ½) * 2 = ¼ ;
P(collide) = 1 – P(won’t collide) = ¾ ;

For n-vertex polygon:

Similar as triangle:
P(won't collide) =(½ * ½ * …* ½) * 2 = $(½)^n * 2$;
P(collide) = 1 – P(won't collide) = 1 – $(½)^{n-1}$;

Question05:

We can first fill 5-quart jug, and filled 3-quart jug with 5-quart's contents, so there will be 2-quarts water left in 5-quart jug.
Then we dumped 3-quart jug and fill this jug with 2-quarts water in the other jug.
Then we fill 5-quart jug once again, and use this jug to fill 3-quart jug, since there are already 2-quartes water exist in the 3-quart jug, it will take out 1 quart from the 5-quart jug, thus there will be four quarts of water left in the 5-quart jug. Done.

Question06:
Base case: one person has blue eyes.
Assume that there are only one person has blue eyes, the blue-eyed person can look around and find out no one else has blue eyes, since everyone know at least one person has blue eyes, so this person must know he is the blue-eyed person, and take the flight that evening.

Case 2: two people have blue eyes.

When the two blue-eyed people see each other, but are unsure whether c is 1 or 2. They know, from the previous case, that if c = 1, the blue-eyed person would leave on the first night. So if on the second day's morning, they find out the blue-eyed person is still there, they will be intelligent enough to recognize that c must be 2, and they will be the blue-eyed people, and both of them will leave at the second day's night.

Case 3: general case.

As we increase c, we can see that this logic continues to apply, if c = 3, then those three people will know that there are either 2 or 3 people with blue eyes, if there are two blue-eyed people, based on the previous analysis, they will leave at the second day's night, if they do not leave, then all of them will realize that there are 3 people with blue eyes, and they will all leave at the third day's night.
This same pattern extends up through any value of c.
Hence, if c men have blue eyes, it will take c nights for the blue-eyed men to leave and all of them will leave at that night.

Question07:

What will the gender ratio of the new generation be?

First, we work out the probability for each gender sequence:

P(G) = 1 / 2, the probability of G is 1 / 2, and in this case, the number of boys * P = 0 * (1/2) = 0;
P(BG) = $(1 / 2)^2$, and in this case, the number of boys * P = 1 * (1/4) = 1/4;
P(BBG) = $(1 / 2)^3$, and in this case, the number of boys * P = 2 * (1/8) = 2/8;
P(BBBG) = $(1 / 2)^4$, and in this case, the number of boys * P = 3 * (1/16) = 3/16;
....
So once we sum the number of boys * P in each situation, we can get:
$i=0\infty i2i\approx 1$ (assume s = $i=0\infty i2i$, and use 2s – s to find the result)

Since when i - > infinite, the sum will -> 1, which means the gender ratio is even, i.e. there will be 50% girl and 50% boys.
The logical simulation is in Solution07(chat6).

Question08：

One thing we are sure is that the egg2 must do a linear search.

The most east approach is that:
We drop egg1 at 10, 20, 30, …, 100 these floors,
Once egg1 breaks, like we say it breaks at 20 floor, then we use egg2 to do linear search from 11, 12, …, to 19 to check N.
However, this approach is not quite even under each situation, for example, if egg1 breaks at 10 floor, the worst case we can find out N will be 1+9=10.
If egg1 breaks at 100, the worst case we can find out N will be 10+9=19.

In order to create a system for dropping egg1 such that the number of drops is as consistent as possible, i.e. drops(egg1) + drops(egg2) is always the same:
Assume we drop egg1 start at floor x.
If egg1 breaks, then egg2 need x-1 time to find N, drops(egg1) + drops(egg2) = 1 + x – 1 = x.
If egg1 doesn't break, then we should drop egg1 at floor x + x-1, then we can make drops(egg1) + drops(egg2) = 2 + x – 2 = x.
So we keep doing this until the floor we drop egg1 reach to 100:

x + (x-1) + (x-2) +(x-3) …+ 1 = 100

We solve x = 13.65

And after simple analysis, we find we should choose 14 instead of 13 in order to balance our content of drops.
So the result is that we drop egg1 at 14, 27, 39, 50…
This will cost 14 steps in the worse case.

I also write a computer simulation of it in Solution08(chat6).

Question09

1.The door is left open if the number of factors (which include 1 and itself) is odd.
2.And we can prove that the number of factor is odd if n is a perfect square, for example, if n is 36, the factor are (1, 36), (2, 18), (3, 12), (4, 9), (6, 6). Before (6, 6), there are always even numbers of factors, since 6 only count once, so the number of factor is odd.

3. Then, we just need find out how many perfect squares are there from 1 to 100:
1*1, 2*2, 3*3, …, 10*10. So there are 10 lockers open at the end of this process.

Question10：

Beside the first approach that:
At 0th day we drop the 0xx, 1xx, … 9xx into strip0, strip1, … strip9 respectively. And the 1th day drop the x0x, x1x, … x9x into strip0, strip1, … strip9 respectively. And the 2th day, xx0, xx1, …, xx9 into strip0, strip1, … strip9 respectively. In order to distinguish situation like 383, 388, we should at the 3th day to drop xx9, xx8, xx7, … xx0 into strip0, strip1, … strip9 respectively. Then we check the color from the 7^{th} to 10^{th} day, and get the bottle number accordingly.

An optimal way to approach this question is that:
Since it is a binary indicator for poisoned or unpoisoned.
Can we map 1000 keys to 10 binary values such that each key is mapped to a unique configuration of values? Yes, that’s how binary number works.

So the optimal way is that: we take each bottle number and look its binary representation. For example, for number 3 bottle, 0011, then we just add a drop of this bottle’s contents to strip0 and strip1. Since 210 is 1024 > 1000, so 10 test strips will be enough to handle 1000 bottles.

After wait seven days, we just check which strips get positive and use their number presented by binary number to get the bottle’s number.

Follow up is in the Solution10 (chat6).