Computational Statistics Assignment 2

by James D. Wilson (University of San Francisco)

1. Suppose that we model observed data $\mathbf{y}$ with the joint probability density function $f(\mathbf{y} \mid \theta)$.
   (a) Describe the assumptions about θ from both the *frequentist* and *Bayesian* standpoint.
   (b) Suppose that we would like to determine whether or not $\theta = 0$. Describe how to go about this from both a *frequentist* and *Bayesian* standpoint.
   (c) When do *frequentist* and *Bayesian* inference provide the same conclusions about θ from your inferential tasks described in (b)?
   (d) What is the difference between a 95% confidence interval and a 95% credible interval for θ?

2. State which methods you would use to perform the following computations. If more than 1 method is needed, provide pseudo-code to describe how to go about the computation. Further, if there are multiple options, please write all options that you can think of down.
   (a) Estimate $\mathbb{E}[\log(|\theta|)|y]$ when

$$p(\theta \mid y) = \frac{1}{\pi y}\left(1 + \left(\frac{\theta - 1}{y}\right)^2\right)$$

   (b) Estimate $\mathbb{E}[\log(|\theta|)|y]$ when $p(\theta \mid y)$ is unknown, but we know that $p(\theta \mid y) \propto q(\theta \mid y)$
   (c) Simulate from an unknown $p(\theta \mid y)$
   (d) Simulate from a known but unrecognizable $p(\theta \mid y)$

3. **Creating animations with MCMC** Read the blog post about creating animations with MCMC here: `https://jankrepl.github.io/creating-animations-with-MCMC/`. Using the source code from the author, create GIF animations for two different images of your choice (example - the USF logo). For each image, run rejection sampling, Gibbs, and Metropolis Hastings on each image with the following parameters
   (a) Run 1, 2, 3, and 4 chains for each with different colors for each chain and run each with 10000 samples.
   (b) Repeat (a) but with 100, 500, 1000, and 3000 samples.

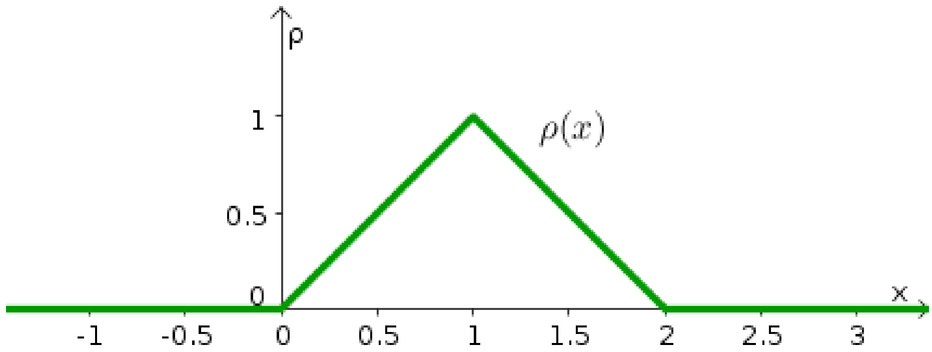
   Now answer the following questions:
   (a) Based on the animation, what is the importance of running multiple chains? What happens if we only use say 1 chain?
   (b) Based on your sampling procedures and sample sizes, which method seems to converge to the distribution of the image the fastest? Which is the slowest?
   (c) Based on your sample size analysis, how many samples and chains would you suggest someone uses for each sampling procedure and each image?

4. In 2015, ABC News conducted a survey of 600 people before a presidential debate between Trump and Clinton, and an independent group of 600 people were polled after the presidential debate. The results were recorded as follows:

Survey	Trump	Clinton	Other
pre-debate	215	310	75
post-debate	280	290	30

Let α_1 be the proportion of people that supported Trump before the presidential debate. And let α_2 be the proportion of people that supported him after the debate. Construct a Bayesian model from which you analyze the value $\alpha_2 - \alpha_1$. State your date generating distribution, and your prior and conduct MCMC to sample from the posterior of α_1, α_2, and $\alpha_2 - \alpha_1$. What is the probability that there was a shift away from Clinton after the debate?

5. Suppose that you have the following density $p(x)$ below (and you have its functional form).



Your goal is to simulate 1000 values from this density. Consider doing this using rejection sampling.

(a) What proposal function would you use to simulate from $p(x)$?

(b) What M would you use based on your proposal function?

(c) Consider testing the efficiency of your sampling procedure based on your choice M. Choose a grid of 100 plausible M values that still satisfy the constraints of rejection sampling. Apply rejection sampling across the values of M to retrieve 1000 samples from $p(x)$. For each M, calculate the rejection rate (i.e., what proportion of samples were rejected) and the time it takes for the sampling to run. Plot these values as a function of M and discuss what you find in terms of efficiency of your algorithm.