本节内容

红黑树

插入操作

红黑树的插入

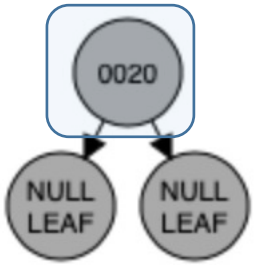
从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18

- 先查找，确定插入位置（原理同二叉排序树），插入新结点
- 新结点是根——染为黑色
- 新结点非根——染为红色
  - 若插入新结点后依然满足红黑树定义，则插入结束
  - 若插入新结点后不满足红黑树定义，需要调整，使其重新满足红黑树定义
    - 黑叔：旋转+染色
      - LL型：右单旋，父换爷+染色
      - RR型：左单旋，父换爷+染色
      - LR型：左、右双旋，儿换爷+染色
      - RL型：右、左双旋，儿换爷+染色
    - 红叔：染色+变新
      - 叔父爷染色，爷变为新结点

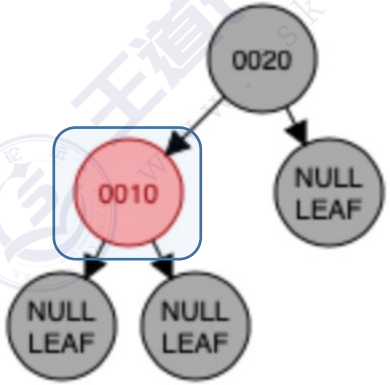
如何调整：看新结点叔叔的脸色

红黑树的插入

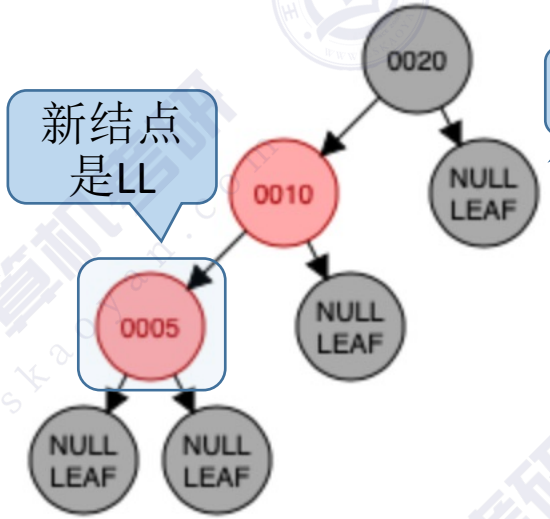
从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18



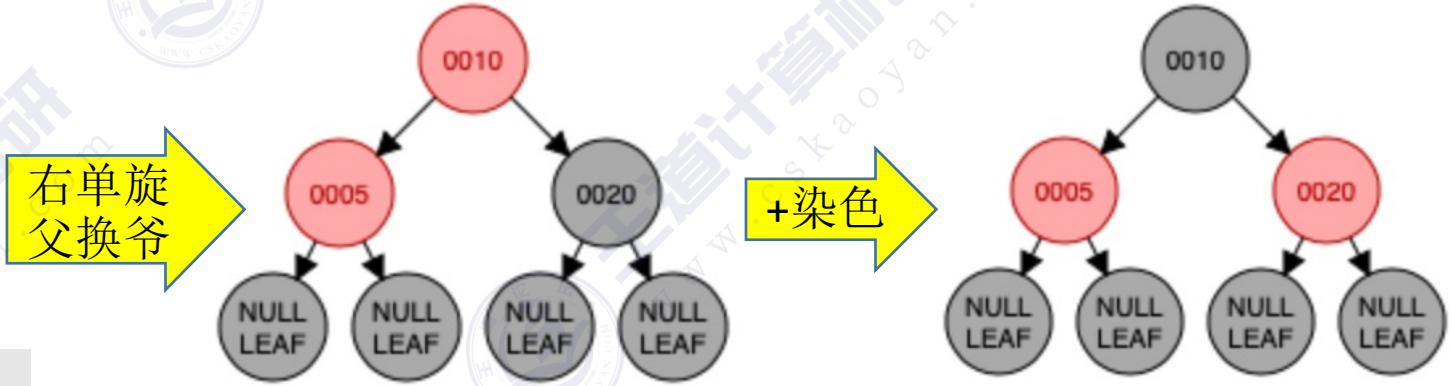
新结点是根，染黑



新结点非根，染红
依然满足红黑树



新结点非根，染红
违反“不红红”



- 黑叔：旋转+染色
  - LL型：右单旋，父换爷+染色

红黑树的插入

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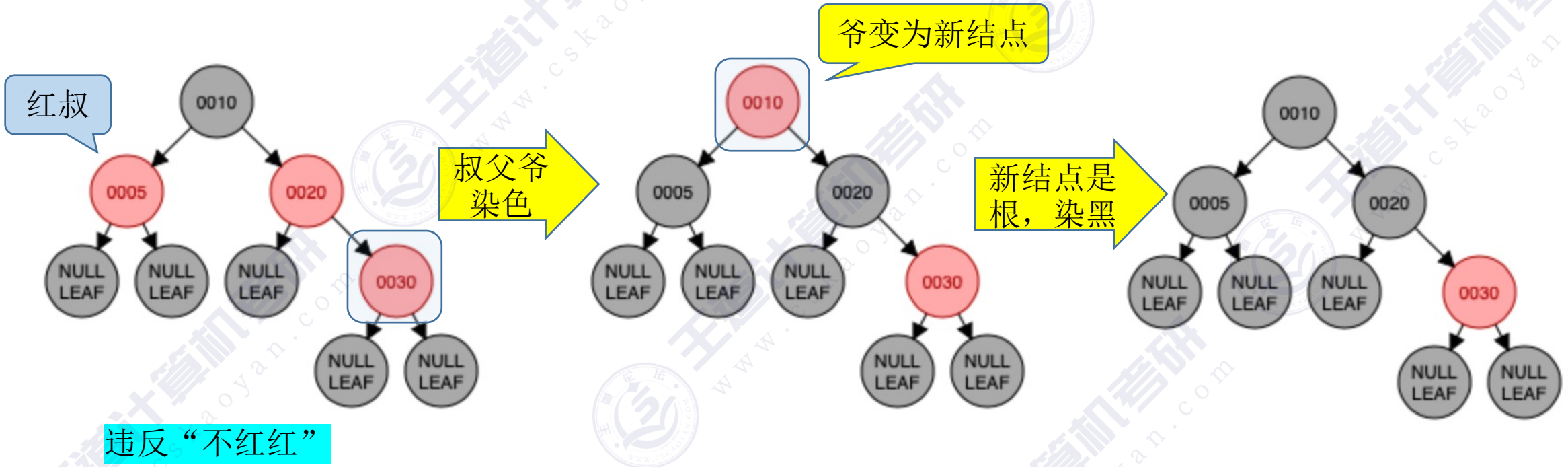

违反“不红红”

- 红叔：染色+变新
  - 叔父爷染色，爷变为新结点

红黑树的插入

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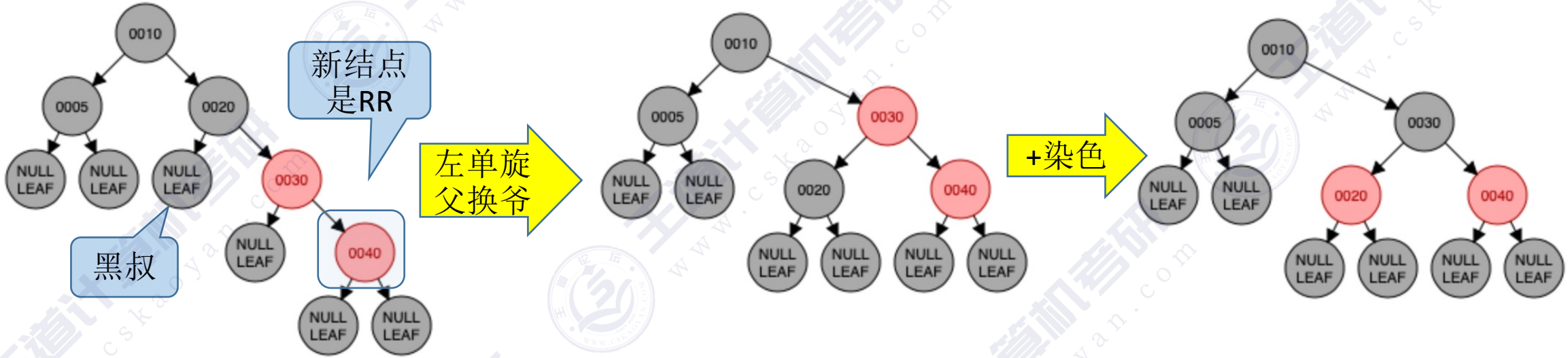

- 黑叔：旋转+染色
  - RR型：左单旋，父换爷+染色

红黑树的插入

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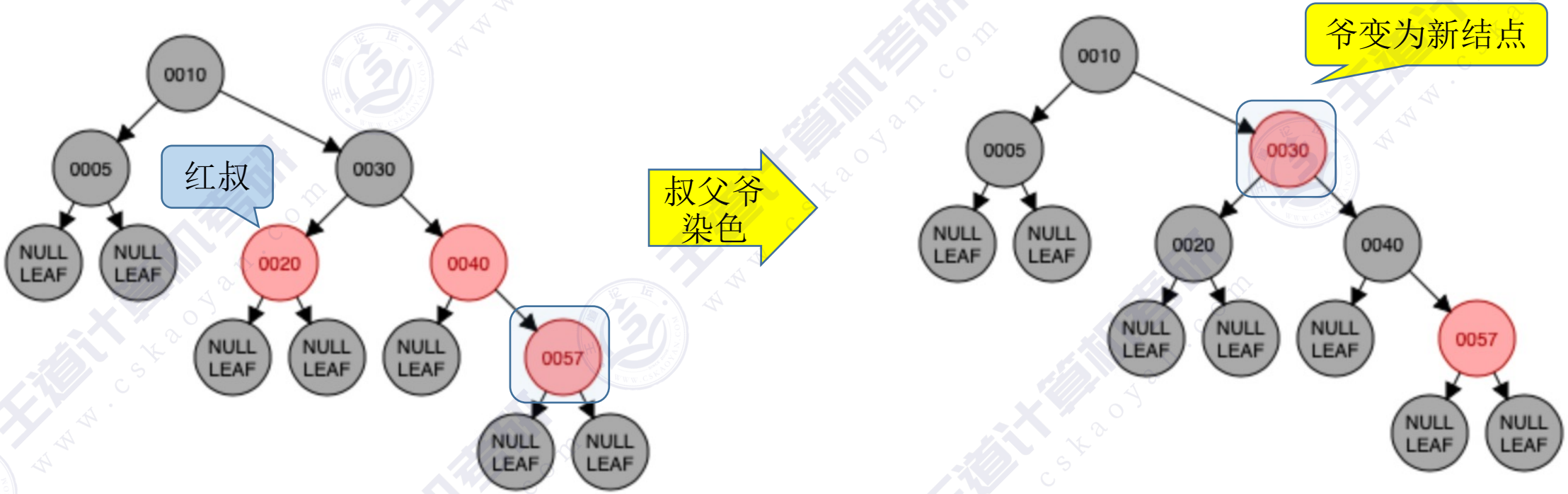

- 红叔：染色+变新
  - 叔父爷染色，爷变为新结点

红黑树的插入

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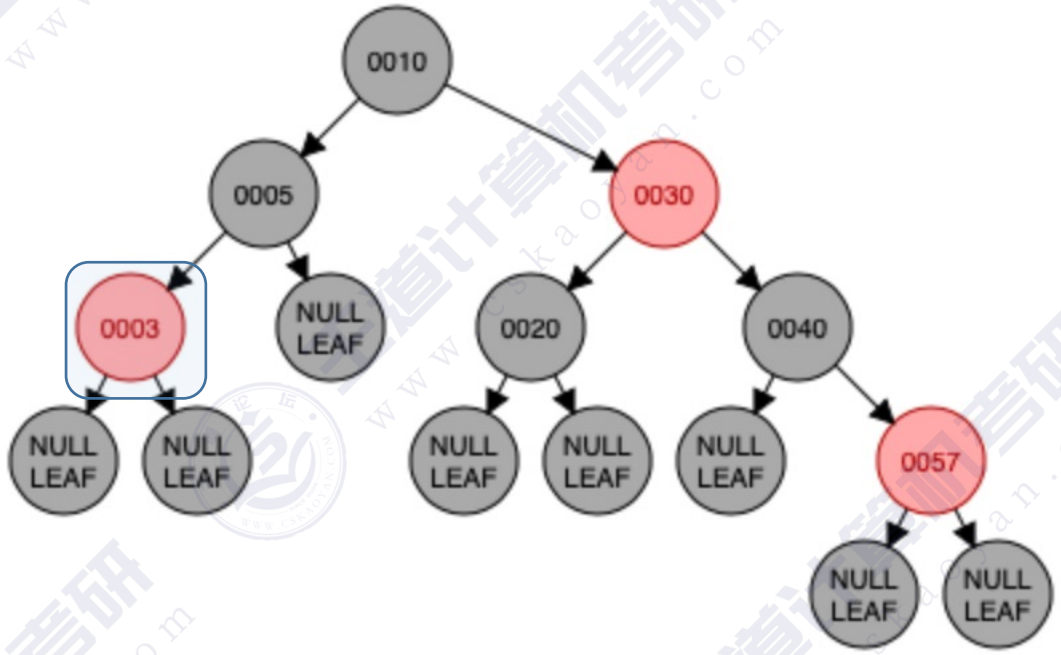

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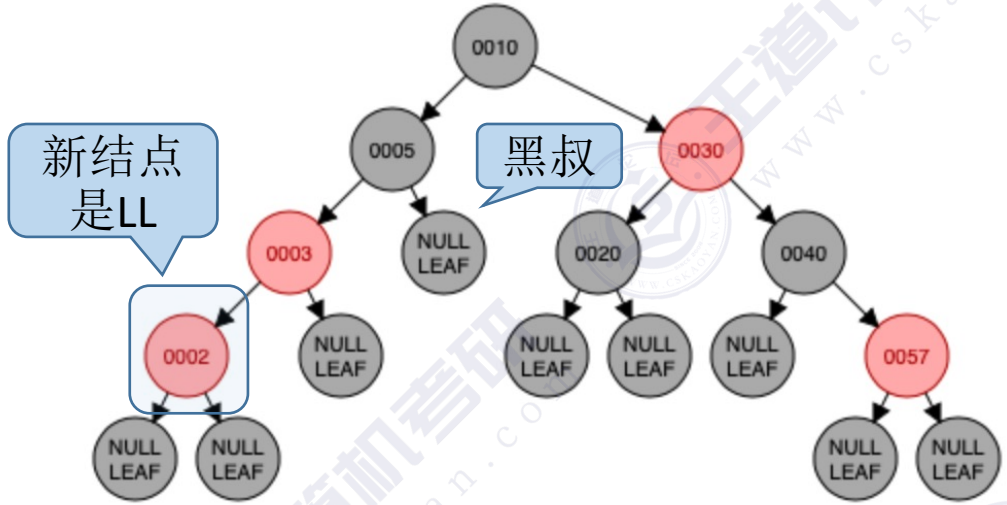
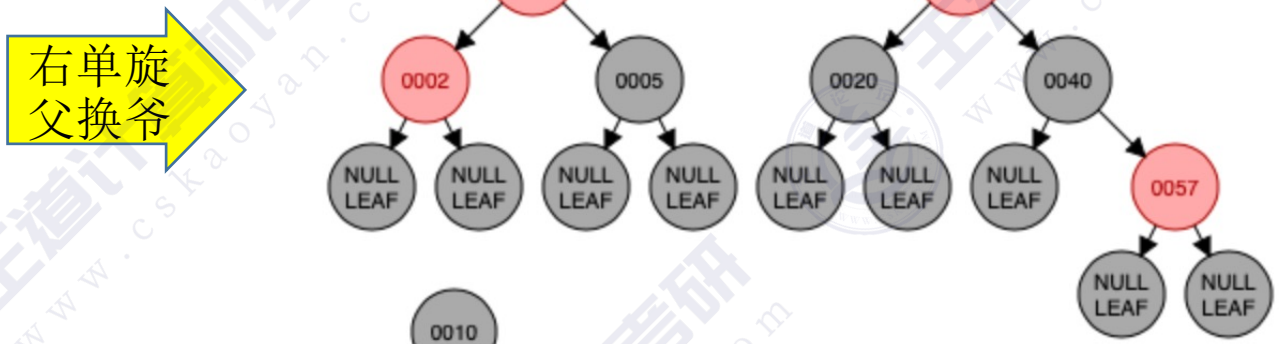
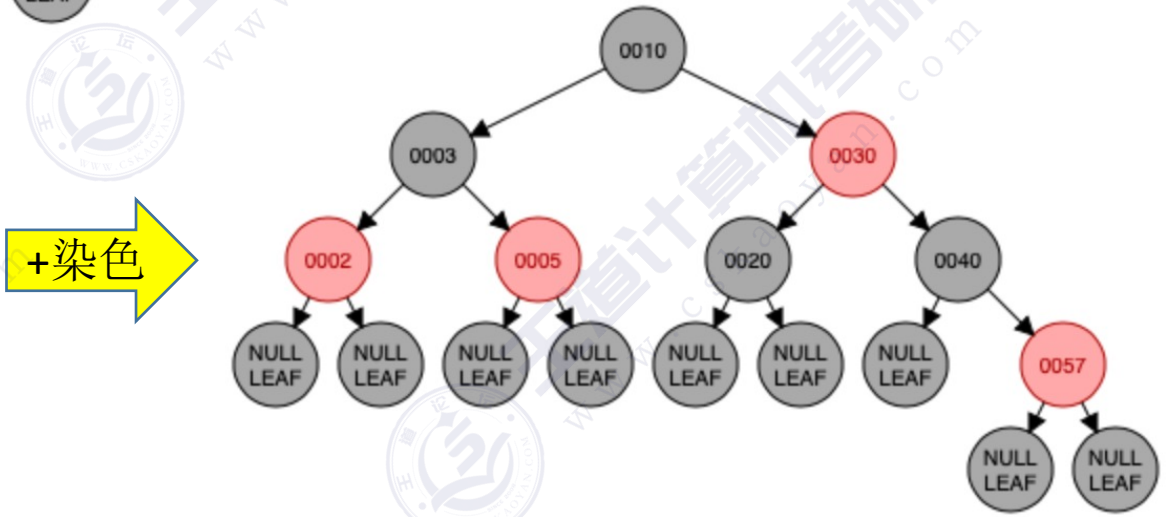


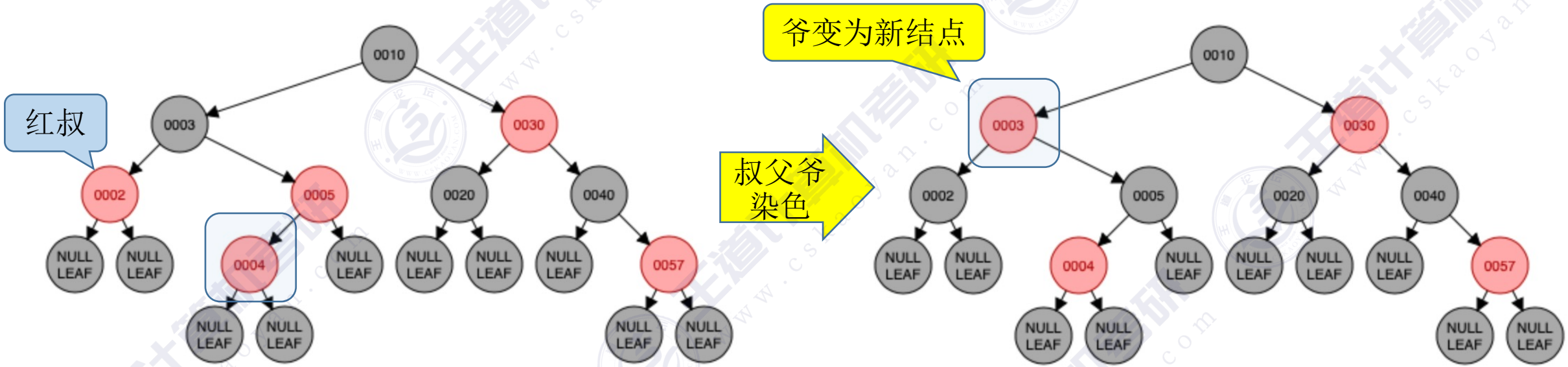




- 黑叔：旋转+染色
  - LL型：右单旋，父换爷+染色

红黑树的插入

从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18



- 红叔：染色+变新
  - 叔父爷染色，爷变为新结点

红黑树的插入

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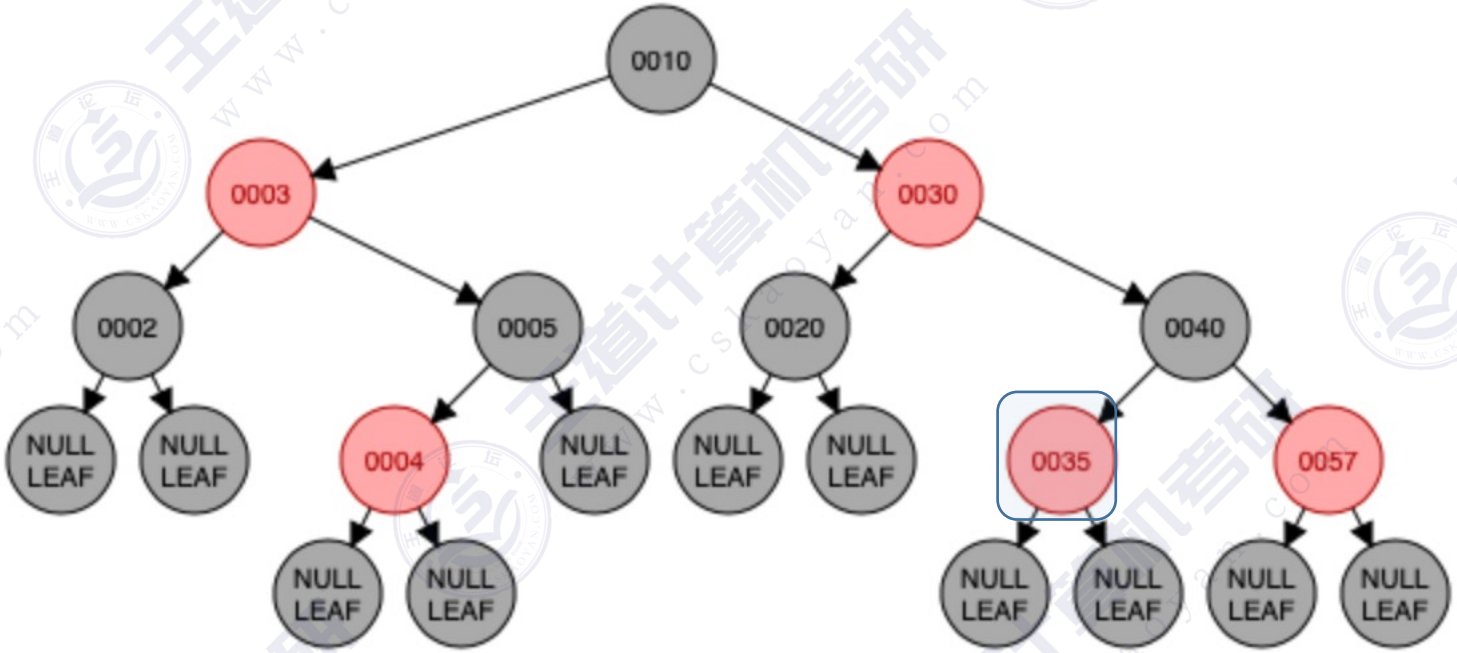

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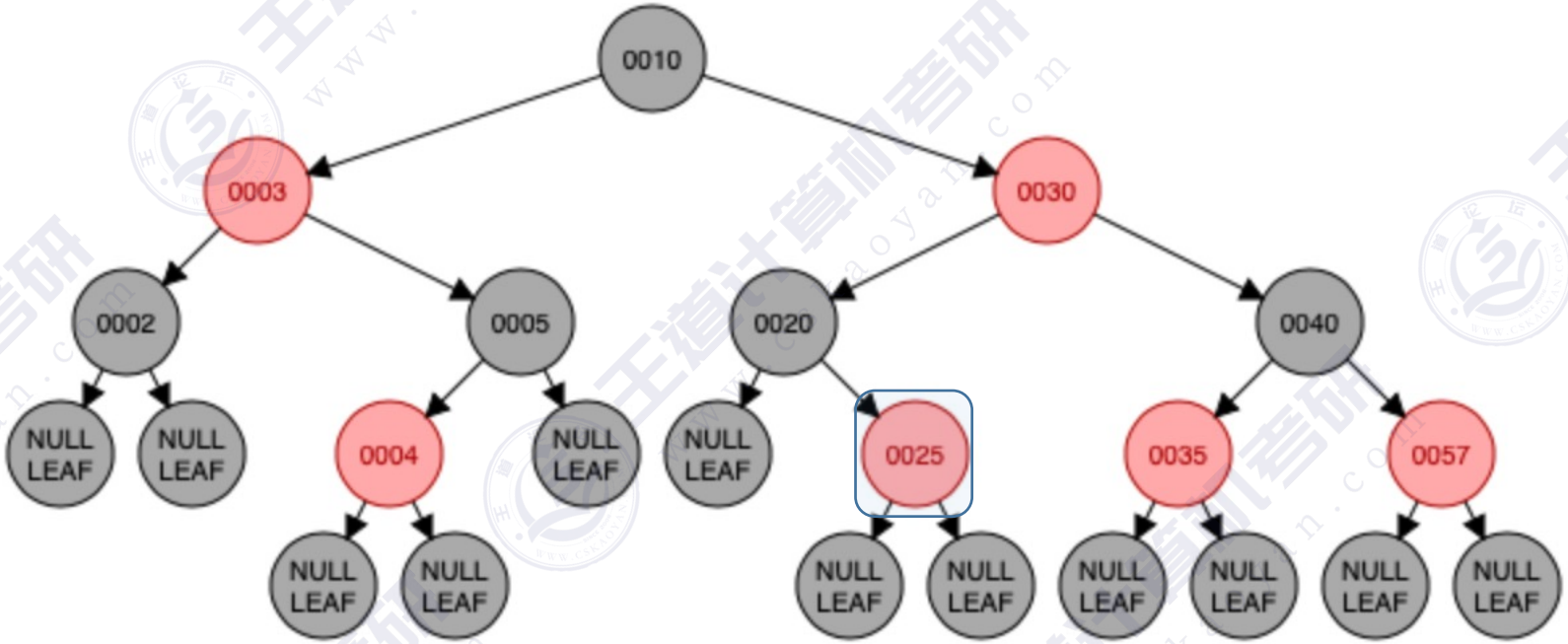

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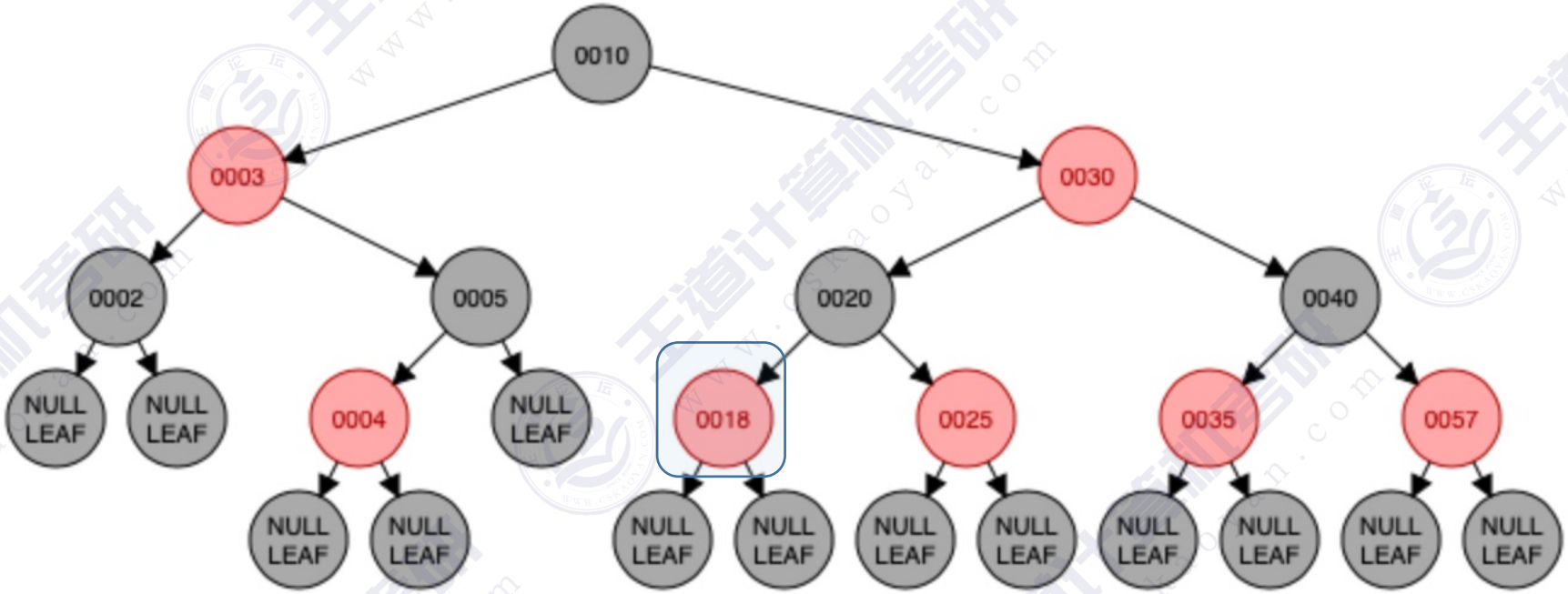

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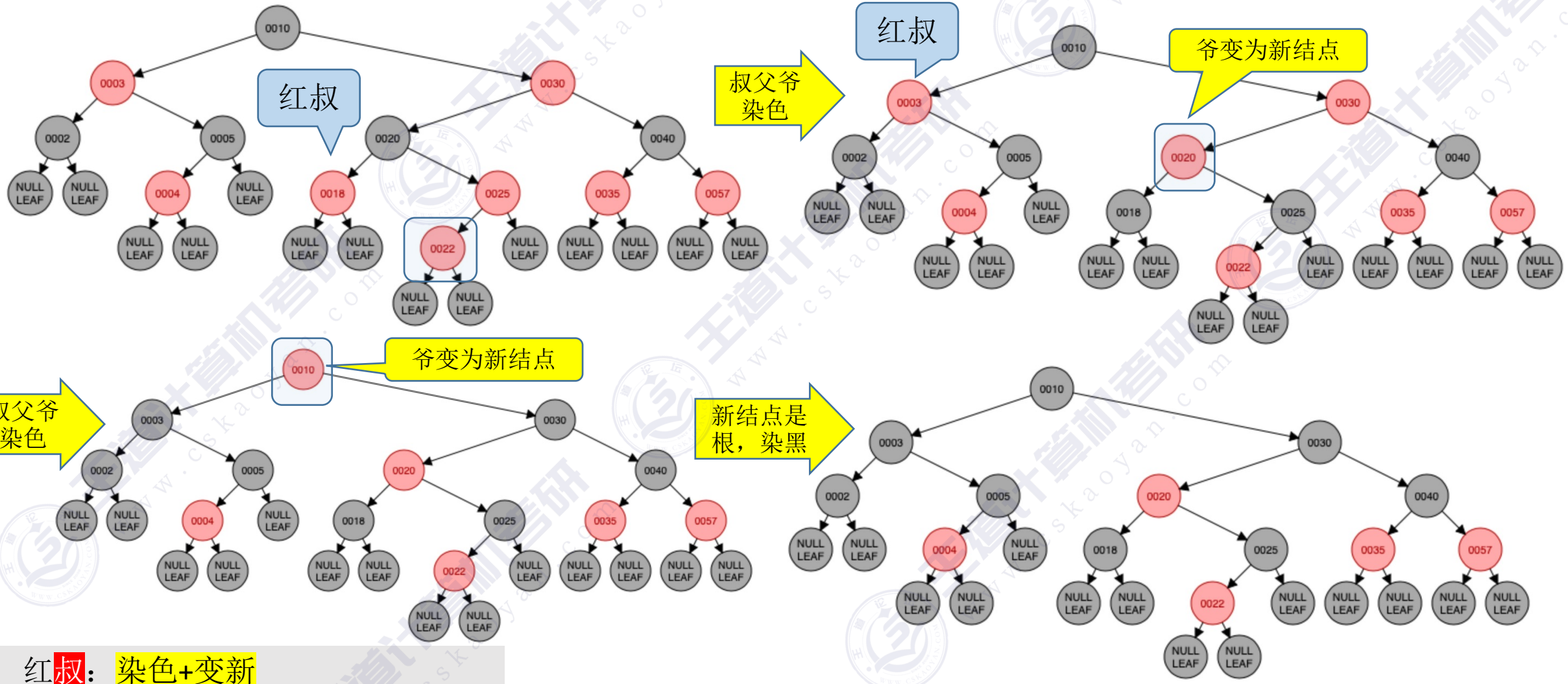

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  - 叔父爷染色，爷变为新结点

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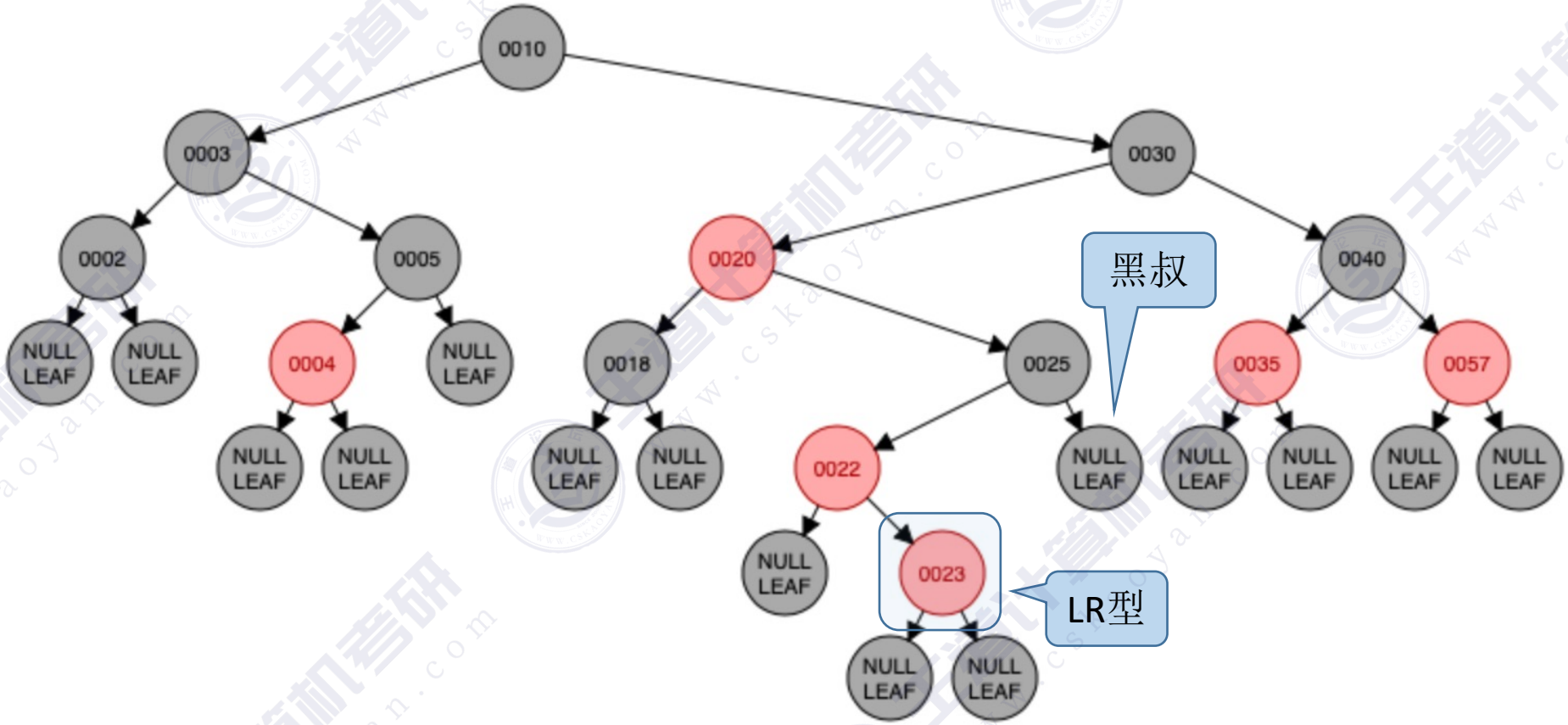
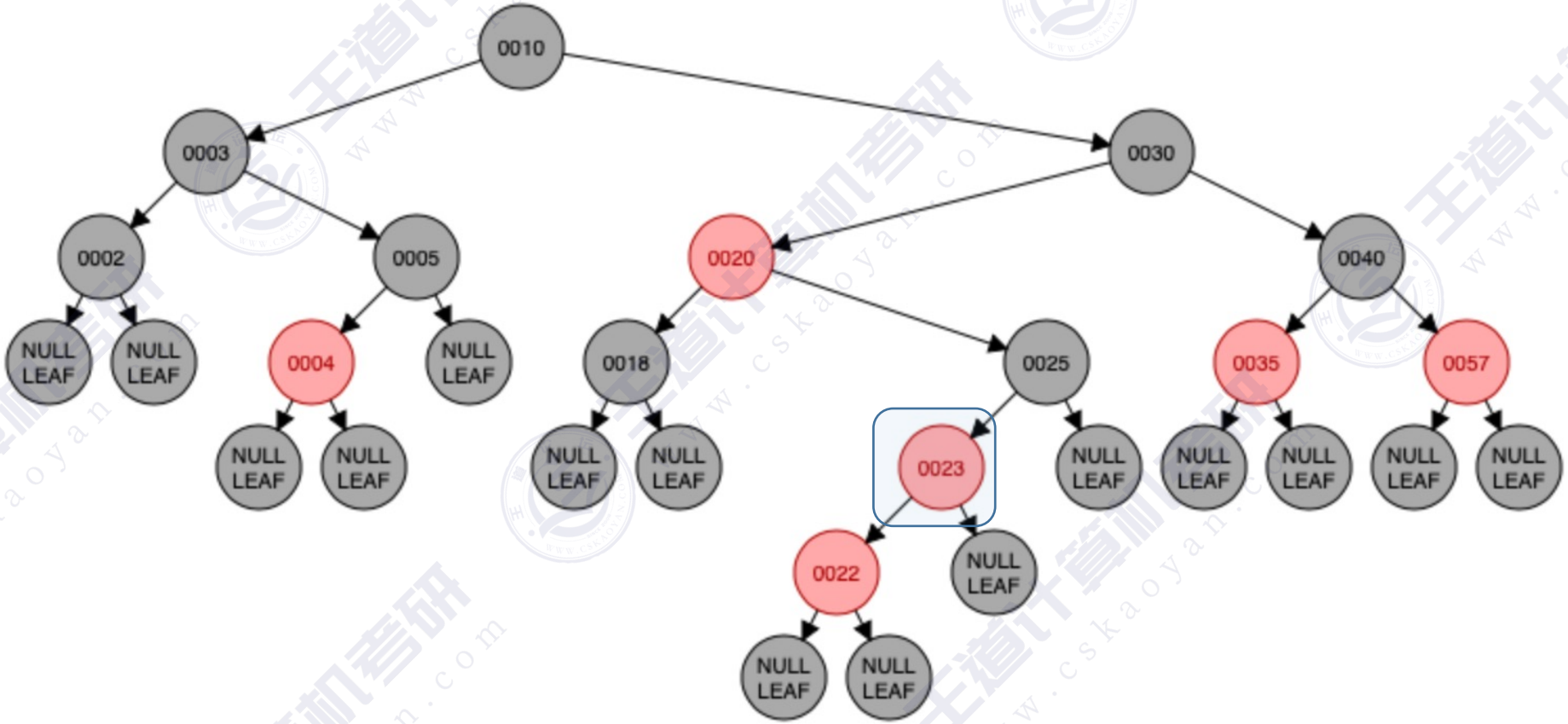


- 黑叔：旋转+染色
  - LR型：左、右双旋，儿换爷+染色

红黑树的插入

从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18

左单旋



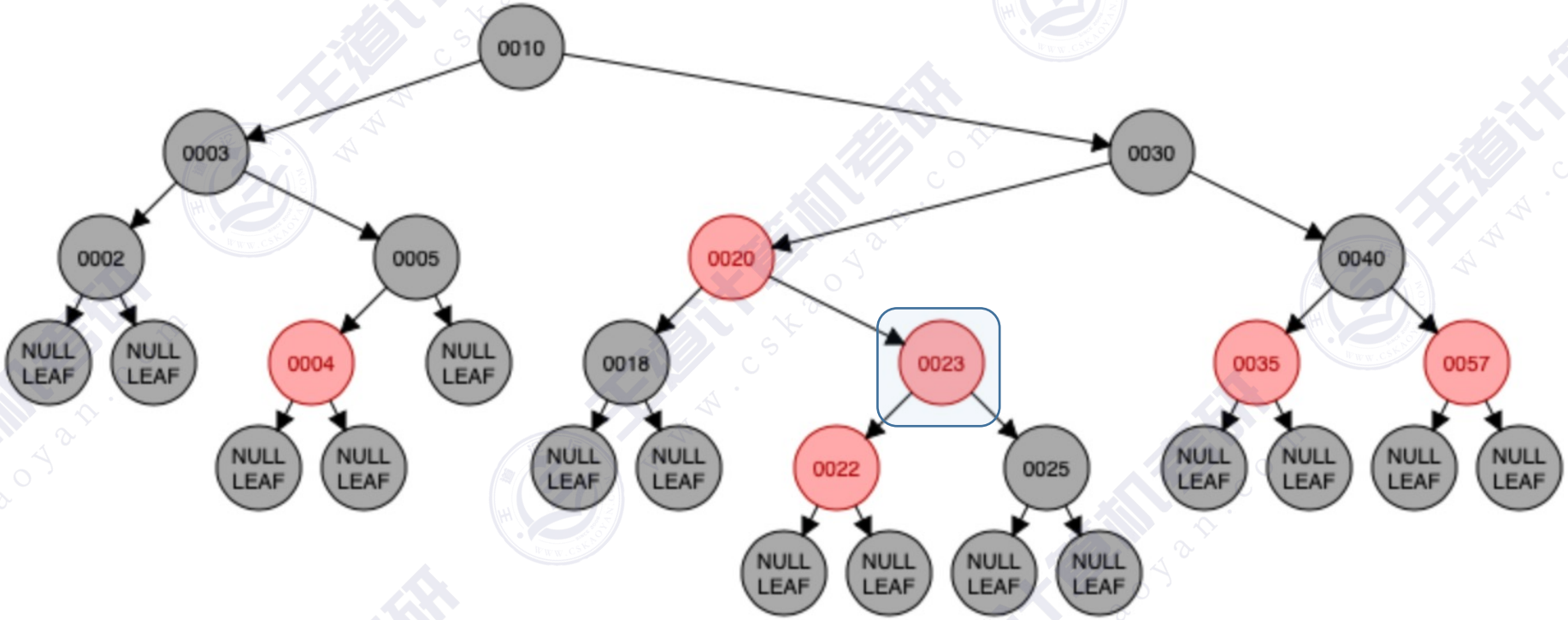
①左单旋的结果：儿子辈分升高

- 黑叔：旋转+染色
  - LR型：左、右双旋，儿换爷+染色

红黑树的插入

从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18

右单旋



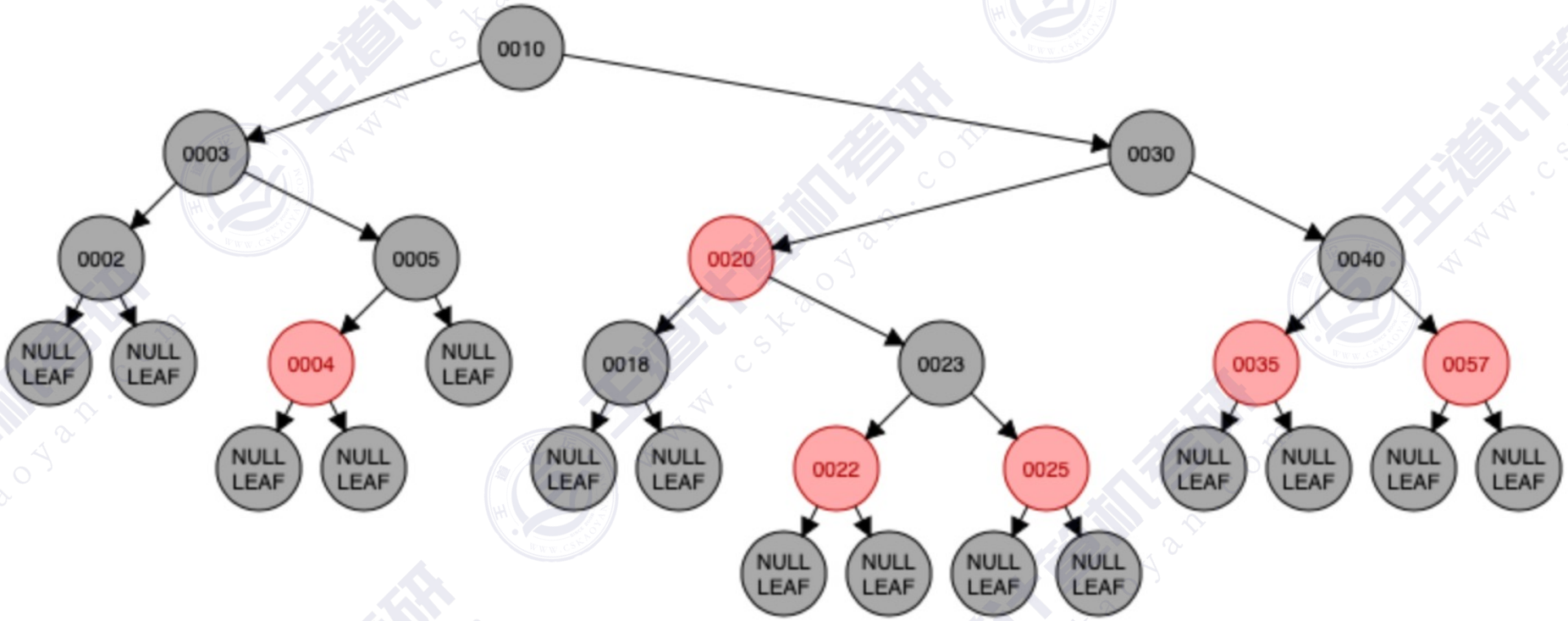
②右单旋的结果：儿子辈分再升高

- 黑叔：旋转+染色
  - LR型：左、右双旋，儿换爷+染色

红黑树的插入

从一棵空的红黑树开始，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18

原来的儿和爷“染色”



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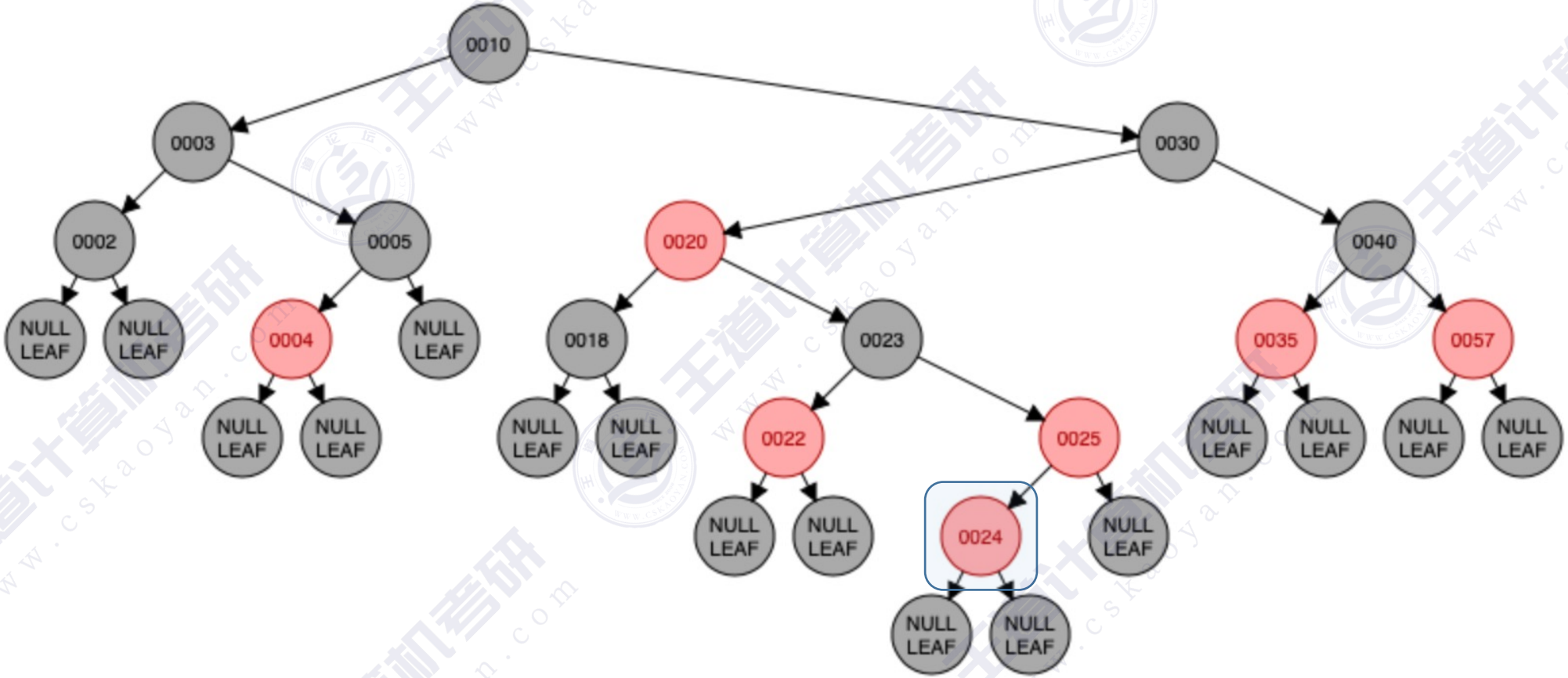

- 红叔：染色+变新
  - 叔父爷染色，爷变为新结点

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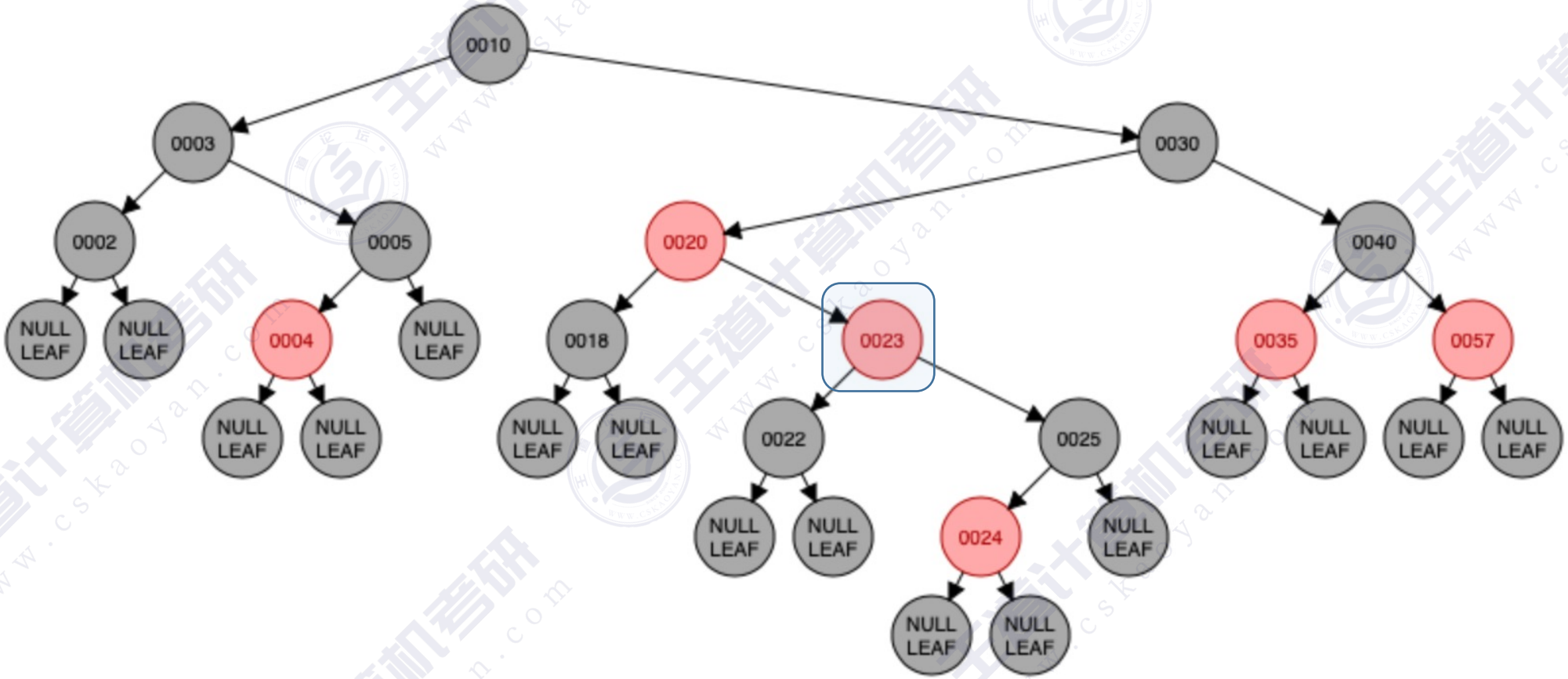

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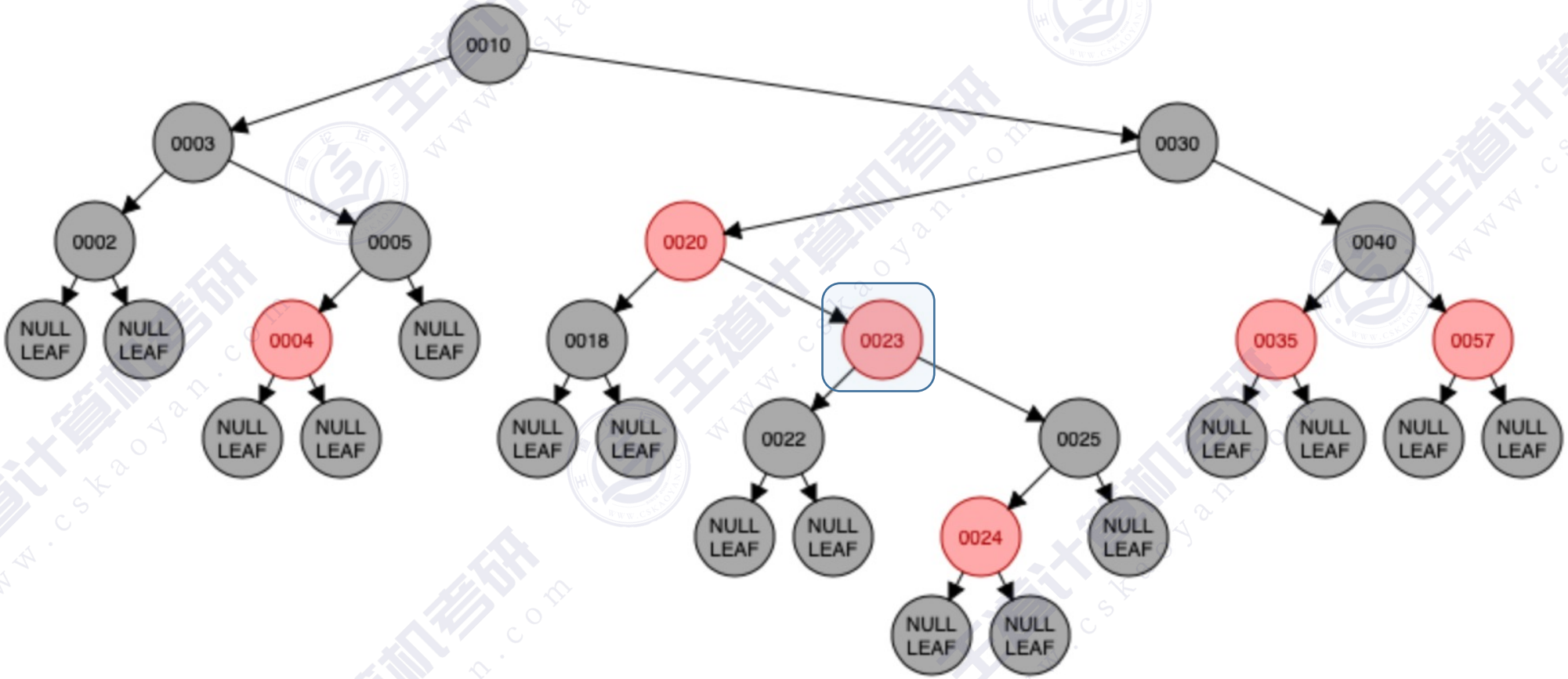

- 黑叔：旋转+染色
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红黑树的插入

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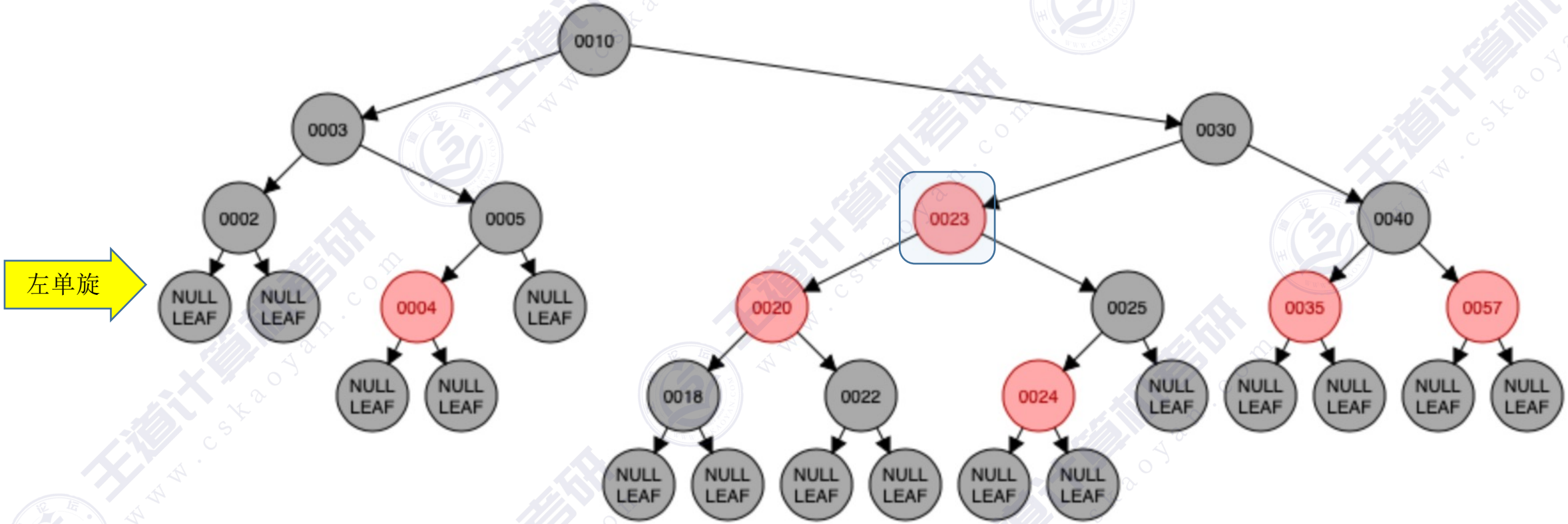

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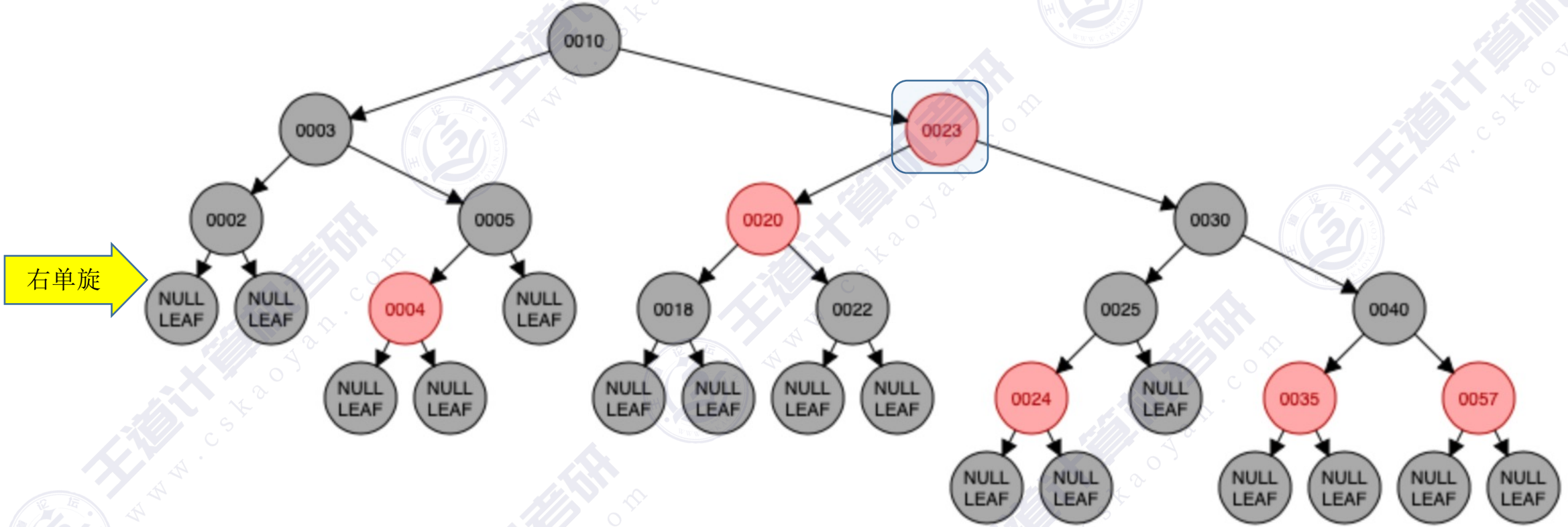

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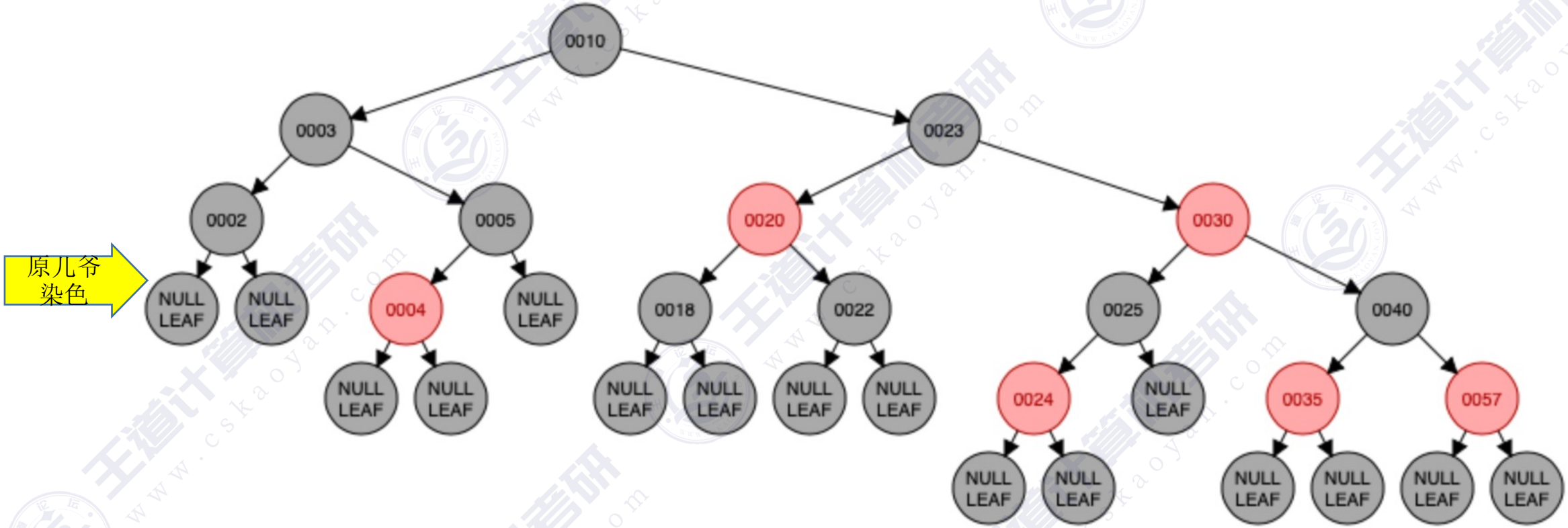

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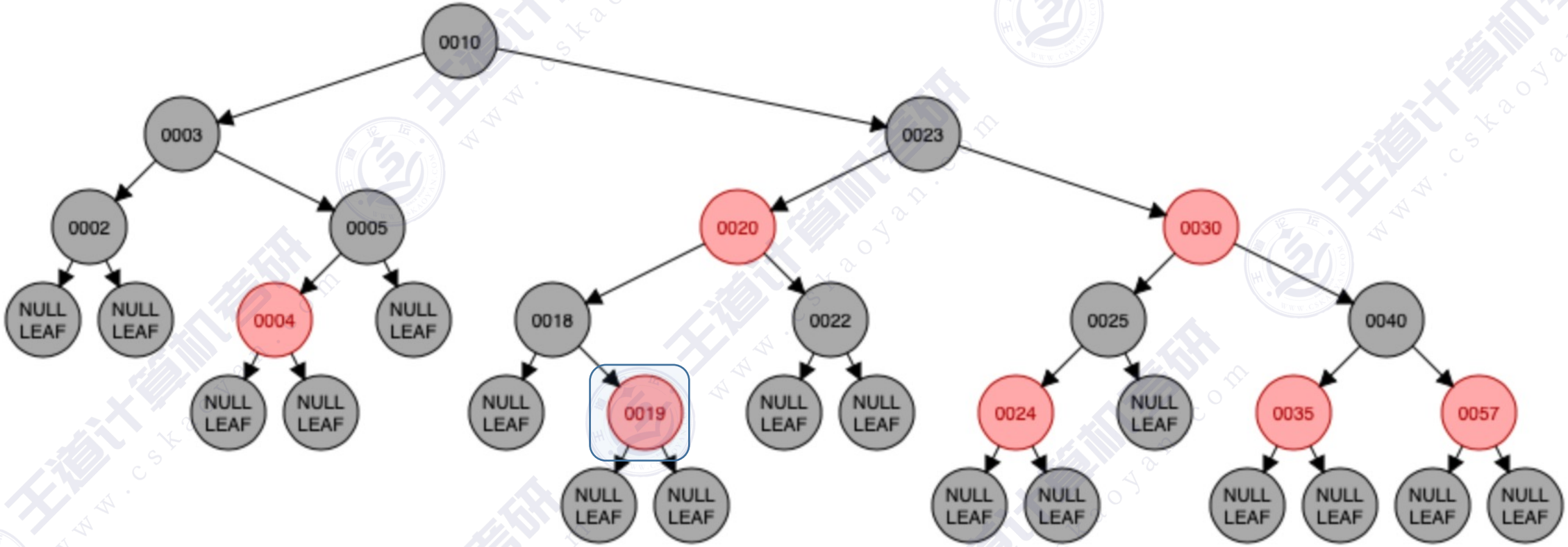

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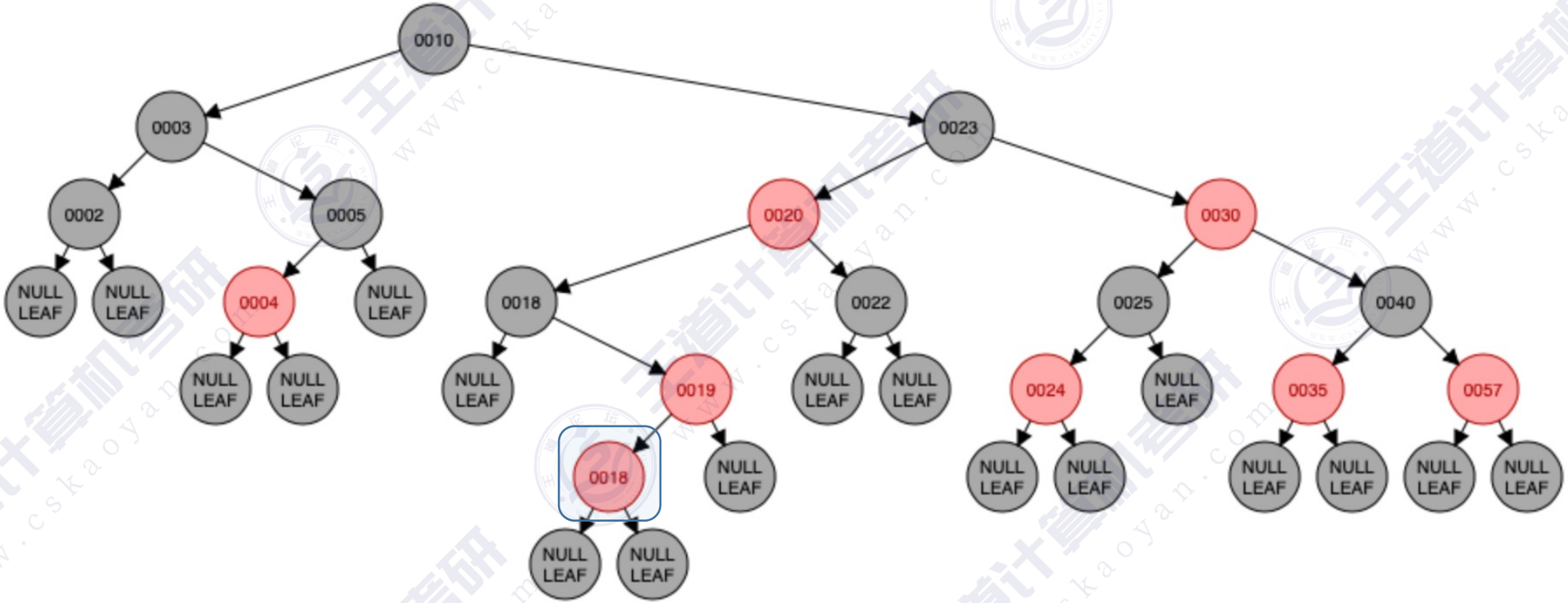

- 黑叔：旋转+染色
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红黑树的插入

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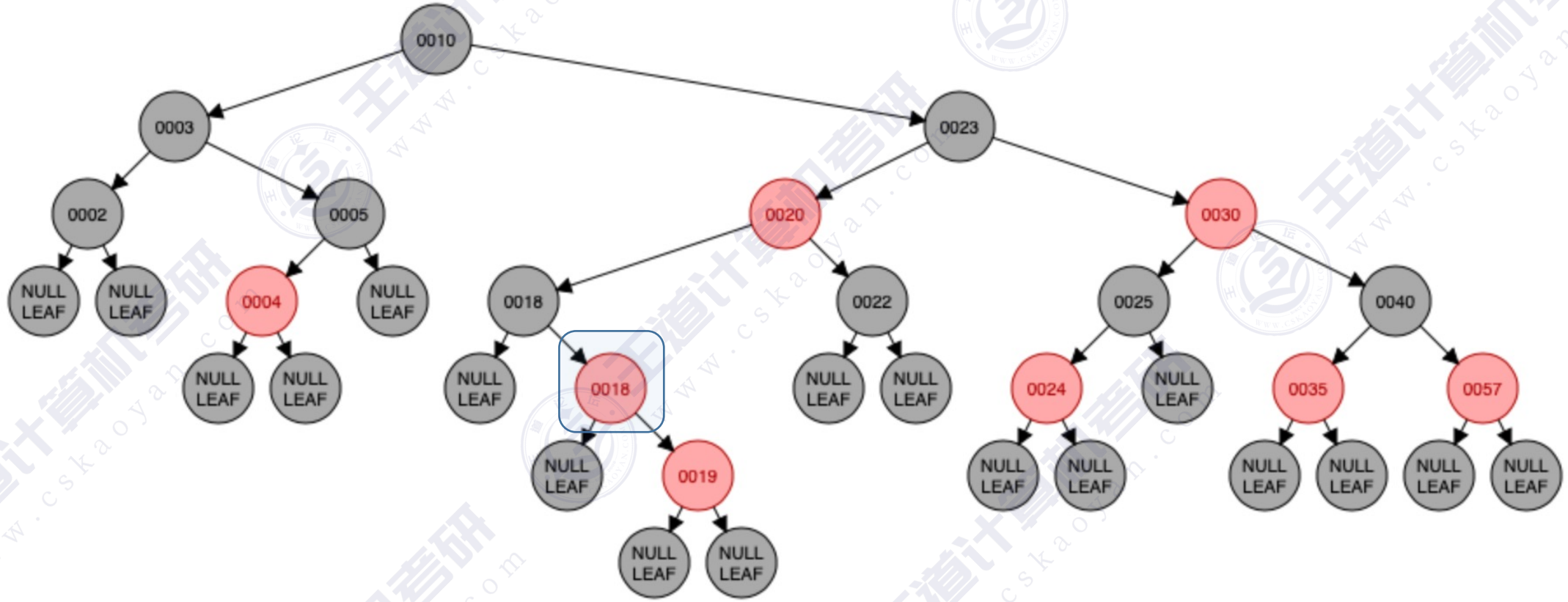
右单旋



- 黑叔：旋转+染色
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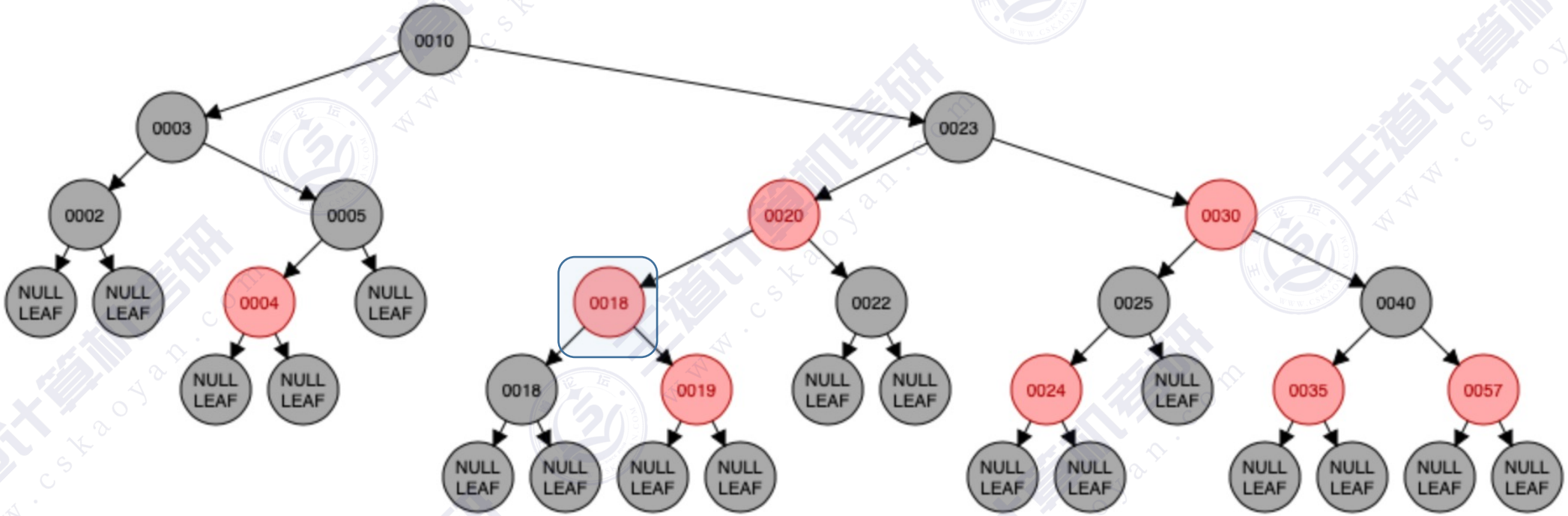
左单旋



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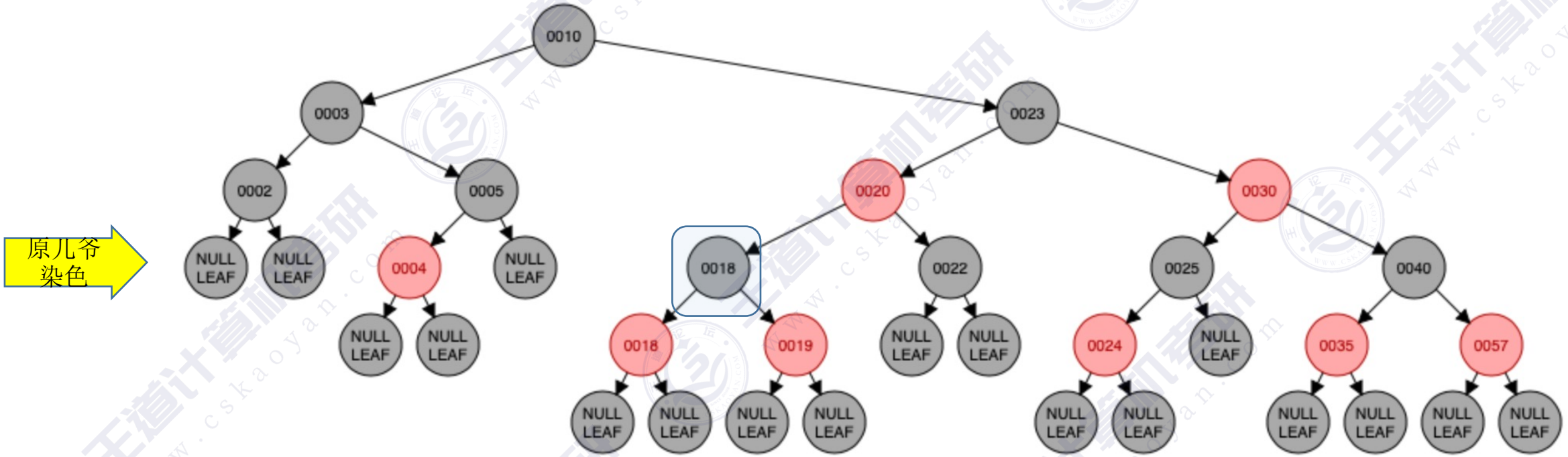


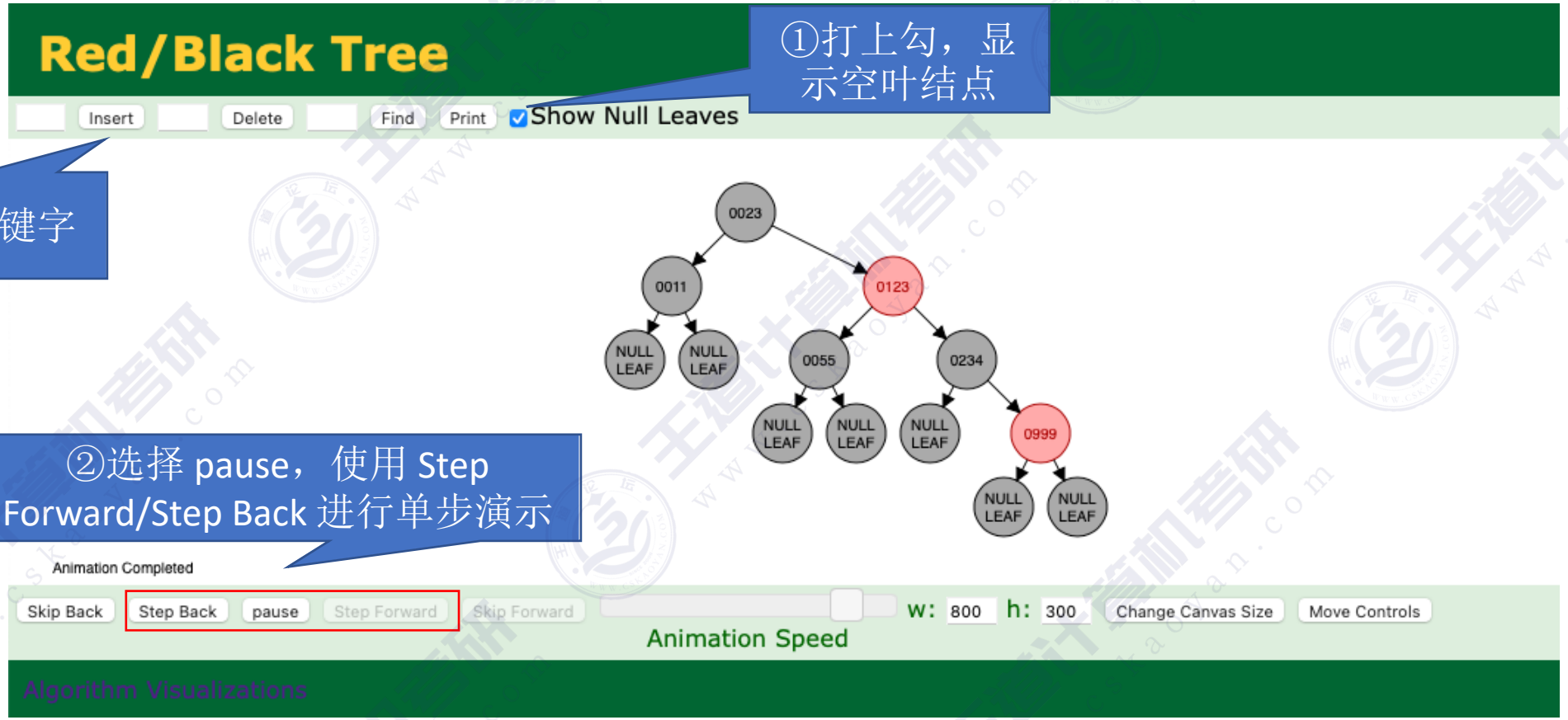
- 黑叔：旋转+染色
  - RL型：右、左双旋，儿换爷+染色

知识回顾与重要考点


红黑树RBT
相关概念
黑高 —— 一个结点的“黑高”：从某结点出发（不含该结点）到达任一空叶结点的路径上黑结点总数
叶结点 —— 在RBT中，“叶结点”通常指“失败结点”（又名：空叶结点/NULL结点/外部结点）
定义
左根右 —— RBT是一种BST，需满足 左≤根≤右
根叶黑 —— 根节点、叶结点一定是黑色
不红红 —— 任何一条查找路径上不能连续出现两个红结点
黑路同 —— 从任一结点出发，到达任一空叶结点的路径上经过的黑结点数量都相同
性质
从根节点到叶结点的最长路径不大于最短路径的2倍
有n个内部节点的红黑树高度 h≤2log (n+1)
插入
新结点为根染黑
新结点非根染红，若插入后违反“不红红”，则看叔叔的脸色
黑叔：旋转+染色
LL —— 右单旋，父换爷+染色
RR —— 左单旋，父换爷+染色
LR —— 左、右双旋，儿换爷+染色
RL —— 右、左双旋，儿换爷+染色
红叔：染色+变新 —— 叔父爷染色，爷变为新结点

红黑树练习方法（插入操作）

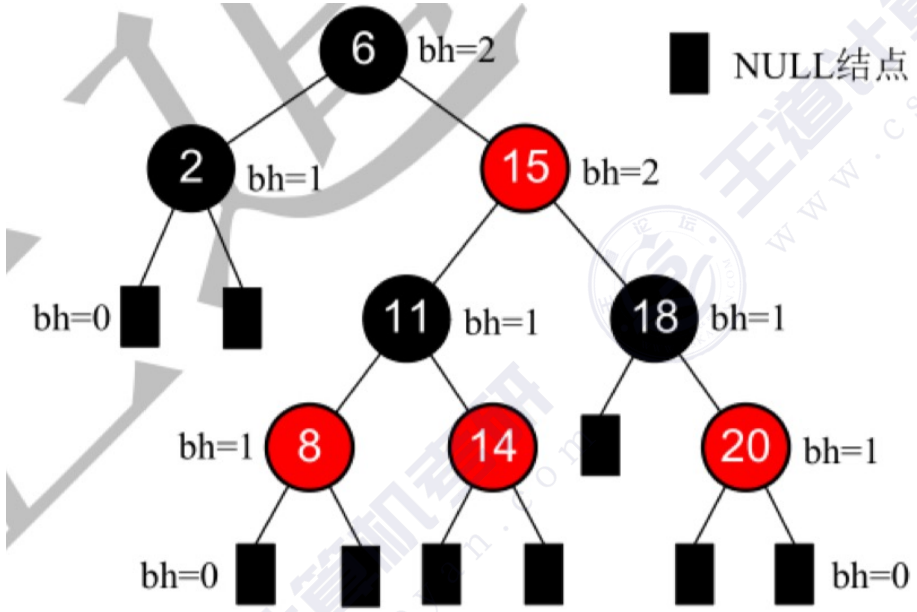
课件中的例子，插入：20, 10, 5, 30, 40, 57, 3, 2, 4, 35, 25, 18, 22, 23, 24, 19, 18

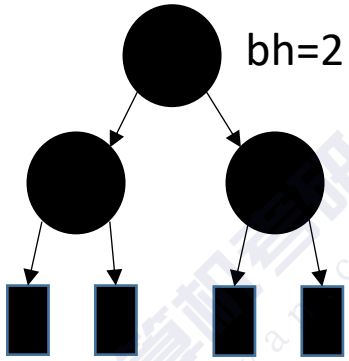


自我训练方法：你可以自己设计一些插入序列，每一次插入后，都先思考应该如何调整，然后在网站中演示验证。插入新元素时，尽可能覆盖 黑叔LL/RR/LR/RL、红叔 的情况。

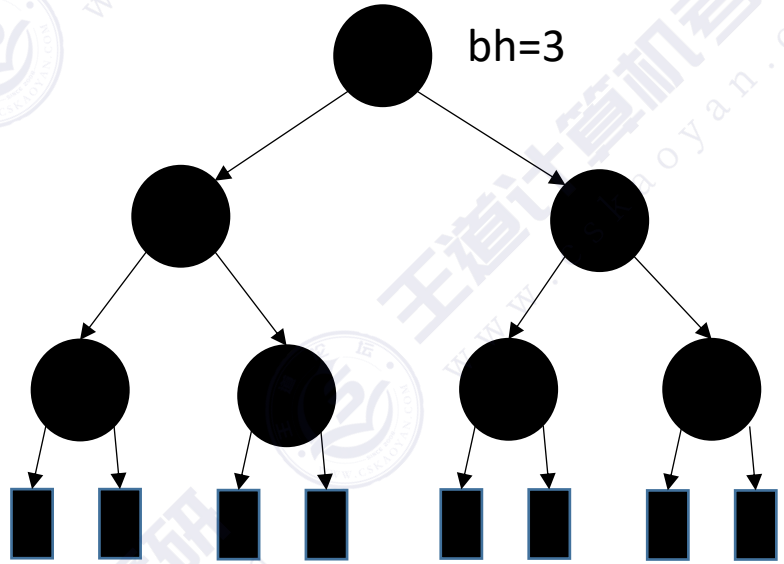
https://www.cs.usfca.edu/~galles/visualization/RedBlack.html

与“黑高”相关的推论





根节点黑高=2，内部结点数最少的情况



根节点黑高=3，内部结点数最少的情况

结点的黑高 bh —— 从某结点出发（不含该结点）到达任一叶结点的路径上黑结点总数

思考：根节点黑高为 h 的红黑树，内部结点数（关键字）至少有多少个？
回答：内部结点数最少的情况——总共h层黑结点的满树形态
结论：若根节点黑高为h，内部结点数（关键字）最少有 2^h-1 个

红黑树的定义→性质

红黑树是二叉排序树 → 左子树结点值 ≤ 根结点值 ≤ 右子树结点值

与普通BST相比，有什么要求 →

①每个结点或是红色，或是黑色的
②根节点是黑色的
③叶结点（外部结点、NULL结点、失败结点）均是黑色的
④不存在两个相邻的红结点（即红结点的父节点和孩子结点均是黑色）
⑤对每个结点，从该节点到任一叶结点的简单路径上，所含黑结点的数目相同

左根右，根叶黑
不红红，黑路同

张口就是freestyle

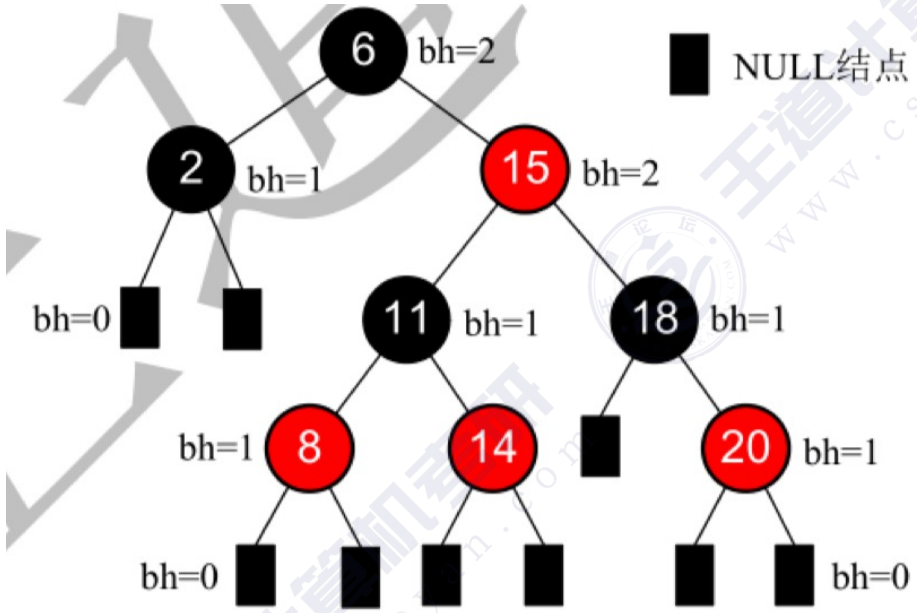
→ 性质1：从根节点到叶结点的最长路径不大于最短路径的2倍
性质2：有n个内部节点的红黑树高度 $h \leq 2log_2(n+1)$

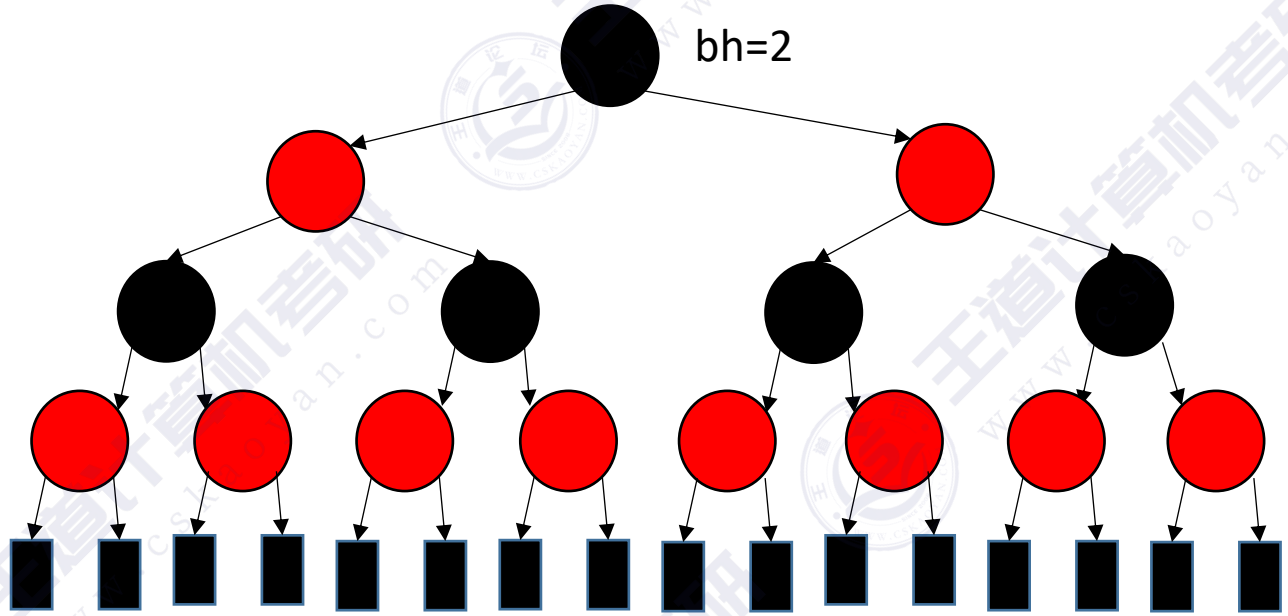
→ 红黑树查找操作时间复杂度 = $O(log_2 n)$

性质1证明：任何一条查找失败路径上黑结点数量都相同，而路径上不能连续出现两个红结点，即红结点只能穿插在各个黑结点中间

性质2证明：若红黑树总高度=h，则根节点黑高 ≥ h/2，因此内部结点数 $n \geq 2^{h/2}-1$，由此推出 $h \leq 2log_2(n+1)$

与“黑高”相关的推论





根节点黑高=2，内部结点数最多的情况

结点的黑高 bh —— 从某结点出发（不含该结点）到达任一叶结点的路径上黑结点总数

思考：根节点黑高为 h 的红黑树，内部结点数（关键字）至多有多少个？
回答：内部结点数最多的情况——h层黑结点，每一层黑结点下面都铺满一层红结点。共2h层的满树形态
结论：若根节点黑高为h，内部结点数（关键字）最多有 $2^{2h}-1$ 个