1 向量

1.1 基本概念

1. n 个数 $a_1, a_2, \ldots, a_n$ 所组成的有序数组 $\alpha = (a_1, a_2, \ldots, a_n)^T$ 或 $\alpha = (a_1, a_2, \ldots, a_n)$ 称为 n 维向量，其中 $a_1, a_2, \ldots, a_n$ 称为向量 α 的分量（或坐标），前一个表示式称为列向量，后者称为行向量

2. 对 n 维向量 $\alpha_1, \alpha_2, \ldots, \alpha_s$，如果存在不全为零的数 k，使得

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

   则称向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 线性相关，否则，称向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 线性无关

   (a) 有零向量

   (b) 两向量成比例

   (c) $n+1$ 个 n 维向量

3. TODO: 向量组 $\alpha_1 = (a_{11}, a_{21}, \ldots, a_{r1})^T, \alpha_2 = (a_{12}, a_{22}, \ldots, a_{r2})^T, \ldots, \alpha_m = (a_{1m}, a_{2m}, \ldots, a_{rm})^T$ 及向量组 $\alpha_1 = (a_{11}, a_{21}, \ldots, a_{r1})^T, \alpha_2 = (a_{12}, a_{22}, \ldots, a_{r2})^T, \ldots, \alpha_m = (a_{1m}, a_{2m}, \ldots, a_{rm})^T$

1.2 定理

1. 向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 线性相关

   (a) $\Leftrightarrow$ □□□□□□□$[\alpha_1, \alpha_2, \ldots, \alpha_s][x_1, x_2, \ldots, x_n]^T = 0$ 有非零解

   (b) $\Leftrightarrow$ □□□□□r$(\alpha_1, \alpha_2, \ldots, \alpha_s) < s$，$s$ 表示**未知数的个数或向量个数**

   (c) $\Leftrightarrow$ □□□□□□□(n□n□□□)□□$|\alpha_1, \alpha_2, \ldots, \alpha_n| = 0$

2. $n+1$ 个 n 维向量一定线性相关

3. 任何部分组 $\alpha_1, \alpha_2, \ldots, \alpha_r$ 相关$\Rightarrow$ 整体组$\alpha_1, \alpha_2, \ldots, \alpha_r, \ldots \alpha_s$ 相关

4. 整体组 $\alpha_1, \alpha_2, \ldots, \alpha_r, \ldots \alpha_s$ 无关$\Rightarrow$ 部分组$\alpha_1, \alpha_2, \ldots, \alpha_r$ 无关

5. $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性无关$\Rightarrow$ 延伸组$\widetilde{\alpha_{\mathbf{1}}}, \widetilde{\alpha_{\mathbf{2}}}, \ldots, \widetilde{\alpha_{\mathbf{n}}}$ 线性无关

6. $\widetilde{\alpha_{\mathbf{1}}}, \widetilde{\alpha_{\mathbf{2}}}, \ldots, \widetilde{\alpha_{\mathbf{n}}}$ 线性相关$\Rightarrow$ 缩短组$\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性相关

1.3 运算

1. 设 n 维向量 $\alpha = (a_1, a_2, \ldots, a_n)^T, \beta = (b_1, b_2, \ldots, b_n)^T$，则

   (a) $\alpha + \beta = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)^T$

   (b) $k\alpha = (ka_1, ka_2, \ldots, ka_n)^T$

   (c) $0\alpha = 0$

   (d) $(\alpha, \beta) = \alpha^T\beta = \beta^T\alpha = a_1b_1 + a_2b_2 + \cdots + a_nb_n$

(e) $\alpha + \beta = \beta + \alpha$

(f) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$

(g) $\alpha + 0 = \alpha$

(h) $\alpha + (-\alpha) = 0$

(i) $1\alpha = \alpha$

(j) $k(l\alpha) = (kl)\alpha$

(k) $k(\alpha + \beta) = k\alpha + k\beta$

(l) $(k + l)\alpha = k\alpha + l\alpha$

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1.4 条件转换思路

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