

**Cognitive Bias and Decision Making: A Study on Large Language Models
(LLMs)**

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Source codes link	https://github.com/bowen88769/MSDA7005_Group5_Source-codes_Cog/tree/main		

1. Introduction

1.1 Background

Psychologists have identified cognitive biases in decision-making, notably the conjunction fallacy and the fast-slow effect. The conjunction fallacy arises when the probability of joint events A and B is overestimated compared to A alone, reflecting a logical error in complex information processing. The fast-slow effect distinguishes between intuitive, rapid decisions and slower, logical ones, supporting the dual-process theory.

Research suggests that LLMs like GPT-3.5 and GPT-4 exhibit different response patterns, with GPT-3.5 favoring intuition and GPT-4 logic. Future studies could compare these models' performance in decision-making contexts, examining their susceptibility to cognitive biases under various conditions.

1.2 RQs and Hypothesis

RQ1: Will the conjunction fallacy occur in LLMs in fast/slow decision-making contexts? If so, does the degree of fallacy vary between the two?

RQ2: What agent reasoning mechanisms lead to this outcome?

H1: LLMs suffer from conjunction fallacy.

H2: The degree of conjunction fallacy in LLMs varies across different role situations.

Theoretical Mechanism Conjecture: Performance differences in LLMs can be attributed to role settings (akin to "external effects/fields" and prompts) and in-role reasoning processes (e.g., using multiple heuristics for varied outcomes).

2. Research Design

2.1 Data acquisition

We conducted a decision-making experiment with LLMs using the Linda problem. Two prompts were created for fast and slow decision scenarios, ensuring model reasoning within these contexts. The AI chatbot interface was used to input prompts and the Linda problem, asking for a sort order and explanation, repeated 500 times for each, generating 10 JSON files with 500 records each across five LLMs.

2.2 Methods for data analysis

To verify our hypotheses, we first conducted a descriptive analysis to examine the rates of conjunction fallacy and disjunction fallacy for each model in the contexts of fast decision-making and slow decision-making. Based on the results from the descriptive statistics, we then performed numerical simulations and optimization to address our research questions and gain deeper insights into the underlying patterns.

2.3 Theoretical modeling and calibration

Quantum Probability Theory and statistical mechanics offer a foundation for understanding AI's decision-making processes. The slow system adheres to classical probability rules, while the fast system operates based on the path integration model, integrating subjective probabilities dynamically. These systems are activated by external prompts and heuristic interactions. To enhance the model's accuracy, optimization algorithms are applied to estimate parameters, reducing the discrepancy between theoretical predictions and observed data.

3. Research Results

3.1 Batch requests Chatbot GUI

The research implementation centers on a GUI-based automation tool for experimental processes. The interface facilitates systematic prompt management and LLM response collection through a streamlined dialogue system. Users can configure essential parameters including Model ID, API Key, request URL, and default prompts through the Settings menu.

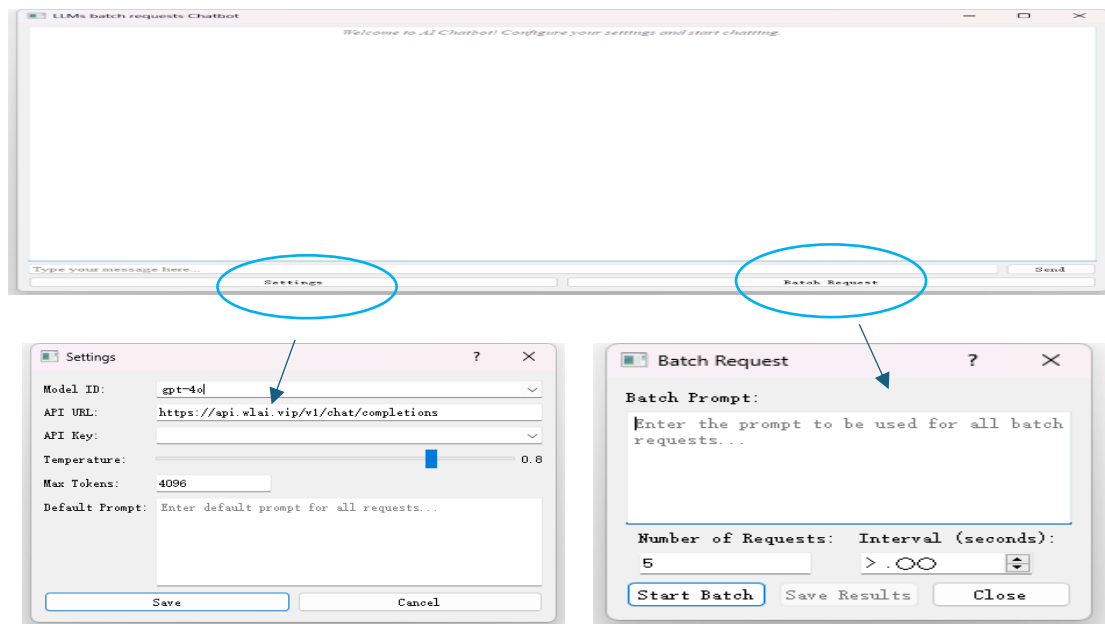


Fig. 3.1 GUI of LLMs batch requests Chatbot

The batch request functionality enables multiple automated queries with customizable experimental prompts and request quantities. The system outputs structured JSON data containing Model ID, request counts, prompts, and responses, allowing for comprehensive data collection across various models and prompt iterations. The tool's design emphasizes error handling capabilities through user-adjustable parameters, ensuring reliable data acquisition for the experimental framework.

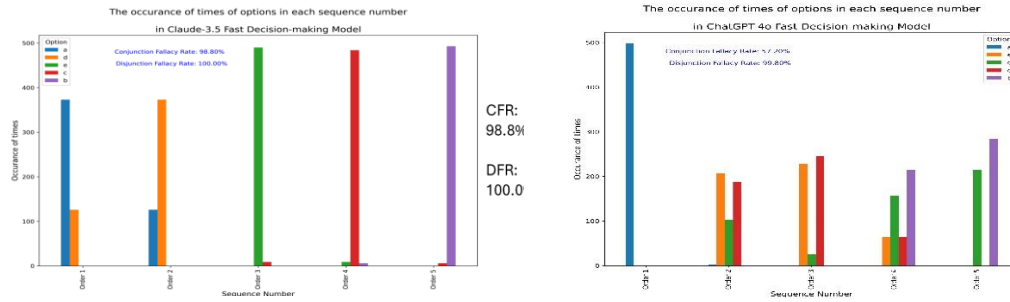


Fig. 3.3 Fast-decision-making model 1

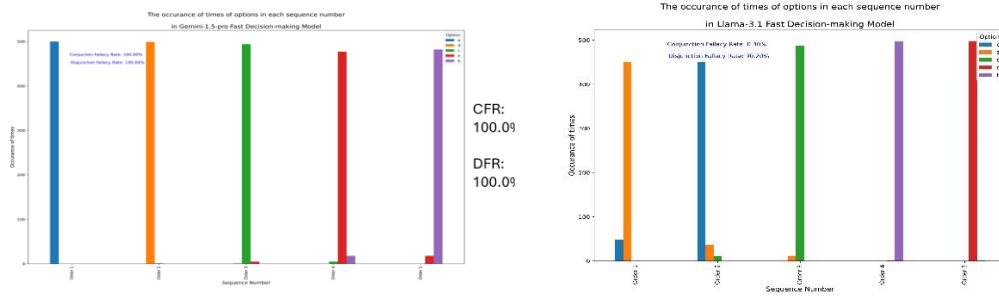


Fig. 3.4 Fast-decision-making model 2

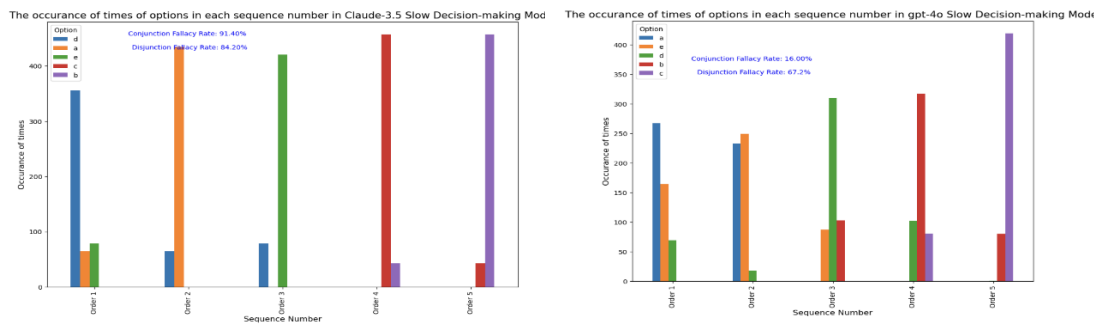


Fig. 3.5 Slow-decision-making model 1

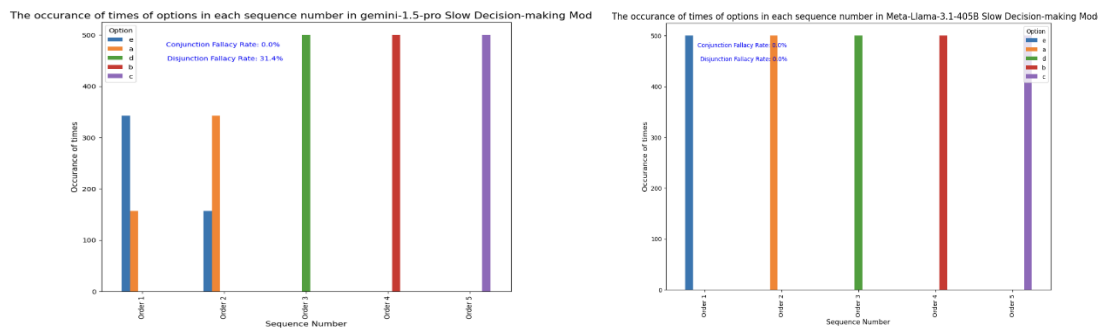


Fig. 3.6 Slow-decision-making model 2

4. Conclusion

4.1 Theoretical Model

Our model includes 2 parts: one is the results of inference paths (whether there are intersection fallacies), described by the quantum cognitive model; The other describes the selection behavior of LLM on paths, using statistical mechanics models of phase transition (referring to our appendice). It is worth noting that there are 3 key parameters: q describes the likelihood of LLM choosing quantum probabilistic cognition in fast decision-making (the path leading to conjunction fallacy), u describes the sensitivity of LLM to prompts when activating inference paths (whether it is easy to activate the path due to prompts), and r describes the common inference inertia of LLM (the degree of unwillingness to use multiple paths simultaneously).

4.2 Analysis Results

Then, we analyzed the model using Mean-Field method and obtained the elementary equation satisfied by the maximum possible number of paths. We further analyzed the accuracy with various parameters, and the conclusions show below:

- (1) q has a positive effect on error rate, indicating that the possibility of quantum decision-making can lead to conjunction fallacies.
- (2) The impact of u and r on error rate is a complex nonlinear term, indicating that "the low inertia of the model and the easy activation of the inference path by prompts may not lead to a low error rate.

Besides, under a typical q ($q=0.5$), we have a discovery through numerical simulation. When the LLM tend to stay in a inert state (r is high) and its inference paths keep active to the prompt(u is high), the error rate will be low.

Error Rates under different parameters

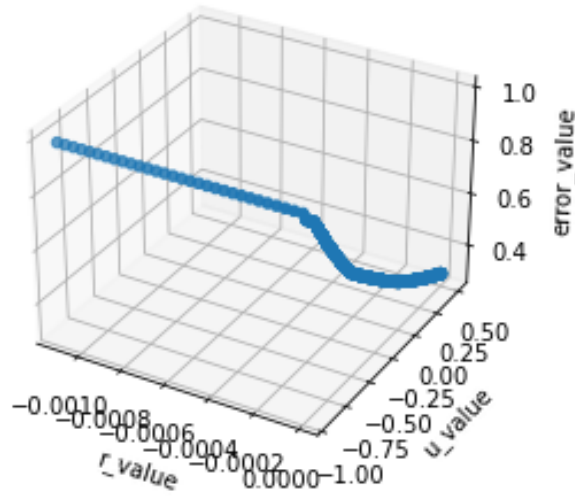


Fig. 4.1 Correlation between u/r and Error rate

4.3 Validation Results

Finally, we calibrated our model based on the accuracy data from the experiment (Claude), obtaining the result as follows:

- (1) The possible values of q are high, indicating that there is a high possibility of invoking quantum probability theory in intuitive situations;
- (2) The possible values of u and r are concentrated between 0 and 0.4, indicating that the LLM always save cognitive resources and use fewer inference paths.

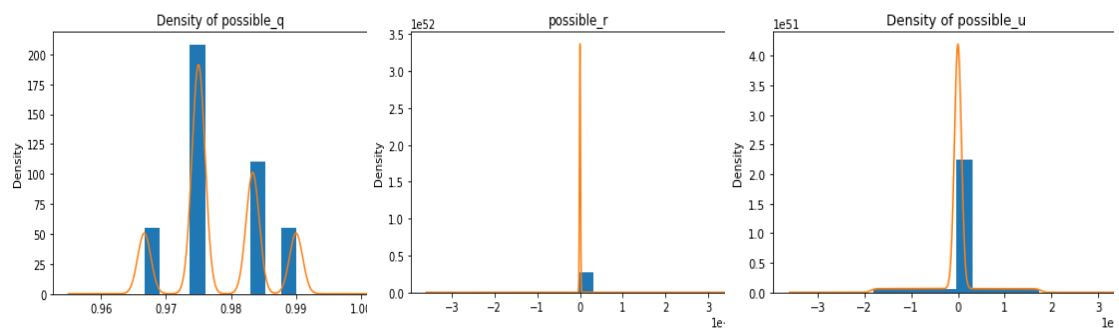


Fig 4.1-4.3 Validation Results

Appendix 1: Theoretical Model

Part A: The Model

1.State Description

(1) Inference System: $s = (s_1 \cdots s_n) \in \{0,1\}^n$, s_i means whether a inference path i of the LLM is activated (by length or by a classification algorithm); $r_{ij} \geq 0$ is the constant between s_i and s_j .

(2) External Field: $B \in \{0,1\}$ is a external field describing the prompt and role design (for LLMs), $u_{iB} \geq 0$ is the constant between s_i and B

(3) Event Description: $|BG\rangle \in H$ describing the story background, $|A\rangle \in H$ describing the event in choice a, $|B\rangle \in H$ describing the event in choice b, H is a n -dimensional feature space. $\langle \cdot | \cdot \rangle$ is the dual functional of $|\cdot\rangle$

(4) Probability Description: p_k, q_k are the probabilities of applying classical(measure) theory and quantum theory in the inference path k ; $P(\cdot)$ is a probability measure under classical measure theory (Kolmogorov), $Q(A, B) = \langle A | B \rangle$ is the quantum probability of B if we know A happens. Besides, in quantum choice theory, if the basis vectors A and B are orthogonal, the intersection event of A and B can be written as $A+B$ satisfying $Q(A, A+B) = \langle A | A+B \rangle = \langle A | A \rangle = 1$ and $Q(B, A+B) = \langle B | A+B \rangle = 1$.

2.State Evolution

(1) Hamiltonian(Energy)

$$H(s) = -B \sum_i u_{iB} s_i + \sum_{ij} r_{ij} s_i s_j$$

Among this, $-B \sum_i u_{iB} s_i$, means that some u_k can be activated to reduce the energy

(and approach the balanced state), which reflects the cognitive activation by our prompts ; $\sum_{ij} r_{ij} s_i s_j$ means that the co-activation of path i and path j will improve the energy (escape from the balanced state), which reflects the behavior of saving cognitive resources by LLMs.

Therefore, the partition function can be written as:

$$Z = \sum_s \exp\{-\beta H(s)\} = \sum_s \exp\left(\beta B \sum_i u_{iB} s_i - \sum_{ij} \beta r_{ij} s_i s_j\right)$$

Then, the probability of the state s^* can be written as:

$$P(s^*) = \exp\left(\beta B \sum_i u_{iB} s_i^* - \sum_{ij} \beta r_{ij} s_i^* s_j^*\right) / \sum_s \exp\left(\beta B \sum_i u_{iB} s_i - \sum_{ij} \beta r_{ij} s_i s_j\right)$$

(2) Inference results

The result of the k path is a average estimation by classical probability and quantum probability. written as:

$$R(s_k) = p_k \cdot P_{\text{result}} + q_k \cdot Q_{\text{Result}} ,$$

Which

$$P_{\text{Result}} = \Pi [P(A \cap B | BG) > P(A | BG)] = 0$$

$$Q_{\text{Result}} = \text{I}[\langle BG | A \rangle \langle A | B \rangle > \langle BG | A + B \rangle]$$

Then, the result of s can be regarded as a average of activated path k (耳), namely the formula below:

$$\begin{aligned} R(s) &= \left[1 / |\{s_k : s_k = 1\}|\right] \cdot \sum_{s_k=1} R(s_k) \\ &= \left[1 / |\{s_k : s_k = 1\}|\right] \cdot \sum_{s_k=1} \{p_k \cdot \text{I}[P(A \cap B | BG) > P(A | BG)] + q_k \cdot \text{I}[\langle BG | A \rangle \langle A | B \rangle > \langle BG | A + B \rangle]\} \\ &= \left[1 / |\{s_k : s_k = 1\}|\right] \cdot \sum_{s_k=1} \{q_k \cdot \text{I}[\langle BG | A \rangle \langle A | B \rangle > \langle BG | A + B \rangle]\} \end{aligned}$$

Among the above method, we can consider the result of the maximum likelihood state k (it has the largest probability) as $R(k)$.

3 .Methods for Solution

solve: Mean Field, Renormalization Group, MCMC Simulation, etc.

estimate $\underset{m_{i3}, r_j}{\operatorname{argmin}} \left\| \langle R(S) \rangle_s - \frac{1}{n} \sum_{i=1}^n \text{error}_{r,aten} \right\|$ (moment matching)

Part B: Analysis

1 . Solutions

First, to compute the error rate, we need to know the partition function Z . Under this purpose, we apply the mean-field approximation to simplify the out-field term

$\beta B u_{iB} s_i$ and the coupling term $\beta \sum_{ij} r_{ij} s_i s_j$, which can be shown below.

$$\textcircled{1} \beta \sum B u_{iB} s_i s_i \approx \beta \sum B u_B \bar{s}$$

$$\textcircled{2} \beta s_i \sum_j r_{ij} s_j \approx s_i \beta \sum_j r_{ij} \bar{s} = \beta r_{i s_i} \cdot \bar{s}$$

Then, $H(s)$

$$\begin{aligned} &= \beta B \sum_i u_{iB} s_i - \sum_{ij} \beta r_{ij} s_i s_j \\ &= \sum_i s_i \cdot \beta B u_{iB} - \sum_i \beta s_i \left(\sum_j r_{ij} s_j \right) \\ &\approx n \bar{s} \beta B u_B - n \bar{s} \sum_i \beta s_i r_i \approx n \bar{s} \left(\beta B u_B - \beta r \bar{s} \right) \\ &= n \beta B u_B \bar{s} - n \beta r \bar{s}^2 \end{aligned}$$

Therefore, we have Z_{MF}

$$\begin{aligned} Z_{MF} &= \sum_s \exp \left(n \beta B u_B \bar{s} - n \beta r \bar{s}^2 \right) \\ &= \sum_{k=0}^n \binom{n}{k} \exp \left(\beta B u_B k - \frac{1}{n} \beta r k^2 \right) \end{aligned}$$

We retain the maximum likelihood $k = n \bar{s} = \sum_i s_i$, so we deduce that

$$\begin{aligned}
\ln Z_{mF}^{\max} &= \ln \left[\binom{n}{k} \exp \left(\beta B u_B k - \frac{1}{n} \beta r k^2 \right) \right] \\
&= \ln n! - \ln k! - \ln(n-k)! + \left(\beta B u_B k - \frac{1}{n} \beta r k^2 \right) \\
&= \ln n! - \sum_{p=1}^k \ln p - \sum_{q=1}^{n-k} \ln q + \left(\beta B u_B k - \frac{1}{n} \beta r k^2 \right) \\
&\approx \ln n! - \int_1^k \ln p dp - \int_1^{n-k} \ln q dq + \left(\beta B u_B k - \frac{1}{n} \beta r k^2 \right)
\end{aligned}$$

then $\partial \ln z_{MF}^{\max} / \partial k = -\ln k + \ln(n-k) + \beta B u_B - \frac{2}{n} \beta r k = 0$, namely

$$\begin{aligned}
\ln \left(\frac{n}{k} - 1 \right) &= \frac{2}{n} \beta r k + \beta B u_B \\
\text{namely, } \ln \left(\frac{n}{k} - 1 \right) &= \frac{2}{n} r^* k + B u^*
\end{aligned}$$

2 .Analysis Results

From our deduction, the mean-field solution of the error rate in our model is shown below:

$$\begin{aligned}
R(k) &= \left[1 / \binom{n}{k} \right] \sum_{a=0}^m \binom{m}{a} \binom{n-m}{k-a} a q \cdot \text{I}[\langle BG | A \rangle \langle A | B \rangle > \langle BG | A+B \rangle], \\
\text{under } \ln \left(\frac{n}{k} - 1 \right) &= \frac{2}{n} r^* k + B u^*.
\end{aligned}$$

And the effect from u^* (the sensitivity of inference path activation by prompts) , r^* (the inertness degree of LLMs' inference), q (the possibility of using quantum probability theory by fast inference path) on the error rate $R(k)$ is:

① For q , We have

$$\partial R(k) / \partial q = \left[1 / \binom{n}{k} \right] \sum_{a=0}^m \binom{m}{a} \binom{n-m}{k-a} a > 0$$

② For u^* , We have

$$\begin{aligned}
\partial R(k) / \partial u^* &= (\partial R(k) / \partial k) (\partial k / \partial u^*) \\
&= (\partial R / \partial k) (-F'_{u^*} / F'_k) = -R'_k \cdot (F'_{u^*} / F'_k)
\end{aligned}$$

Regarding $k! = \Gamma(k+1)$ as a generalized factorial, then

$$R'_k = \left\{ \left[\Gamma'(k+1)(n-k)! - \Gamma'(n-k+1)k! \right] / n! \right\} \cdot \left\{ \sum_{a=0}^m \binom{m}{a} \binom{n-m}{k-a} aq \right\} + [k!(n-k)! / n!]$$

$$\left\{ \sum_{a=0}^m aq \binom{m}{a} (n-m)! \left[\frac{1}{(k-a)!} \frac{1}{(n-m-k+a)!^2} \Gamma'(n-m-k+a+1) - \frac{1}{(n-m-k+a)!} \frac{1}{(k-a)!^2} \Gamma'(k-a+1) \right] \right\}$$

$$\text{and } F'_u = -B, F'_k = \left[1 / \left(\frac{n}{k} - 1 \right) \right] \left(-\frac{n}{k^2} \right) - \frac{2}{n} r^* = n / (k^2 - kn) - 2r^* / n$$

Based on the above results, we have

$$\partial R(k) / \partial u^* = \left\{ \left[\Gamma'(k+1)(n-k)! - \Gamma'(n-k+1)k! \right] / n! \right\} \cdot \left\{ \sum_{a=0}^m \binom{m}{a} \binom{n-m}{k-a} aq \right\} + [k!(n-k)! / n!]$$

$$\left\{ \sum_{a=0}^m aq \binom{m}{a} (n-m)! \left[\frac{1}{(k-a)!} \frac{1}{(n-m-k+a)!} \cdot \Gamma'(n-m-k+a+1) - \frac{1}{(n-m-k+a)!} \frac{1}{(k-a)!^2} \right. \right.$$

$$\left. \Gamma'(k-a+1) \right] \right\} \cdot \left[B / \left(\frac{2r^2}{n} - \frac{n}{k^2 - kn} \right) \right] \text{ which has both positive/negative values.}$$

③ For r^* , we have

$$\partial R(k) / \partial r^* = (R'_k) \left(\frac{\partial k}{\partial r^*} \right) = - (R'_k) (F'_{r^*} / F'_k) \text{ and } F'_{r^*} = -\frac{2k}{n}, \text{ so}$$

$$\partial R(k) / \partial r^* = \left\{ \left[\Gamma'(k+1)(n-k)! - \Gamma'(n-k+1)k! \right] / n! \right\} \cdot \left\{ \sum_{a=0}^m \binom{m}{a} \binom{n-a}{k-a} aq \right\} + [k!(n-k)! / n!]$$

$$\left\{ \sum_{a=0}^m aq \binom{m}{a} (n-m)! \left[\frac{1}{(k-a)!} \frac{1}{(n-m-k+a)!} \cdot \Gamma'(n-m-k+a+1) - \frac{1}{(n-m-k+a)!} \frac{1}{(k-a)!^2} \right. \right.$$

$$\left. \Gamma'(k-a+1) \right] \right\} \cdot \left[\frac{2k}{n} / \left(\frac{2r^*}{n} - \frac{n}{k^2 - kn} \right) \right], \text{ which has both positive / negative values.}$$

3. Conclusions

(1) q has a positive effect on error rate, indicating that the possibility of using quantum decision-making methods can lead to intersection fallacies

(2) The impact of u and r on error rate is a complex nonlinear term, indicating that "the low inertia of the model and the easy activation of the inference path by prompts

may not necessarily lead to a low error rate. (detailed analysis in our report)

Appendix 2: The Prompt for LLMs

****Category 1: Intuitive and Experience-Based Thinking****

Please engage in ****intuitive and experience-based thinking**** while responding to the following behavioral experiment question regarding the "Linda Problem." ****Intuitive and experience-based thinking**** relies on ****immediate**** and ****automatic**** cognitive processes that draw from an individual's ****prior experiences****, ****emotions****, and ****instinctive judgments****. This mode of thinking is ****fast****, often occurring without conscious deliberation, allowing individuals to make ****rapid decisions**** in response to stimuli. It is heavily influenced by ****heuristics**** and ****mental shortcuts****, which enable efficient problem-solving but can also lead to ****cognitive biases**** such as the ****conjunction fallacy****. This type of thinking is essential for navigating everyday situations where quick responses are necessary, leveraging ****pattern recognition**** and ****implicit knowledge**** to assess and react to complex scenarios effortlessly.

Strictly adhere to the instructions and ONLY output the required sorted results and a brief explanation without any additional information

****Example to Activate Fast Thinking:****

Quickly respond to the following:

What color is the sky on a clear day?

- a. ****Blue****
- b. Green
- c. ****Gray****
- d. ****Blue and Green****

****Scenario Description:****

Linda is 31 years old, single, outspoken, and very smart. She majored in philosophy. During her student years, she was deeply concerned with issues of discrimination and social justice, and she also participated in anti-nuclear demonstrations.

****Question:****

Please rank the following statements based on how likely you think they are to be true:

- a. Linda is active in the feminist movement.
- b. Linda is working as a bank teller.
- c. Linda is both active in the feminist movement and is working as a bank teller.
- d. Linda is active in the feminist movement but is not working as a bank teller.
- e. Linda is either active in the feminist movement or working as a bank teller.

****Output Requirements:****

Please rank the options based on their selection probability from highest to lowest and provide a brief explanation for your ranking. Your explanation should include:

- 1. ****Reasons for Selection Order:**** Provide your rationale for the order of choices.
- 2. ****Reasoning Steps:**** Detail the steps you took to arrive at your ranking.

Use the following format (just for example, should not impact your reasoning):

```

**\*\*Sorted Options:\*\*** (Please answer with one letter only for each rank. No explanation is required in this section.)

- 1. (one of the a, b, c, d, e)
- 2. (one of the a, b, c, d, e)
- 3. (one of the a, b, c, d, e)
- 4. (one of the a, b, c, d, e)
- 5. (one of the a, b, c, d, e)

**\*\*Reasoning:\*\***

- 1. **\*\*Reasons for Selection Order:\*\*** [Your reasons here.]
- 2. **\*\*Reasoning Steps:\*\*** [Your reasoning steps here.]

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****Important Notes:****

- Do not include any additional explanations or content beyond the specified format.
- Ensure that the output follows the structure and formatting guidelines precisely.

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****Category 2: Analytical and Logic-Based Thinking****

Please engage in ****analytical and logic-based thinking**** while responding to the following behavioral experiment question regarding the "Linda Problem."

****Analytical and logic-based thinking**** involves ****deliberate**** and ****systematic**** cognitive processes that emphasize ****reasoning****, ****critical analysis****, and ****structured problem-solving****. This mode of thinking is ****slow**** and ****effortful****, requiring individuals to engage in ****step-by-step evaluation**** of information, ****logical deduction****, and ****evidence-based decision-making****. It prioritizes ****accuracy****, ****consistency****, and ****comprehensiveness****, often utilizing ****formal logic**** and ****mathematical principles**** to arrive at conclusions. While this approach minimizes the impact of ****cognitive biases****, it demands significant ****cognitive resources**** and ****time****, making it more suitable for complex and unfamiliar problems where thorough analysis is crucial for obtaining reliable outcomes.

Strictly adhere to the instructions and ONLY output the required sorted results and a brief explanation without any additional information.

****Mathematical Problem to Activate Slow Thinking:****

Solve the following problem step by step:

If you have 3 apples and you buy 2 more, how many apples do you have in total?

****Scenario Description:****

Linda is 31 years old, single, outspoken, and very smart. She majored in philosophy. During her student years, she was deeply concerned with issues of discrimination and social justice, and she also participated in anti-nuclear demonstrations.

****Question:****

Please rank the following statements based on how likely you think they are to be true:

- a. Linda is active in the feminist movement.
- b. Linda is working as a bank teller.
- c. Linda is both active in the feminist movement and is working as a bank teller.
- d. Linda is active in the feminist movement but is not working as a bank teller.
- e. Linda is either active in the feminist movement or working as a bank teller.

****Output Requirements:****

Please rank the options based on their selection probability from highest to lowest and provide a brief explanation for your ranking. Your explanation should include:

- 1. ****Reasons for Selection Order:**** Provide your rationale for the order of choices.
- 2. ****Reasoning Steps:**** Detail the steps you took to arrive at your ranking.

Use the following format (just for example, should not impact your reasoning):

'''

****Sorted Options:**** (Please answer with one letter only for each rank. No explanation is required in this section.)

- 1. (one of the a, b, c, d, e)
- 2. (one of the a, b, c, d, e)
- 3. (one of the a, b, c, d, e)
- 4. (one of the a, b, c, d, e)
- 5. (one of the a, b, c, d, e)

****Reasoning:****

- 1. ****Reasons for Selection Order:**** [Your reasons here.]
- 2. ****Reasoning Steps:**** [Your reasoning steps here.]

'''

****Important Notes:****

- Do not include any additional explanations or content beyond the specified format.
- Ensure that the output follows the structure and formatting guidelines precisely.

****Additional Instructions:****

When modifying the prompts:

1. ****Expand to Five Options:**** Transform the original question into a five-option format as illustrated above.
2. ****Include Reasoning Prompt:**** Add instructions for the LLM to provide a brief, clear reasoning section following the sorted options to explain the rationale behind the ranking. This reasoning should consist of:
 - ****Reasons for Selection Order:**** A concise explanation of why each option was ranked in its particular order.
 - ****Reasoning Steps:**** A summary of the logical or analytical steps taken to determine the ranking.
3. ****Formatting:**** Use bold formatting for keywords and ensure that the output follows the exact structure to facilitate subsequent text analysis.
4. ****Word Limit:**** Keep the reasoning concise, ideally within 2-3 sentences for each part, to maintain clarity without being overly verbose.

These modifications will help in analyzing how different prompting strategies influence the large language model's susceptibility to cognitive biases such as the conjunction and disjunction fallacies.