Project 7

Public Finance in Macroeconomics

Handed in by the **Heterogeneous Geeks**

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Public Finance in Macroeconomics: Heterogenous Agent Models
at the Graduate School of Economics, Finance, and Management

1 Simple Variant of Krusell-Smith Algorithm

1.1 Characterization of Equilibrium Dynamics

The proof of Proposition 3 uses a Guess and Verify.

1. Guess that households will save a constant share s of (disposable) wage,

$$a_{2,t+1} = s(1-\tau)w_t \tag{1}$$

Since the firm optimization problem

$$\max \Pi = \zeta_t F(K_t, \Upsilon_t L) - (\bar{\delta} + r_t)\varrho_t^{-1} K_t - w_t L,$$

where ζ_t and ϱ_t are aggregate shocks, imply the f.o.c.

$$w_t = (1-\alpha)\Upsilon_t k_t^{\alpha} \zeta_t$$

where

$$k_t = \frac{K_t}{\Upsilon_t L}, \tag{2}$$

then 1 becomes

$$a_{2,t+1} = s(1-\tau)(1-\alpha)\Upsilon_t k_t^{\alpha} \zeta_t \tag{3}$$

Market clearing conditions $K_t = a_{2,t}$ and $L = 1 + \lambda$, growth rate of labor productivity $\Upsilon_{t+1} = (1+g)\Upsilon_t$ and 2 imply that

$$k_{t+1} = \frac{s(1-\tau)(1-\alpha)k_t^{\alpha}\zeta_t}{(1+g)(1+\lambda)} \tag{4}$$

which is the equilibrium dynamics implied by our guess s.

2. We know that, given s, 4 solves the Firm´s problem and the market clering condition. Then to verify that our guess indeed an equilibrium solution we need to check for which value of s the household problem is solved.

   The agent budget constraints, substituting for w_t, w_{t+1} and $\alpha_{2,t+1}$ using 3 implies consumption levels

$$\begin{aligned} c_{1,t} &= (1-s)w_t = (1-s)(1-\tau)\Upsilon_t k_t^{\alpha}\zeta_t \\ c_{2,t+1} &= s(1-\tau)(1-\alpha)\zeta_t \Upsilon_t k_t^{\alpha} \alpha \zeta_{t+1} \varrho_{t+1} k_{t+1}^{\alpha-1} \\ &\quad + (1-\alpha)\Upsilon_{t+1}\zeta_{t+1}k_{t+1}^{\alpha}(\lambda\eta_{2,t+1} + \tau(1+\lambda(1-\eta_{2,t+1}))) \end{aligned}$$

in the two periods lived by the agent, where the interest rate is provided by the second f.o.c of the firm,

$$1 + r_t = \alpha k_t^{\alpha-1} \zeta_t \varrho_t$$

Substituting 4 yields

$$\begin{aligned} c_{2,t+1} = (&\alpha\varrho_{t+1}(1+\lambda) + (1+\alpha)(\lambda\eta_{2,t+1} \\ &+ \tau(1+\lambda(1-\eta_{2,t+1}))))\Upsilon_{t+1}\zeta_{t+1}k_{t+1}^{\alpha}. \end{aligned}$$

Substituting the consumption levels $c_{1,t}$ and $c_{2,t+1}$ into the consumer Euler Equation

$$1 = \beta\mathbb{E}_t\left[\frac{c_{1,t}(1+r_{t+1})}{c_{2,t+1}}\right]$$

implies

$$1 = \frac{\beta(1-s)}{s}\Phi(\tau)$$

where $\Phi(\tau)$ is a function of pension system contribution τ as defined in Proposition 3. Therefore, optimal saving rate in general quilibrium is also a function of τ:

$$s(\tau) = \frac{\beta\Phi(\tau)}{1+\beta\Phi(\tau)}. \tag{5}$$

3. Due to convexity, the solution represented by 4 and 5 is unique. Q.e.d.