


Project 5

Public Finance in Macroeconomics

Handed in by the **Heterogeneous Geeks**

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Project in the context of Prof. Ludwig's course:

Public Finance in Macroeconomics: Heterogenous Agent Models
at the Graduate School of Economics, Finance, and Management

1 Solution

1.1 Explain all sections of the code

The main function presented in `towards_olg` are:

```
% -----%
% use all functions below to print graphs
function towards_olg
% -----%

% -----%
% define the values of parameters
function func_calibr(opt_det,opt_nosr,opt_ny)
% ----- %

% ----- %
% solution of the household problem
% prints:
%   - state variable matrix (gridx)
%   - saving grid (gridsav)
%   - optimal consumption function (cfun)
%   - optimal value function (vfun)
function [gridx,gridsav,gridass,cfun,vfun] = func_hh
% ----- %

% ----- %
% aggregation and cross-sectional measure
% prints:
%   - distribution of assets conditional by age and shock (Phi)
%   - distribution of assets (PhiAss)
function [Phi,PhiAss,ass]=func_aggr(gridx,gridsav,cfun,gridass)
% ----- %

% ----- %
% compute average life-cycle profiles
function [labinclife,inclife,asslife,conslife,vallife] = lcprofile(Phi,gridass,cfun,vfun)
% ----- %

% ----- %
% utility function
function u = U(c)
% ----- %
```

```

% ----- %
% marginal utility function
function muc=MUc(c)
% ----- %

% ----- %
% construction of the Markov chain process
% prints:
%     - transition matrix first eigenvector (pini)
%     - shocks values (gridy)
%     - transition matrix (pi)
function [pini,pi,gridy]=mchain(rhoeta,epsil)
% ----- %

% ----- %
% this function computes the inverted of the utility function
% it is required in func_hh to retrieve optimal consumption from the value function
function invut=invut(marg)
% ----- %

% ----- %
% create curved grid using curvature parameter c
function grd = makegrid(x1,x2,n,c)
% ----- %

```

For a more detailed description, please refer to `toward_olg_commented`.

1.2

The solution procedure in `func_hh` uses the "endogenous gridpoints approach" for solving dynamic stochastic optimization problems, as introduced by [Carroll \(2002, 2006\)](#). Instead of creating using an "exogenous" - in that it is given by the coder - grid for the state variable cash-on-hand (x), the "endogenous" approach exploit the definition

$$s = x - c$$

- that is, (end of period) savings are equal to cash-on-hand minus consumption - to compute a new cash-on-hand grid in each period of iteration (over time and possible shock realization), given a fixed (and "exogenous") grid for s . Furthermore, substituting s into the Euler equation allows to solve easily for consumption for each point of the saving grid. That is, for each iteration, a pair s and $c(s)$ is defined. The endogenous cash-on-hand grid is then $x(s) = s + c(s)$, and represent the level of cash-on-hand that are required for each level of end-of-period savings (and optimal consumption given saving target). Then the value function in current period is updated for each value of the new endogenous cash-on-hand grid (given age and shock, still) by interpolation.

The exogenous grid approach, on the other hand, does both the derivation of the conditional policy function and the updating of the value function at the end of the iteration on the endogenous grid. The advantages of the exogenous grid method are twofold: first of all, it does not require a nonlinear rootfinding since the Euler equation is solved for c given s , instead of x , thus removing consumption from the RHS. Second, updating the value function over the endogenous grid for x is more efficient in that the value function (V') is interpolated only for values of x' that have been used to compute c' in the first instance, while the exogenous approach loops over *all* possible x' .

1.3

Please refer to `toward_olg_exogenous`

1.4

Please refer to `toward_olg_not_first_order`

1.5

What is the role of the objects TT and Phi?

Firstly, for this exercise, we need to understand even more clearly how `pi`, `pini` and `gridy` look like. They are explained in the `mchain` function.

```
[pini,pi,gridy]=mchain(rhoeta,epsil);

% pi are Transition Probabilities. You always have 1- rhoeta probability of remaining in
    you current state.. although i find it weird that the off-diagonal elements are also
    one
pi=rhoeta*ones(2,2);
pi(1,2)=1.0-rhoeta;
pi(2,1)=1.0-rhoeta;

% gridy are rescaled income shocks such that mean is one
gridy=zeros(2,1);
gridy(1)=exp(1.0-epsil);
gridy(2)=exp(1.0+epsil);
gridy=2.0*gridy/(sum(gridy));

% pini is Initial Distribution
pini=0.5*ones(2,1);
```

Okay, so now that this is clear, let's try to explain the role of TT and Phi! Both these objects are defined and used in lines 402-434 in the code, which is why inserted the corresponding section below:

```
function [Phi,PhiAss,ass]=func_aggr(gridx,gridsav,cfun,gridass)

global r nj nx ny pi gridy netw pens sr epsi pini frac totpop
```

```

disp('aggregation and cross-sectional measure');

% Compute Cross sectional distributions and aggregate variables
Phi = zeros(nj,ny,nx);          % distribution of assets conditional by age and shock
PhiAss = zeros(nx,1);           % distribution of assets

% Distribution of newborns over cash at hand
for yc=1:ny

    % income (wages and pensions) in current period/age:
    inc=epsi(1)*netw*gridy(yc)+(1-epsi(1))*pens;

    % initial cash-on-hand:
    cahini=inc;

    [vals,inds]=basefun(gridx(1,yc,:),cahini,nx);
    Phi(1,yc,inds(1))=vals(1)*pini(yc)*frac(1);
    Phi(1,yc,inds(2))=vals(2)*pini(yc)*frac(1);
end;

for jc=2:nj
    TT = zeros(ny,nx,ny,nx); % transfer function

    for xc=1:nx
        for yc=1:ny
            for ycc=1:ny

                % income (wages and pensions) in current period/age:
                inc=epsi(jc)*netw*gridy(ycc)+(1-epsi(jc))*pens;

                % cash on hand:  $x=a*(1+r)+y = s(-1)*(1+r)+y$ ;
                cah=inc+(1.0+r)*gridsav(xc);

                [vals,inds]=basefun(gridx(jc,ycc,:),cah,nx);

                TT(ycc,inds(1),yc,xc)=vals(1)*pi(yc,ycc);
                TT(ycc,inds(2),yc,xc)=vals(2)*pi(yc,ycc);
            end;
        end;
    end;

    for xc=1:nx
        for yc=1:ny
            for xcc=1:nx

```

```

        for ycc=1:ny
            % transfer distribution:
            Phi(jc,ycc,xcc)=Phi(jc,ycc,xcc)+Phi(jc-1,yc,xc)*TT(ycc,xcc,yc,xc)*sr(jc-1);
        end;
    end;
end;
end; % end for jc

```

First notice that Phi has three dimensions: It accounts for age, the current state of income shocks and the level of cah that individuals hold. In the first period of life we know how assets are distributed, as we assume that initial assets are generated with the same process as income. Basically, everyone starts off with some assets that amount to the income realization people had in the past period, i.e. $t = 0 - 1$. The initial distribution at age 1 is again state dependent; i.e. we could start off with any of these two states; which explains that there could be differences in the aggregate asset distribution over the entire life cycle due to the (randomly selected) state you start off in.

Then, next TT is the transfer function that is defined for most periods in time, or - more precisely - for the $n_j - 1$ times that the people transition into the next age group. TT is build up in 4 layers. To explain them, will denote them as follows $TT(ny, nx, ny, nx) = TT(ycc, xc_new, yc, xc_old)$.

We need to define TT for all age groups, which is done by simply creating a new TT for all current ages. Given each age, one might be holding different levels of cah, which is what we do through the xc_old index. In the last period, each person's income was dependent on a shock yc, which needs to be accounted for as the past shock will determine the probability of one's next shock (remember that these estimates are stored in $\pi(y_c, y_{cc})$ which gives the transition probability from state y_c to y_{cc}). Given a past shock, we can compute the income for any of the new shocks ycc that might hit us. That income times the interest rate plus the savings associated with the given level of cah xc_old will determine the cah in the next period. In the transition matrix we will now store the expected probability of transitioning into state xc_new in the coming period.

Note: The last part is done by relying on the basefun and lookup functions. What they basically do here, is to estimate the cah level you will get in next period. As this will usually not be a precise gridpoint of gridx, we assign a positive probability to the gridpoint just above and just below the new cah estimate. :)

In a final step for each age you fill out the new Phi distribution for all past states of shock realizations and cah holdings. The probability of dying early is also accounted for.

Compare this with the corresponding definitions in the lecture notes:

This is actually really hard to do, because our transition matrix in the Markov process is very simple. We can basically apply the same transition process at all ages, in the sense that multiplying it

with the current shock distribution, will give us a matrix with the new distribution of income shock realizations.

In terms of Φ , we do not really have an assumption about the initial state of assets in the lecture notes. Rather, it is suggested that we may run simulations and average over them.

Code modification The project instructions ask us to implement a more efficient way for looping over all the different states. we did so, as you can see in the attached matlab file named "Solution_1.5" The time elapsed now was:

```
time elapsed: 9.3518
```

vs. the time before:

```
time elapsed: 11.5845
```

(It is to note however, that the milliseconds vary across different times. The above times are more or less a medium measure of how long each script takes.)

1.6

Comparison of Models

In the attached PDF (Comparison model outcomes) we plotted all required scenarios in a way that (hopefully) allows an easier comparison¹. We have classified the models as follows, whereby the red coloured part identifies the deviations from the standard model:

- Standard: `opt_det = false, opt_nosr = true, tetta=2, r=0.04, rho=0.04`
- Variation 1: `opt_det = true, opt_nosr = true, tetta=2, r=0.04, rho=0.04`
- Variation 2: `opt_det = true, opt_nosr = false, tetta=2, r=0.04, rho=0.04`
- Variation 3: `opt_det = false, opt_nosr = false, tetta=2, r=0.04, rho=0.04`
- Variation 4: `opt_det = false, opt_nosr = true, tetta=1, r=0.04, rho=0.04`
- Variation 5: `opt_det = false, opt_nosr = true, tetta=5, r=0.04, rho=0.04`
- Variation 6: `opt_det = false, opt_nosr = true, tetta=2, r=0.04, rho=0.4`
- Variation 7: `opt_det = false, opt_nosr = true, tetta=2, r=0.4, rho=0.04`

The standard model interest rates and impatience parameter balance each other out, s.t. the left and RHS of the standard Euler equation (w.o. accounting for possible borrowing constraints) simplify to the marginal utility of consumption today having to equal the expected marginal utility of consumption tomorrow. However, there is a precautionary motive turned on given the positive third derivative (presence of prudence) and variability in the income process (it is NOT deterministic but

¹The asset distribution at different ages was cut down to just display it at age 80 out of space constraints.

rather modelled by a Markov process). People know for certain when they will die which explains the shape of assets that monotonically approaches zero from above (last part of the asset function over the life cycle is concave). the intuition is, that people do not want to die with left-over assets as they still derive utility from them.

Variation 1 makes income deterministic, which will turn of the precautionary savings behavior. The distribution of assets is suddenly more dense, i.e. people do not behave so differently in their savings behavior, as they do not experience different income states. They also build up the assets more slowly than they did under the standard specifications. Fianlly, it is to note that there is a massive change in the consumption profile of agents. Looking at the scale that corresponds to the consumption level graph, the changes in absolute consumption are smaller than those in the standard model. The consume their income almost fully until they are about to retire. In the last years of the working life the agents start saving a larger portion (as they become concerned with their retirement time) and during the retirement their consumption remains constant (as it did in the standard model).

Variation 3 (we will come to Variation 2 next), uses the standard model but now changes besides having stochastic income allows for survival risk (i.e. people never know when they will exactly die). A 100 is however the moment, when one dies with absolute certainty. This survival risk is another source of risk, which can change the consumption pattern. As in the standard model, the asset distribution is rather wide, although a bit lower than in the standard case (people apparently don't want to save as much as before, as they never know whether this means having wasted consumption during the precious time alive ;) Accordingly savings are also a bit lower, and consumption a bit higher than in the standard case. In line with the logic described above, the shape of the consumption function over the life cycle decreases over age (and does not stay constant and high as in the standard model w/o survival risk).

Variation 2 combines survival risk and a deterministic income process: i.e. we have more risk of dying unpredictably, but save earnings. Again, like in variation 1, the distribution of assets is much more narrow, but this time focuses even more at an early age (when it is still pretty unlikely that one will die). In comparison to Variation 1, which has no survival risk, we see that the consumption profile is hump-shaped, in line with the idea that one knows that it is ever more unlikely that one will survive and doe snot want to leave too many assets behind without consuming them. In contrast to Variation 3, where income is not deterministic, people only start saving more out of current income as they approach the retirement age. This is in line with the idea, that the labor income during working age (conditional on surviving) will come in in its full/ constant amount for sure.

Variation 4 and 5. Looking at Variations 4 and 5, we can make some observations on how tetta, the coefficient of relative risk aversion, changes consumption/ savings behavior in a model of stochastic income but with no survival risk. A higher tetta implies that the RHS of the Euler equation will increase with uncertainty, and that we will want to save more. This is hard to see in the graphs, but there is indeed some differences over the assets accumulated over the life cycle, when one pays close attention to the savings and the assets graph. Also, and perhaps more clearly, the graph on the consumption function over the life cycle reveals that during the retirement age, people with a

higher θ are able to consume more (given their savings).

Also note, that $1/\theta = \text{IES}$, the intertemporal elasticity of substitution. The latter makes people more willing to adapt their consumption intertemporally when they see an important gain (e.g. in interest rates) from doing so. Here, the interest rate and the intertemporal discount factor equal each other out, but we can still see that people with low IES (high θ) have a somewhat wider asset distribution over the life cycle, while the ones with low θ (high IES) take advantage of the compounded interest effects, i.e. they focus the distribution of assets more on the younger ages to take more advantage of it later.

Variation 6 and 7 show how the patterns change when (given a constant θ) the relationship between interest rates and discount factor change. Call $A = \frac{1+r}{1+\rho}$; a high A should lead to more savings today (i.e. lower consumption, higher marginal utility from consumption today). Variation 6 entails a very low A and indeed has relatively low savings when comparing it to the Standard case. This is exactly opposite in the Variation 7, where savings reach over 20 times the level of variation 6. Instead of the humped shaped consumption that follows for Variation 6, in 7 we observe a steady and almost exponential increase in consumption of the life cycle. From a highly negative value function in the early years, this rises to significantly higher levels than those observed in variation 6 at approx. the age of 30.

@someone: Please note that the labor income in Variation 7 looks really odd- maybe we chose parameters that are not consistent at some point. I still think we can use these graphs here, but I would recommend explaining/ understanding why this happened.

Problem 2: Calibration

Problem 3: Analysis

Problem 4: Summary of De Nardi (2004)

Since the distribution of wealth is much more concentrated than the distribution of (labour) earnings, De Nardi (2004) estimates a quantitative, general equilibrium, incomplete markets, overlapping-generations model to match this observation from the data. In doing so, the author links parents and children by accidental as well as voluntary bequests - but not by inter vivo transfers - and allows children to inherit some of their parents' productivity. Further, she calibrates the model to match important features of, first, the data of the U.S. and, second, the data of Sweden. Both countries have different wealth distributions even though their Gini coefficients are relatively similar. This paper's framework makes it possible for intergenerational links to induce saving behaviour that generates a more concentrated distribution of wealth than that of earnings due to allowing two different saving motives: self-insurance against labour earnings shocks and life-span risk, i.e. saving for retirement, which both can be seen as precautionary motives, and the preference to leave bequests to their offspring, which is an altruistic motive.

De Nardi finds that saving for precautionary purposes and saving for retirement are the primary factors for wealth accumulation at the lower tail of the distribution, while saving to leave bequests

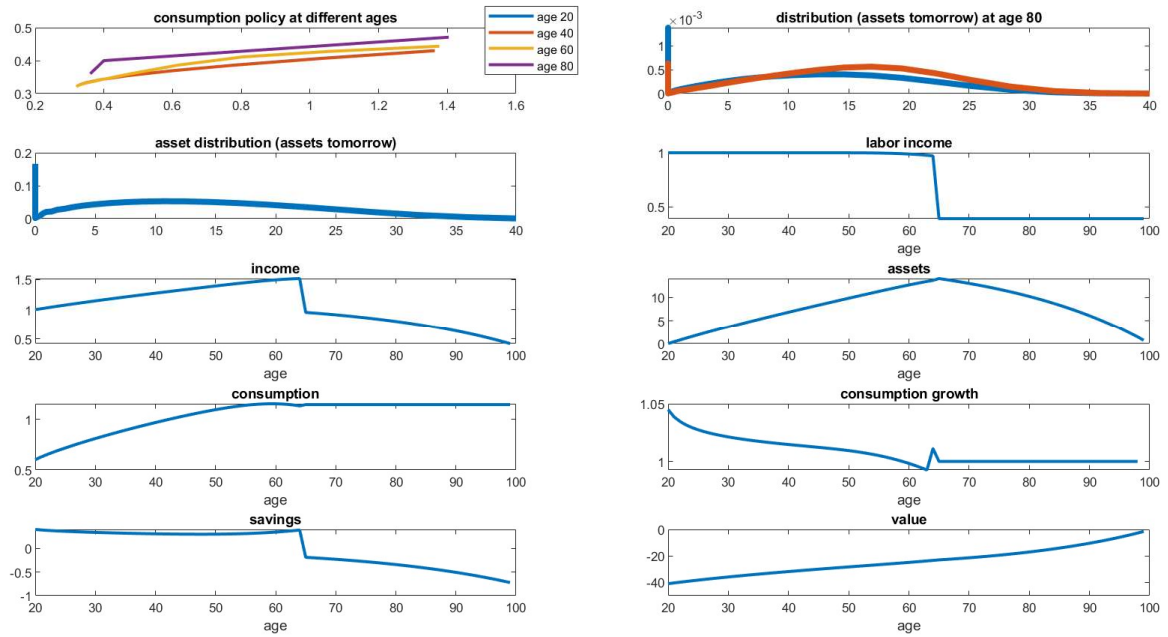
significantly affects the shape of the upper tail.

References

- Christopher D Carroll. Lecture notes on solution methods for microeconomic dynamic stochastic optimization problems”, mimeo. 2002.
- Christopher D Carroll. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics letters*, 91(3):312–320, 2006.
- Mariacristina De Nardi. Wealth inequality and intergenerational links. *The Review of Economic Studies*, 71(3):743–768, 2004.

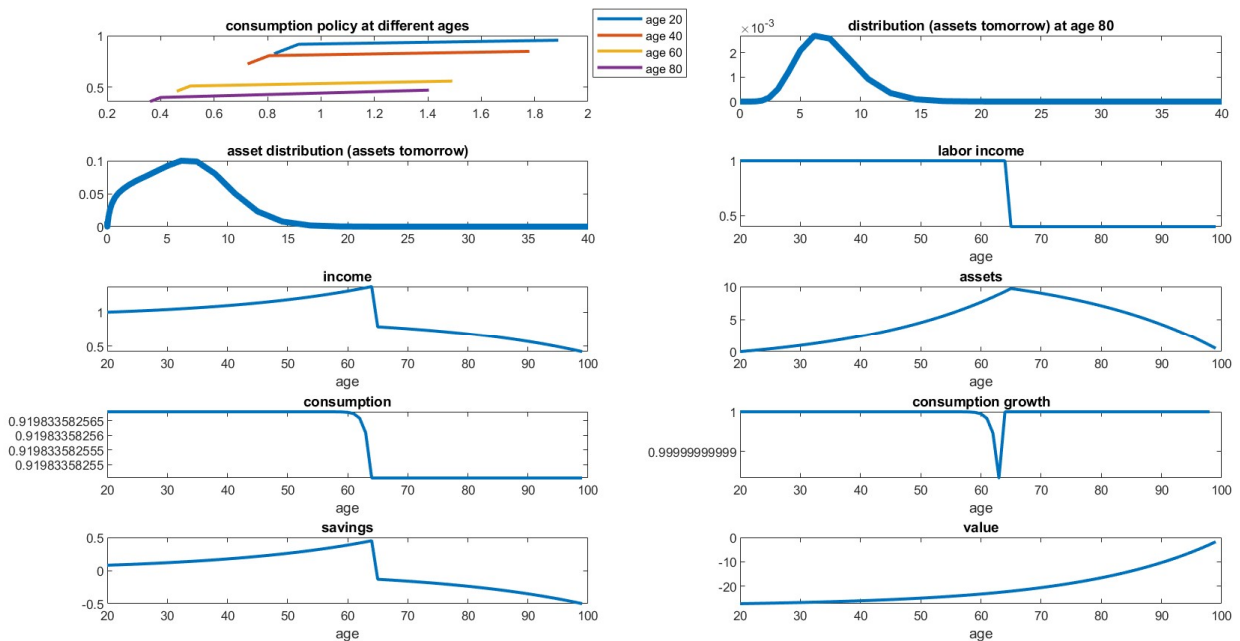
Standard: `opt_det = false`, `opt_nosr = true`, `tetta=2`, `r=0.04`, `rho=0.04`

(the model is not deterministic w.r.t. income processes; hence there is income risk represented by the Markov process & The agents know when they will die)

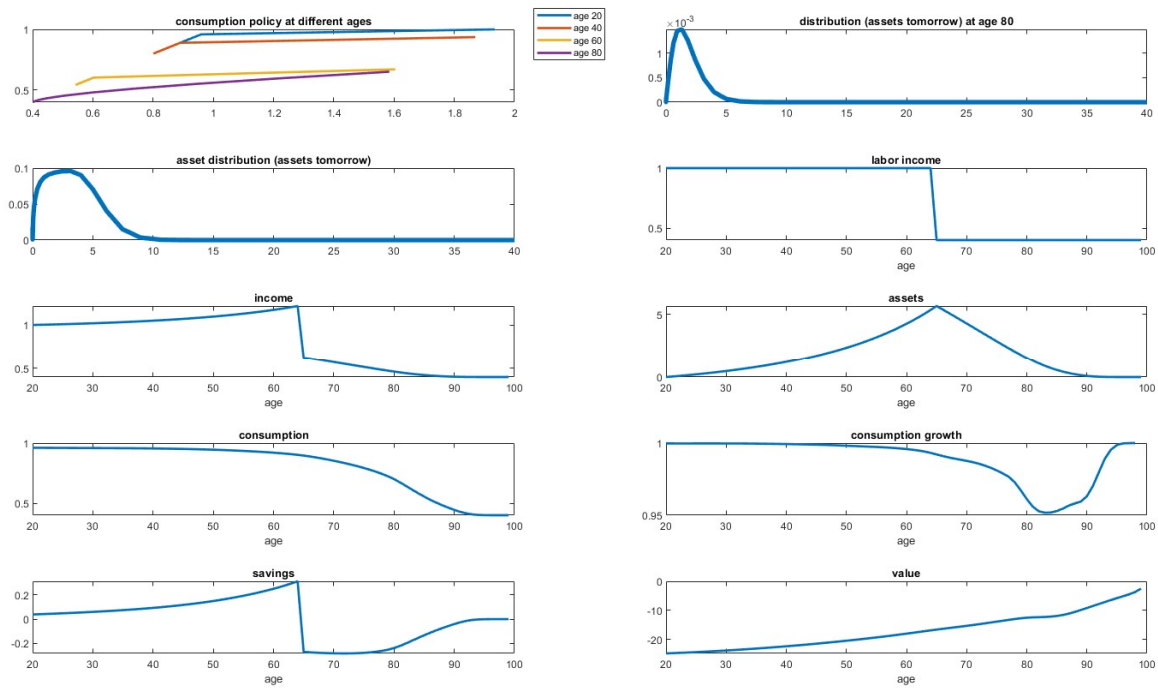


Variation 1: `opt_det = true`, `opt_nosr = true`, `tetta=2`, `r=0.04`, `rho=0.04`

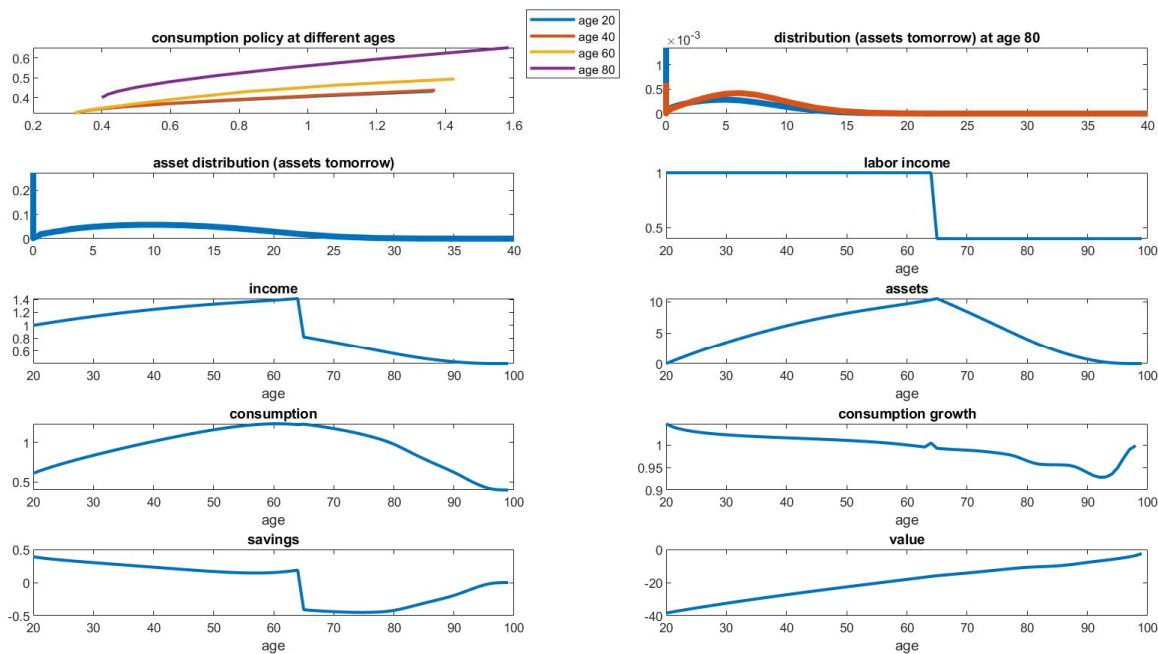
(the model is deterministic w.r.t. income; no income risk; & the agents know when they will die)



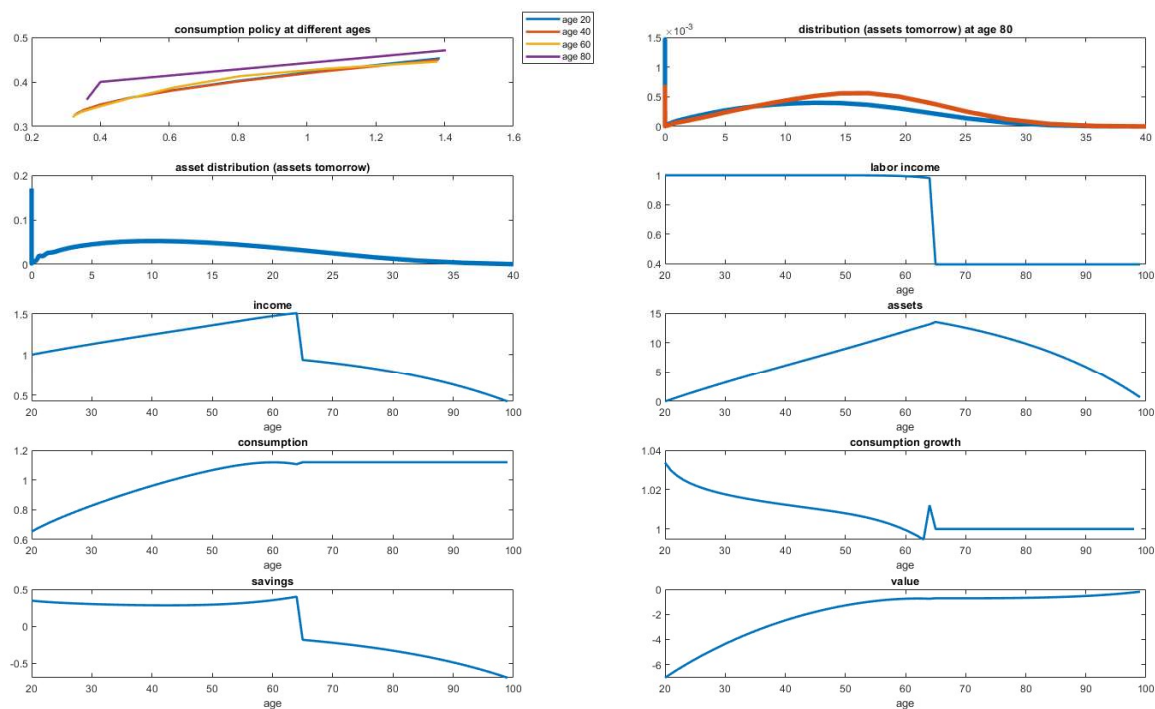
Variation 2: `opt_det = true`, `opt_nosr = false`, `tetta=2`, `r=0.04`, `rho=0.04`
 (the model is deterministic w.r.t. income; no income risk; & the agents do NOT know when they will die)



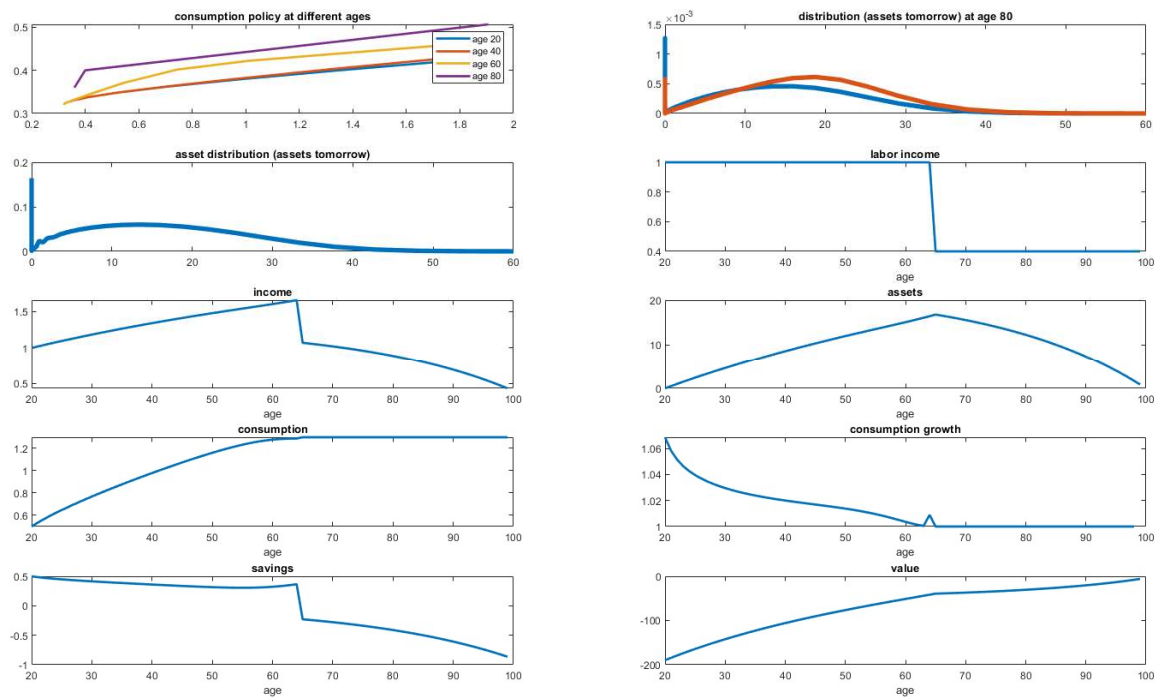
Variation 3: `opt_det = false`, `opt_nosr = false`, `tetta=2`, `r=0.04`, `rho=0.04`
 (the model is not deterministic w.r.t. income processes; hence there is income risk represented by the Markov process & The agents do NOT know when they will die)



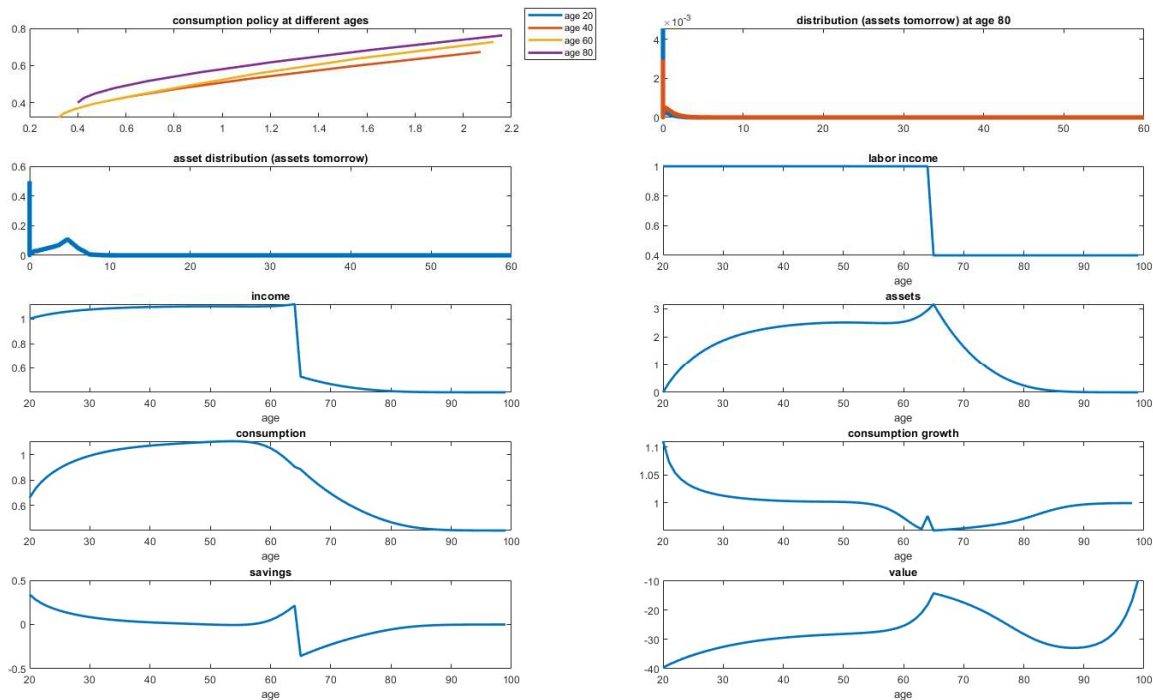
Variation 4: $\text{opt_det} = \text{false}$, $\text{opt_nosr} = \text{true}$, $\text{tetta}=1$, $r=0.04$, $\rho=0.04$
 (low relative risk aversion = high IES)



Variation 5: $\text{opt_det} = \text{false}$, $\text{opt_nosr} = \text{true}$, $\text{tetta}=5$, $r=0.04$, $\rho=0.04$
 (high relative risk aversion = low IES)



Variation 6: $\text{opt_det} = \text{false}$, $\text{opt_nosr} = \text{true}$, $\text{tetta}=2$, $r=0.04$, $\text{rho}=0.4$
 (rel. low return given high discount factor betta)



Variation 7: $\text{opt_det} = \text{false}$, $\text{opt_nosr} = \text{true}$, $\text{tetta}=2$, $r=0.4$, $\text{rho}=0.04$
 (rel. higher return than discount factor betta)

