CS280 Assignment 3
Joel Bowen S-ID: 303-216-3187
Baladitya Yellapragada S-ID: 21124445
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1 Fundamental Matrix

1.1 Procedure

We utilized the 8-point algorithm as descried in the handout to estimate the fundamental matrix F. More specifically, given corresponding points $\mathbf{x}_1$ and $\mathbf{x}_2$, the fundamental matrix is defined as:

$$\mathbf{x}_2^t F \mathbf{x}_1 = 0.$$

1.1.1 Normalization

We first start by normalizing our corresponding points to minimize the effects of noise. We can achieve this by translating by μ (so points are shifted to have a mean at the origin) and scaling by σ (so the mean distance to the origin is $\sqrt{2}$). This is achieved by transforming the points by a linear system T, where T is defined as the following:

$$T = \begin{bmatrix} \sigma & 0 & -\sigma\mu \\ 0 & \sigma & -\sigma\mu \\ 0 & 0 & 1 \end{bmatrix}$$

1.1.2 Optimization

As seen in the handout, the above formulation for F (or f as a row-wise reshaped column vector) can be approximated by optimizing the following:

$$\min_f \|Af\|_2 \quad s.t. \quad \|f\|_2 = 1 \tag{1}$$

We can now find f^* which solves the above optimization problem by using the singular value decomposition of matrix A. Using SVD, $A = \tilde{U}\tilde{S}\tilde{V}^T$, where $\tilde{S}$ is a diagonal matrix with descending singular values along its diagonal, and $\tilde{U}$ and $\tilde{V}^T$ are orthogonal matrices. The rows of $\tilde{V}^T$ (or equivalently columns of $\tilde{V}$) correspond to the singular values of S. Therefore, the f^* which solves equation (1) is the right singular vector (column in $\tilde{V}$) that corresponds to the smallest singular value in $\tilde{S}$. Then the estimate of the fundamental matrix, F^* is achieved by reshaping the column vector f^* row-wise into a $3x3$ matrix.

Furthermore, as seen in the handout, we can constrain our estimate of F to be rank two by first taking the SVD of F^*: $F^* = USV^T$. Then, a rank of 2 is achieved by recomposing with

$$\hat{S} = diag(s1, s2, 0)$$

where we set the least singular value of F^* to zero. Then we achieve a approximate F of rank 2 by recomposing:

$$F = U\hat{S}V^T$$

1.1.3 Denormalization

Finally we need to denormalize our result. This is achieved by the following:

$$F \leftarrow T_2^t F T_1$$

1.2 Results

The following are the fundamental matrices for the corresponding points of the 'house' and the 'library':

$$F_{house} = \begin{bmatrix} -4.37x10^{-8} & 1.01x10^{-6} & 9.42x10^{-5} \\ 4.46x10^{-6} & -3.84x10^{-7} & -1.5x10^{-2} \\ -7.24x10^{-5} & 1/05x10^{-2} & -7.33x10^{-3} \end{bmatrix}, \quad F_{library} = \begin{bmatrix} -3.66x10^{-8} & 7.80x10^{-7} & -1.18x10^{-4} \\ -4.74x10^{-6} & -4.86x10^{-8} & 8.79x10^{-3} \\ 1.13x10^{-3} & -7.9x10^{-3} & -2.14x10^{-1} \end{bmatrix}$$

1.3 Residuals

The following table gives the residuals (mean squared distance between the points in the two images and the corresponding epipolar lines) for 'house' and 'library':

Name	Residual Error
house	0.00468
library	0.03483

Table 1: residual error

1.4 Additional Questions

Let R_i be the epipolar contraint for corresponding point pair i. Then the objective function we are minimizing using the SVD is equivalent to

$$\sum_i R_i^2$$

Now let l and $l^{'}$ be the epipolar lines in each image. Then the residual error calculated here is related to the objective by the following:

$$\frac{1}{2n}\left(\frac{1}{\|l\|_2} + \frac{1}{\|l'\|_2}\right)\sum_i R_i^2$$

So what we are minimizing is not exactly the residual but is related by the above.

2 Extrinsic Camera Parameters

We can calculate the essential matrix E with the given calibration matrices K_1 and K_2 and the fundamental matrix F we calculated in the first section.

$$E = K_2^T F K_1$$

We also know that $E = [t]_x R$ and we are told that $[t]_x = SZR_{90^\circ}S^T$
so

$$E = SZR_{90^\circ}S^T R$$

and since the SVD of $E = U\Sigma V^T$, we can claim that

$$U\Sigma V^T = SZR_{90^\circ}S^T R$$

We can see that U = S, and in noise-free situations $\Sigma = S$, so

$$R = UR_{90^\circ}^T V^T$$

Also, even though

$$[t]_x = U\Sigma R_{90^\circ} U^T$$

We will actually just use $\hat{t}$ = 3rd left singular vector in U. However, when choosing a correct translation vector t and rotation matrix R, the sign of the two elements is arbitrary. Therefore we must consider both $\pm t$ and $\pm R$. Additionally, we must choose R's for both $R_{\pm 90^\circ}$. Therefore, we are left with 2 possible t's and 4 possible R's. However, we only consider R's with determinant 1.

2.1 House

We find 4 Rs:

With determinant = 1

$$R_1 = \begin{bmatrix} 0.99647234 & -0.0114259 & -0.0913558 \\ 0.00201953 & -0.67279431 & 0.1169276 \\ -0.18253558 & -0.86324618 & -1.32428279 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.98752415 & -0.05126811 & -0.14643536 \\ 0.10158247 & 1.32430174 & 0.12450397 \\ 0.0678056 & -0.8843725 & 0.65910293 \end{bmatrix}$$

With determinant = -1

$$R_3 = \begin{bmatrix} -0.99647234 & 0.0114259 & 0.0913558 \\ -0.00201953 & 0.67279431 & -0.1169276 \\ 0.18253558 & 0.86324618 & 1.32428279 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} -0.98752415 & 0.05126811 & 0.14643536 \\ -0.10158247 & -1.32430174 & -0.12450397 \\ -0.0678056 & 0.8843725 & -0.65910293 \end{bmatrix}$$

And we find 2 ts:

$$t_1 = \begin{bmatrix} -0.99941217 & -0.02023113 & -0.02767687 \end{bmatrix}$$

$$t_2 = \begin{bmatrix} 0.99941217 & 0.02023113 & 0.02767687 \end{bmatrix}$$

2.2 Library

We find 4 Rs:

With determinant = 1

$$R_1 = \begin{bmatrix} 0.95702867 & 0.08293273 & -0.2851699 \\ -0.02558023 & 0.94867095 & 0.00377596 \\ 0.28484123 & 0.99694234 & 1.01006073 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.91835894 & 0.07225599 & -0.39201305 \\ 0.01617008 & -1.05084429 & -0.00559828 \\ -0.39835351 & 0.99206368 & -0.86618664 \end{bmatrix}$$

With determinant = -1

$$R_3 = \begin{bmatrix} -0.95702867 & -0.08293273 & 0.2851699 \\ 0.02558023 & -0.94867095 & -0.00377596 \\ -0.28484123 & -0.99694234 & -1.01006073 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} -0.91835894 & -0.07225599 & 0.39201305 \\ -0.01617008 & 1.05084429 & 0.00559828 \\ 0.39835351 & -0.99206368 & 0.86618664 \end{bmatrix}$$

And we find 2 ts:

$$t_1 = \begin{bmatrix}-0.99837057 & 0.00519231 & 0.0568264\end{bmatrix}$$

$$t_2 = \begin{bmatrix}0.99837057 & -0.00519231 & -0.0568264\end{bmatrix}$$

3 Triangulation

3.1 3D Point Cloud Reconstruction

The reconstruction of the 3D point cloud, $X_{1,\ldots,n}$ was accomplished by reformulating equations (12) and (13) in the handout to be a linear system of equations. Given the camera matrices $P^{(j)}$ and corresponding points $\mathbf{x}_j$ for $j = 1, 2$ (two view points), we can reduce the system to $H\mathbf{X} = 0$ where,

$$H = \begin{bmatrix} P_{11}^{(j)} - x_j P_{31}^{(j)} & P_{12}^{(j)} - x_j P_{32}^{(j)} & P_{13}^{(j)} - x_j P_{33}^{(j)} & P_{14}^{(j)} - x_j P_{34}^{(j)} \\ P_{21}^{(j)} - y_j P_{31}^{(j)} & P_{22}^{(j)} - y_j P_{32}^{(j)} & P_{23}^{(j)} - y_j P_{33}^{(j)} & P_{24}^{(j)} - y_j P_{34}^{(j)} \\ \vdots & \vdots & \vdots & \end{bmatrix},$$

In this case, H is a (4x4) matrix. Then we can solve for the $\mathbf{X}^*$ which minimizes $H\mathbf{X}$ by picking the right singular vector of H corresponding to the samllest singular value (similar to the Fundamental matrix approximation).

3.2 Reconstruction Error

The reconstruction error is defined as the the mean distance between the 2D points and the projected 3D points in the two images. Mathematically, given the reconstructed 3D point $\tilde{X}_i$ for match i, the reconstruction error is the following:

$$error = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{2} \left\| P_j \tilde{X}_i - \mathbf{x}_j \right\|_2$$

The reconstruction errors for the 'house' and 'library' are summarized in the table below.

Name	Reconstruction Error
house	19.87
library	7.67

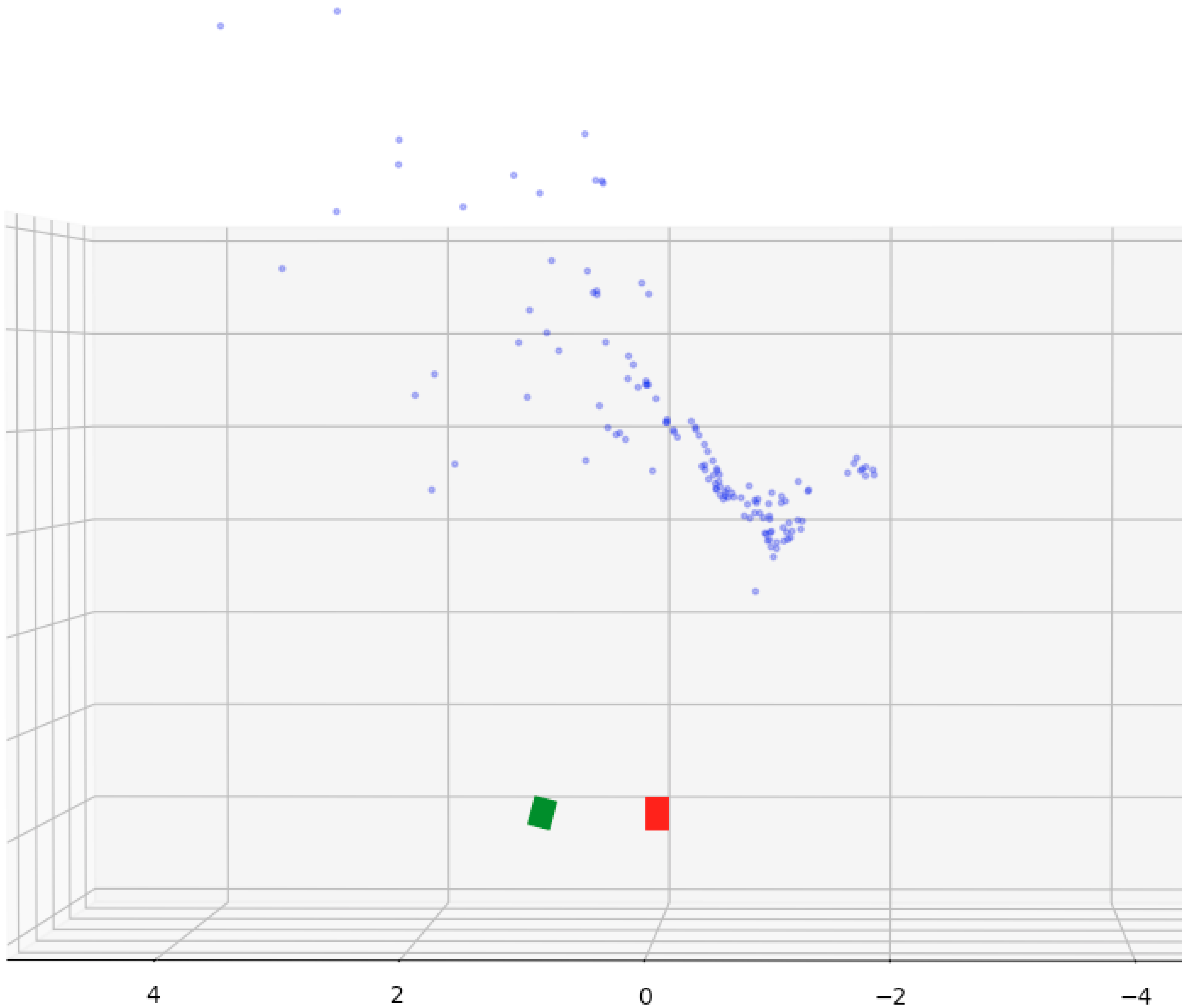
Table 2: reconstruction error

4 3D Reconstruction

4.1 House

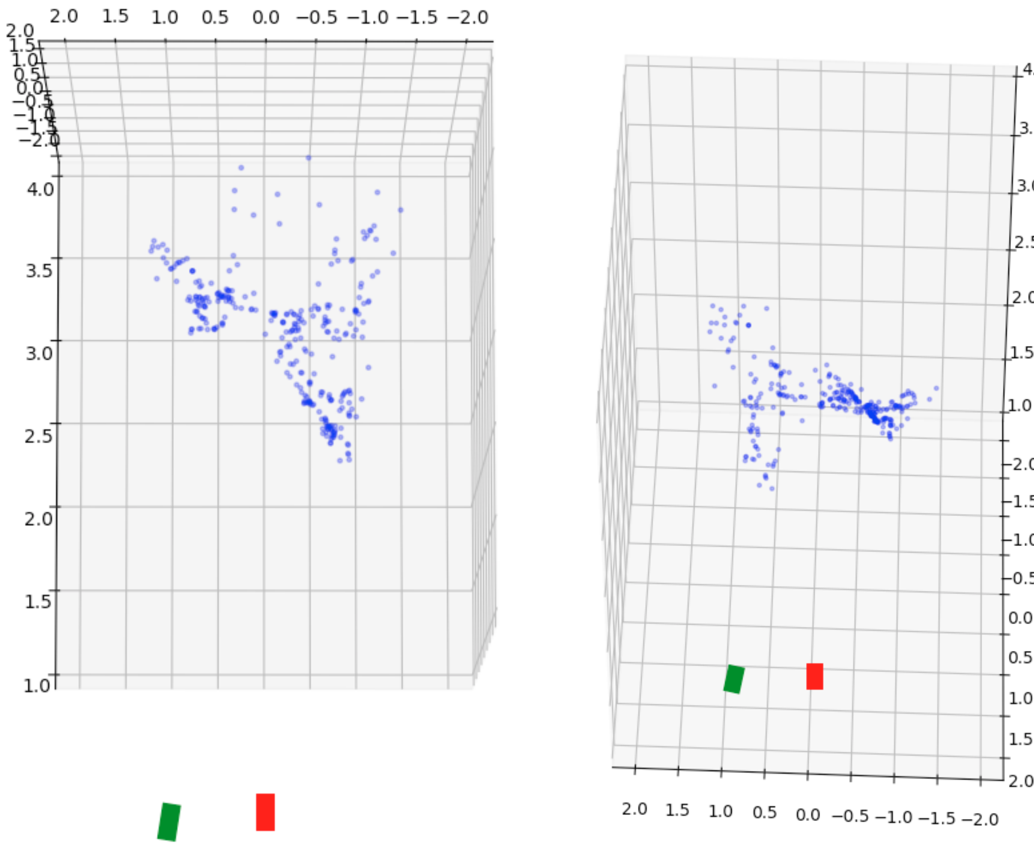
Here is the 3d point cloud plot for the house from one perspective (top-down). We can see the sharp corner of the house is detected on the right side of the visual field.

There are some outliers that are cropped from the window.



4.2 Library

Here are the 3d point cloud plots for the library from two perspectives (top-down and angled from top-down). We can see the angle of the library is captured in the point cloud, and there is a piece of the point cloud that juts out that corresponds to the tower in the left sides of the images.


2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
4.0 3.5 3.0 2.5 2.0 1.5 1.0
4.0 3.5 3.0 2.5 2.0 1.5 1.0
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0