Hamiltonian Report

1. introduction

This experiment hopes to establish a Matlab simulation model to represent the pendubot to obtain better physical parameters for real system and gain understanding of dynamics of the real system.

2. method

This section should include the theory, mathematical equations, and complete description of how and why the chosen method is a valid approach to the problem.

A. Hamiltonian

In Hamiltonian mechanics, physical systems are described by q and p, where q is position of components and p is generalized momentum of components. From external robot, we can find position and velocity of components use sensors. Momentum is also easy to get use velocity and mass of components.

In Hamiltonian dynamics, Hamiltonian H represents total energy of system, which is the sum of kinetic energy K and potential energy V.

$$H = K + V$$

Hamiltonian function:

$$H(q(t), \dot{q}(t)) = \sum_i h_i\big(q(t), \dot{q(t)}\big)\dot{\theta}_i + m_1 g z_{c1}$$

We calculate Hamiltonian use the usual definition of H as Legendre transformation of L :

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h1(i) = 0.5*q1d(i)^2;
h2(i) = 0.5*sin(q2(i))^2*q1d(i)^2+0.5*q2d(i)^2;
h3(i) = cos(q2(i))*q1d(i)*q2d(i);
h4(i) = 9.802*cos(q2(i));
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Represent parameters as follows：

$$\theta = A^{-1}d = \begin{bmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_6\end{bmatrix}$$

$$d = \begin{bmatrix}\int_{t_0}^{t_1} v(s)\dot{q}_1(s)ds\\ \int_{t_0}^{t_2} v(s)\dot{q}_1(s)ds\\ \vdots\\ \int_{t_0}^{t_n} v(s)\dot{q}_1(s)ds\end{bmatrix} \quad , \quad A = [\bar{H}\ \bar{F}]$$

$$\bar{H}(t, t_0) = \begin{bmatrix} h_1\big(q(t_1), \dot{q}(t_1)\big) - h_0\big(q(t_1), \dot{q}(t_1)\big) & \cdots & h_4\big(q(t_1), \dot{q}(t_1)\big) - h_0\big(q(t_1), \dot{q}(t_1)\big)\\ \vdots & \ddots & \vdots\\ h_1\big(q(t_n), \dot{q}(t_1)\big) - h_0\big(q(t_n), \dot{q}(t_1)\big) & \cdots & h_4\big(q(t_n), \dot{q}(t_1)\big) - h_0\big(q(t_n), \dot{q}(t_1)\big)\end{bmatrix}$$

$$\bar{F}(t,t_0)=\begin{bmatrix}\int_{t_0}^{t_1}\dot{q}_1(s)^2ds & \int_{t_0}^{t_1}\dot{q}_2(s)^2ds\\ \vdots & \vdots\\ \int_{t_0}^{t_n}\dot{q}_1(s)^2ds & \int_{t_0}^{t_n}\dot{q}_2(s)^2ds\end{bmatrix}$$

Tips:

A is not square matrix, so should not use 'inv()' to get its inverse matrix. We can use 'pinv()' or use devide.

B. Lagrangian

We calculate Hamiltonian from Lagragian. Lagrain also means conservation of energy, that is, Mechanical energy change equals external input minus internal consumption.

Lagrangian equation:

$$\mathcal{L}=K-V$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_j}-\frac{\partial\mathcal{L}}{\partial q_j}=\tau_j, j=1,\ldots,m$$

Link 1 Kinetic Energy:

$$K_1=\tfrac{1}{2}J_1\dot{q}_1^2$$

Link 1 Potential Energy:

$$V_1=m_1gz_{c1}$$

For link 2, use q1 and q2 to map the position to original frame:

$$p(x_2)=\begin{pmatrix}(l_1+l_2')\cos(q_1)-x_2\sin(q_2)\sin(q_1)\\ (l_1+l_2')\sin(q_1)+x_2\sin(q_2)\cos(q_1)\\ x_2\cos(q_2)\end{pmatrix}$$

$$v(x_2)=\begin{pmatrix}-(l_1+l_2')\sin(q_1)\dot{q}_1-x_2\cos(q_2)\sin(q_1)\dot{q}_2-x_2\sin(q_2)\cos(q_1)\dot{q}_1\\ (l_1+l_2')\cos(q_1)\dot{q}_1+x_2\cos(q_2)\cos(q_1)\dot{q}_2-x_2\sin(q_2)\sin(q_1)\dot{q}_1\\ -x_2\sin(q_2)\dot{q}_2\end{pmatrix}$$

$$K_1=\tfrac{1}{2}J_1\dot{q}_1^2$$

$$K_2 = \frac{1}{2}\int_0^{l_2} \rho_2 A_2 |v(x_2)|^2 \, dx_2$$

Total Potential Energy:

$$V = m_1 g z_{c_1} + m_2 g l_{c_2} \cos(q_2)$$

Express torque using voltage:

$$v = R_a i_a + k_v \dot{q}_m$$

Consider friction as internal consumption and revise function for joint 1 and joint 2:

$$\tau_1 \to \tau_1 - \beta_1 \dot{q}_1$$

$$0 \to \beta_2 \dot{q}_2$$

Get Langrangian Dynamics:

$$[\theta_1 + \theta_2 \sin^2(q_2)]\ddot{q}_1 + \theta_3 \cos(q_2)\ddot{q}_2 + 2\theta_2 \sin(q_2)\cos(q_2)\dot{q}_1\dot{q}_2 - \theta_3 \sin(q_2)\dot{q}_2^2 + \theta_5 \dot{q}_1$$
$$\theta_3 \cos(q_2)\ddot{q}_1 + \theta_2 \ddot{q}_2 - \theta_2 \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4 g \sin(q_2) + \theta_6 \dot{q}_2 = 0$$

3. procedure

This experiment can be summarized into four steps, the first step is the experimental preparation work, the second step is to establish the Hamiltonian Dynamic equation to represent the internal dynamics of the robot, and the third step is to collect the data calculation system from the real robot system and calculate system parameters.

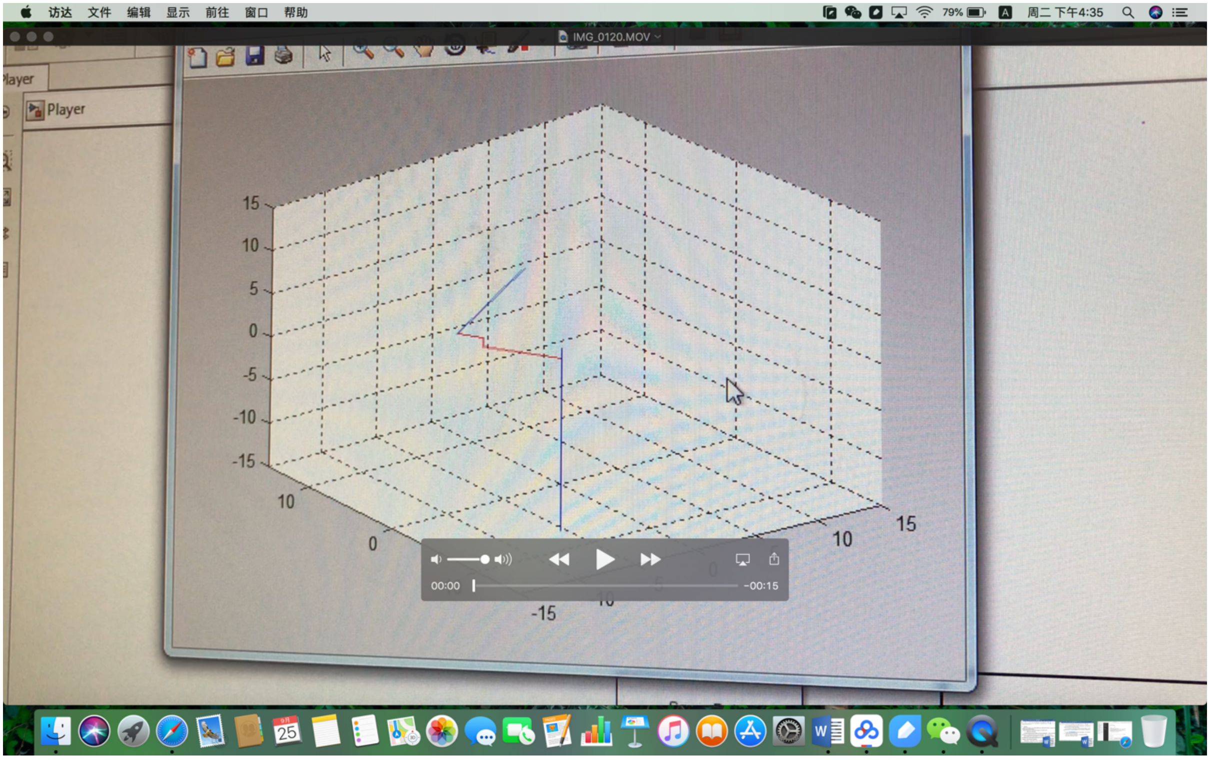
A. First Step

Preparation is to represent the two parts of the robot to the same coordinate system and draw the robot in Matlab.

First, when observing the two arms of the robot, we observe their respective positions (angles). Because of the different reference coordinate systems, they need to unify their coordinate systems.

Secondly, in order to prove that the collected data is correctly represented, we manually move links and record the position data, and draw the robot motion with Matlab. The comparison should be the same as the external.

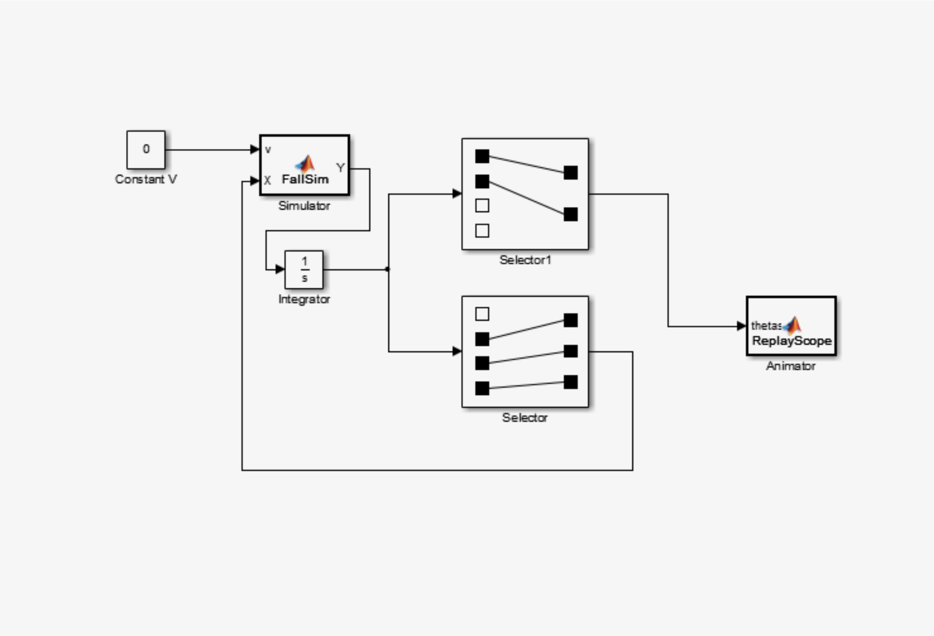
Result:



B. Second Step

First, establish the system Lagrangian dynamic equation, input the initial state (position and velocity of the two arms) and speed, and simulate the real system motion in Matlab.

The simulation model is shown below：

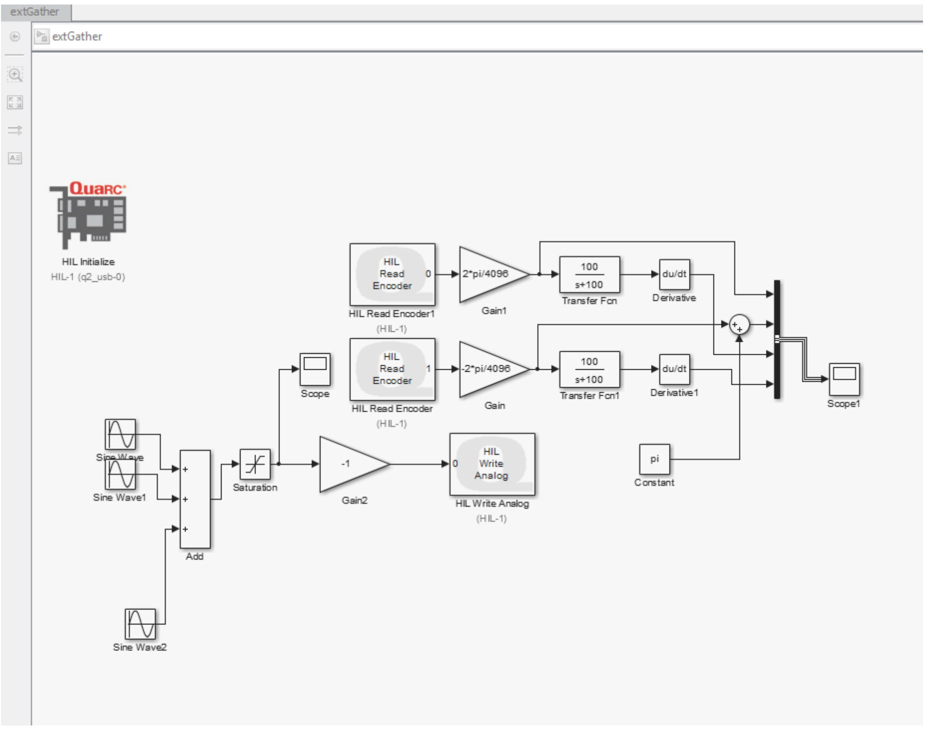


Next, the Hamilton dynamic equation is established. Enter the series of motion states of the two arms calculated by Lagrange dynamics based on the given parameters generated from previous step to calculate the system parameters. In this step, we make sure our Hamilton dynamic equation is established correctly from getting the same parameter values as the previous step.

C. Third Step

We collect data and calculate parameters from real robot systems and analyze the parameter values.

The model that implements this step is as follows：



There are three points worth noting:

First, when building the model, in the right-handed coordinate system, the counterclockwise rotation direction is the positive direction, but inputting a positive voltage in the real system causes the link 1 to rotate clockwise, so the input voltage should be changed to negative.

Second, in the established model, the position of link 2 at the highest point is pi, and the position at the lowest point is 0. However, when the potential energy is calculated, the position of pi corresponds to the negative potential energy, and the position of 0 corresponds to the positive potential energy, so the position representation of link 2 needs to be corrected.

Third, we need to correct the direction of link 2.

To collect the data, we input three sinewave signals. It should be noted that in order for the collected data to represent all the properties of the system (friction, gravity of the two arms, etc.), the input signal should be able to make the link 2 flip over.

4. results

We run our model and collect data from 4 different stations. And then use the data to calculate thetas. We use the table below to compare results from different stations or using different input.

station number	8	6	5	12	
theta1	0.0672	0.0758	0.0805	0.0732	ours
theta2	0.0231	0.0245	0.029	0.024	
theta3	0.0207	0.0244	0.0244	0.024	
theta4	0.1088	0.1178	0.132	0.0993	
theta5	0.5031	0.4879	0.4762	0.5	
theta6	0.0148	0.0124	0.0072	0.0153	
theta1	0.078	0.08	0.0785	0.0758	others
theta2	0.0298	0.33	0.0301	0.0264	
theta3	0.0292	0.0275	0.028	0.0254	
theta4	0.1477	0.1429	0.1404	0.1325	
theta5	0.5754	0.5563	0.5391	0.5432	
theta6	-0.0012	0.0051	0.0094	0.0123	

5. conclusion

The parameters in each real system are similar but not identical to the parameters in the simulation system. The reasons for analysis are as follows:

First, here is an expression for the parameters not given earlier：

$$\begin{aligned}
\theta_1' &= J_1 + m_2(l_1 + l_2')^2 \\
\theta_2' &= \tfrac{1}{3} m_2 (l_2)^2 \\
\theta_3' &= \tfrac{1}{2} m_2 (l_1 + l_2') l_2
\end{aligned}$$

$$\theta_4' = m_2 l_{c_2}$$

$$\begin{aligned}
\theta_i &= \theta_i' \frac{R_a}{k_r k_t} \quad i = 1, \ldots, 4 \\
\theta_5 &= \beta_1 \frac{R_a}{k_r k_t} + k_r k_v, \\
\theta_6 &= \beta_2 \frac{R_a}{k_r k_t}.
\end{aligned}$$

Analyze:

1. The friction is small, so theta5 difference in each station is mainly due to krkv. Kr is gear transfer, each system is the same, so theta5 is mainly the difference of kv, theta5 is larger, it proves that the larger kv, the higher the motor efficiency. Adjusting theta5 has a big impact on link1.
2. Theta6 is small compared to other parameters, proving that the friction of joint 2 is small
3. Theta2 and theta4 mainly reflect the difference in link2 mass. The bigger the mass, the larger the parameter value. These two parameters determine the time required to reach steady state under constant energy input.
4. Adjusting theta1 has little effect on the system