Fintech 545 Project 2

Problem 1:

This analysis examines the demeaned arithmetic and logarithmic returns for three stocks: SPY (S&P 500 ETF), AAPL (Apple Inc.), and EQIX (Equinix Inc.). Using the DailyPrices.csv dataset, I calculated both types of returns, removed their respective means to create zero-mean series, and analyzed their standard deviations and recent performance patterns.

A: Arithmetic Returns Analysis

Arithmetic returns were calculated using the percentage change formula: (P_t / P_{t-1}) - 1, which represents the simple percentage change in price from one period to the next. After calculating these returns, I subtracted the mean from each stock's return series to create demeaned series with zero mean.

The last five rows of the demeaned arithmetic returns reveal some interesting patterns. In late December 2024 through early January 2025, SPY and AAPL experienced predominantly negative returns, with SPY showing values of -0.011492, -0.012377, -0.004603, and -0.003422 before turning positive (0.011538) on January 3, 2025. AAPL demonstrated consistently negative returns throughout this period, with a particularly significant negative return of -0.027671 on January 2, 2025. In contrast, EQIX showed mixed performance, with negative returns in late December but positive returns of 0.006512 on December 31 and 0.015745 on January 3.

The standard deviations of the demeaned arithmetic returns provide insight into each stock's volatility. SPY exhibited the lowest standard deviation at 0.00807675, indicating it was the least volatile of the three stocks. AAPL showed moderate volatility with a standard deviation of 0.01348288, while EQIX demonstrated the highest volatility at 0.01536057. This suggests that EQIX experienced more significant price fluctuations than the other two stocks during the observed period.

B: Log Returns Analysis

Log returns were calculated using the natural logarithm of price ratios: ln(P_t / P_{t-1}). This approach offers certain theoretical advantages, including time-additivity and better alignment with the assumption of normally distributed returns. As with the arithmetic returns, the mean was subtracted from each log return series to create zero-mean series.

Examining the last five rows of demeaned log returns shows patterns very similar to the arithmetic returns, which is expected for small percentage changes. SPY's log returns ranged from -0.012410 to 0.011494, AAPL's ranged from -0.027930 to -0.003356 (remaining consistently negative), and EQIX's ranged from -0.007972 to 0.015725.

The standard deviations of the demeaned log returns were 0.00807822 for SPY, 0.01344643 for AAPL, and 0.01527023 for EQIX. These values are very close to but slightly different from the arithmetic return standard deviations. SPY again showed the lowest volatility, AAPL moderate volatility, and EQIX the highest volatility.

When comparing the two return calculation methods, we observe that the standard deviations are remarkably similar between arithmetic and log returns for each stock. SPY's standard deviation was 0.00807675 for arithmetic returns versus 0.00807822 for log returns, a negligible difference of about 0.00000147. For AAPL, the standard deviation was 0.01348288 for arithmetic returns versus 0.01344643 for log returns, a slightly larger but still minimal difference of 0.00003645. For EQIX, the difference was more noticeable at 0.00009034 (0.01536057 for arithmetic versus 0.01527023 for log returns).

These small differences are consistent with financial theory, which suggests that for small returns, arithmetic and log returns are approximately equal. The log return standard deviations tend to be slightly lower for stocks with higher volatility (like AAPL and EQIX), which is also consistent with theoretical expectations.

In conclusion, this analysis confirms that SPY, representing a broad market index, demonstrates lower volatility than individual stocks AAPL and EQIX. The recent return patterns show mixed performance across the three securities, with EQIX showing more positive returns in early 2025 compared to SPY and AAPL. Both arithmetic and logarithmic return calculations provide similar insights, with the latter offering theoretical advantages for further statistical analysis.

Problem 2: Portfolio Value and Risk Assessment Analysis

A. Portfolio Value Calculation

Based on the portfolio composition of 100 shares of SPY, 200 shares of AAPL, and 150 shares of EQIX, I calculated the total portfolio value as of January 3, 2025. The portfolio has a total value of $251,862.50. This calculation was made by multiplying the number of shares of each stock by their respective prices on January 3, 2025, and summing the results.

The portfolio weights reveal the relative contribution of each stock to the total portfolio value:

- SPY: 23.50%

- AAPL: 19.32%

- EQIX: 57.17%

These weights indicate that EQIX constitutes more than half of the portfolio's value despite having fewer shares than AAPL. This is due to EQIX's significantly higher price per share compared to the other two stocks. The portfolio's concentration in EQIX has important implications for its risk profile, as will be discussed in the risk metric analysis.

B. Value at Risk (VaR) and Expected Shortfall (ES) Analysis

I calculated the Value at Risk (VaR) and Expected Shortfall (ES) at the 5% alpha level for each stock and the entire portfolio using three different methodologies. All calculations assume arithmetic returns with zero mean.

a. Normal Distribution with Exponentially Weighted Covariance ($\lambda=0.97$)

Using the normal distribution model with exponentially weighted moving average (EWMA) covariance, I obtained the following results:

VaR (5%):

- SPY: $826.58

- AAPL: $947.04

- EQIX: $2,950.05

- Portfolio: $3,880.54

ES (5%):

- SPY: $1,863.14

- AAPL: $2,134.68

- EQIX: $6,649.53

- Portfolio: $8,746.90

This method places more weight on recent observations through the exponential weighting factor λ=0.97. The portfolio's VaR is significantly lower than the sum of individual stock VaRs ($4,723.67), demonstrating the diversification benefit of approximately 17.85%. This indicates that even though the stocks may be correlated, they don't all experience their worst outcomes simultaneously, thus reducing the portfolio's overall risk.

b. T-Distribution using a Gaussian Copula

The T-distribution approach accounts for fat tails in the return distributions. Based on the kurtosis of the return series, I estimated 5.81 degrees of freedom, indicating significantly fatter tails than a normal distribution would predict.

VaR (5%):

- SPY: $982.13

- AAPL: $1,125.26

- EQIX: $3,505.20

- Portfolio: $4,610.80

ES (5%):

- SPY: $59,923.96

- AAPL: $68,657.35

- EQIX: $213,868.07

- Portfolio: $281,325.55

The T-distribution VaR values are higher than those from the normal distribution model, reflecting the increased probability of extreme events. However, the ES values are exceptionally high, which may indicate an implementation issue in the ES calculation for the T-distribution. Typically, we'd expect higher, but not astronomically higher, ES values compared to the normal distribution.

c. Historical Simulation

The historical simulation method uses actual historical returns to estimate potential future losses without assuming any specific distribution.

VaR (5%):

- SPY: $815.29

- AAPL: $999.47

- EQIX: $3,517.90

- Portfolio: $4,317.17

ES (5%):

- SPY: $1,022.97

- AAPL: $1,367.92

- EQIX: $4,581.80

- Portfolio: $5,799.29

Historical simulation produced VaR estimates that fall between the normal and T-distribution estimates for most assets. The historical ES values are significantly lower than those from the T-distribution and also lower than the normal distribution ES values, suggesting a more moderate assessment of tail risk.

C. Discussion of Differences Between Methods

The three methods for calculating VaR and ES yield different results due to their underlying assumptions and approaches to modeling risk:

1. Distributional Assumptions:

The normal distribution model assumes returns follow a normal (Gaussian) distribution, which often underestimates the probability of extreme events. The T-distribution accommodates fatter tails but requires an appropriate estimate of degrees of freedom. Historical simulation makes no distributional assumptions, instead relying on the empirical distribution of past returns.

2. Treatment of Recent vs. Historical Data:

The EWMA approach in the normal distribution model places greater emphasis on recent observations, making it more responsive to changing market conditions. The decay factor $\lambda=0.97$ means that observations from roughly 23 trading days ago receive half the weight of the most recent observations. In contrast, historical simulation treats all historical observations equally, potentially including data that may no longer be relevant to current market conditions.

3. Estimation of Extreme Risk:

The differences in ES values across methods are particularly striking. The normal distribution likely underestimates extreme risk due to its thin tails. The T-distribution theoretically provides a better estimate for ES, though the extremely high values in our implementation suggest potential calculation issues. Historical simulation's ES represents the average of actual historical losses beyond VaR, providing a data-driven estimate that doesn't rely on distributional assumptions.

4. Diversification Benefits:

All three methods show diversification benefits, but to varying degrees. The normal distribution with EWMA and the T-distribution show similar diversification benefits (around 17.85%), while historical simulation shows a slightly higher benefit (approximately 19.04%). This suggests that historical correlations during stress periods may differ from those implied by parametric models.

5. Computational and Implementation Considerations:

The normal distribution model is the most straightforward to implement but makes the strongest assumptions. The T-distribution approach requires estimating degrees of freedom, which can be sensitive to outliers. Historical simulation is intuitive and easy to understand but may not capture future scenarios that haven't occurred in the historical data.

In conclusion, no single method provides a complete picture of portfolio risk. The normal distribution model with EWMA offers a balanced approach that adapts to changing market conditions but may underestimate extreme events. The T-distribution better accounts for fat tails but requires careful calibration. Historical simulation makes the fewest assumptions but is limited by available historical data. A prudent risk management approach would consider all three methods, potentially placing more weight on the higher risk estimates from the T-distribution and historical simulation when making conservative risk assessments.

The exceptionally high ES values from the T-distribution method warrant further investigation and may indicate a need to reconsider the specific implementation of the ES calculation for this distribution.

Problem 3: European Option Analysis and Portfolio Risk Assessment

A. Implied Volatility Calculation

For the European call option with a time to maturity of 3 months (0.25 years), a market price of $3.00, stock price of $31, strike price of $30, and risk-free rate of 10%, I calculated the implied volatility using the Newton-Raphson method to iteratively solve the Black-Scholes-Merton equation.

The resulting implied volatility is 33.51%.

This value represents the market's expectation of future volatility over the life of the option. It's worth noting that this implied volatility is significantly higher than a typical market index volatility, suggesting the market anticipates substantial price fluctuations for this particular stock.

B. Option Greeks and Price Sensitivity to Volatility

Using the calculated implied volatility of 33.51%, I determined the following option Greeks:

- Delta: 0.6659 - This indicates that for a $1 increase in the stock price, the call option price will increase by approximately $0.6659. The delta of 0.6659 also suggests that the option is moderately in-the-money.

- Vega: 0.0564 per 1% change in volatility - This means that for each percentage point increase in implied volatility, the option price is expected to increase by $0.0564.

- Theta: -0.0152 daily - This negative value indicates that the option loses approximately $0.0152 in value each day due to time decay, all else being equal.

To verify the accuracy of Vega, I calculated the new option price after increasing the implied volatility by 1 percentage point (from 33.51% to 34.51%):

- New option price: $3.0565

- Actual price change: $0.0565

- Expected change based on Vega: $0.0564

The actual price change ($0.0565) is virtually identical to what was predicted by Vega ($0.0564), with only a minimal difference of $0.0001 due to second-order effects. This confirms that Vega accurately predicts the price sensitivity to small changes in volatility.

C. Put Price Calculation and Put-Call Parity Verification

Using the Black-Scholes-Merton formula with the derived implied volatility, I calculated the price of the corresponding European put option:

Put Price: $1.2593

To verify whether put-call parity holds, I checked the following relationship:

- Left-hand side (C - P): $3.00 - $1.2593 = $1.7407

- Right-hand side (S - K·e^(-rT)): $31 - $30·e^(-0.10·0.25) = $1.7407

The difference between the two sides is effectively zero (within rounding error), confirming that put-call parity holds perfectly. This is expected in an arbitrage-free market for European options on non-dividend-paying stocks.

D. Portfolio Risk Calculation

I analyzed a portfolio consisting of:

1. One call option (same as above)

2. One put option (same strike and maturity)

3. One share of the underlying stock

Initial values of the portfolio components:

- Stock: $31.00

- Call option: $3.0001

- Put option: $1.2594

- Total portfolio value: $35.2595

The portfolio's Greeks:

- Portfolio Delta: 1.3318 (indicating the portfolio has greater sensitivity to stock price movements than the stock alone)

- Portfolio Gamma: 0.140129

- Portfolio Theta: -$0.0224 daily

For the risk calculations, I used the following assumptions:

- Annual stock volatility: 25%

- Expected annual return: 0%

- 255 trading days in a year

- 20-day holding period

- 5% confidence level for VaR and ES

- Constant implied volatility

Delta Normal Approximation Results

- Holding Period Volatility: 7.00%

- Portfolio Value Standard Deviation: $2.8907

- Time Decay Effect: -$0.4473

- Value at Risk (VaR, 5%): $5.2021

- Expected Shortfall (ES, 5%): $6.0235

Monte Carlo Simulation Results

Using 10,000 simulations of stock price paths:

- Value at Risk (VaR, 5%): $4.1428

- Expected Shortfall (ES, 5%): $4.5853

E. Comparison of Risk Measurement Methods

The two methods for calculating VaR and ES yielded different results due to their underlying assumptions and approaches:

1. Delta Normal Approximation:

   - This method assumes a linear relationship between changes in the stock price and changes in the portfolio value, based on the portfolio's delta.

   - It calculates VaR by applying a normal distribution quantile to the standard deviation of portfolio value changes, adjusted for time decay.

   - The Delta Normal method estimated a higher VaR ($5.2021) compared to Monte Carlo simulation ($4.1428), representing a difference of approximately 25%.

   - The method simplifies the non-linear behavior of options, which limits its accuracy for portfolios with significant option components.

2. Monte Carlo Simulation:

   - This approach simulates thousands of possible stock price paths and calculates the portfolio value for each scenario.

   - It captures the full non-linear relationship between stock price changes and portfolio value changes by revaluing options at each simulated price point.

   - Monte Carlo includes the interaction between price changes and time decay naturally through the Black-Scholes-Merton revaluation at each step.

- The resulting VaR estimate is likely more accurate for portfolios with options, as it accounts for their non-linear payoff structure.

3. Key Differences:

- The portfolio value versus stock price graph reveals a non-linear relationship that the Delta Normal method approximates as linear, while Monte Carlo captures fully.

- The Delta Normal method shows higher risk estimates because it doesn't account for the convexity benefit provided by the options in the portfolio.

- The portfolio has a positive gamma (0.140129), meaning its delta increases as the stock price rises and decreases as the stock price falls. This convexity actually reduces downside risk, which Monte Carlo correctly reflects but Delta Normal ignores.

- The time decay effect is handled as a constant adjustment in Delta Normal but interacts dynamically with price movements in Monte Carlo.

4. Practical Implications:

- For this particular portfolio with a delta greater than 1 and positive gamma, the Delta Normal method tends to overestimate risk.

- Monte Carlo provides a more realistic risk assessment by accounting for the full distribution of potential outcomes and the non-linear behavior of options.

- The difference between the methods becomes more significant for longer holding periods or portfolios with more complex option structures.

In conclusion, while the Delta Normal approximation provides a computationally efficient way to estimate risk, Monte Carlo simulation offers a more accurate assessment by capturing the non-linear characteristics of option-based portfolios. The graphs of portfolio value versus stock price clearly illustrate how the linear approximation differs from the actual non-linear relationship, explaining the different risk estimates produced by the two methods.