

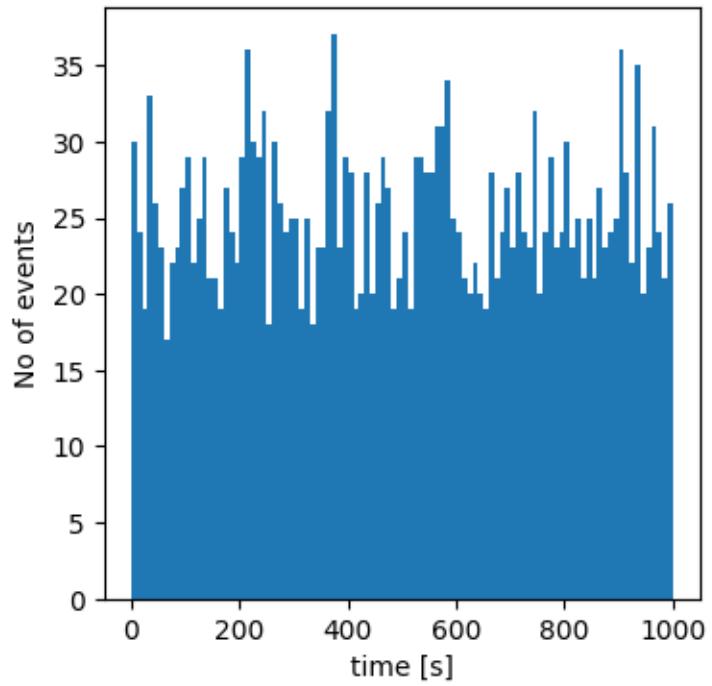
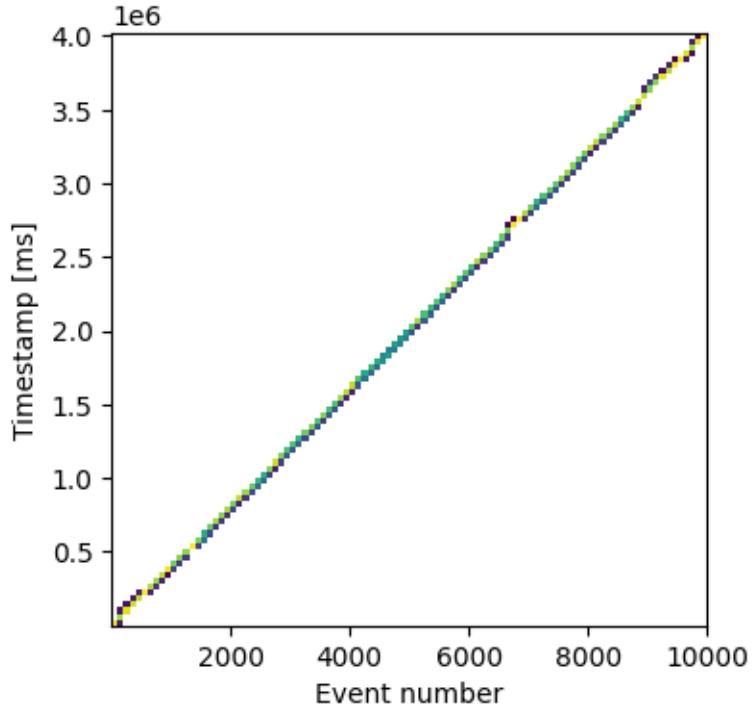
poisson

February 16, 2026

```
[1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

```
[ ]: data_path = "data/FileC015.txt"
df = pd.read_csv(data_path, skiprows=7, delim_whitespace=True, names=[
    'Event', 'Time', 'Date', 'TimeStamp', 'ADC1', 'ADC2', 'SiPM',
    'Temp', 'Pressure', 'DeadTime', 'Coincident', 'ID'
])
event = df["Event"].array
timestamp = df["TimeStamp"].array
print("First time stamp is ", timestamp[0])
print("Last time stamp is ", timestamp[-1])
plt.figure(figsize = (4,4))
counts, e1, e2, xxx = plt.hist2d(event[0:10000], timestamp[0:
    ↪10000], bins=(100,100), cmin=0.1)
plt.ylabel("Timestamp [ms]")
plt.xlabel("Event number")
plt.show()
# make histogram with 100 10s bins
plt.figure(figsize = (4, 4))
counts, e2, xxx = plt.hist(timestamp / 1000, range=(0, 1000), bins=100)
plt.xlabel("time [s]")
plt.ylabel("No of events")
plt.show()
```

```
/var/folders/j7/3qgnxg7x0mdgf8_bcqjqgyh80000gp/T/ipykernel_84114/839595825.py:2:
FutureWarning: The 'delim_whitespace' keyword in pd.read_csv is deprecated and
will be removed in a future version. Use ``sep='\\s+'`` instead
  df = pd.read_csv(data_path,skiprows=7,delim_whitespace=True,
First time stamp is  287
Last time stamp is  162659216
```



```
[3]: df.head(10)
```

```
[3]:   Event      Time      Date TimeStamp  ADC1  ADC2  SiPM  Temp  Pressure \
 0      1  0:00:07  1/1/2019       287  141.0    7.0   7.76  30.82  101315.64
 1      2  0:00:07  1/1/2019       349  789.0   36.0  21.02  30.82  101315.64
 2      3  0:00:07  1/1/2019       679  960.0  184.0  43.80  30.82  101315.82
 3      4  0:00:07  1/1/2019       789   87.0    7.0   6.28  30.82  101315.82
 4      5  0:00:09  1/1/2019      2105  927.0  142.0  37.97  30.81  101316.94
 5      6  0:00:09  1/1/2019      2415  102.0    8.0   6.65  30.81  101316.94
 6      7  0:00:09  1/1/2019      2755  204.0    8.0   9.39  30.81  101317.23
 7      8  0:00:09  1/1/2019      2990  577.0    8.0  13.57  30.81  101317.23
 8      9  0:00:10  1/1/2019      2999  725.0   16.0  16.99  30.81  101317.23
 9     10  0:00:10  1/1/2019      3611  377.0    8.0  11.92  30.81  101317.32
```

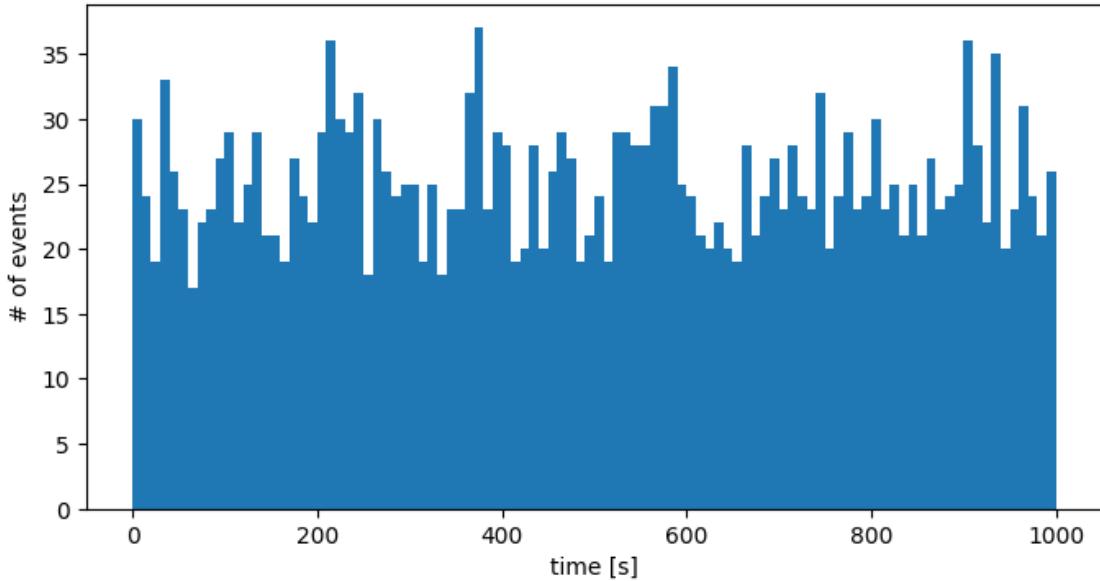


```
   DeadTime  Coincident        ID
 0     47968          0 ShirleyAnnJackson
 1     5975          0 ShirleyAnnJackson
 2     49804          0 ShirleyAnnJackson
 3     1834          0 ShirleyAnnJackson
 4     49802          0 ShirleyAnnJackson
 5     1835          0 ShirleyAnnJackson
 6     49803          0 ShirleyAnnJackson
 7     3823          0 ShirleyAnnJackson
 8     1835          0 ShirleyAnnJackson
 9     49804          0 ShirleyAnnJackson
```

0.1 1. Measuring the mean

0.1.1 (1) First 100 10-second bins

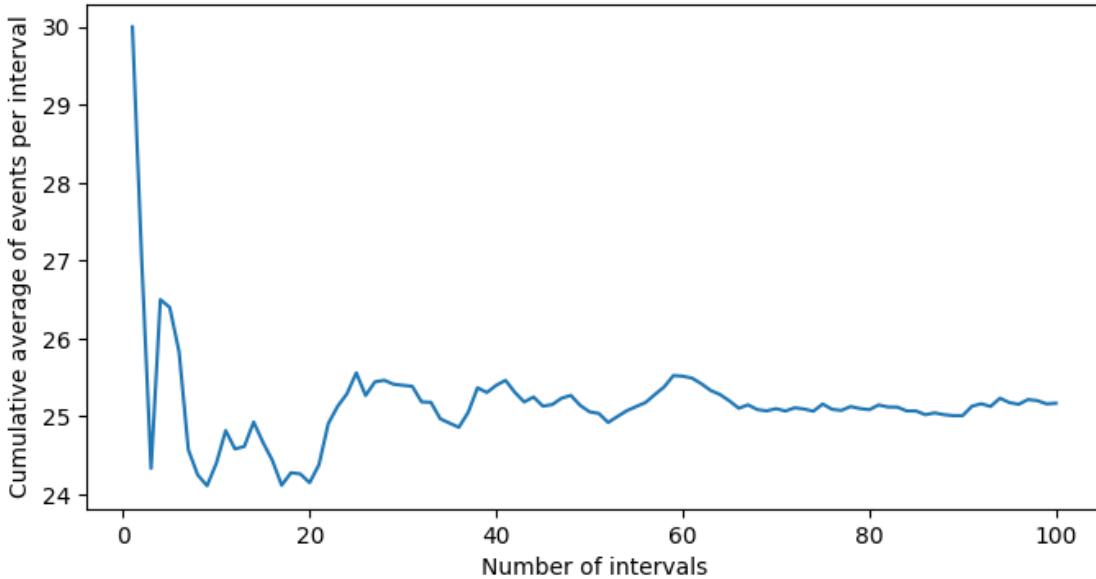
```
[ ]: plt.figure(figsize=(8, 4))
counts, _, _ = plt.hist(timestamp / 1000, range=(0, 1000), bins=100)
plt.xlabel("time [s]")
plt.ylabel("# of events")
plt.show()
print("Counts in the first 100 10-second bins: ", list(counts.astype(int)))
```



Counts in the first 100 10-second bins: [30, 24, 19, 33, 26, 23, 17, 22, 23, 27, 29, 22, 25, 29, 21, 21, 19, 27, 24, 22, 29, 36, 30, 29, 32, 18, 30, 26, 24, 25, 25, 19, 25, 18, 23, 23, 32, 37, 23, 29, 28, 19, 20, 28, 20, 26, 29, 27, 19, 21, 24, 19, 29, 29, 28, 28, 31, 31, 34, 25, 24, 21, 20, 22, 20, 19, 28, 21, 24, 27, 23, 28, 24, 23, 32, 20, 24, 29, 23, 24, 30, 23, 25, 21, 25, 21, 27, 23, 24, 25, 36, 28, 22, 35, 20, 23, 31, 24, 21, 26]

0.1.2 (2) Cumulative average for the 100 intervals

```
[ ]: cumulative_counts = np.cumsum(counts)
cumulative_avg = cumulative_counts / np.arange(1, 101)
plt.figure(figsize=(8, 4))
# plt.ylim(0, 30)
plt.plot(np.arange(1, 101), cumulative_avg)
plt.xlabel("Number of intervals")
plt.ylabel("Cumulative average of events per interval")
plt.show()
```

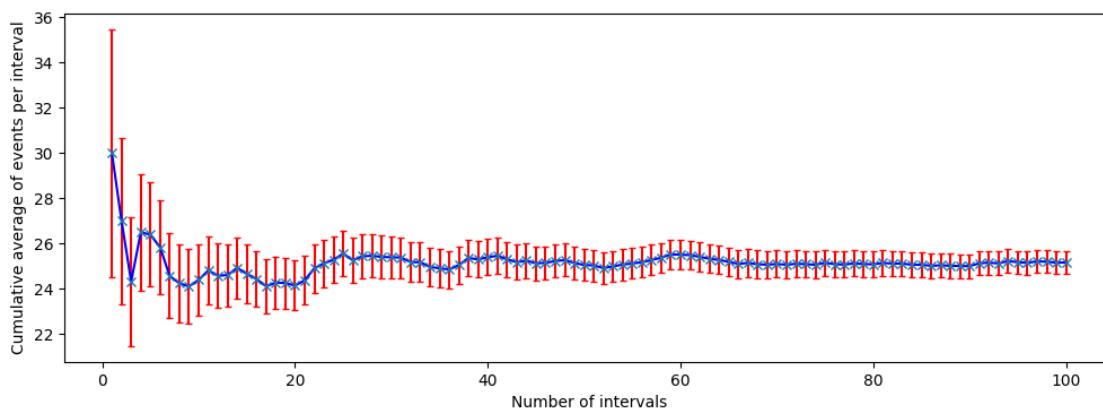


0.1.3 (3) Error estimation

$$\text{Var}(\mu_c(x_j)) = \frac{1}{j^2} \sum_{i=1}^j \text{Var}(x_i) = \frac{1}{j^2} \sum_{i=1}^j x_i = \frac{1}{j} \mu_c(x_j)$$

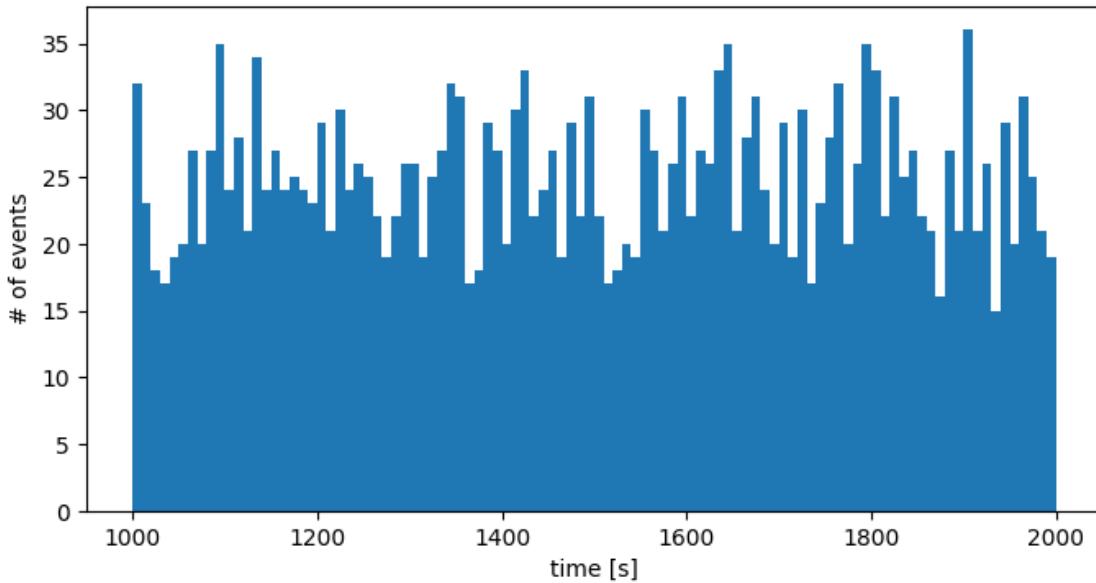
Therefore the standard deviation of $\mu_c(x_j)$ is $\sqrt{\frac{1}{j} \mu_c(x_j)}$.

```
[18]: errors = np.sqrt(cumulative_counts) / np.arange(1, 101)
plt.figure(figsize=(12, 4))
plt.errorbar(np.arange(1, 101), cumulative_avg, yerr=errors, fmt='x', ecolor='red', capsize=2)
plt.plot(np.arange(1, 101), cumulative_avg, color='blue')
plt.xlabel("Number of intervals")
plt.ylabel("Cumulative average of events per interval")
plt.show()
```



0.1.4 (4) A fresh set of 100 10-second bins

```
[19]: plt.figure(figsize=(8, 4))
counts_2, _, _ = plt.hist(timestamp / 1000, range=(1000, 2000), bins=100)
plt.xlabel("time [s]")
plt.ylabel("# of events")
plt.show()
print("Counts in the second 100 10-second bins: ", list(counts.astype(int)))
```



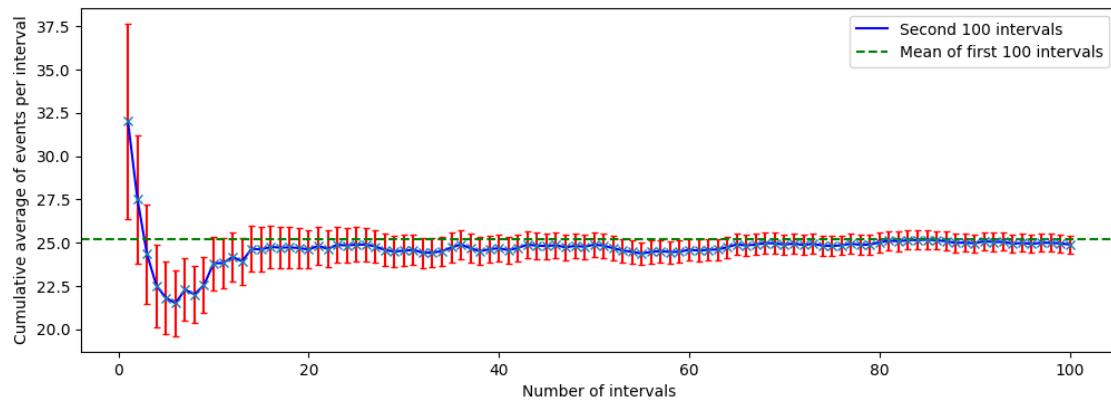
Counts in the second 100 10-second bins: [30, 24, 19, 33, 26, 23, 17, 22, 23, 27, 29, 22, 25, 29, 21, 21, 19, 27, 24, 22, 29, 36, 30, 29, 32, 18, 30, 26, 24, 25, 25, 19, 25, 18, 23, 23, 32, 37, 23, 29, 28, 19, 20, 28, 20, 26, 29, 27, 19, 21, 24, 19, 29, 29, 28, 28, 31, 31, 34, 25, 24, 21, 20, 22, 20, 19, 28, 21, 24, 27, 23, 28, 24, 23, 32, 20, 24, 29, 23, 24, 30, 23, 25, 21, 25, 21, 27, 23, 24, 25, 36, 28, 22, 35, 20, 23, 31, 24, 21, 26]

```
[20]: cumulative_counts_2 = np.cumsum(counts_2)
cumulative_avg_2 = cumulative_counts_2 / np.arange(1, 101)
# For reference, calculate the mean from the first set
mean_1 = cumulative_avg[-1]
plt.figure(figsize=(12, 4))
plt.errorbar(np.arange(1, 101), cumulative_avg_2, yerr=np.
    ↪sqrt(cumulative_counts_2) / np.arange(1, 101), fmt='x', ecolor='red', ↪
    ↪capsize=2)
```

```

plt.plot(np.arange(1, 101), cumulative_avg_2, color='blue', label='Second 100 intervals')
plt.axhline(mean_1, color='green', linestyle='--', label='Mean of first 100 intervals')
plt.xlabel("Number of intervals")
plt.ylabel("Cumulative average of events per interval")
plt.legend()
plt.show()

```



χ^2 test for the mean:

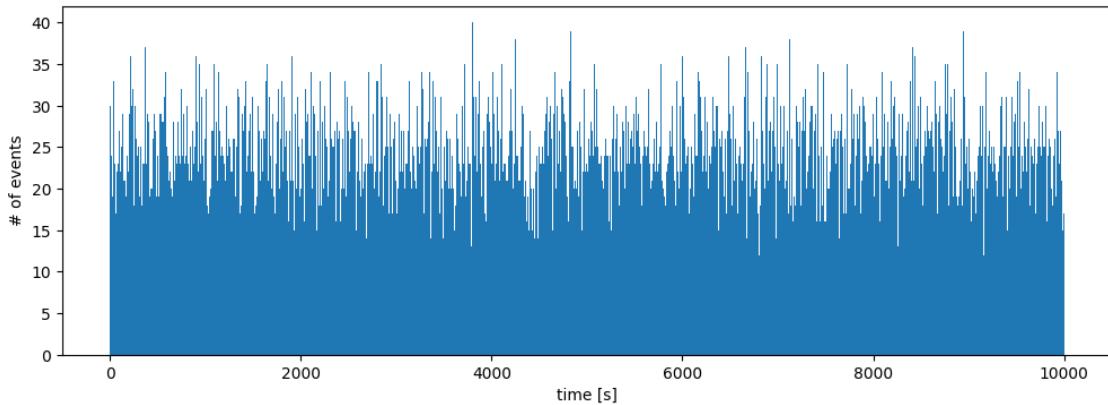
$$\chi^2 = \sum_{i=1}^n \frac{(x_i - \mu)^2}{\sigma_i^2}$$

```
[27]: # Do chi^2 test for the first 100 intervals
chi_square = np.sum((counts - mean_1) ** 2 / counts)
degrees_of_freedom = len(counts) - 1
print(f"\chi^2 / ndf = {chi_square:.2f} / {degrees_of_freedom}")

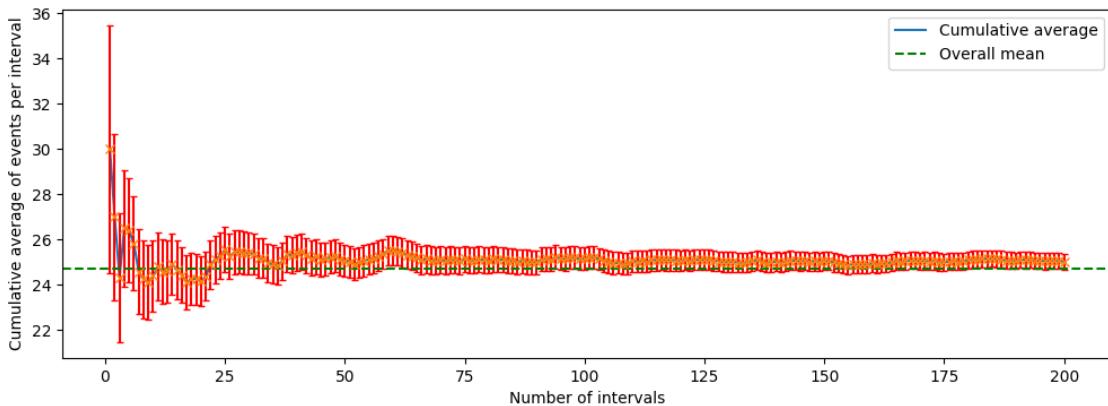
\chi^2 / ndf = 78.74 / 99
```

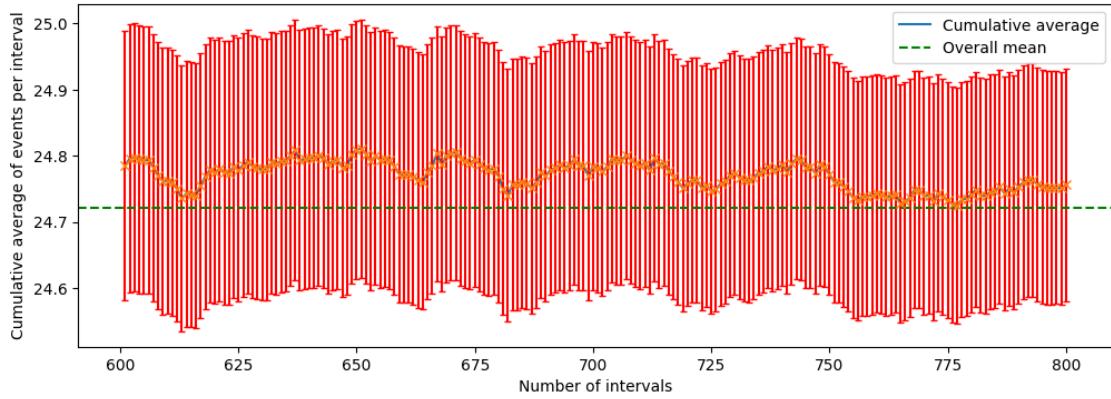
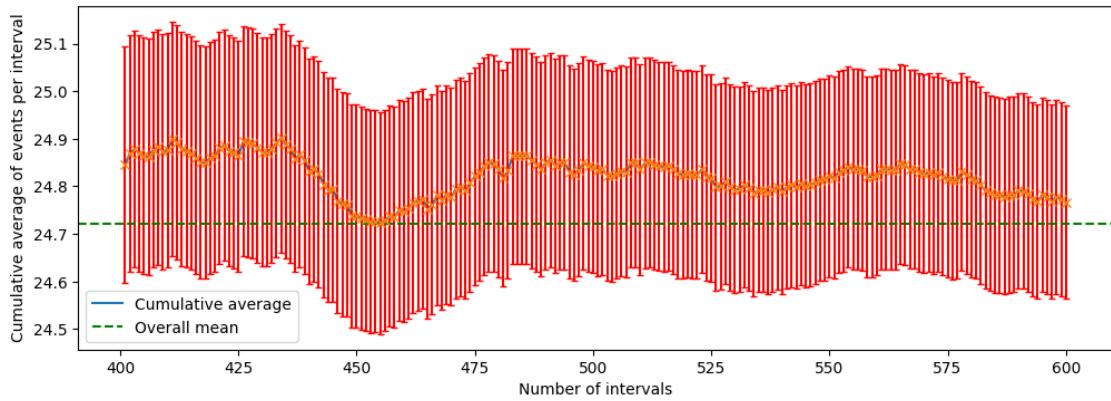
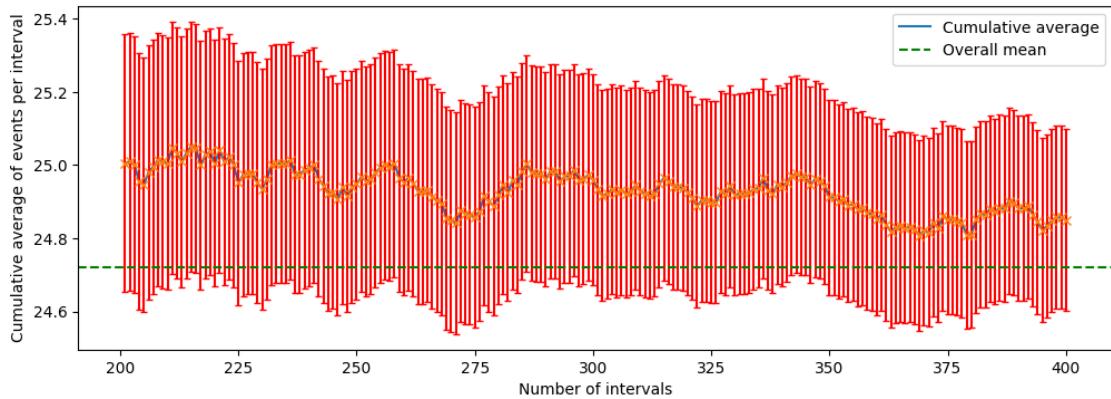
0.1.5 (5) Just some extension: let's do 1000 10-second bins

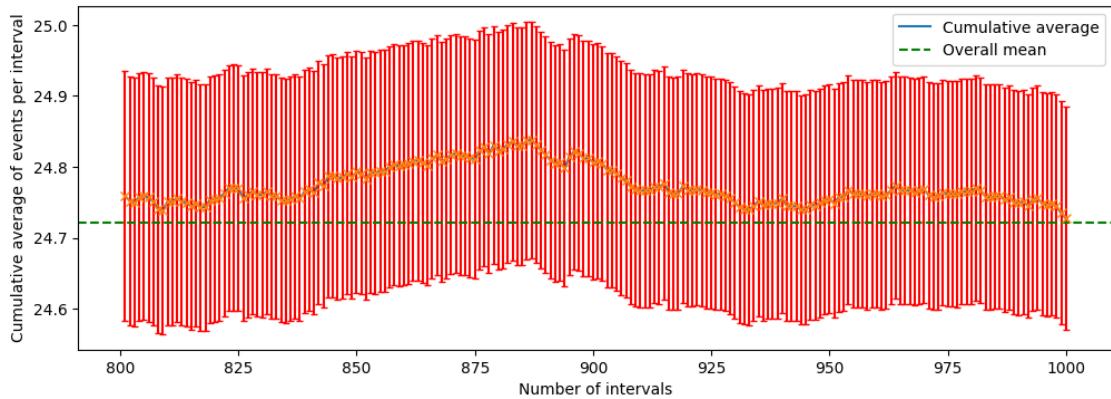
```
[29]: plt.figure(figsize=(12, 4))
counts_3, _, _ = plt.hist(timestamp / 1000, range=(0, 10000), bins=1000)
plt.xlabel("time [s]")
plt.ylabel("# of events")
plt.show()
```



```
[35]: mean = np.mean(counts_3[1:])
cumulative_counts_3 = np.cumsum(counts_3)
cumulative_avg_3 = cumulative_counts_3 / np.arange(1, 1001)
for end in [200, 400, 600, 800, 1000]:
    plt.figure(figsize=(12, 4))
    plt.plot(np.arange(end - 199, end + 1), cumulative_avg_3[end - 200:end], color='red', label='Cumulative average')
    plt.errorbar(np.arange(end - 199, end + 1), cumulative_avg_3[end - 200:end], yerr=np.sqrt(cumulative_counts_3[end-200:end]) / np.arange(end - 199, end + 1), fmt='x', ecolor='red', capsized=2)
    plt.axhline(mean, color='green', linestyle='--', label='Overall mean')
    plt.xlabel("Number of intervals")
    plt.ylabel("Cumulative average of events per interval")
    plt.legend()
    plt.show()
```







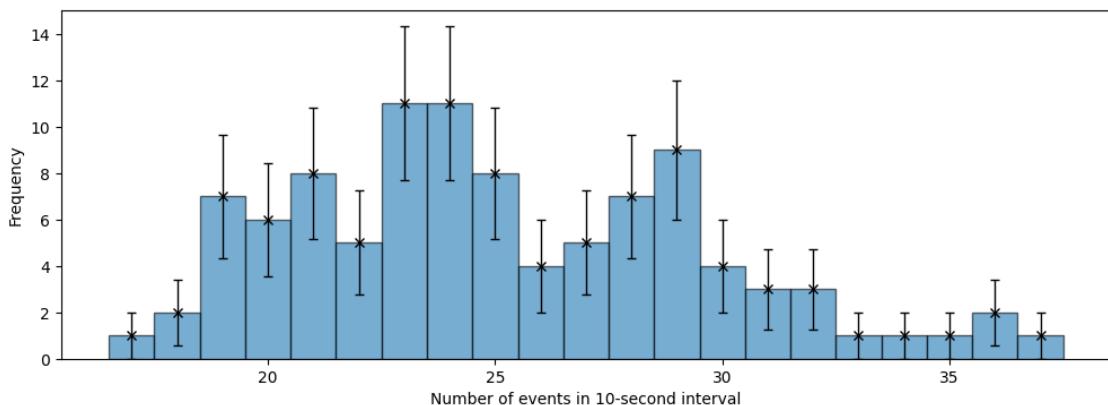
```
[34]: # Do chi^2 test for the first 100 intervals
chi_square = np.sum((counts_3 - mean) ** 2 / counts_3)
degrees_of_freedom = len(counts_3) - 1
print(f"\chi^2 / ndf = {chi_square:.2f} / {degrees_of_freedom}")

\chi^2 / ndf = 1092.76 / 999
```

0.2 2. Poisson versus Gaussian

0.2.1 (1) Frequency versus the Number of Counts

```
[52]: plt.figure(figsize=(12, 4))
n, bins, _ = plt.hist(counts, bins=np.arange(min(counts) - 0.5, max(counts) + 1.5, 1), alpha=0.6, color="CO", edgecolor="k")
centers = 0.5 * (bins[:-1] + bins[1:])
yerr = np.sqrt(n) # Poisson error on counts per bin
plt.errorbar(centers, n, yerr=yerr, fmt="x", color="k", capsize=3, lw=1)
plt.xlabel("Number of events in 10-second interval")
plt.ylabel("Frequency")
plt.show()
```



0.2.2 (2) Poisson Distribution Fit, from $\mu_c(100)$

The Poisson distribution is given by:

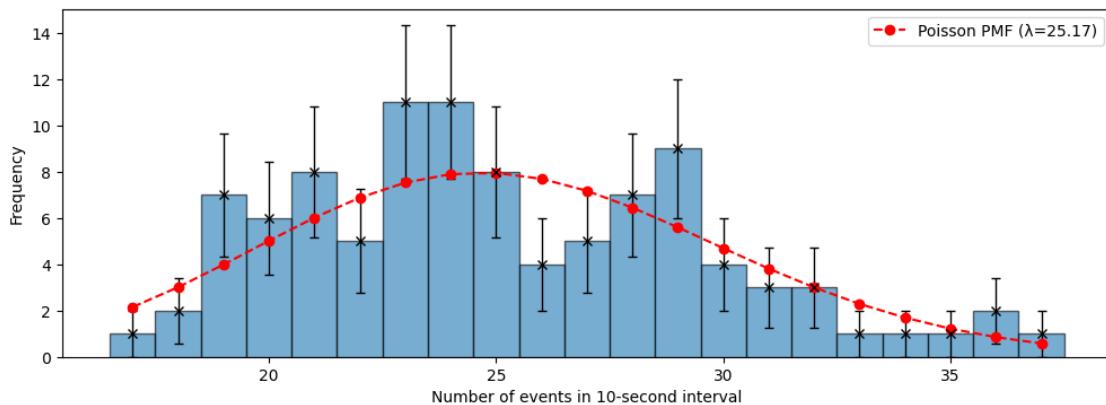
$$P(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Here, since the mean count is $\mu_c(100)$, $\lambda = \mu_c(100)$.

```
[55]: import math
def poisson_pmf(k, lam):
    return (lam ** k * np.exp(-lam)) / math.factorial(k)

lam = np.mean(counts)
k_values = np.arange(min(counts), max(counts) + 1).astype(int)
p_values = [poisson_pmf(k, lam) * 100 for k in k_values]

plt.figure(figsize=(12, 4))
n, bins, _ = plt.hist(counts, bins=np.arange(min(counts) - 0.5, max(counts) + 1.5, 1), alpha=0.6, color="C0", edgecolor="k")
centers = 0.5 * (bins[:-1] + bins[1:])
yerr = np.sqrt(n) # Poisson error on counts per bin
plt.errorbar(centers, n, yerr=yerr, fmt="x", color="k", capsize=3, lw=1)
plt.plot(k_values, p_values, color="red", marker="o", linestyle="--", label=f"Poisson PMF (=lam:.2f})")
plt.xlabel("Number of events in 10-second interval")
plt.ylabel("Frequency")
plt.legend()
plt.show()
```



```
[63]: # chi^2 test for goodness of fit
expected_counts = [poisson_pmf(k, lam) * len(counts) for k in k_values]
chi_square = np.sum((n - expected_counts) ** 2 / expected_counts)
ndf = len(k_values) - 1

print(f"\chi^2 / ndf = {chi_square:.2f} / {ndf}")

from scipy.stats import chi2
p = 1 - chi2.cdf(chi_square, ndf)
print(f"p-value = {p:.4f}")
```

\chi^2 / ndf = 15.15 / 20
p-value = 0.7677

0.2.3 (3) Gaussian Distribution Fit

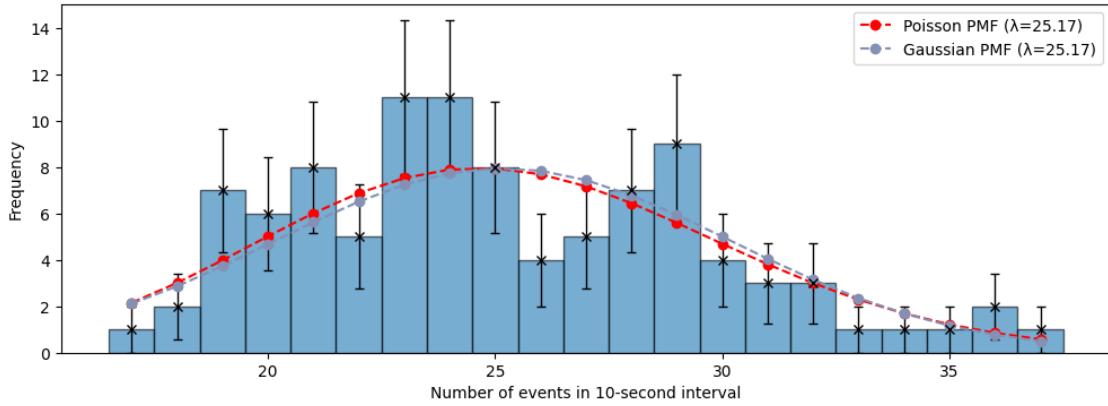
When μ is large,

$$\lim_{\mu \rightarrow \infty} P(k; \mu) = \frac{1}{\sqrt{2\pi\mu}} e^{-\frac{(k-\mu)^2}{2\mu}}$$

```
[ ]: def gaussian_pmf(k, mu):
    return (1 / (np.sqrt(2 * np.pi * mu))) * np.exp(-0.5 * ((k - mu) ** 2) / mu)

gaussian_values = [gaussian_pmf(k, lam) for k in k_values]

plt.figure(figsize=(12, 4))
n, bins, _ = plt.hist(counts, bins=np.arange(min(counts) - 0.5, max(counts) + 1.5, 1), alpha=0.6, color="#C0", edgecolor="k")
centers = 0.5 * (bins[:-1] + bins[1:])
yerr = np.sqrt(n) # Poisson error on counts per bin
plt.errorbar(centers, n, yerr=yerr, fmt="x", color="k", capsize=3, lw=1)
plt.plot(k_values, p_values, color="red", marker="o", linestyle="--", label=f"Poisson PMF (=lam:.2f))")
plt.plot(k_values, gaussian_values, color="#8491B4", marker="o", linestyle="--", label=f"Gaussian PMF (=lam:.2f))")
plt.xlabel("Number of events in 10-second interval")
plt.ylabel("Frequency")
plt.legend()
plt.show()
```



```
[65]: # again, chi^2 test for goodness of fit
expected_counts = [gaussian_pmf(k, lam) for k in k_values]
chi_square = np.sum((n - expected_counts) ** 2 / expected_counts)
ndf = len(k_values) - 1

print(f"\chi^2 / ndf = {chi_square:.2f} / {ndf}")

p = 1 - chi2.cdf(chi_square, ndf)
print(f"p-value = {p:.4f}")
```

χ^2 / $ndf = 17.05 / 20$

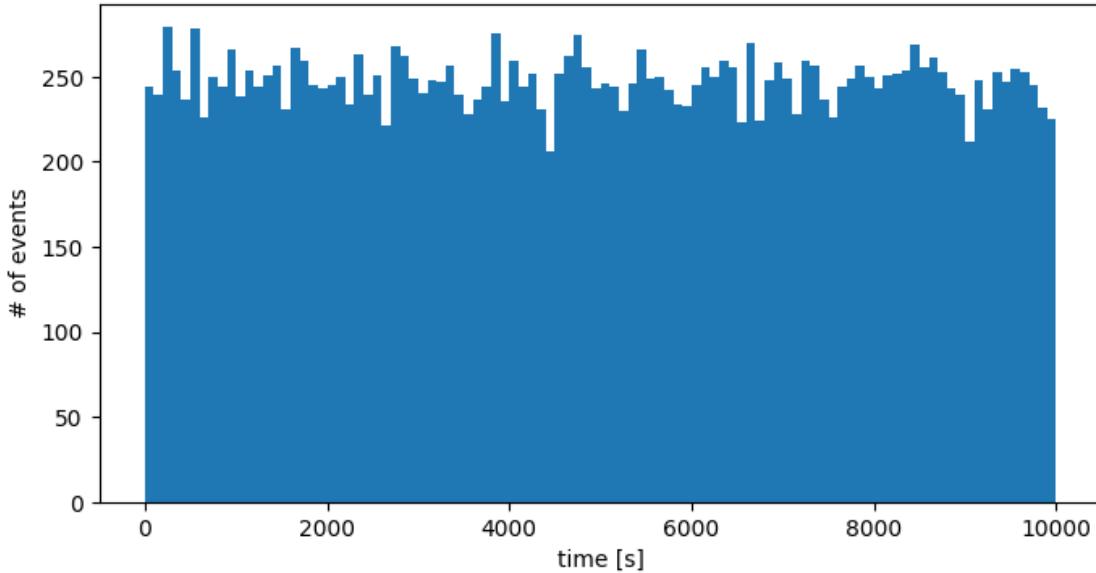
p-value = 0.6500

χ^2 test for the Poisson/Gaussian fit:

- Poisson: $\chi^2 / ndf = 15.15 / 20, p = 0.7677$
- Gaussian: $\chi^2 / ndf = 17.05 / 20, p = 0.6500$

0.2.4 (4) Same Process, But with 100 100-Second Bins

```
[66]: plt.figure(figsize=(8, 4))
counts, _, _ = plt.hist(timestamp / 1000, range=(0, 10000), bins=100)
plt.xlabel("time [s]")
plt.ylabel("# of events")
plt.show()
print("Counts in the first 100 100-second bins: ", list(counts.astype(int)))
```



Counts in the first 100 100-second bins: [244, 239, 279, 254, 237, 278, 226, 250, 244, 266, 238, 254, 244, 251, 257, 231, 267, 259, 245, 243, 245, 250, 234, 263, 239, 251, 221, 268, 262, 249, 240, 248, 247, 257, 239, 228, 237, 244, 276, 236, 259, 244, 252, 231, 206, 252, 262, 275, 256, 243, 246, 244, 230, 246, 266, 249, 250, 242, 234, 233, 245, 256, 250, 259, 256, 223, 270, 224, 248, 258, 249, 228, 259, 257, 237, 226, 244, 249, 257, 250, 243, 251, 252, 254, 269, 256, 261, 253, 243, 239, 212, 248, 231, 253, 247, 255, 253, 245, 232, 225]

```
[71]: mean_new = np.mean(counts)
       mean_new, min(counts), max(counts)
```

[71]: (247.27, 206.0, 279.0)

```
[76]: def poisson_pmf(k, lam):
        try:
            from scipy.stats import poisson
            return poisson.pmf(k, lam)
        except ImportError:
            return (lam ** k * np.exp(-lam)) / math.factorial(k)

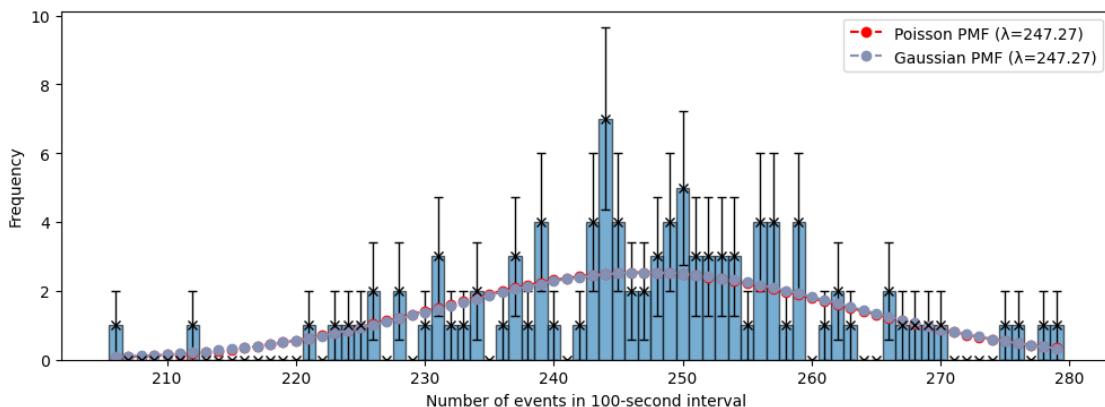
def gaussian_pmf(k, mu, bin_width):
    try:
        from scipy.stats import norm
        return norm.pdf(k, loc=mu, scale=np.sqrt(mu)) * bin_width
    except ImportError:
        return (1 / (np.sqrt(2 * np.pi * mu))) * np.exp(-0.5 * ((k - mu) ** 2) /
        mu) * bin_width
```

```

k_values_new = np.arange(int(min(counts)), int(max(counts)) + 1)
poisson_values_new = [poisson_pmf(k, mean_new) * len(counts) for k in
    ↪k_values_new]
gaussian_values_new = [gaussian_pmf(k, mean_new, bin_width=10000/100) for k in
    ↪k_values_new]

plt.figure(figsize=(12, 4))
n, bins, _ = plt.hist(counts, bins=np.arange(min(counts) - 0.5, max(counts) + 1.
    ↪5, 1), alpha=0.6, color="#C0E0F0", edgecolor="k")
centers = 0.5 * (bins[:-1] + bins[1:])
yerr = np.sqrt(n) # Poisson error on counts per bin
plt.errorbar(centers, n, yerr=yerr, fmt="x", color="k", capsize=3, lw=1)
plt.plot(k_values_new, poisson_values_new, color="red", marker="o",
    ↪linestyle="--", label=f"Poisson PMF (={mean_new:.2f})")
plt.plot(k_values_new, gaussian_values_new, color="#8491B4", marker="o",
    ↪linestyle="--", label=f"Gaussian PMF (={mean_new:.2f})")
plt.xlabel("Number of events in 100-second interval")
plt.ylabel("Frequency")
plt.legend()
plt.show()

```



```
[77]: chi_square_poisson = np.sum((n - poisson_values_new) ** 2 / poisson_values_new)
chi_square_gaussian = np.sum((n - gaussian_values_new) ** 2 / ↪
    ↪gaussian_values_new)
ndf = len(k_values_new) - 1
print(f"Poisson fit: \chi^2 / ndf = {chi_square_poisson:.2f} / {ndf}")
print(f"Gaussian fit: \chi^2 / ndf = {chi_square_gaussian:.2f} / {ndf}")

Poisson fit: \chi^2 / ndf = 68.09 / 73
Gaussian fit: \chi^2 / ndf = 66.43 / 73
```