Covariance adjustment in randomized experiments

Thomas Leavitt

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Covariance adjustment in randomized experiments

- We often have information contained in baseline covariates
  - I.e., variables measured **before** treatment assignment
- If baseline covariates related to POs, we want to use covariates to increase
  1. Precision of estimators
  2. Power of tests

Covariance adjustment in randomized experiments

- Two primary approaches to covariate adjustment

    1. Random assignment within blocks of units similar in covariates

    2. Rescaling outcome to make its variance smaller

Block random assignment

Block random assignment

- Construct blocks of units similar in baseline covariates related to POs
- By blocking we reduce number of possible assignments
    - Goal is to exclude assignments yielding estimates far from truth
      so that estimator will be closer to truth, on average

Block random assignment: Example

Example: Experiment with $N = 6$ units, $n_1 = 3$ and $n_0 = 3$

$\boldsymbol{y}(\mathbf{0})$	$\boldsymbol{y}(\mathbf{0})$	$\boldsymbol{\tau}$	$\boldsymbol{x}$
20	22	2	1
8	12	4	1
11	11	0	0
10	15	5	1
14	18	4	1
1	4	3	0

Table 1: True values of $\boldsymbol{y}(\mathbf{0})$, $\boldsymbol{y}(\mathbf{0})$, $\boldsymbol{\tau}$ and baseline covariate $\boldsymbol{x}$

Stratify on $\boldsymbol{x}$ and assign half of units to treatment within strata

In this case, instead of $\binom{6}{3} = 20$ assignments, we have $\prod_{b=1}^{B} \binom{N_b}{n_{1,b}} = 12$ assignments, where $b = 1, \ldots, B$ indexes the blocks

Block random assignment: Example

- Set of assignments under block random assignment
- Units with $x_i = 1$; units with $x_i = 0$

$$\Omega = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

- Only 12 assignments (instead of 20) because half of units w/ $x_i = 1$ treated and half of units w/ $x_i = 0$ treated

Block random assignment: Example

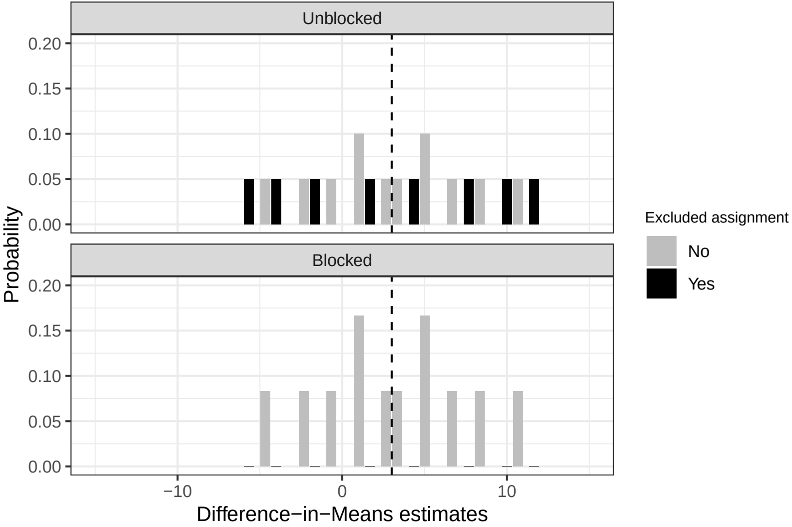

Figure 1: Difference-in-Means distribution under unblocked and blocked assignment

Rescaling outcomes

Rescaling outcomes

- Variance of Diff-in-Means

$$\mathrm{Var}\left[\hat{\tau}\left(\boldsymbol{Z}, \boldsymbol{Y}\right)\right] = \frac{1}{N-1}\left(\frac{n_1 \sigma^2_{\boldsymbol{y}(\mathbf{0})}}{n_0} + \frac{n_0 \sigma^2_{\boldsymbol{y}(\mathbf{1})}}{n_1} + 2\sigma_{\boldsymbol{y}(\mathbf{0}),\boldsymbol{y}(\mathbf{1})}\right)$$

- Can we rescale outcomes so that $\sigma^2_{\boldsymbol{y}(\mathbf{0})}$ and $\sigma^2_{\boldsymbol{y}(\mathbf{1})}$ are smaller?

⋆ Residualizing cannot alter individual treatment effects

$$\begin{aligned} y_i(1) - f(\boldsymbol{x}_i) - [y_i(0) - f(\boldsymbol{x}_i)] &= y_i(1) - f(\boldsymbol{x}_i) - y_i(0) + f(\boldsymbol{x}_i) \\ &= y_i(1) - y_i(0) \\ &= \tau_i, \end{aligned}$$

where $f(\cdot)$ is function that predicts outcome from $\boldsymbol{x}_i \in \mathbb{R}^K$ (Rosenbaum, 2002)

- Ideally, $f(\cdot)$ fit to historical data or set-aside sample not in experiment

Rescaling outcomes

- Example: Acorn GOTV experiment (Arceneaux, 2005)

	GOTV?	vote03 (%)	vote03 - vote02 (%)
1	0	38	-36
⋮	⋮	⋮	⋮
13	0	19	-38
14	0	34	-27
15	1	49	-25
16	1	38	-28
⋮	⋮	⋮	
28	1	29	-32

Conduct analysis on rescaled vote03 - vote02 (%) outcome

Rescaling outcomes

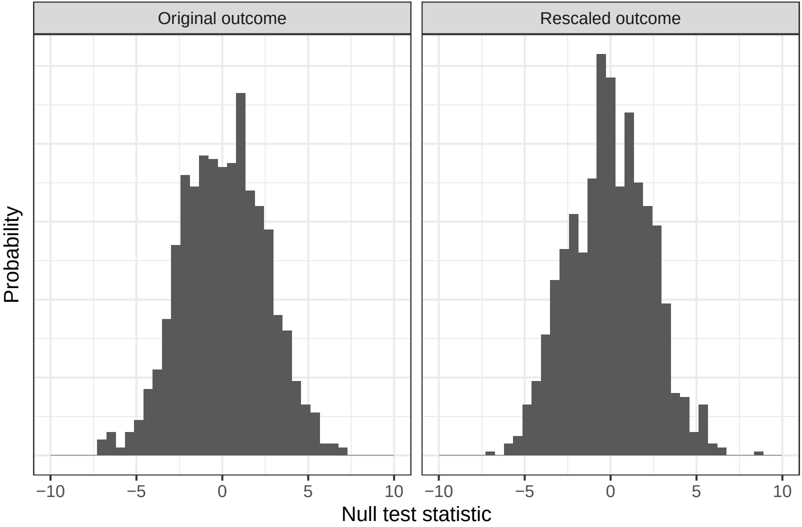

Figure 2: Difference-in-Means under sharp null on original and rescaled outcomes

Regression Adjustment

Linear regression

- Common tool for incorporating info from covariates
- Usual regression assumptions
  - Independent and identically distributed observations
  - Outcome conditional on predictors, $Y \mid X$, is Normally distributed
- In our setting,
  - Potential outcomes fixed (non-random) quantities
  - Experimental subjects **not** sampled from superpopulation
  - Randomness due only to assignment process
- Is regression w/out standard assumptions still useful?

Regression: No covariate adjustment

- Suppose only SUTVA and complete random assignment (not the usual regression assumptions)

  - W/out covariate adjustment, z's estimated coeff equivalent to Diff-in-Means

    ```
    coef(lm(formula = y ~ z, data = data))["z"]
    ```

  - HC2 standard error equivalent to Neyman's conservative variance estimator

    ```
    library(sandwich)
    diag(vcovHC(x = mod, type = "HC2"))["z"]
    ```

- How does regression perform with covariate adjustment? (Freedman, 2008a,b; Lin, 2013; Cohen and Fogarty, 2023)

Regression: With covariate adjustment

On regression adjustments to experimental data

David A. Freedman

Statistics Department, University of California, Berkeley, CA 94720-3860, USA

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Abstract

Regression adjustments are often made to experimental data. Since randomization does not justify the models, almost anything can happen. Here, we evaluate results using Neyman's non-parametric model, where each subject has two potential responses, one if treated and the other if untreated. Only one of the two responses is observed. Regression estimates are generally biased, but the bias is small with large samples. Adjustment may improve precision, or make precision worse; standard errors computed according to usual procedures may overstate the precision, or understate, by quite large factors. Asymptotic expansions make these ideas more precise.

MSC: 62A01; 62J99

Keywords: Models; Randomization; Multiple regression; Balance; Intention-to-treat

Figure 3: Freedman (2008), p. 180

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AGNOSTIC NOTES ON REGRESSION ADJUSTMENTS TO EXPERIMENTAL DATA: REEXAMINING FREEDMAN'S CRITIQUE

BY WINSTON LIN

University of California, Berkeley

Freedman [*Adv. in Appl. Math.* **40** (2008) 180–193; *Ann. Appl. Stat.* **2** (2008) 176–196] critiqued ordinary least squares regression adjustment of estimated treatment effects in randomized experiments, using Neyman's model for randomization inference. Contrary to conventional wisdom, he argued that adjustment can lead to worsened asymptotic precision, invalid measures of precision, and small-sample bias. This paper shows that in sufficiently large samples, those problems are either minor or easily fixed. OLS adjustment cannot hurt asymptotic precision when a full set of treatment–covariate interactions is included. Asymptotically valid confidence intervals can be constructed with the Huber–White sandwich standard error estimator. Checks on the asymptotic approximations are illustrated with data from Angrist, Lang, and Oreopoulos's [*Am. Econ. J.: Appl. Econ.* **1:1** (2009) 136–163] evaluation of strategies to improve college students' achievement. The strongest reasons to support Freedman's preference for unadjusted estimates are transparency and the dangers of specification search.

Figure 4: Lin (2013), p. 295

Linear regression

- Lin (2013) shows regression w/ full set of treatment-covariate interactions
  - Consistent for average treatment effect
    (but may have bias in small experiments)
  - Robust HC2 variance estimator satisfies "do-no-harm" property
    - W/ large N, regression cannot make variance larger than Diff-in-Means' variance
      (Usually regression will decrease variance)

- To implement Lin estimator in `R`

```
library(estimatr)
lm_lin(y ~ z, covariates = x_1 + x_2, data = data)
```

- Equivalent to

```
lm(formula = y ~ z + x_1_cent + x_2_cent + z * x_1_cent + z
* x_2_cent, data = data)
```

Nonlinear regression

- "The logit model is often used to analyze experimental data. However, randomization does not justify the model" (Freedman, 2008b, p. 237).
- Argument analogous to Lin (2013) applies for nonlinear covariate adjustment
  - E.g., logistic regression, Poisson regression, etc.
- See Cohen and Fogarty (2023) for details

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