Causal Inference I Day 6 In-Class Exercise 1

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Observed control areas:	**Total**	**# polio cases**	**Polio rate**
Vaccinated	221,998	76	34
Controls	725,173	439	61
Grade 2 not inoculated	123,605	66	53
Incomplete vaccinations	9,904	4	40

Table 1: Adapted from Francis (1955), Tables 2 and 3.

Let's reorganize the table:

Group	Z	D	Y	n	**Polio rate**
Vaccinated	Treatment	1	76	221,998	34
Grade 2 not inoculated	Treatment	0	66	123,605	53
Incomplete vaccinations	Treatment	0	4	9,904	40
Controls*	Control	0	439	725,173	61

* Controls are first- and third-grade total population.

Table 2: Polio study data reorganized to show treatment assignment (D) and outcome (Y). Adapted from Francis (1955).

Why reorganize the table in this way?

The original table summarizes polio incidence and vaccination status for different groups, but it does not clearly distinguish between *treatment assignment* and *actual treatment received*. For causal inference, especially when estimating intention-to-treat (ITT) effects and complier average causal effects (CACE), it is crucial to differentiate:

- The **treatment assignment** (whether a subject was assigned to be offered the vaccine, i.e., second graders vs. controls),
- The **treatment received** (D; whether a subject was actually vaccinated), and
- The **outcome** (Y; whether a subject contracted polio).

1. **Estimate the ITT_Y.**

   The Intention-To-Treat effect (ITT_Y) is the difference in polio rates between those assigned to be vaccinated and the controls:

   $$\text{ITT}_Y = \text{Polio rate}_{\text{Vaccinated}} - \text{Polio rate}_{\text{Controls}} = 34 - 61 = -27$$

   Interpretation: Assigning second graders to be vaccinated reduces polio incidence by 27 cases per 100,000 compared to controls.

2. **Estimate ITT_D.**

   ITT_D is the difference in the proportion actually vaccinated between the treatment and control groups:

   $$\text{ITT}_D = \text{Proportion vaccinated}_{\text{treatment}} - \text{Proportion vaccinated}_{\text{control}}$$

   In the treatment group (all 2nd graders), the proportion vaccinated is:

   $$\begin{aligned}\text{Proportion vaccinated}_{\text{treatment}} &= \frac{221,998}{221,998 + 123,605 + 9,904} \\ &= \frac{221,998}{355,507} \\ &\approx 0.625\end{aligned}$$

   In the control group, none were vaccinated, so:

   $$\text{ITT}_D = 0.625 - 0 = 0.625$$

   Interpretation: About 62.5% of those assigned to treatment were actually vaccinated.

3. **Were there/could there have been "never-takers"?**

   Yes. The "Grade 2 not inoculated" group (123,605 children) were assigned to treatment but did not receive the vaccine, so they are never-takers.

4. **Were there/could there have been "defiers"?**

   No.

   A **defier** is someone who does the opposite of their assignment:

   * If assigned treatment ($Z = 1$), they do not take it ($D = 0$). * If assigned control ($Z = 0$), they somehow get the treatment anyway ($D = 1$). * All controls ($Z = 0$, i.e., first and third graders) have $D = 0$ (no vaccine). * There is no evidence in the table of anyone in the control group receiving the vaccine (i.e., $D = 1$ when $Z = 0$). * The study notes that "Controls are first- and third-grade total population," and there is no mention of vaccine access for these grades.

5. **Were there/could there have been "always-takers"?**

   No.

   An **always-taker** is someone who gets the treatment **no matter what their assignment**:

   * If assigned treatment ($Z = 1$), $D = 1$. * If assigned control ($Z = 0$), $D = 1$.

   * If there were always-takers, there would be at least some vaccinated children in the control group ($D = 1$ for $Z = 0$), but none exist in the table. * **Thus, the data strongly support that there are no always-takers.**

   Exceptions? Given that there existed no polio vaccines before this trail, it was highly unlikely that anyone could get vaccines outside the experiment.

6. **Use Bloom's method to estimate the CACE. How does your answer compare to the RCT's estimate of the ACE,** $41 - 81 = -40$ **cases per 100K inoculations?**

   Bloom's method estimates the Complier Average Causal Effect (CACE) as:

$$\text{CACE} = \frac{\text{ITT}_Y}{\text{ITT}_D} = \frac{-27}{0.625} = -43.2$$

   So, the estimated effect of vaccination among compliers is -43 cases per 100,000.

   This is close to the RCT's direct estimate of the Average Causal Effect (ACE), which was -40 cases per 100,000 inoculations.