CRETIN

Session 7

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Session topics

Line radiation transport

- Physics considerations
  - what is important / different about lines?
  - material coupling and convergence
  - important physical effects
- Code options
  - setting up frequency meshes
  - options for including physical effects
  - methods for achieving convergence

What is important about lines?

- Transitions between levels in the same charge state
- Narrow frequency range → strong localized absorption / emission
- Radiation is strongly coupled to bound electrons
- Most radiation emission from multi-electron atoms is due to lines
- At low densities, line effects can dominate the kinetics

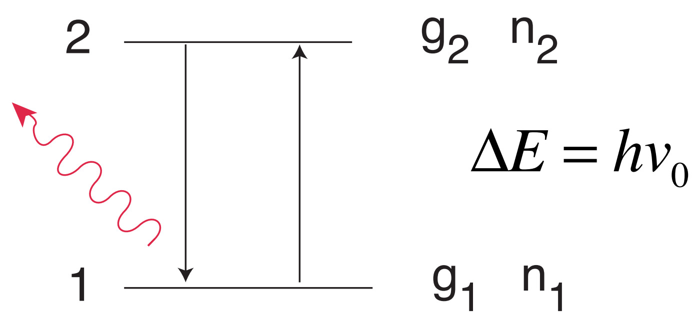
Probability (per unit time) of

- Spontaneous emission: A_{21}
- Absorption: B_{12}
- Stimulated emission: B_{21}

$$g_1 B_{21} = g_2 B_{12} \quad , \quad A_{21} = \frac{2h\nu_0^{\,3}}{c^2} B_{21}$$

A_{21}, B_{12}, B_{21} are Einstein coefficients

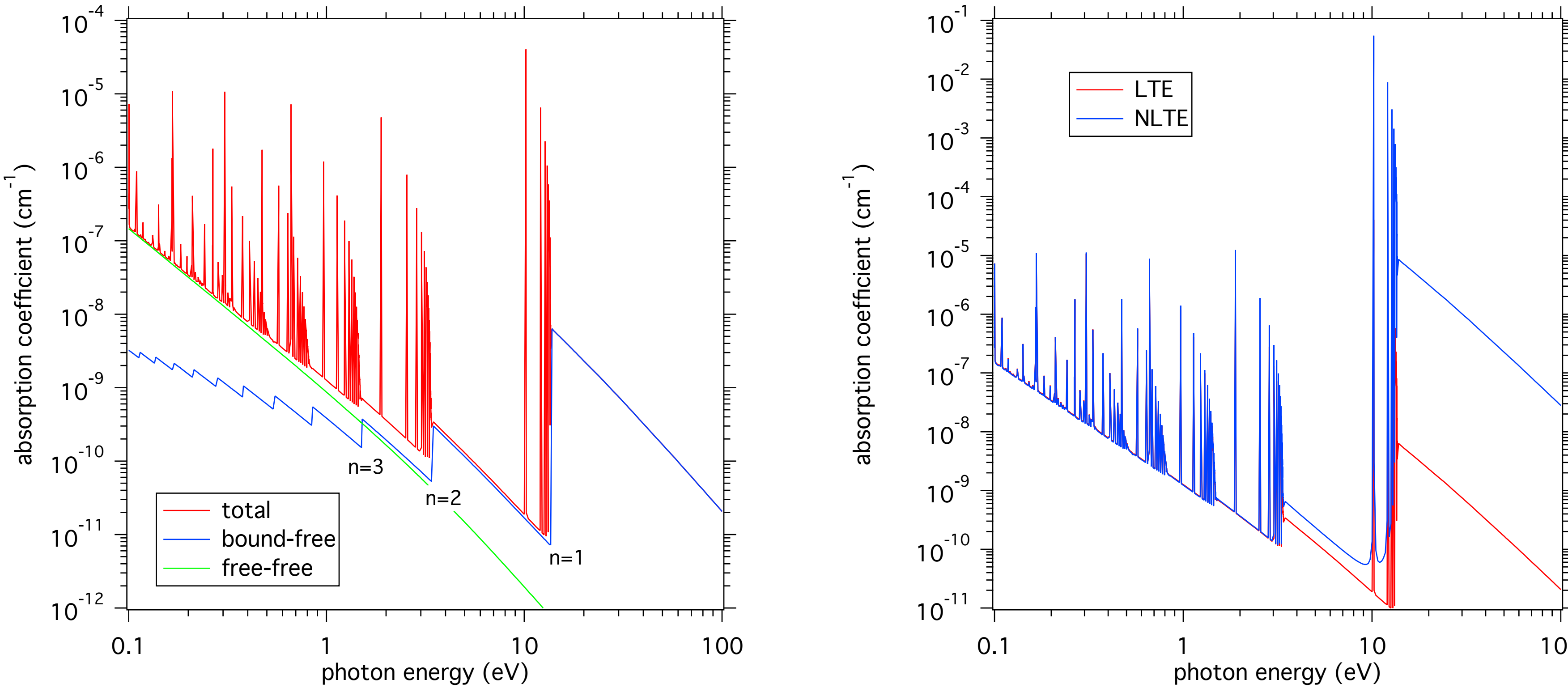
Two energy level system



Line radiation is critical to non-LTE physics

A simple example – low density H

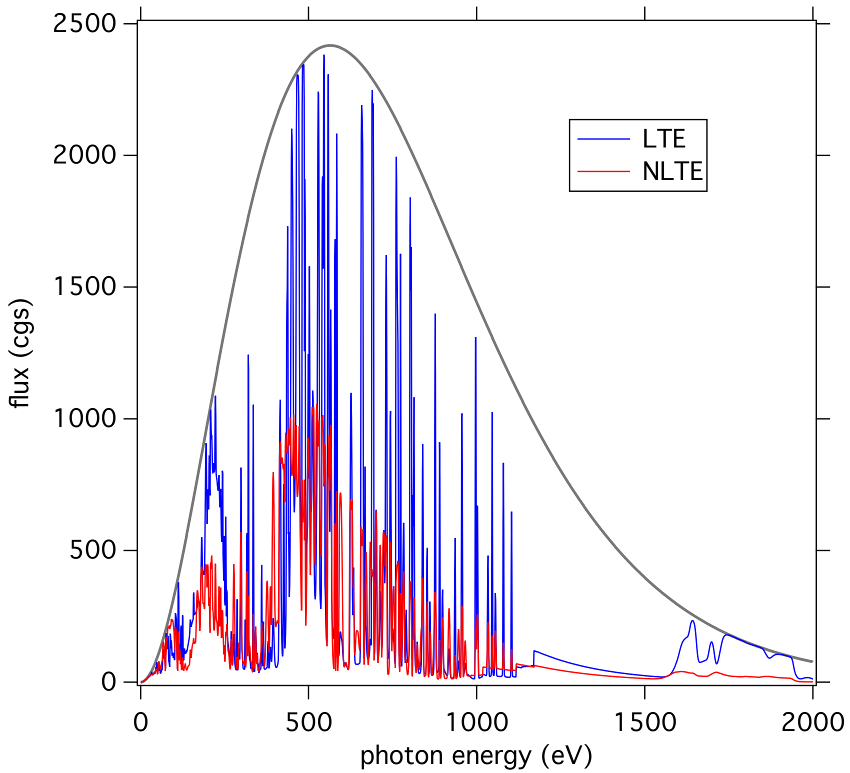
$T_e = 2\text{ eV},\ N_H = 10^{14}\text{ cm}^{-3}$



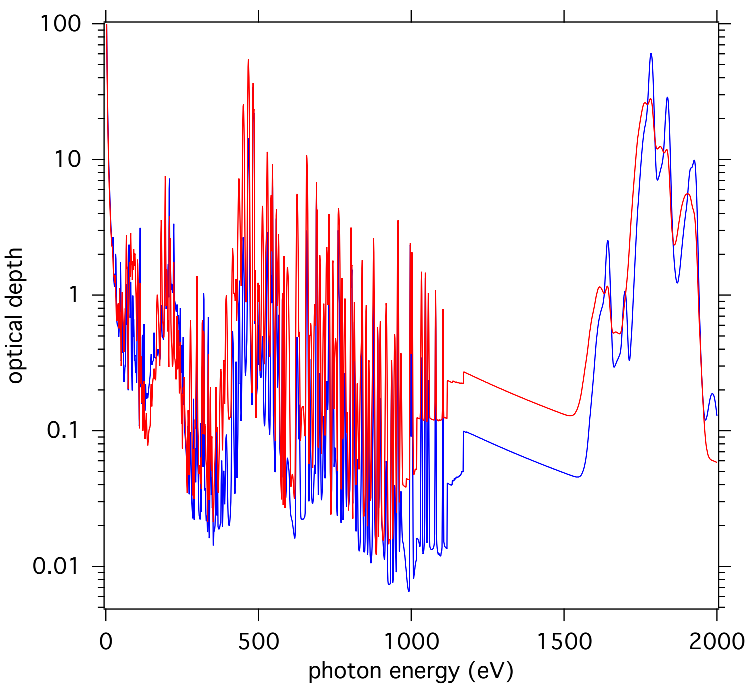
Lines become optically thick before the continuum

A mid-Z example – Kr sphere

T_e = 200 eV, ρ = 0.01 g/cm^3



R = 0.1 cm

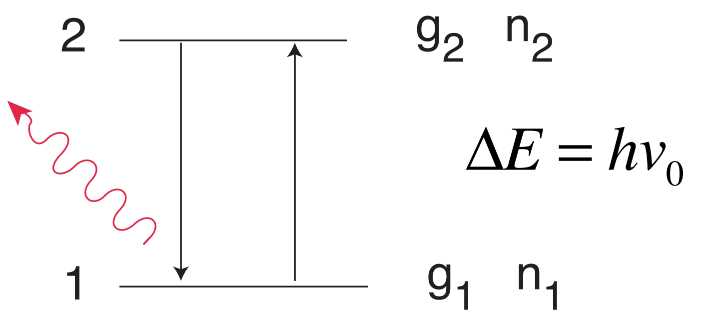


Line radiation dominates the escaping flux

Coupling to material – effective scattering

- Line radiation couples directly to just the two levels involved
- Line radiation "scatters" by resonant absorption / emission
- Upper level decays
  - Radiatively with rate A_{21}
  - Collisionally with rate $n_e C_{21}$
- The fraction $\varepsilon \approx n_e C_{21} / A_{21}$ of photons are destroyed / thermalized
- The fraction $(1\text{-}\varepsilon)$ of photons are "scattered"
  → energy changes only slightly (mostly Doppler shifts)
  → undergo many "scatterings" before being thermalized
- Convergence by lambda iteration takes $\sim \tau^2$ iterations

Two energy level system



$\varepsilon \ll 1$ is the condition for a strongly non-LTE transition and is easily satisfied for low density or high ΔE !

Line profiles

- Bound-bound absorption cross section:

$$\sigma(\nu)=\frac{h\nu_0}{4\pi}B_{21}\phi(\nu)=\frac{\pi e^2}{mc}f_{12}\phi(\nu) \qquad \frac{\pi e^2}{mc}=0.02654\ \mathrm{cm^2/s}$$

- Oscillator strength f_{12} relates the quantum mechanical result to the classical treatment of a harmonic oscillator
  - Strong transitions have f~1

$$\int_0^\infty \phi(\nu)\,d\nu=1$$

The line profile $\phi(\nu)$ describes the frequency dependence of the absorption coefficient

$$\alpha_\nu = n_1\frac{\pi e^2}{mc^2}f_{12}\phi(\nu)\left[1-\frac{g_1 n_2}{g_2 n_1}\right]$$

$$\eta_\nu = \left(\frac{2h\nu^3}{c^2}\right)n_2\frac{\pi e^2}{mc^2}f_{12}\phi(\nu)$$

- This assumes absorption and emission profiles are the same
- If line width is small ($\Delta\nu \ll \nu_0$), can replace ν by ν_0

Cretin line transport assumes $\Delta\nu \ll \nu_0$

Line profiles

Line profiles are determined by multiple effects:

- Natural broadening (A_{12}) - Lorentzian
- Collisional broadening (n_e, T_e) - Lorentzian
- Doppler broadening (T_i) - Gaussian
- Stark effect (plasma microfields) - complex

$$\phi(\nu) = \frac{\Gamma/4\pi^2}{(\nu - \nu_0)^2 + (\Gamma/4\pi)^2}$$

Γ = destruction rate

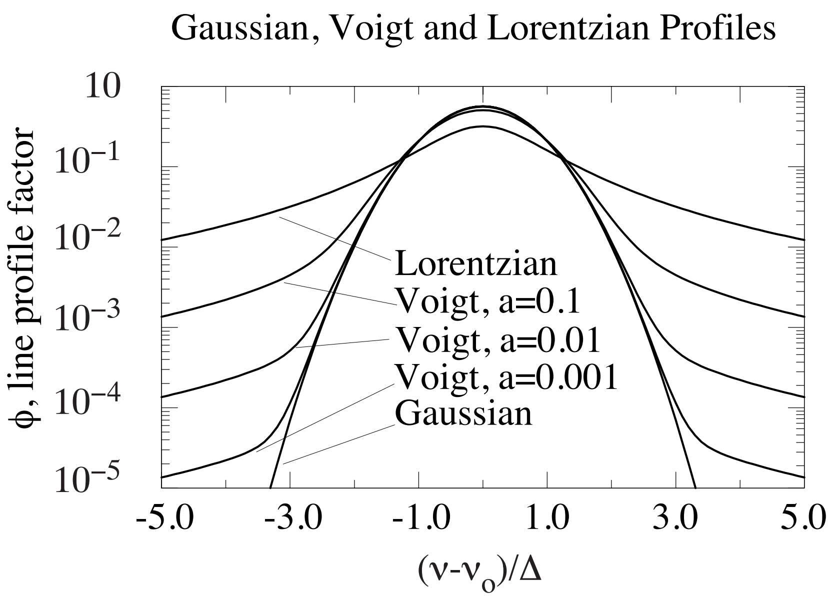
The convolution of Gaussian and Lorentzian shapes gives the Voigt profile:

- Gaussian core
- Lorentzian wings

$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}} H(a,x)\ ,\ H(a,x) = \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{e^{-y^2}}{(x-y)^2 + a^2}\,dy$$

$$a = \Gamma\big/4\pi\Delta\nu_D\ ,\ \Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT_i}{m_i}}$$

a = Voigt parameter



Configuration "broadening"

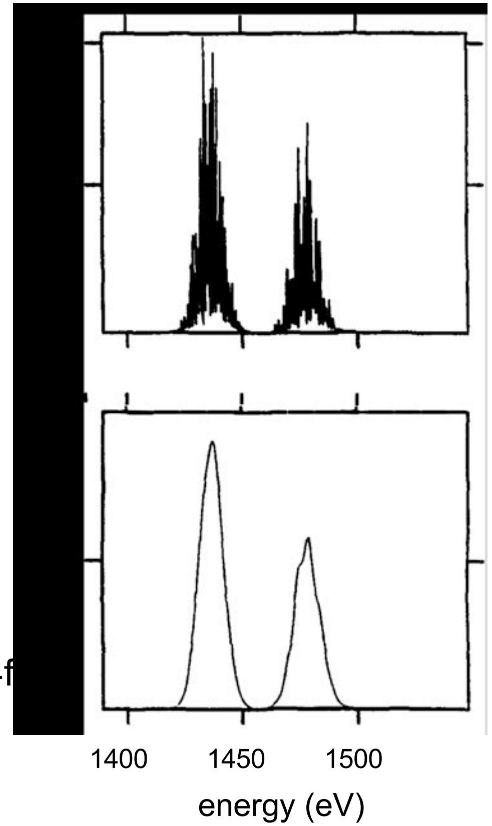
For multi-electron ions, multiple atomic states are combined into "levels" representing

- LSJ-configurations $1s^2 2p^6\ 3s^2\ 3p_{3/2}$
- LS-configurations $1s^2 2p^6\ 3s^2\ 3p$
- Superconfigurations $(1)^2(2)^6(3)^3$

A single transition between levels represents many transitions between states

- UTA – unresolved transition array

The screened-hydrogenic models dca_*xx* use super-configurations for levels + multiple UTAs between levels for transitions

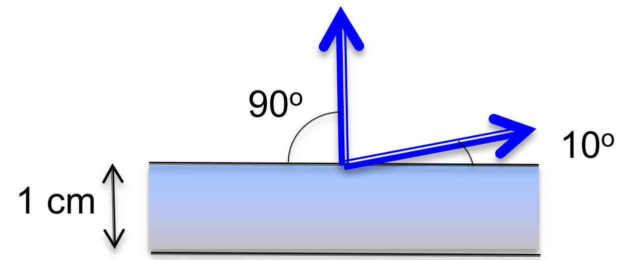


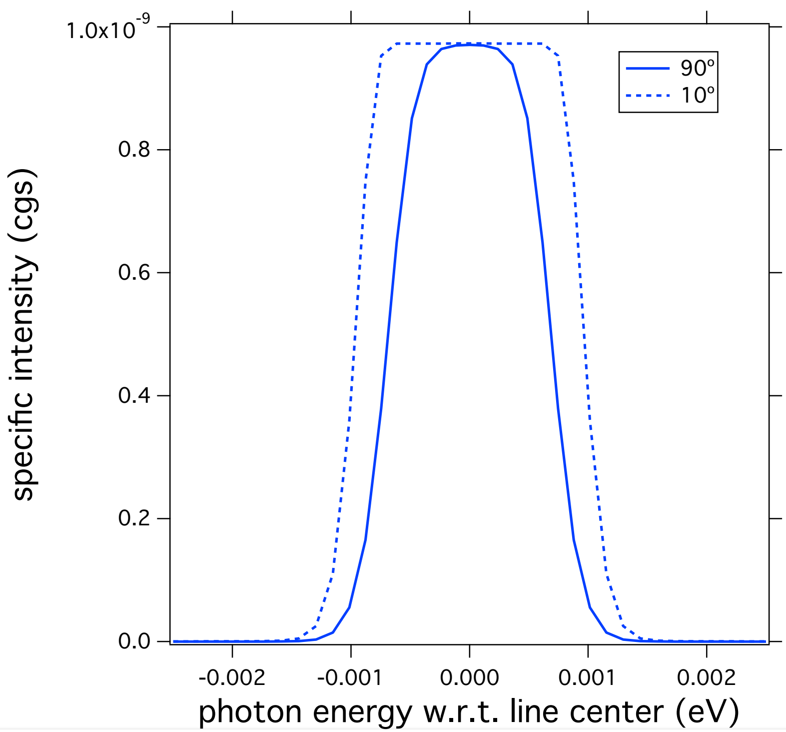
Tm (Z=69) 3d-4f
C. Smith (1995)

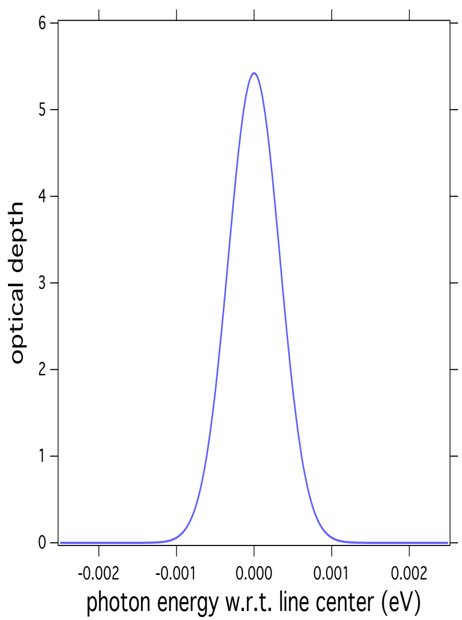
Configuration broadening often dominates high-Z spectra

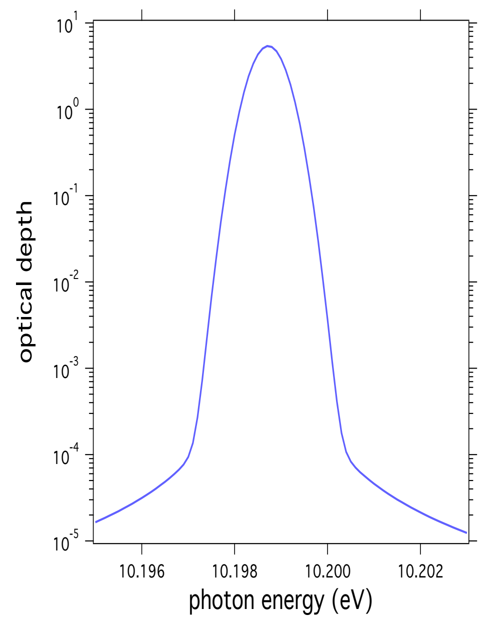
Example – Hydrogen Ly-α

- Uniform conditions: T_e = 1 eV, n_e = 10^{14} cm^{-3}
- Moderate optical depth $\tau \sim 5$
- Viewing angles 90° and 10° show optical depth broadening



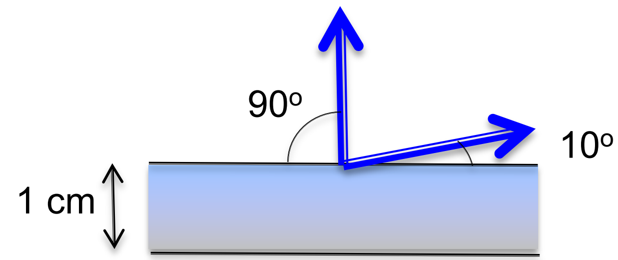


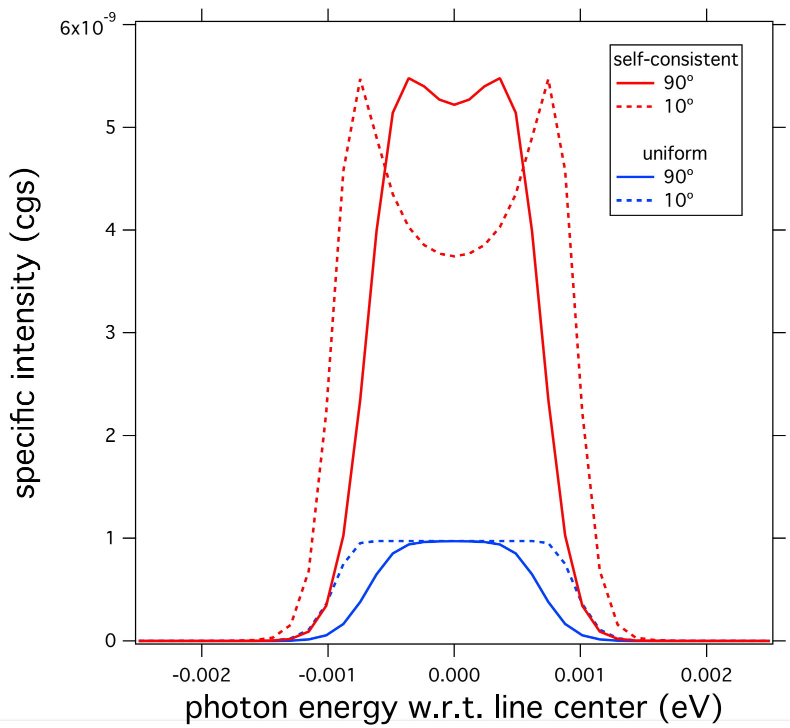


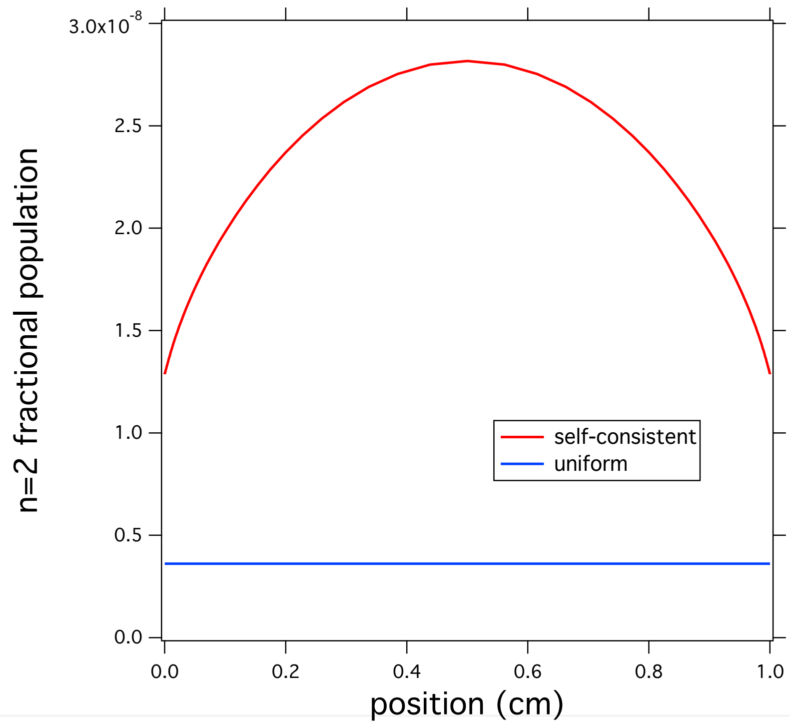


Example – Hydrogen Ly-α

- Uniform conditions: T_e = 1 eV, $n_e = 10^{14}$ cm^{-3}
- Self-consistent solution displays effects of
  - Radiation trapping / pumping
  - Non-uniformity due to boundaries







Redistribution

- The emission profile ψ_ν is determined by multiple effects:
  - coherent scattering, elastic scattering, Doppler broadening
- It is related to the absorption profile through the redistribution function

$$\int_0^\infty R(\nu,\nu')\,d\nu = \phi(\nu') \quad , \quad \psi(\nu) = \int_0^\infty R(\nu,\nu')J(\nu')\,d\nu' \Bigg/ \int_0^\infty \phi(\nu')J(\nu')\,d\nu'$$

- Complete redistribution (CRD): $\psi_\nu = \phi_\nu$
- Doppler broadening is only slightly different from CRD, while coherent scattering gives $R(\nu,\nu') = \phi(\nu)\delta(\nu-\nu')$
- A good approximation for partial redistribution (PRD) is often

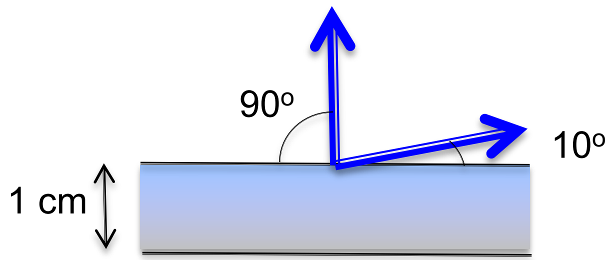
$$R(\nu,\nu') = (1-f)\phi(\nu')\phi(\nu) + f\,R_{II}(\nu,\nu')$$

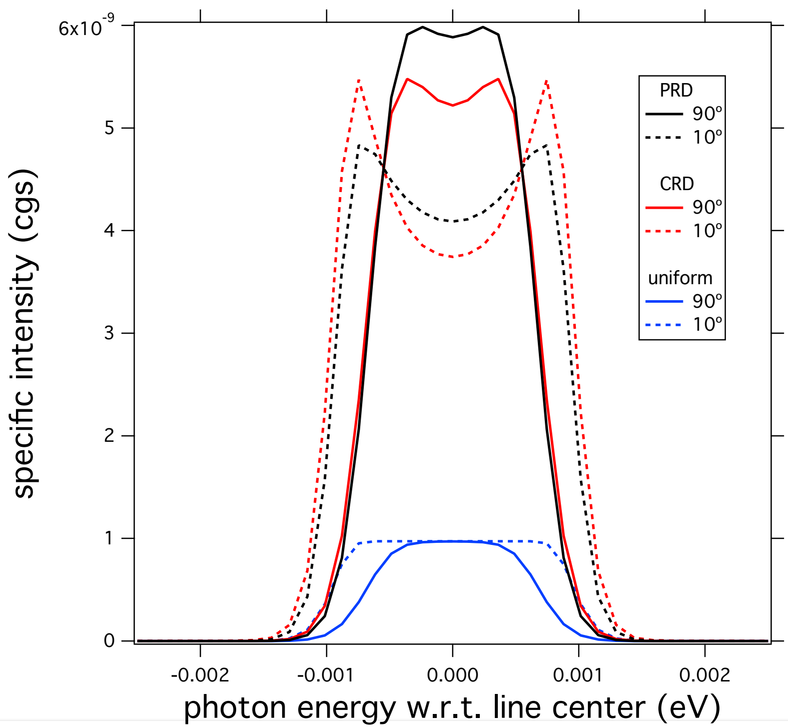
- where f (<<1 for X-rays) is the ratio of elastic scattering and de-excitation rates, R_{II} includes coherent scattering and Doppler broadening

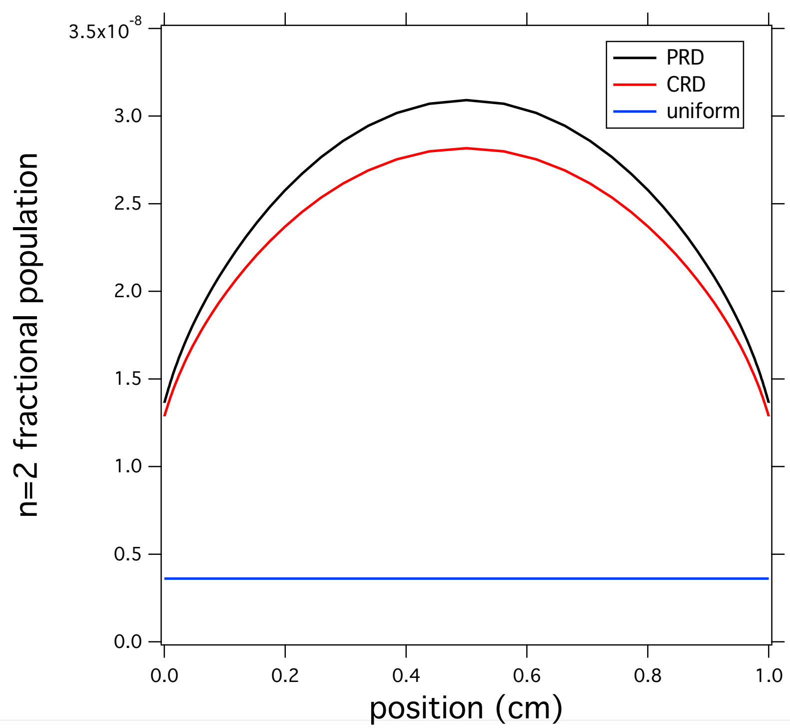
PRD effects become stronger with increasing optical depth

Example – Hydrogen Ly-α with Partial Redistribution

- Uniform conditions: T_e = 1 eV, n_e = 10^{14} cm^{-3}
- Optical depth $\tau \sim 5$
- Voigt parameter a ~ 0.0003







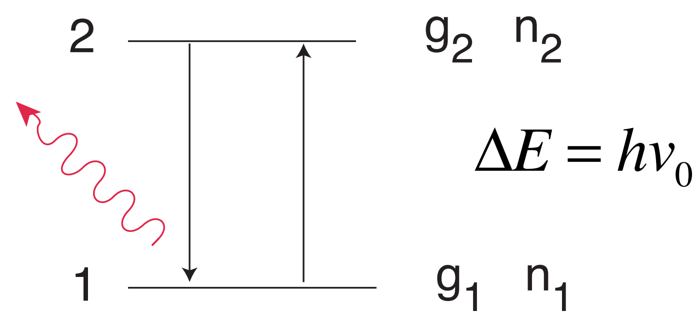
Radiation transport for 2-level atom

- Rate equation for two levels in steady state:

$$n_1(B_{12}\bar{J}_{12}+C_{12}) = n_2(A_{21}+B_{21}\bar{J}_{12}+C_{21})$$

$$\bar{J}_{12} = \int_0^\infty J_\nu \phi_{12}(\nu)\,d\nu \;,\; C_{12} = \tfrac{g_2}{g_1} e^{-h\nu_0/kT} C_{21}$$

Two energy level system



- Absorption / Emission:

$$\alpha_\nu = \frac{h\nu}{4\pi}(n_1 B_{12} - n_2 B_{21})\phi_{12}(\nu) \;,\; \eta_\nu = \frac{h\nu}{4\pi} n_2 A_{21}\phi_{12}(\nu)$$

- Source function:

$$S_\nu = {n_2 A_{21}}\Big/{n_1 B_{12} - n_2 B_{21}} = (1-\varepsilon)\bar{J}_{12} + \varepsilon B_\nu \;\;,\;\; \frac{\varepsilon}{1-\varepsilon} = \frac{C_{21}}{A_{21}}\left(1 - e^{-h\nu_0/kT}\right)$$

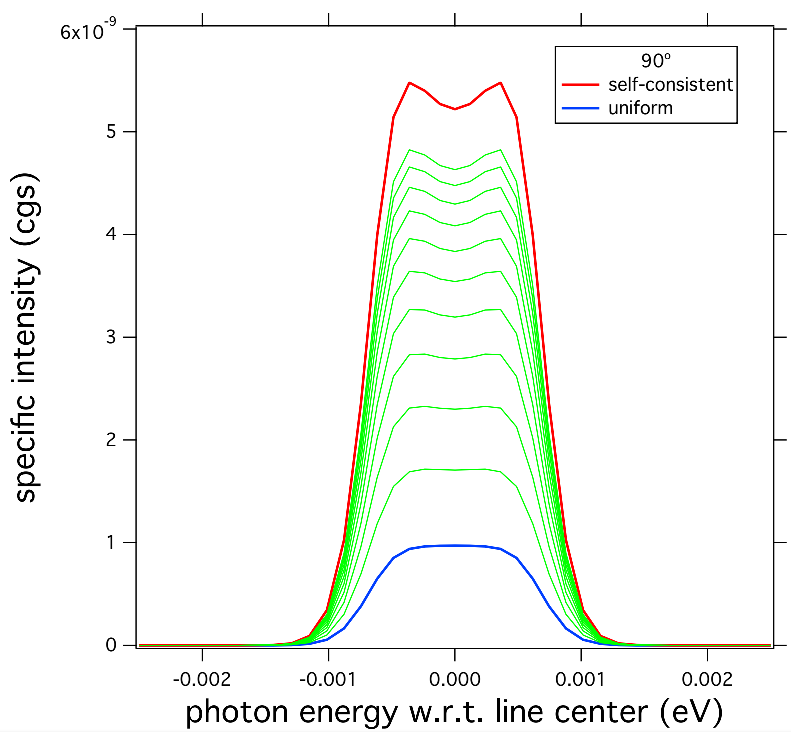
- Including multiple transitions and separate emission profiles is straightforward

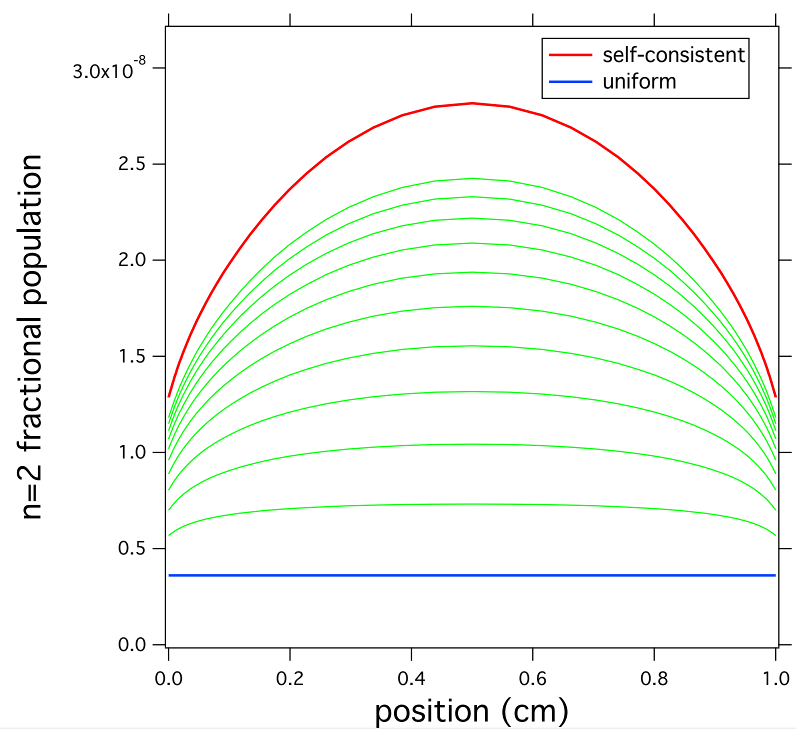
Calculating $\bar{J}$ is the focus of line radiation transport

Example – Hydrogen Ly-α

- Source iteration (green curves) approaches self-consistent solution slowly
- Linearization achieves convergence in 1 iteration since the source function is a linear function of $\bar{J}$

$$S_{ij} = a + b\bar{J}_{ij}, \bar{J}_{ij} = \int J_\nu \phi_\nu \, d\nu$$





Velocity gradients

- Absorption / emission is (usually) isotropic in the fluid frame
- Doppler shifts make these anisotropic in the laboratory frame
- This effect becomes significant when $\mathrm{v/c} \sim \mathrm{dE} / \mathrm{E}_0$
- For thermal velocities: $\frac{\mathrm{v}}{\mathrm{c}} \sim \sqrt{\frac{2kT_i}{m_i c^2}} \sim 0.0015\sqrt{\frac{T_i}{A_i}}$ T_i in keV
- For Doppler-broadening lines, sound speed bulk velocities become important

Uniformly expanding spherical annulus

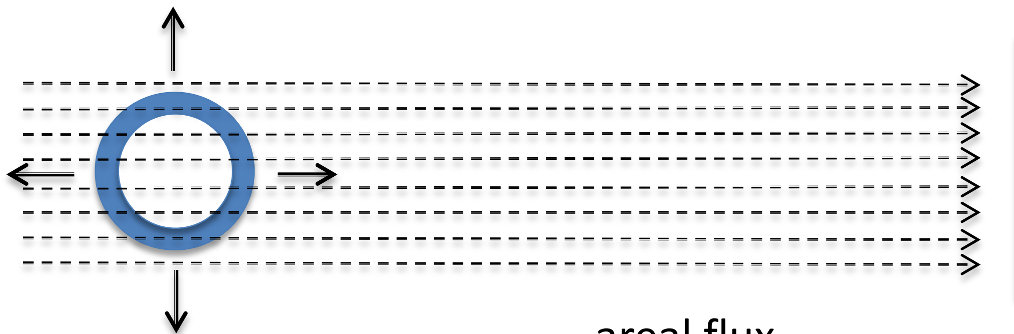
Al conditions:

$T_e = 500$ eV

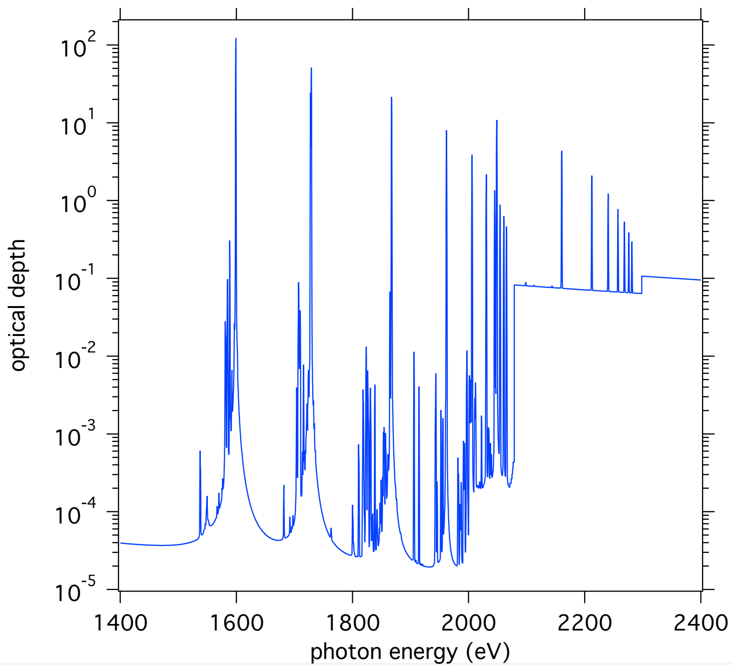
$\rho = 10^{-3}$ g/cm^3

$R_{outer} = 1.0$ cm

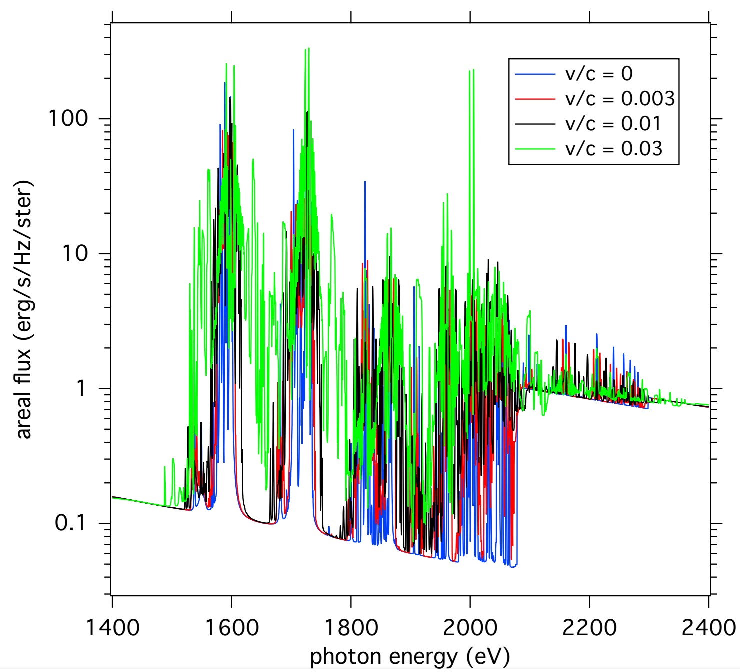
$R_{inner} = 0.9$ cm



optical depth



areal flux



Uniformly expanding spherical annulus