Attempts by Fan

1.

(a) $\pm(1+f)\times 2^n$, $f=\sum_{i=1}^{d} b_i\cdot 2^{-i}$

$f=2^{-d}z$, $z\in\{0,1,\dots 2^d-1\}$

$\Rightarrow$ $f=\frac{z}{16}$, $z\in\{0,1,\dots 15\}$

First find possible n:

$$\begin{cases} 4\geq(1+f)\cdot 2^n\geq 2^n \Rightarrow n\leq 2 \\ \frac{1}{2}\leq(1+f)\cdot 2^n < 2^{n+1} \Rightarrow n>-2 \end{cases}$$

So: ① $n=-1$. $\frac{1}{2}\leq(1+f)\cdot 2^n=\frac{1+f}{2}\leq 1 \Rightarrow 16$

② $n=2$, $(1+f)\cdot 2^n=4+4f$

only when $f=0$. $(z=0)$

③ $n=1$ $\frac{1}{2}\leq(1+f)\cdot 2=2+2f\leq 4 \Rightarrow 16$

So conclusion: n: -1 0 1 2

16 16 16 1

Total: $16+16+16+1=49$

(b) $1/10\in[\frac{1}{16},\frac{1}{8})=[2^{-4},2^{-3})$

$(1+f)\cdot 2^{-4}$. $f=\frac{z}{16}$, $z\in\{0,1,2\dots 15\}$

let: $(1+f)\cdot 2^{-4}=\frac{1}{10} \Rightarrow 1+f=\frac{16}{10} \Rightarrow f=\frac{3}{5}$

let: $\frac{3}{5}=\frac{z}{16}$ $\check{z}=9.6$. Consider $z=9$. 10.

when $z=9$, $|\check{z}-0.1|=0.0023$

$z=10$, $|\check{z}-0.1|=0.0016$, So it's $(1+\frac{10}{16})\times 2^{-4}$

(c) $2^{-d} \cdot 2^n > 1$

$\Rightarrow \quad 2^n > 2^4 \quad \Rightarrow \quad n > 4$

Consider $n=5$, $\quad (1+f) \cdot 2^5$

$f=0, \Rightarrow 32$. $\quad f=\frac{1}{16} \quad \Rightarrow 34$

So. 33 is the smallest positive integer $\notin \mathbb{F}$

2. ① 1.1.4 $\Rightarrow$ 1.1.5

by 1.1.4: $|fl(x) - x| \leq \frac{1}{2} \varepsilon_{mach} |x|$

$\Rightarrow \quad |fl(x) - |x|| \leq ||fl(x)| - |x|| \leq \frac{1}{2} \varepsilon_{mach} |x|$

$\Leftrightarrow (-\frac{1}{2}\varepsilon_{mach}+1)|x| \leq |fl(x)| \leq (\frac{1}{2}\varepsilon_{mach}+1)|x|$

So $\exists\ \varepsilon$, s.t. $|\varepsilon| \leq \frac{1}{2}\varepsilon_{mach}$, and $|fl(x)| = (1+\varepsilon)|x|$

$\Leftrightarrow fl(x) = (1+\varepsilon)x$, since $fl(x)$ and x have the same sign

② 1.1.5 $\Rightarrow$ 1.1.4

$\exists\ \varepsilon$. $|\varepsilon| \leq \frac{1}{2}\varepsilon_{mach}$, $\quad fl(x) = x(1+\varepsilon)$

$fl(x) - x = x\varepsilon$

$\Rightarrow |fl(x) - x| = |x||\varepsilon|$

$\Rightarrow \quad \frac{|fl(x) - x|}{|x|} = |\varepsilon| \leq \frac{1}{2}\varepsilon_{mac}$

3.

(a). exponent: $32-23-1=8$, $[0, 255]$

significant: $2^{-23} \cdot f$. $f \in \{0, \ldots 2^{23}-1\}$

So: it's $1+2^{-23}$

(b) $2^{-23} \cdot 2^{n} > 1 \Rightarrow n > 23$

let $n=24$, $(1+z \cdot 2^{-23}) \cdot 2^{24} = 2^{24} + 2z$

So it's $2^{24}+1 = 16777217$