Section 1.1

1. (a) $\pm(1+f)\times 2^{n}$

$d=4$

$$f=\sum_{i=1}^{d} b_i 2^{-i}$$

$n=\{-1,0,2\}$

$|f| = 2^{d}$

$3\cdot 2^{d} = 3\times 2^{4} = \underline{48}$

$48+1=49$

(b). $\frac{1}{10}\in[\frac{1}{2^4},\frac{1}{2^3})$.

$n=4$

$(1+f)2^{-4} \quad \frac{1}{10}$

$1+f \quad 1.6$

$f \quad 0.6$

$\underline{0.5} \quad 0.25 \quad 0.125 \quad \underline{0.0625}$

$= \underline{0.5625}$

$(1+0.5625)\times 2^{-4} = 0.09765625$

(c) $\epsilon_{mach} = 2^{-4} = \frac{1}{16}$

$\epsilon_{mach} \times 2^{n} > 1$

$2^{n} > 16$

$n > 4$

$n = 5$

$[2^{5}, 2^{6})$ $+2$.

$2^{5}+1$

2. $0 \le \frac{|fl(x) - x|}{|x|} \le \frac{\frac{1}{2}(2^{n-d})}{|x|}$

$\le \frac{2^{n-d-1}}{2^{n}}$

$= 2^{-d-1} = \frac{1}{2}\epsilon_{mach}$

$\left|\frac{fl(x) - x}{x}\right| \le \frac{1}{2}\epsilon_{mach}$

$-\frac{1}{2}\epsilon_{mach} \le \frac{fl(x) - x}{x} \le \frac{1}{2}\epsilon_{mach}$

$x(1 - \frac{1}{2}\epsilon_{mach}) \le fl(x) \le x(1 + \frac{1}{2}\epsilon_{mach})$

$fl(x) = x(1+\epsilon)$

$|\epsilon| \le \frac{1}{2}\epsilon_{mach}$.

3. (a) $355/113 = 3.14159292 0 \ldots$

7 digits

(b) 3.1415926520961532

4. (a). $1 \in [2^0, 2^1)$. $\quad \epsilon_{mach} = 2^{-d} = 2^{-23}$

$1 + \epsilon_{mach} \cdot 2^0 = 1 + 2^{-23}$

(b). $\epsilon_{mach} \cdot 2^n > 1$

$2^{-23} \cdot 2^n > 1$

$n > 23$

$n = 24$

$2^{24} + 1$

5 $\rightarrow$ Inf

$\rightarrow$ float min().

Section 1.2

1 (a) $f(x) = x^p$

$$\kappa_f(x) = \left| \frac{x \cdot p \cdot x^{p-1}}{x^p} \right| = |p|$$

(b) $$\kappa_f(x) = \left| \frac{x \cdot \frac{1}{x}}{\log(x)} \right| = \left| \frac{1}{\log(x)} \right|$$

(c). $$\kappa_f(x) = \left| \frac{x \cdot (-\sin x)}{\cos(x)} \right| = |x \tan x|$$

(d). $$\kappa_f(x) = \left| \frac{x \cdot e^x}{e^x} \right| = |x|$$

2. (a). $t = g(x) = x+5 \qquad f(t) = \sqrt{t}$

$$\kappa_f(x) = \kappa_f(t) \cdot \kappa_g(x)$$
$$= \left| \frac{t \cdot \frac{1}{2} t^{-\frac{1}{2}}}{t^{\frac{1}{2}}} \right| \cdot \left| \frac{x \cdot 1}{x+5} \right|$$
$$= \left| \frac{x}{2x+10} \right|$$

$$\kappa_f(x) = \left| \frac{x \cdot \frac{1}{2} (x+5)^{-\frac{1}{2}}}{(x+5)^{\frac{1}{2}}} \right|$$

$$= \left| \frac{x}{2x+10} \right|$$

3\. (a) $k_f(x) = \left| \dfrac{x \cdot (1-\tanh^2 x)}{\tanh x} \right|$

$= \left| \dfrac{x}{\tanh x} - \tanh x \right|$

$x=0, \quad k_f(x) \to \infty$

(b) $k_f(x) = \left| \dfrac{\cancel{x} \cdot \dfrac{e^x \cdot x - e^x + 1}{\cancel{x^2}}}{\dfrac{e^x - 1}{\cancel{x}}} \right|$

$= \left| \dfrac{xe^x - e^x + 1}{e^x - 1} \right|$

$x \to \infty$

4\. $k_h(x) = \left| \dfrac{x \cdot h'(x)}{h(x)} \right|$

$= \left| \dfrac{x \cdot f'(g(x)) \cdot g'(x)}{f(g(x))} \right|$

$= \left| \dfrac{g(x)\, f'(g(x))}{f(g(x))} \right| \left| \dfrac{x g'(x)}{g(x)} \right|$

$= k_f(g(x)) \cdot k_g(x)$

5. $y = f(x)$, $x = g(y) = f^{-1}(y)$

$$1 = \frac{dg}{dy} \cdot \frac{dy}{dx}$$

$$\frac{dg}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{f'(x)}$$

$$g'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$k_{f^{-1}}(x) = \left| \frac{x \cdot \frac{1}{f'(f^{-1}(x))}}{f^{-1}(x)} \right|$$

$$= \left| \frac{x}{f^{-1}(x) \cdot f'(f^{-1}(x))} \right|$$

$$= \left| \frac{1}{k_f(f^{-1}(x))} \right|$$

6. $ax^2 + bx + c = 0$ $\quad r_1 = f(b)$

$$ar_1^2 + br_1 + c = 0$$

$$a \cdot 2 \cdot r_1 \cdot \frac{dr_1}{db} + r_1 + b \cdot \frac{dr_1}{b} = 0$$

$$\frac{dr_1}{db} = \frac{-r_1}{2ar_1 + b}$$

$$k_f(b) = \left| \frac{b \cdot \frac{-r_1}{2ar_1 + b}}{r_1} \right|$$

$$= \left| \frac{b}{2ar_1 + b} \right| = \left| \frac{b}{r_1 - r_2} \right|$$

7. (b)

$$\cancel{|r(\epsilon)-1| \approx C\cdot\epsilon^{q}}$$

$$\cancel{-C\cdot\epsilon^{q} < r(\epsilon)-1 < C\cdot\epsilon^{q}}$$

$$\cancel{1-C\cdot\epsilon^{q} < r(\epsilon) < 1+C\cdot\epsilon^{q}}$$

~~for~~ $\cancel{0<q<1}$

$$x^2-(2+\epsilon)x+1=0.$$

$$\Delta = (2+\epsilon)^2-4$$

$$= \cancel{4}+4\epsilon+\epsilon^2-\cancel{4}$$

$$= \epsilon^2+4\epsilon$$

$$x = \frac{2+\epsilon \pm \sqrt{\epsilon^2+4\epsilon}}{2}$$

$$|r(\epsilon)-1| = |x-1|$$

$$= \left|\frac{\epsilon \pm \sqrt{\epsilon^2+4\epsilon}}{2}\right|$$

$$= \left|\frac{\epsilon \pm 2\sqrt{\epsilon}\sqrt{1+\frac{\epsilon}{4}}}{2}\right|$$

$$\approx \left|\frac{\epsilon \pm 2\sqrt{\epsilon}\left(1+\frac{\epsilon}{8}\right)}{2}\right|$$

$$= \left|\frac{\epsilon}{2} \pm \sqrt{\epsilon} \pm \frac{\epsilon\sqrt{\epsilon}}{8}\right|$$

$$\approx |\sqrt{\epsilon}| = C\cdot\epsilon^{q},\quad q=\frac{1}{2}$$

8. $r = f([a_0, a_1, \cdots, a_n])$

$$a_n r^n + \cdots + a_1 r + a_0 = 0.$$

$$\sum_{i=0}^{n} a_i \cdot r^i = 0$$

$$\sum_{i=0}^{n} a_i \cdot i \cdot r^{i-1} \cdot \frac{dr}{da_k} + r^k = 0$$

$$\frac{dr}{da_k} \sum_{i=0}^{n} i a_i r^{i-1} = -r^k$$

$$\frac{dr}{da_k} = \frac{-r^k}{\sum_{i=0}^{n} i a_i r^{i-1}}$$

$$K_r(a_k) = \left| \frac{\frac{dr}{da_k} \cdot a_k}{r} \right|$$

$$= \left| \frac{a_k \cdot r^{k-1}}{p'(r)} \right|$$

section 1.4.

1 (a) $f'(x) = \dfrac{\sin^2 x - \cos x(1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin^2 x}$

$K_f(x) = \left| \dfrac{f'(x)x}{f(x)} \right|$

$= \left| \dfrac{x}{\sin x} \right|$

(d). g more acurrate.

2. (a) $f'(x) = \dfrac{e^x \cdot x - e^x + 1}{x^2}$

$K_f(x) = \left| \dfrac{\dfrac{e^x x - e^x + 1}{\cancel{x}} \cdot \cancel{x}}{e^x - 1} \right|$

$= \left| \dfrac{xe^x - e^x + 1}{e^x - 1} \right|$

$K_f'(x) = \dfrac{xe^x \quad (e^x - 1) - (xe^x - e^x + 1)e^x}{} = 0.$

$\cancel{xe^{2x}} - xe^x + \cancel{xe^{2x}} - e^{2x} + e^x = 0$

$-x - e^x + 1 = 0$

$e^x = 1 - x$

$x=0$.

$$\kappa_f(0) = \left|\frac{e^x + xe^x - e^x}{e^x}\right|_{x=0}$$

$$= 0$$

3. (a). $f(x) = a\cosh(x)$

$$y = a\cosh(x)$$

$$\cosh(y) = x$$

$$x = \frac{e^y + e^{-y}}{2}$$

$$1 = \frac{1}{2}\left(e^y \cdot \frac{dy}{dx} - e^{-y} \cdot \frac{dy}{dx}\right)$$

$$2 = \left(e^y - e^{-y}\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2}{e^y - e^{-y}}$$

$$\kappa_f(x) = \left|\frac{f'(x)\cdot x}{f(x)}\right|$$

$$= \frac{\frac{2}{e^y - e^{-y}} \cdot x}{y}$$

$$= \frac{2\cosh(y)}{y(e^y - e^{-y})}$$