1.1

$\tilde{l}_1 = [a_1, b_1, c_1]^T \quad a_1x + b_1y + c_1 = 0$

$\tilde{l}_2 = [a_2, b_2, c_2]^T \quad a_2x + b_2y + c_2 = 0$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -c_1 \\ -c_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, \quad b = \begin{bmatrix} -c_1 \\ -c_2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot A^* = \frac{1}{a_1b_2 - b_1a_2} \cdot \begin{bmatrix} b_2 & -a_2 \\ -b_1 & a_1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot b = \frac{1}{a_1b_2 - b_1a_2} \cdot \begin{bmatrix} -c_1b_2 + c_2a_2 \\ b_1c_1 - a_1c_2 \end{bmatrix}$$

$$x = \frac{a_2c_2 - b_2c_1}{a_1b_2 - b_1a_2}, \quad y = \frac{b_1c_1 - a_1c_2}{a_1b_2 - b_1a_2}$$

$$\tilde{l}_1 \times \tilde{l}_2 = [\tilde{l}_1]_\times \cdot \tilde{l}_2$$

$$= \begin{bmatrix} 0 & -c_1 & b_1 \\ c_1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$$

$$= \begin{bmatrix} -b_2c_1 + b_1c_2 \\ a_2c_1 - a_1c_2 \\ -a_2b_1 + a_1b_2 \end{bmatrix}$$

(b). $x = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$, $y = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$\tilde{x}_1 = \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$, $\tilde{x}_2 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$

$\tilde{l} = \tilde{x}_1 \times \tilde{x}_2 = [\tilde{x}_1]_\times \cdot \tilde{x}_2$

$$= \begin{bmatrix} 0 & -1 & y_0 \\ 1 & 0 & -x_0 \\ -y_0 & x_0 & 0 \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -y_1 + y_0 \\ x_1 - x_0 \\ -x_1 y_0 + x_0 y_1 \end{pmatrix}$$

$$\frac{y - y_0}{x - x_0} = \frac{y_0 - y_1}{x_0 - x_1}$$

$$(x_0 - x_1)\, y - x_0 y_0 + x_1 y_0 = (y_0 - y_1)\, x - x_0 y_0 + x_0 y_1$$

$$(y_0 - y_1)\, x + (x_1 - x_0)\, y + x_0 y_1 - x_1 y_0 .$$

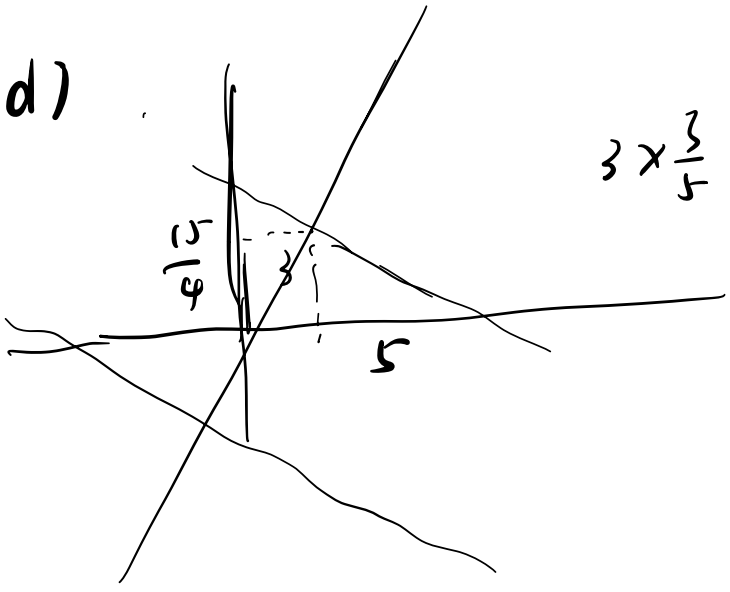
(c). $u_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

$u_2 = \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix}$.

$x = u_1 \times u_2 = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} 13 \\ -10 \\ -1 \end{bmatrix} = \begin{bmatrix} -13 \\ 10 \\ 1 \end{bmatrix}$

(d)



$3 \times \frac{3}{5}$ $\left[\frac{9}{5},\right.$

$\frac{x}{5} + \frac{4y}{15} = 1$

$3x + 4y = 15$

$3x + 4y = -15$

$ax + by + c = 0$

$a(x_1 - x_0) + b(y_1 - y_0) = 0$

$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \end{pmatrix} = 0$

$ax + by = -c$

$\frac{x}{\frac{-c}{a}} + \frac{y}{\frac{-c}{b}} = 1$

$d \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}$

$= \frac{c^2}{ab}$

$\sqrt{a^2+b^2} = 1$ satisfy

$$d = \frac{\frac{c}{ab}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$d = \frac{c}{\sqrt{a^2+b^2}}$$

(e).

$$\tilde{l} = \begin{pmatrix} \frac{2}{\sqrt{29}} \\ \frac{5}{\sqrt{29}} \\ \frac{1}{5} \end{pmatrix}$$

1.2

(a).

(b). $T = \begin{bmatrix} t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 \end{bmatrix}$ $\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$

$$T \cdot \bar{x}_i = \begin{pmatrix} t_1 x_1 + t_2 x_1 + t_3 \\ t_4 x_2 + t_5 x_2 + t_6 \end{pmatrix}$$

$$E(T) = \| T \cdot \bar{x}_i - y_i \|^2 = \frac{[(t_1+t_2)x_1 + t_3 - y_1]^2 + [(t_4+t_5)x_2 + t_6 - y_2]^2}{2}$$

$$\frac{\partial E(T)}{\partial t_1} = (t_1 x_1 + t_2 x_1 + t_3 - y_1) \cdot x_1$$

$$= 0$$

$$t_1 x_1 + t_2 x_1 + t_3 - y_1 = 0$$

$$\begin{cases} y_1 = t_1 x_1 + t_2 x_2 + t_3 \\ y_2 = t_4 x_2 + t_5 x_2 + t_6 \end{cases}$$

1.3 (a) $R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

$$T = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$K = \begin{pmatrix} 100 & 0 & 25 \\ 0 & 100 & 25 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P = K \cdot R.$$

(b)

(c). (i) $K = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$

$$\tilde{x}_s = K \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5x + 10z \\ 5y + 10z \\ z \end{bmatrix}$$

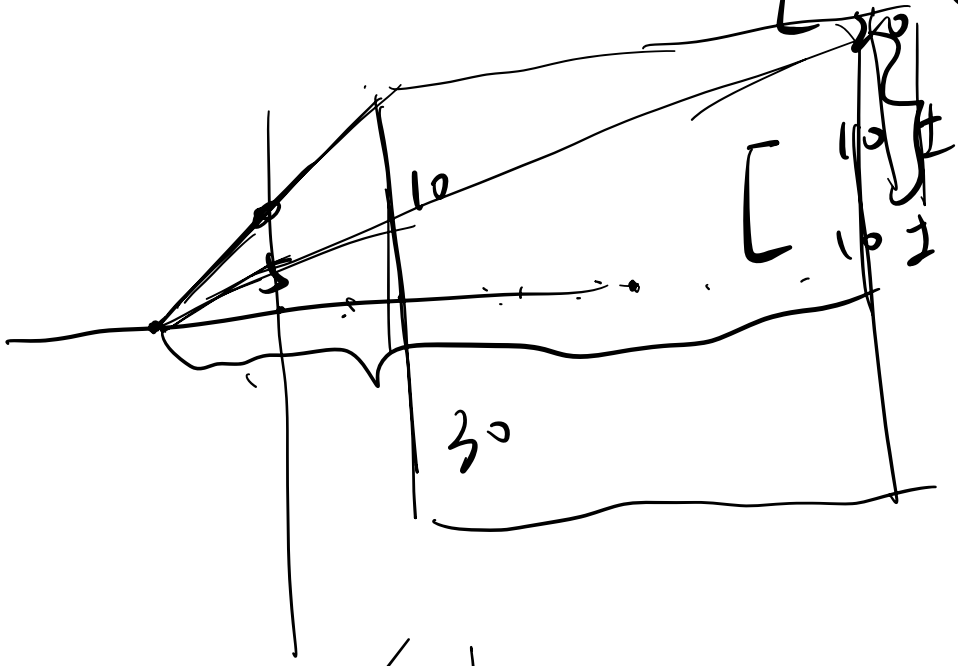
$$\bar{x}_s = 5 \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix} + \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$$= 5 \begin{bmatrix} \frac{\pm 10}{5,25} \\ \frac{+10}{5,25} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\pm 10}{10 \text{ or } 5} \\ \frac{\pm 10}{10 \text{ or } 5} \end{bmatrix} \quad \begin{bmatrix} 20 \\ 0 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

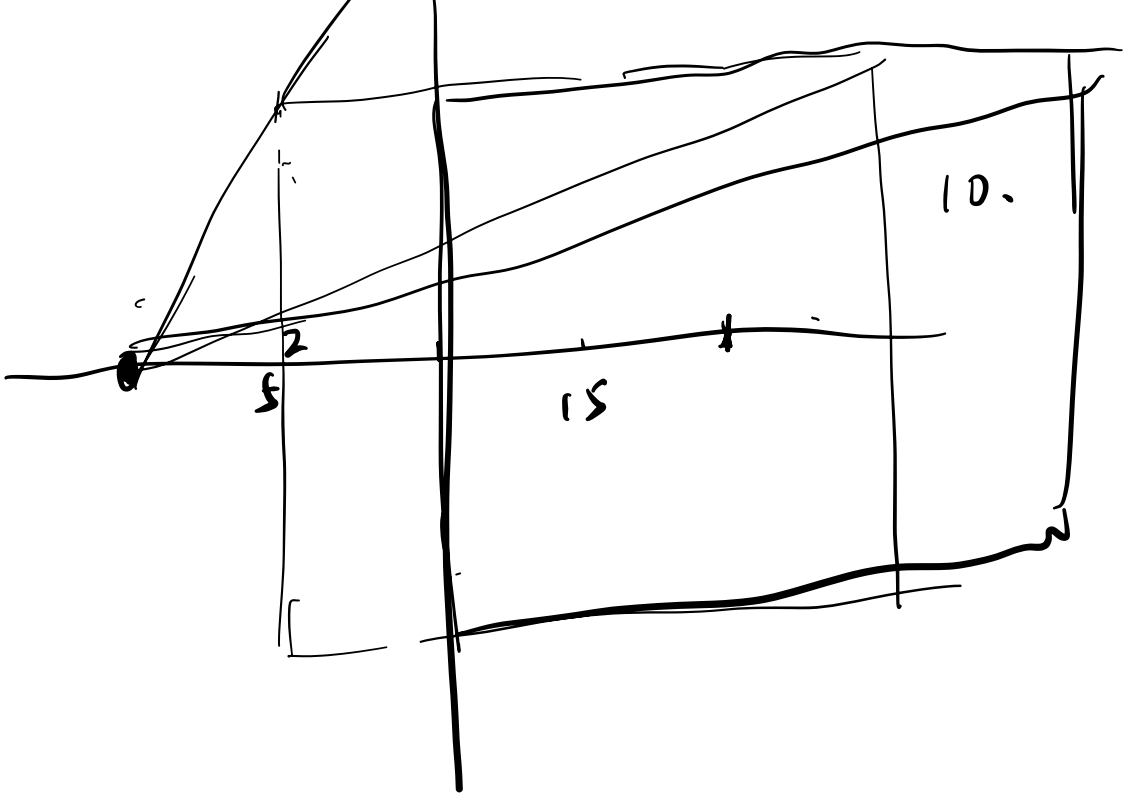
$$\begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 10 \pm 2 \\ 10 \pm 2 \end{bmatrix}$$



$\left(\frac{1}{6}\right) \times 10$

$\left(\frac{5}{3}\right)$



1.4 (a)

(b)