5 B _ F Đ L Ÿ b W

k y k k @ k ë ÿ k Ÿ } ” A > £ Đ ù ¯ q [

ñ B t v ? r m n B b ! x D m X 2 / m X + M

k y k M R R e

R P p 2 ` p B 2 r

线性空间及其子空间的概念	要求掌握线性空间及其子空间的定义与验证，掌握线性组合、线性表示、线性扩张的概念与性质。
线性相关性	掌握线性相关、线性无关的概念与判断及相关定理。
向量组的秩与线性空间的维数 向量组的坐标	要求掌握向量组的秩与线性空间维数的概念及其关系，要求掌握极大线性无关组的概念与计算，要求掌握基的扩张，要求掌握等价向量组的概念与判断，掌握向量坐标的定义与计算。
*线性空间的交与和 （带星号章节，但不排除考察可能）	要求掌握线性空间的交、和的概念与计算，掌握线性空间直和的概念与等价命题。

k L Ÿ b W # 0 b W ¥ À Q

k X R L Ÿ b W ¥ ç l

L Ÿ b W ^ á Ì ¤ (¥ » B ñ 1 × 1 ¥ À Q ñ ^ ç l d b " † V „ ” × 6 ¥ i O ç l V Ï ¥ F E „ V 6 ' ” × Ï ¥ ” Đ " † Ï _ - W ¥ ” ð Ø b 9 ² Ÿ ü ^ d b " † Y ” × Y Ø î ¥ b i O Ø ¡ @ [/ Ÿ É

F E î Œ Đ • h V : + i - : • n • ‹ R X R y l k B î , 1 p

R X † p a + (b + c) = (a + b) + c

k X F E † Ê í 90 2 V P ¤ 8 2 V µ a + 0 = 0 + a

j X l í 8 2 V ; 9 b 2 V µ a + b = b + a = 0 : b = a

9 X Œ Đ p 8 ; 2 V ; + = + X

ÿ i F E , í „ l í ^ · B ¥ á Ì V [ç l h E Ø ¹ F B ñ í Í ¥ l b

”ð Ø9¡@¹HŸÉ 8; 2V;8; 2 6[# 6 ¥ðE†Êí RµRX =

kX( )=( )

jX(+) = +

9X(+)= + X

ÿi 8† •ŸÉáÌµZñ + 1 1+ 2 2+ + r r=0 60 H¥³¹ = ¹ 1 1+ ¹ 2 2+ + ¹ r rX

[çlh A¹:¿• IkV?1pF £LŸbWb:k4 9i,v - :•¹HŸÉûµ ëSÿ ”ð Ø9^²†p„s¥qF†Êíb

ÿiLŸbWÎµBñ×1¥ÀQ^ Ø•> 'LŸbWï¥íÍÉ›FE ”ð Øª ¤ž¥íÍ¯–^‹¿LŸbW¥bƒBÄ^çl1p¥ FE•>^- :•¥1p ”ðh•l•‹çl¥1pb

kXkLŸbW è

áÌº ØñÈn¥LŸbW

RX [T 6[x]n+1 = fa0 + a1x + + anxn j ai 2 6g îLŸbW Œ

6[x]n+1 = fa0 + a1x + + anxn j ai 2 6; ai 60g

, îLŸbWb

ÿ € È| [T:¹ 6[x]n+1 VUQ”,ÑV n ¥ [T¥"†b

Ènn:| (k1p1 + k2p2)(x) = k1px(x) + k2p2(x) b

kX¯”ÐL” V[£ †8¯” î¥"†^”× */_ ¥LŸbWbN)B çÿi¯” * N)]H C "†„”×Ïb

ÿi ƒBè0Vü]B"†V[,]”× î,]¥LŸbWb+Y^V[£ LŸbW *(*) »”¹ R,]¿LŸbW *(_) »”¹ kb

'– ,]¥"†9V[ ]Bñ”× î,]¥LŸbW èÂ *(_) „ _(_)b

jX LŸZñF¥³ V[£ QLŸZñF AX =0 ¥³"^LŸbW 6M¥B ñ0bW Œd QLŸZñF¥³, îLŸbW y¹FE Ø,•>b 8n• ‹ Sek¥ kXk7h¥è0[# S3e5 j jb

kXj LŸ0bW

áÌn5ALŸ0bW¥çl

çl RX W ^LŸbW V(6) ¥db0" ÂT W V Ï¥ Ø9 î× 6 ¥LŸbW 5ë W ^ V ¥LŸ0bW eë0bW X

hBçÿiçlÏ¥db0" yp £0bWH5 £dbb¤/Ÿ^ £0bW¥
BîZE

çØ RXLŸbW V(6) ¥db0" W ¹ V ¥0bW¥ sA1Hq^ W ¿ V(6) ¥
LŸ Ø•> X

ƒVü°10bWÏ¥íÍ¡@ ðbW¥FE„”ð Ø•>'Vb
ÿiLŸbWµ ñ0bWë¹ÜO0bW 'Çc,í¥0" f0g„ 1& Vb
ñ0bWë¹dÜO0bWb

è RX R ªü _[x]2 ^ _[x]3 ¥0bW

k ‘ $W_1 = f(x;y;z)\ j\ \frac{x}{3} = \frac{y}{2} = zg; W_2 = f(x;;y;z)\ j\ x + y + z = 1; x \quad y + z = 1g$
^Ž¹ _3 ¥0bW X

»=lÙVüVðÄ¥°L fÜë îØ»bW¥0bW ,VðÄ¥íE ûLŸŸb
¤/ŸáÌ),LŸ”f# ŸÉ áÌn5ŸALŸF†„LŸVU¥ÀQ

çl kX! V(6) ^BñLŸbW $_i \in V; _i \in 6(i = 1;2; \ ;m)$ 5_ = $_1 \ _1 +$
$_2 \ _2 + \ + \ _m \ _m$ ë¹_ F $f_{\ 1;\ 2;\ \ ;\ m}g \times$ 6¥LŸF† ª × 6 V
¨_ F $f_{\ 1;\ 2;\ \ ;\ m}g$ LŸVU X

¿N áÌó LŸ”f¥çl

çl jX! S ^LŸbW V(6) ¥db0" áÌë

$$L(S) = f_{\ 1\ 1} + \ + \ _{k\ k}\ j\ _{1;\ \ ;\ k} \in 6(_{1;\ \ ;\ k} \in S; k \in L g$$

¹ S ¥LŸ”f' S ÏîµµK0" × 6 ¥BMLŸF†Fî¥ V(6) ¥0" X

/ë¥çØáÔáÌV[YVLŸ”f /0bW

çØ kXLŸbW V(6) ¥db0" S ¥LŸ”f L(S) ^ V Ï c S ¥Kl0bW X

ƒBçØ¥£ün5£üLŸ”f^0bW ƒ^¸^¥ –ªªüKlº³1ªü
L(S) ^ V Ï c S ¥©i0bW¥0"'Vb
KªáÌ ªüµK»LŸbW„íK»LŸbW¥çl '[ñùî¥=¸û µ
K»LŸbW

çl 9XV(6) ë¹µK»LŸbW ÂT V Ïi BñµK0" S P¤ L(S) = V Q-
ë¹íK»LŸbW X

è kX£ü _[x]3 ^µK»LŸbW _[x] ^íK»LŸbW X

k X 9 5

F

R X / " † · ç ¥ F E „ ” ð Ø ^ Ž î L ” × ¥ L Ÿ b W X

R µ Ø ” " Z ô Y ¥ ” ¥ F E „ ð E

k " † $\_^2$ Y È ¥ _ F E „ Â / ç l ¥ ” ð E (x; y) = (x; y)

j $\_^n_+$ ' n í ž L ” _ Â / ç l ¥ F E „ ” ð Ø

$$(a_1; \ ; a_n) + (b_1; \ ; b_n) = (a_1 b_1; \ ; a_n b_n);$$

$$(a_1; \ ; a_n) = (a_1; \ ; a_n):$$

9 h ? Ã › î • ‹ S 3 e » = c 5 » B 5 N @ RÙR 1 ¿ f ” ¥ F E ” ð ç l L Ÿ b W ¥ Ù 5 X

k X h › î • ‹ S 3 e @ » 3 = œ c 5 » Ø 5 ¥ † † l Ù X » 8 l Ù Ü È Ù 5 L = ü ^ 1 ‘ ¡ @ B ç H q ¥ [T ^ Ž î 0 b W X

" F

R X V ^ B ñ L Ÿ b W W ^ V ¥ 0 " £ ü W ^ V ¥ 0 b W () L(W) = W X

* F

R X E ^ × F ¥ B ñ 0 × X

R £ ü F 1 ¿ 1 & ¥ F E „ ð E î B ñ E ¥ _ b W i B è

k è ª ü E(E 6 F) , ^ F ¥ L Ÿ b W

j £ ü Ï V ^ F ¥ B ñ L Ÿ b W 5 V 9 ^ E ¥ B ñ L Ÿ b W X

j L Ÿ M 1 Ÿ

j X R L Ÿ M 1 Ÿ ¥ ç l

ù î L Ÿ M 1 Ÿ Ÿ ÷ ¿ á Ì ¬ © ¡ µ K » L Ÿ b W À ³ 1 ñ _ f î [/ ^ ç l

ç l 8 X V (6) ^ B ñ L Ÿ b W $_1; \ _2; \ ; \ _m$ 2 V Â T i , † ¹ 0 ¥ $_1; \ _2; \ ; \ _m$ 2 6 P ¤

$$_1 \ 1 + \ _2 \ 2 + \ + \ _m \ _m = 0$$

î ë 5 ë $_1; \ _2; \ ; \ _m$ L Ÿ M 1 Ž 5 ë L Ÿ í 1 ' " ” º ? ¹ 0 X

L Ÿ M 1 „ L Ÿ í 1 ¥ £ ü ü ^ ¿ ƒ ñ ç l h A ¹ ¹ g ä b
° ¤ ® ç l á Ì V [• [/ ² ,
R X Ł Ÿ b W Ï † ñ _ L Ÿ M 1 ¥ 1 H q ^ ¹ , _
k X © … c , _ ¥ _ F û L Ÿ M 1
á Ì Ÿ A + ñ ' ¥ è 0

è j X R ‘ (1; 1; 0); (0; 1; 1); (1; 0; 1) ¥ L Ÿ M 1 Ÿ
k ‘ (1; 3; 1); (1; 2; 2); (1; 1; 3) ¥ L Ÿ M 1 Ÿ
j ‘ $p_1(x) = 1 + x; p_2(x) = 1 \quad x; p_3(x) = x + x^2$ ¥ L Ÿ M 1 Ÿ
9 ‘ 1; b B ª Y M + Q x b ¥ L Ÿ M 1 Ÿ
8 ‘ $1; 2^x; 2^{\,x}$ ¥ L Ÿ M 1 Ÿ X

ÿ i • j @ ß ¹ , ? } Æ + y ¥ x ´ Ÿ ª ü è Â j 7 $x = 0$ ¤ ž L Ÿ M 1 ¥ S E ^ p ¥ y ¹ j Ï L Ÿ b W ü ^ [T î ¥ L Ÿ b W Ï ¥ í Í ü ^ [T , ? } Æ ´ b ÿ i 8 V ^ + y 5 ˜ ³ 1 / ÷ ¥ Z ñ Ÿ p ³ ƒ B Ù 5 b

j X k L Ÿ M 1 Ÿ ¥ ç Ø

' « = ¸ E s × 1 ^ Ø ³ L Ÿ b W ² ¥ $ ¬] Ð Ì [/ ç Ø # £ ü E s
ƒ i O 1 µ ' Y ¥ Ø ³ b á Ì ¥ ö L ± ^ ^ V ,] Z ë Ø ³ L Ÿ M 1 Ÿ

R X V L Ÿ F † A ç l

_ F L Ÿ M 1 () ñ Ì µ " " , † ¹ y ¥ L Ÿ F † © ¿ , _

_ F L Ÿ í 1 () ñ Ì º µ " " † ¹ y ¥ L Ÿ F † Œ ö © ¿ , _ b

k X V L Ÿ V U A • ‹ ç Ø k X j

ç Ø j X L Ÿ b W V (6) Ï ¥ _ F $_1; _2; \ldots; _m$ (m 2) L Ÿ M 1 ¥ s A 1 H q ^ $_1; _2; \ldots; _m$ Ï µ B ñ _ V ® Å _ × 6 L Ÿ V U X

ƒ B ç Ø © N í • ¹ _ F L Ÿ í 1 ¥ s A 1 H q ^ Ï ¥ _ í E º M V U b
ƒ ^ A – ¥ y ¹ _ F ? º M V U æ ¨ ç l V [^ ¾ • d , " " ¥ L Ÿ V U b 9
² B / ' ¹

_ F L Ÿ M 1 () Ï À µ B ñ _ V [® Å _ L Ÿ V U

_ F L Ÿ í 1 () Ï Ä B ñ _ û , ? ® Å _ L Ÿ V b

j X V Q L Ÿ Z ñ F A • ‹ S e è j L = ƒ B Ä Ð ç l E s Ë »

_ F $_1; _2; \ldots; _m$ L Ÿ M 1 () Q L Ÿ Z ñ F $x_1\ _1 + x_2\ _2 + \ldots + x_m\ _m = 0$ µ d , ³ c

_ F $_1; _2; \ldots; _m$ LŸí1 () QLŸZñF $x_1\ _1 + x_2\ _2 + \ldots + x_m\ _m = 0$
º µ , ³ b

9XV_ FLŸVUBñ_ ¥ZTA •‹çØ kX9

çØ 9XÏ_ F $_1; _2; \ldots; _m$ LŸí1 7_ F ; $_1; _2; \ldots; _m$ LŸM1 5
V® $_1; _2; \ldots; _m$ LŸVU OVUE·B X

9²Â/ Ï_ F“6B_ V®ƒBF_ LŸVU 5

_ FLŸí1 () V ZT·B

_ FLŸM1 () VUZTµík Õb

VUZT·B¥£ü^ÜÅ¥ '!µ6BÕVUZT –ªæ¨LŸí1¥çlª
üƒ ÕVUZTAçM]'Vb

8XV_ FÐñ¥†sF¥1"A •‹ Sedæe

ÂT_ F¥Bñ†sFLŸM1 *¹œñ_ F9LŸM1

ÂT_ FLŸí1 *¹ñ¥©…Bñ†sF9LŸí1b

"N-“ áÌÎV[V "¥Ç[#› T^Ž¹ y¥˜ Ø³ ƒt=¸ªÃöµº
b

KªáÌÎ1º BñçØ ƒBçØ÷F×1 £üV? ¹¯ Œ²,Bç1ƒ
gä

çØ 8X! V(6) Ï_ F f $_1; _2; \ldots; _s$g ¥Äñ_ V®6B_ F $_1; _2; \ldots; _r$ LŸ
VUXÏ s>r 5 f $_1; _2; \ldots; _s$g LŸM1 X

ƒBçØ¥©N 5¹ f $_1; _2; \ldots; _s$g LŸí15Aµ s rb
ƒBçØVYÌÀ“¹ ¥_ FV[$ ¥_ FLŸVU ¥BçLŸM1b
QVŸª LŸí1¥_ º?$©É ÷É¥_ FLŸVUb

jXj 5

F

RXh5›î•‹ S3d@»3=3c 5» Ry5¥‘ 5

kX£ü ÂT_ FLŸM1 ÜÄñ_ €Õ m ñÊÂBÁ¥s ¤ž¥ê F¯
LŸM1 ÂT_ FLŸí1 ÜÄñ_ @F m ñÊÂBÁ¥s ¤ž¥ê
F¯LŸí1

jXa |…´H $_1 = (1;3;6;2)^h; _2 = (2;1;2;\ 1)^h; _3 = (1;\ 1;a;\ 2)^h$ LŸí1$

e

9 X $_1$; $_2$; ; $_n$ 2 6 £ü $_1$; $_2$; ; $_n$ LŸí1¥ 1Hq^ 6 Ï©B_ ûV[®ñÌLŸVU X

8 X $S_1 = f_{\ 1}$; ; $_s$g;$S_2 = f_{\ 1}$; ; $_t$g ^_ bW V ¥ ñLŸí1¥0" £ü $_1$; ; $_s$; $_1$; ; $_t$ LŸí1 () $L(S_1) \setminus L(S_2) = f0g$X

" F

RXX© $_1$6 0 5 $_1$; $_2$; ; $_n$ LŸM1¥ 1Hq^i i(2 i n) P¤ $_i$ V® $_1$; $_2$; ; $_{i\ 1}$ LŸVU OVUE·B X

kX! ^LŸbW V ¥LŸMÐ ÂT $^{k\ 1}$( )6 0 Œ ^{k}( )=0 £ü ; ( ); : : : ; $^{k\ 1}$( )(k > 0) LŸí1 '5Îµ ‹¥ "ñ' ³E 'BÁ X

ÿ [51•¨V·_P fVP_·_°»Bñ,¹ y¥""¥ZE ƒÕ±X^ ×1¥ X

jX!_ F $_1$; $_2$; ; $_n$ LŸí1 _ F ; $_1$; $_2$; ; $_n$ ÏÀ µBñ_ $_i$(1 i r) V$ -ë¥ i ñ_ ; $_1$; $_2$; ; $_{i\ 1}$ LŸVU X

9 X£ü 1;$e^{\ 1}$X;$e^{\ 2}$X($_1$6 $_2$ O(,¹ y) LŸí1 X

* F

RXÂT n ¨Z" A_1;A_2; ;A_m ¡@ $A_i^2$6 0(i = 1;2; ;m) O' i6 j H A_iA_j = O £ü m n X

kXX© m ñ_ $_1$; $_2$; ; $_m$ LŸM1 Œ Ï©i m 1 ñûLŸí1 £ü

RÏ $k_{1\ 1}$ + + $k_{m\ m}$ = 0 5 k_1; ;k_m †¹ y †,¹ y

kÏi ñ©T

$$k_{1\ 1} + \quad + k_{m\ m} = 0;$$

$$l_{1\ 1} + \quad + l_{m\ m} = 0;$$

Ï $l_1$6 0 £ü $\frac{k_1}{l_1} = \quad = \frac{k_m}{l_m}$X

9 _ F¥Ç LŸbW¥»" _ F¥US

9 X RÇÐ»"¥ÀQ

çl eXÏLŸbW V(6) ¥db0" S Ïi LŸí1¥_ F B = f $_1$; $_2$; ; $_m$g OSÏÄñ_ ûV[® B LŸVU 5 B Ï_ ¥ñ" r ¡S S¥Ç :T r(S) = r L = '¹ vLŸí1F¥É X

çl dXÏLŸbW V(6) ¥µK0" B = f $_1$; $_2$; ; $_n$g LŸí1 O L(B) = V 5 ë B ¹ V ¥BF ië n ¹ V ¥»" :T /BKX = nX

L= 8† • ñçláÌV[Až ÂT S ¹µK»LŸbW V(6) ¥0bW *¹ S ¥Çü^ S ¥»"b

V»"¥çlÏáÌV[Ø³ž BñLŸbW¥ ¥É Aç^·B¥ Ž5]Bñ LŸbW|ö C ñ,]¥»" ƒ^,†Ø¥b£üV[°¤æ¨ Bl«Ï¥çØ 8 Q£E'Vb'–áÌ9V[,æ¨çØ 8°¤YVLŸí1¥çl[#LŸZñF¥³ ¥fƒ),¤žLŸbW»"·B¥², vEV[1›Çk L= á¥ùÏ¯ kykR ñÏ9µƒ³b

áÌ³14#BñÀQ '1– bèÂØ»bW¥1– ¹ (1;0;0);(0;1;0);(0;0;1) n »bW9µË»¥w< ' n ñºµBÊ¹ y Å†¹ R¥_ b ¿ [TáÌ5| 1;x;x²; ë¹1– "af"©9µM1¥È¨¥ b

9XkM1çØÐŸÉ

ô ÇÐ»"¥ÀQÐçØ 8 •‹çØ kX8áÌV[¤ž[/°¤¥², RX(S) = n 5 SÏ n+1 ñ_ ALŸM1 SÏ©...LŸí1_ FÀ c n ñ _ O|c n ñLŸí1_ ¥_ Fë¹ S¥ vLŸí1Fb]Ø n »LŸbW Ï n+1 ñ_ ALŸM1 Ïc n ñLŸí1_ ¥_ Fë¹LŸbW¥BF

ÿi vLŸí1Fµ ñ1oM LŸí1ÐfîbW áÌÎµ ñ²,

Ç!_ F¥Ç¹ r 5¥©i r ñLŸí1¥_ û îñ¥Bñ vLŸí1F

Ç!_ F¥Ç¹ r 5Ï_ FV[® Ï¥ r ñ_ LŸV *¹ƒ r ñ_ ü ^ð_ F¥Bñ vLŸí1Fb

kXLŸbW¥ ¥ñ" »" ^·B¥ Œ Ï_ Ê|,·B

jXÏ r(S) = r B = f $_1$; $_2$; ; $_r$g ^ S ¥ vLŸí1F 5 L(S) = L(B) ' /BK(S) = r(S) ÇÐ»"dB X

¤/ŸáÌº ©N_ F¥ÀQbáÌëV[ºMLŸVU¥ ñ_ F¹©N_ FbV[ík•‹ RX«j©N1" V©©N_ FA¶¡@1QŸa ëŸ[#.¾Ÿb ƒBÄ^¸^£ü¥bÿi ©N_ FAçÇM© ƒ^®çØ 8•‹çØ kX8V[°¤• ¥b1¿©N_ F ªÃ 5V "ÏÎµ÷ ¥),b

L= •²,|_ F¥LŸ"f9Ð¹LŸbW _ F¥Ç9Ð¹LŸbW¥ »" _ F¥ vLŸí1F9Ð¹LŸbW¥ áÌ?¤žv ©N¥²,b"N- "Îµ„LŸM1Ÿ[#Çµ1¥" ², áÌ ƒ « 5ÏZUb

è 9X£ü[/ ñ²,

R ! U „ W û^ V ¥d,0bW ÂT U W *¹ /BkØ /BkW
k ! U „ W û^ V ¥d,0bW U W O /BkØ= /BkW 5 U=W X

Kª^Bñ“×1¥çØ ƒBçØ -ªv ¥çØ£üÏûµ ¨

çØ eXÂT W ^ n »LŸbW V ¥Bñ0bW 5 W ¥ V[” ¹ V ¥ X

ƒBçØVüáÌV[ªBñ_ F@F_ ” ¹ --áÌYVp vLŸí
1FéÕBt_ P¤áÌ ¤BF bçØ£ü^e†¥ L= ƒü^•‹çØ kXbe

9Xjp³ vLŸí1F

áÌn5I³Bñ¨0 ” A báÌ,4?C ¨0 ”¥öí d,›¥»Bñd,
íÍ î ¥ î A ¥ _ ¥Bñ vLŸí1Fb£üV[æ¨› T¥Ç¥M1
ÀQ | ”¥0TIn › Tb
ŒáÌBî¥5",ö°¤ó ? î¨0 ”¥_ ŒáÌ©¡ ”ÜV ©›
MÐ,¿M -W¥LŸ1" ±I £ü î[áÌp vLŸí1F¥ZEü^ |
_ ¿ † î ” –ª ”S ©›MÐ Ä¹¨0™ sžöíî ¥ ‹¥
ð_ 'FîBñ vLŸí1Fb
ÿi vLŸí1F^,·B¥ Œ ëó BññTÄ¥ZEbL= ÂT?B
A ²T¥9,AÂN•P ’–5"°¤1p vLŸí1FÎ^‹’• 8Vñ¥b

è 8Xp_ F f $_1$=(1; 1;2;4); $_2$=(0;3;1;2); $_3$=(3;0;7;14); $_4$=(1; 1;2;0); $_5$=(2;1;5;6)g ¥ vLŸí1F„Ç X

9X9_ F¥US

US¥ÀQL= áÌXÜƒ èÂÚÏÐV¥Üë_ Üë_ ¥USVUü^
=»Üë¥ (0;1);(1;0) /¥USVUbáÌC |ƒñÀQ‰Zž÷Bî¥LŸbW

çl 3X! B=f $_1$; $_2$; ; $_n$g ^ n »LŸbW V(6) ¥BF ÂT V ÏíÍ VU
¹ = a_1 $_1$+ a_2 $_2$+ + a_n $_n$ 5 ””F a_1;a_2; ;a_n ë¹ B /¥US :
¹ $_B$=(a_1;a_2; ;a_n)X

áÌ“¸^£ü (+)$_B$= $_B$+ $_B$ „ ( )$_B$= $_B$ îë ƒVüUS9 û ð
bW¥LŸ Ø1",MbiOáÌ,4£üUSÐ_ ^BB ‹¥ iOUS)¿ 6ⁿ
bWÏ î[áÌV[| BîLŸbW¥ùî ¹ùîe†¥ 6ⁿ bW ƒ9^/BcÏ
]ö4#¥b
³1ÿi¥^ _ⁿ Ï¥_ 1– /¥USL=ü^_ '& ³1¹:b

è eXsYp p(x)=a_0+a_1x+a_2x² B_1=f1;x;x²g „ B_2=f1;x 1;(x 1)²g /¥USX

9 X 8 5

F

R XX © $_1$ = (1 ; 2; 4; 3); $_2$ = (1 ; 1; 6; 6); $_3$ = (2; 1; 2; 9); $_4$ = (1 ; 1; 2; 7); = (4; 2; 4; a) X

R p 0 b W L($_1$; $_2$; $_3$; $_4$) ¥ » ” „ B F

k p a ¥ ´ P ¤ 2 W i p R î Ê / ¥ U S X

k X£ ü B = f 1; x a; (x a)^{2}g(a 6 0) ^ _[x]$_3$ ¥ B F i p _[x]$_3$ ¥ 1 – f 1; x; x^{2}g Ï Ä ñ _ 1 ¿ B ¥ U S X

j XX © _ F A = f $_1$; $_2$; $_3$g; B = f $_1$; $_2$; $_3$; $_4$g; C = f $_1$; $_2$; $_3$; $_5$g ¥ Ç s Y ¹ r(A) = r(B) = 3 ; r(C) = 4 £ ü f $_1$; $_2$; $_3$; $_5$ $_4$g ¥ Ç ¹ 9 X

9 X _ F $_1$; $_2$; ; $_s$ ¥ Ç ¹ r Ï © | m ñ _ $_{i1}$; $_{i2}$; ; $_{im}$ £ ü _ F $_{i1}$; $_{i2}$; ; $_{im}$ ¥ Ç r + m s X

8 XX © $_1$; $_2$; ; $_n$ L Ÿ í 1 O $_1$; $_2$; ; $_n$; ; L Ÿ M 1 £ ü 1 ¹ ; V [® $_1$; $_2$; ; $_n$ L Ÿ V U 1 ¹ $_1$; $_2$; ; $_n$; Ð $_1$; $_2$; ; $_n$; © N X

" F

R X L Ÿ b W V(6) Ï _ ^ $_1$; ; $_r$ ¥ L Ÿ F † Œ , ^ $_1$; ; $_{r\ 1}$ ¥ L Ÿ F † £ ü L($_1$; ; $_{r\ 1}$; $_r$) = L($_1$; ; $_{r\ 1}$;) X

k X æ ¨ _ L Ÿ M 1 Ÿ £ ü ” Ç , © T jr(A) r(B)j r(A B) r(A)+ r(B) X

j X M$_n$(R) V U L n ¨ Z ” † 8 î ¥ " † ! W = fA 2 M$_n$(R) j a$_{ji}$ = ka$_{ij}$; i j g p ’ k = 0; 1; 2 H W ¥ B F „ » ” X

9 X _[x]$_3$ ^ Q ” l ¿ j ¥ L " ” [T „ † 8 , [T B F î ¥ " † 1 ¿ [T F E „ ” ð [T Ø î ¥ L ” × ¥ L Ÿ b W X

R £ ü W = ff (x) 2 _[x]$_3$ j f (1) = 0 ^ _[x]$_3$ ¥ B ñ 0 b W i p W ¥ » ” „ B F

k ç l V _[x]$_3$ ž _ ¥ L Ÿ ˜ (f (x)) = f (1) £ ü ¹ L Ÿ ˜ i p A K „ / B K F 2 `

j ! f; g; h 2 _[x]$_3$ O f (1) = g(1) = h(1) £ ü f; g; h L Ÿ M 1 X

8 X V_1; V_2 ^ ” × 6 ¥ L Ÿ b W 8($_1$; $_2$); ($_1$; $_2$) 2 V_1 V_2; 8k 2 6 ? ç

($_1$; $_2$) + ($_1$; $_2$) = ($_1$ + $_1$; $_2$ + $_2$);

R y

$$k(\ _1;\ _2) = (k\ _1; k\ _2):$$

R £ ü V_1 V_2 1 ¿ [Ø î " × 6 ¥ L Ÿ b W

k Ï / B ₭ $_1$ = m; / B ₭ $_2$ = n p / B (₭ V_1 V_2) X

e X ¤ î ' 5 - V [5 í $ 5 Ø " ð E Œ Đ Ù 5 B « ! A 2 6^n ^{n} 7 C(A) = f B 2 6^n ^{n} j AB = BA g X

R £ ü C(A) ¹ 6^n ^{n} ¥ B ñ 0 b W

k ' A = E H p C(A)

j ' A ¹ ~ L í Í º , M © ¥ ~ " H p C(A) ¥ » " „ B F X

d X ‹ S(A) = f B 2 6^n ^{n} j AB = 0 g X

R £ ü S(A) ¹ 6^n ^{n} ¥ 0 b W

k ! r(A) = r p S(A) ¥ B F „ » " X

* F

R X Ø Đ ç Ø ! $_1$; $_2$; ; $_r$ L Ÿ í 1 O V [$ f $_1$; $_2$; ; $_n$ g L Ÿ V U 5 V [V f $_1$; $_2$; ; $_n$ g Ê r ñ _ 9 Đ î $_1$; $_2$; ; $_r$ ª ¤ ž Đ f $_1$; $_2$; ; $_n$ g © N ¥ • _ F ÿ V [P ¨ " Đ B , E £ ü X

k X ‹ L Ÿ b W V = 6^M £ ü

R i V ¥ 0 b W W P ¤ W ¥ © B d , _ ¥ s (, ¹ y

k Ï V ¥ 0 b W W ¥ © B d , _ ¥ s (, ¹ y 5 / B ₭ W = 1

j Ï V ¥ 0 b W W ¥ © B d , _ ¥ , s ñ " (, Ñ V r 5 / B ₭ W r + 1 X

8 L Ÿ b W ¥ Œ Đ „

8 X R L Ÿ b W ¥ Œ Đ „ ¥ À Q

ç l N X ‹ W_1; W_2 ^ L Ÿ b W V (6) ¥ ñ 0 b W 5

$$W_1 \setminus W_2 = f\ j\ 2\ W_1\ M /\ 2\ W_2 g;$$

$$W_1 [\ W_2 = f\ j\ 2\ W_1\ Q `\ 2\ W_2 g;$$

$$W_1 + W_2 = f\ _1 +\ _2\ j\ _1 2\ W_1;\ _2 2\ W_2 g$$

s Y ë ¹ W_1 „ W_2 ¥ Œ a i a „ X

á Ì 1 ÿ i L Ÿ b W ¥ Œ Ð „ ¯ – ^ V ¥ 0 b W h ò Ê] Ð 1 › £ ü b i O V ¥ µ K ñ 0 b W ¥ Œ Ð „ ¯ – ^ V ¥ 0 b W b

1 ¿ L Ÿ b W ¥ i á Ì A ¶ ÿ i L Ÿ b W ¥ i , B ç ^ L Ÿ b W ƒ “ ¸ ^ Ø ³ y ¹ ñ L Ÿ b W í Í F † B ñ L Ÿ b W ò | B ñ í Í p „ A – , B ç i " Ï v E V [1 › Q è b á Ì ó [/ ², W_1 [W_2 ¹ L Ÿ b W () W_1 W_2 W_2 W_1 () W_1 [$W_2 = W_1 + W_2$:

ƒ B ², £ ü i , ¯ ¬ ò Ê] Ð g ä b V ¥ µ K ñ 0 b W ¥ i ¯ ¹ V ¥ 0 b W ¥ 1 H q ^ Ï µ B ñ 0 b W ? c ð î µ 0 b W b

á Ì V [V + ... ° 4 Ø ³ ƒ t À Q è Â Ø » b W Ï ñ ,] ¥ V ð Ä ¥ Ü ë î ¥ L Ÿ b W ¥ Œ ^ Œ L Œ L 9 V ð Ä î ¥ L Ÿ b W „ ¹ œ ñ Ø » b W b Ø » b W Ï B ñ Ü ë Ð , ¾ Ü ë ¥ ° L ¥ Œ º µ , í „ ¹ œ ñ Ø » b W b I k H á Ì Û ž Q è Ù 5 V [n 5 I n ƒ t e † ¥ + ... m ™ ’ – í E ³ % H V [I n (1;0);(0;1);(1;1) N Ë e † ¥ _ ¹ î ¥ b W b

1 ¿ L Ÿ b W ¥ i á Ì Î µ B ñ × 1 ¥ - Â ç Ø

ç Ø d X! $V_1;V_2;$ $;V_s$ ^ L Ÿ b W V ¥ s ñ d Ü O 0 b W £ ü V Ï À i B ñ _ , ‹ ¿ $V_1;V_2;$ $;V_s$ Ï ¥ © ... B ñ ' V_1 [V_2 [[V_s (V:

ƒ B ç Ø V ü © ... B ñ L Ÿ b W û , ? $ 1 & µ K ñ d Ü O 0 b W Y V i ¤ ž b è Â µ L H ° L ¥ i , V ? ^ B ñ Ü ë b ç Ø ¥ £ ü V [P ¨ ” Ð B , E / ë ^ B ñ ‹ ¨ ¥ è 0

è d X! $V_1;V_2;$ $;V_s$ ^ L Ÿ b W V ¥ s ñ d Ü O 0 b W £ ü i V ¥ B F $_1;$ $_2;$; $_n$ û , $V_1;V_2;$ $;V_s$ Ï X

8 X k » ” T

ç Ø 3 X! $W_1;W_2$ ^ L Ÿ b W V (6) ¥ ñ 0 b W 5

$$/ B k W_1 + / B k W_2 = / B (k W_1 + W_2) + / B (k W_1 \setminus W_2):$$

T ë ¹ 0 b W ¥ » ” T u Y ¿ / B 5 Ï ¥ L Ÿ ~ ' ç Ø ¥ » ” T b ƒ B ç Ø ¥ £ ü ± X ^ × 1 ¥ æ ¨ ¥ ” f © / F ³ 1] Ð Ì ƒ g ä / ë ^ B ñ £ ü ± X Ë » ¥ è 0

è 3 X X © A ; B s Y ^ ” × 6 ¥ s k „ k n ” X ^ n 1 ¥ _ £ ü î µ ¡ @ ABX = 0 ¥ BX î B ñ L Ÿ b W V O » ” ¹ r(B) r(AB):

8 X j ° „ ¥ À Q Ð © N H q

á Ì £ ü € „ b W “ H © û ^ æ ¨ „ b W ç l É › _ s ³ ƒ Õ s ³ · B H '
¹ ° „ b á Ì µ Â / ç l

ç l R y X $W_1; W_2$ ^ L Ÿ b W V (6) ¥ ñ 0 b W Ï $W_1 \setminus W_2 = \{0\}$ 5 $W_1 + W_2$ ¡ S W_1 Ð W_2 ¥ ° „ : T W_1 W_2 X N H ë $W_1; W_2$ ¹ º € 0 b W W_1 ^ W_2 ¥ € b W W_2 ^ W_1 ¥ € b W X

á Ì ³ 1 ÿ i B ñ L Ÿ 0 b W ¥ € b W i , · B h] Ð Ì ó M ‹ ¥ è 0 b
° „ µ [/ © N ¥ 5 á Ì £ ü € æ ¨ ° „ û V [© i Ê 4

ç Ø N X 0 b W $W_1; W_2$ / 5 © N
R $W_1 + W_2$ ^ ° „ ' $W_1 \setminus W_2 = \{0\}$
k $W_1 + W_2$ Ï ¥ Ä ñ _ ¥ s ³ T = $_1 +$ $_2 (\ _1 2 W_1;\ _2 2 W_2)$ · B
j , _ ¥ s ³ T 0 = $_1 +$ $_2 (\ _1 2 W_1;\ _2 2 W_2)$ Ç ' $_1 = \ _2 = 0$ H î ë
9 / B k $W_1 + W_2$) = / B k W_1 + / B k W_2 X

á Ì 9 V [ç l µ K ñ 0 b W ¥ ° „ ' $V = W_1 + W_2$ W_n () $W_i \setminus \sum_{j \neq i} W_j =$ $\{0\}$ b © N 5 9 ^ • ç Ø ¥ w < è Â · B s ³ a y ¥ s ³ [# » ” T w < b á Ì µ B ñ Ð b W ° „ M 1 ¥ ç Ø

ç Ø R y Ÿ X $V = V_1 \quad V_2$ $V_1 = V_{11}$ V_{1s} $V_2 = V_{21}$ V_{2t} 5

$$V = V_{11} \quad V_{1s} \quad V_{21} \quad V_{2t}:$$

á Ì £ ü ° „ B î µ Õ ± ^ B Õ ^ 5 £ „ £ ° „ á Ì Ÿ A B ñ è 0

è N X × 6 î µ n) ” F î ¥ L Ÿ b W $V = M_n (\ 6)$ V_1 V U î µ ë ” F î ¥ "
† V_2 V U î µ Q ë ” F î ¥ " † £ ü $V_1; V_2$ û ^ V ¥ 0 b W O $V = V_1 \quad V_2$ X

Î µ B Õ £ ü $V = V_1 \quad V_2$ ¥ Z T ^ 5 7 $W = V_1 + V_2$ 5 £ ü „ ¹ ° „ ' Œ ¹ $\{0\}$ £ $W = V$ ' V / ë ^ B ñ è 0

è R y X A ^ ” × 6 ¥ B ñ n ¨ V I Z ” A ¥ - r ñ › _ F î ¥ ” ¹ B ª n r ñ › _ F î ¥ ” ¹ C n í L Ÿ Z ñ F $BX = 0$ Ð $CX = 0$ ¥ ³ b W s Y ¹ $V_1; V_2$ £ ü 6 $^n = V_1 \quad V_2$ X

8 X 9 5

F

R X V = $f(a_1; a_2; a_3; a_4)$ j $a_1 + a_2 + a_3 + a_4 = 0$ g W = $f(a_1; a_2; a_3; a_4)$ j $a_1 \quad a_2 \quad a_3 + a_4 = 0; a_1 + a_2 + a_3 \quad a_4 = 0$ g X

R j

R £ ü V „ W ¹ _ ⁴ ¥ 0 b W

k s Y p V \ W V + W [# W ¥ € b W ¥ » ” Ð B F X

k X! $f_1 = 1 + x;\ f_2 = 1 - x^2;\ f_3 = 1 - x^3;\ g_1 = x - x^2;\ g_2 = x + x^3;\ V_1 = L(f_1; f_2; f_3);\ V_2 = L(g_1; g_2)$ p

R $V_1 + V_2$ ¥ „ » ”

k $V_1 \setminus V_2$ ¥ „ » ”

j V_2 _[x]₄ b W ¥ € b

j X ” × 6 X © $V_1; V_2$ s Y ¹ Z ñ F $x_1 + x_2 + \ldots + x_n = 0$ Ð $x_1 = x_2 = \ldots = x_n$ ¥ ³ b W p £ $\theta^n = V_1 \ldots V_2$ X

" F

R X $W_1 = \left\{ \begin{pmatrix} x & x \\ y & z \end{pmatrix} \mid x; y; z \in \mathbb{R} \right\}; W_2 = \left\{ \begin{pmatrix} a & b \\ a & c \end{pmatrix} \mid a; b; c \in \mathbb{R} \right\}.$

R £ ü $W_1; W_2 \subset M_2(\mathbb{R})$ ¥ 0 b W i p / B k W_1; / B k W_2; / B (k $W_1 + W_2$); / B (k $W_1 \cap W_2$) c

k p $W_1 \cap W_2$ ¥ B F i p $A = \begin{pmatrix} 3 & 3 \\ 3 & 1 \end{pmatrix}$ 1 ¿ ƒ F ¥ U S X

k X! V ^ × F ¥ n » L Ÿ b W $f_1; \ldots; f_n\} \subset V$ ¥ B F O

$$V_1 = L(f_1 + 2f_2 + \ldots + nf_n);$$

$$V_2 = \left\{ k_1 f_1 + k_2 f_2 + \ldots + k_n f_n \mid k_1 + \frac{k_2}{2} + \ldots + \frac{k_n}{n} = 0 \right\}.$$

£ ü

R $V_2 \subset V$ ¥ 0 b W

k $V = V_1 \ldots V_2$ X

j X! 6 ¹ ” × $V_1 = \{A \in \theta^n \times n \mid A^h = A\}$ $V_2 = \{A \in \theta^n \times n \mid A^h = -A\}$ $V_3 = \{A \in \theta^n \times n \mid A$ ¹ Ø ˜ ” g X

R £ ü $V_1; V_2; V_3$ û ^ $\theta^n \times n$ ¥ 0 b W

k £ ü $\theta^n \times n = V_1 + V_3$ Œ , ¹ ° „ $\theta^n \times n = V_2 \ldots V_3$ X

9 X X © $V_1; V_2$ ^ µ K » L Ÿ b W V ¥ 0 b W O / B (k $V_1 + V_2$) = / B (k $V_1 \cap V_2$) + 1 £ ü 1 ¹ $V_1 \subset V_2$ 1 ¹ $V_2 \subset V_1$ X

8 X£ ü „ $\sum_{i=1}^{s} V_i$ ¹ ° „ ¥ 1 H q ^ $V_i \setminus \sum_{j=1}^{i-1} V_j = f0g (i = 1; 2; \quad ; s)$ X

e X / ª E ^ Ž ž ’

R Ï V $V_1 [V_2 [\quad [V_s$ 5 $V = (V_1 \setminus V) [(V_2 \setminus V) [\quad [(V_s \setminus V)$

k Ï V $V_1 + V_2 + \quad + V_s$ 5 $V = (V_1 \setminus V) + (V_2 \setminus V) + \quad + (V_s \setminus V)$ X

d X V ¹ µ K » L Ÿ b W V_1 ¹ d , 0 b W £ ü i · B ¥ 0 b W V_2 P ¤ V = V_1 V_2 ¥ 1 H q ¹ V_1 = V X

* F

R X V ^ × F ¥ n ¨ ë ”1 ¿ ”F E „ ”ð Ø î ¥ L Ÿ b W 7

$$U = fA 2 V j i(A) = 0g; \quad W = f E j \quad 2 6g:$$

R £ ü U;W ¹ V ¥ 0 b W

k s Y p U;W ¥ B F „ » ”

j £ ü V = U W X

k X $W_0; W_1; W_2; \quad ; W_s$ ^ L Ÿ b W V ¥ s + 1 ñ d Ü O 0 b W O W_0 $W_1 [W_2 [$ [W_s £ ü A i i P ¤ W_0 W_i X

j X X © $_1; _2; \quad ; _s$ ^ L Ÿ b W V ¥ s ñ ,] ¥ L Ÿ M Ð £ ü V Ï A i _ P ¤ $_1( ); _2( ); \quad ; _s( )$ 9 ,] X

9 X ^ ”× 6 ¥ n ¨ Z ” $f_1(x); f_2(x); \quad ; f_s(x)$ 2 6[x] O º Í O f (x) = $f_1(x)$ $f_s(x)$: n í Q L Ÿ Z ñ F $f(A)X = 0; f_1(A)X = 0; \quad ; f_s(A)X = 0$ ¥ ³ b W s Y ¹ V; V_1; ; V_s £ ü V = V_1 V_s X

ÿ ƒ B 5 ^ B ñ d È × 1 ¥ ², á Ì + ˜ ´ 5 Î ? n ž T ¨ be † ¥ ‹ ¨
^ á Ì V [° ¤ £ ü $A^2 = A$ () r(A) + r(E A) = n $A^2 = E$ () r(A + E) = r(A E) = n . À ÷ Ú ¨ ¥ Ë » ¥ Ç © T b