EXTENDS Naturals, Sequences, FiniteSets, TLC

The set of node identifiers CONSTANT Node

The set of available device identifiers CONSTANT Device

The set of available flows CONSTANT Flow

The total number of buckets CONSTANT NumBuckets

A constant value CONSTANT Nil

Message type constants CONSTANTS

DigestsRequest,

Digests Response,

BucketsRequest,

BucketsResponse,

BackupRequest,

BackupResponse

A sequence of messages for each node

VARIABLE messages

States (terms and mastership) for each node

VARIABLE states

The highest term for each device, used to ensure terms are monotonically increasing VARIABLES terms

A queue of mastership changes for each device on each node VARIABLE masterships

The last logical backup time for each bucket/device/node VARIABLE backups

The local logical clock for each node

VARIABLE *clocks*

The flow buckets for each device on each node VARIABLE flows

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An implementation of lamport clocks for causal ordering of events
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Increments the logical clock for the given node
TickClock(n) \triangleq states' = [states \ EXCEPT \ ![n].timestamp = states[n].timestamp + 1]
 Updates the logical clock for the given node using the given timestamp
UpdateClock(n, t) \triangleq
   \lor \land states[n].timestamp < t
       \land states' = [states \ EXCEPT \ ![n].timestamp = t + 1]
   \lor \land states[n].timestamp \ge t
       \land states' = [states \ EXCEPT \ ![n].timestamp = states[n].timestamp + 1]
Messages are modelled as queues for consistency with TCP semantics. Each node has a separate
channel for all requests and responses. The logical clock is managed on each send/receive by
attaching a timestamp to all outgoing messages and updating the node's clock on receive.
 Returns a sequence with the head removed
Pop(q) \triangleq SubSeq(q, 2, Len(q))
 Sends a request on the given node's channel
SendMessage(n, m) \stackrel{\Delta}{=}
     \wedge TickClock(n)
    \land LET message \stackrel{\triangle}{=} [m \text{ EXCEPT } !.clock = clocks'[n]]
       IN messages' = [messages \ EXCEPT \ ![n] = Append(messages[n], message)]
 Removes a message from the given node's channel
ReceiveMessage(n, m) \stackrel{\triangle}{=}
   \land UpdateClock(n, m.clock)
   \land messages' = [messages \ EXCEPT \ ![n] = Pop(messages[n])]
 This section models flows arbitrarily added to/removed from each node
 Returns the bucket ID for the given flow ID by hashing the flow ID to the number of buckets
GetBucket(fid) \stackrel{\Delta}{=} (fid\%NumBuckets) + 1
 Adds a flow 'f' to node 'n' if it believes itself to be the master
 The given flow is hashed to the appropriate bucket within the device table on node 'n'.
AddFlow(n, f) \triangleq
   \land states[n][f.did].master
   \wedge TickClock(n)
   \land flows' = [flows \ EXCEPT \ ![n][f.did][GetBucket(f.fid)] = [
                                  \mapsto states[n][f.did].term,
                     timestamp \mapsto clocks'[n],
                                  \mapsto flows[n][f.did][GetBucket(f.fid)] \cup \{f\}]]
   \land UNCHANGED \langle messages, states, terms, masterships <math>\rangle
```

Mastership terms are modelled as a queue of monotonically increasing term/master notifications. Each node has a separate notification queue, and mastership terms are added to all queues in the same order. This models the fact that different nodes can learn of mastership changes at different times, but each node sees terms increase with the same master for each term.

One significant difference from the spec and the implementation is that the spec does not use limited numbers of backup nodes. If a node is a master it considers all other nodes to be backups.

This section models the replication protocol. The protocol includes a simple backup mechanism which uses logical clocks to determine when buckets need to be replicated. Additionally, an anti-entropy protocol is used to detect out-of-date buckets on backup nodes.

Sends a backup request for device 'd' bucket 'b' from node 'n' to node 'm' if the bucket has been updated since the last backup

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\begin{array}{ll} Backup(n,\,d,\,b,\,m) \; \stackrel{\triangle}{=} \\ & \wedge \quad n \neq m \\ & \wedge \quad \text{LET } bucket \; \stackrel{\triangle}{=} \; flows[n][d][b] \\ & \text{IN} \\ & \quad \wedge \; backups[n][d][b][m] < bucket.timestamp \\ & \quad \wedge \; SendMessage(m,\,[type \;\; \mapsto BackupRequest,\,[theorem][d][b][m]) \\ & \quad \wedge \; SendMessage(m,\,[theorem][d][b][m]) \end{array}
```

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did
                                           \mapsto d,
                                          \mapsto b,
                                    bid
                                    bucket \mapsto bucket,
                                    src
                                           \mapsto n
        UNCHANGED (states, terms, masterships, backups, flows)
 Handles a backup request 'm' on node 'n'
 If the bucket contained in the backup request is more up-to-date than the same bucket
 on node 'n', node 'n's flows will be updated with the newer bucket and a successful
 response will be sent. Otherwise, a failed response will be sent.
HandleBackupRequest(n, m) \stackrel{\Delta}{=}
  IF
      \land states[n][m.did].term = m.term
      \land flows[n][m.did][m.bid].term \leq m.bucket.term
      \land flows[n][m.did][m.bid].timestamp < m.bucket.timestamp
      \land flows' = [flows \ EXCEPT \ ![n][m.did][m.bid] = m.bucket]
      \land SendMessage(m.src, [type
                                               \mapsto BackupResponse,
                                               \mapsto m.did,
                                   did
                                               \mapsto m.bid,
                                  timestamp \mapsto m.bucket.timestamp,
                                  succeeded \mapsto TRUE)
      \land UNCHANGED \langle states, terms, masterships, backups \rangle
   ELSE
      \land SendMessage(m.src, [type \mapsto BackupResponse, succeeded \mapsto FALSE])
      \land UNCHANGED \langle states, terms, masterships, backups, flows <math>\rangle
 Handles a backup response 'm' on node 'n'
 If the backup succeeded, the last backup timestamp on node 'n' is updated from the response.
 Note the backup timestamp is sent in the response message. This is just a product of the language
 and not an actual implementation detail.
HandleBackupResponse(n, m) \stackrel{\Delta}{=}
  _{
m IF}
      \land m.succeeded
      \land backups[n][m.did][m.bid][m.src] < m.timestamp
      \land backups' = [backups \ EXCEPT \ ![n][m.did][m.bid][m.src] = m.timestamp]
      ∧ UNCHANGED ⟨states, terms, masterships, flows⟩
       UNCHANGED (states, terms, masterships, backups, flows)
 Sends a digest request for device 'd' from node 'n' to node 'm'.
 The digest request is part of the anti-entropy protocol which requests bucket timestamps from a
 remote node 'm' to determine whether any flows are missing from the local node 'n'.
RequestDigests(n, d, m) \stackrel{\Delta}{=}
   \wedge n \neq m
```

```
\land SendMessage(m, [type \mapsto DigestsRequest,
                            did \mapsto d,
                            src \mapsto n])
   ∧ UNCHANGED ⟨states, terms, masterships, backups, flows⟩
 Handles a digest request 'm' on node 'n'
 When the digest request is received, a function of buckets is returned containing the bucket digests.
 Digests include the last term and logical time at which the bucket was updated on node 'n'.
HandleDigestsRequest(n, m) \triangleq
   \land LET digests \stackrel{\triangle}{=} [bucket \in DOMAIN flows[n][m.did] \mapsto [
                                          \mapsto flows[n][m.did][bucket].term,
                             timestamp \mapsto flows[n][m.did][bucket].timestamp]]
           SendMessage(m.src, [type])
                                             \mapsto DigestsResponse,
                                      did
                                               \mapsto m.did,
                                      src
                                               \mapsto n,
                                      digests \mapsto digests)
   \land UNCHANGED \langle states, terms, masterships, backups, flows <math>\rangle
 Handles a digest response 'm' on node 'n'
 Digests are tuples of the term and timestamp at which the bucket was last updated.
 This implementation defines a function for which the domain is buckets that are more up-to-date
 than on local node 'n' according to the digests.
HandleDigestsResponse(n, m) \stackrel{\Delta}{=}
   \land LET buckets \stackrel{\triangle}{=} \{bucket \in DOMAIN flows[n][m.did]:
                              \lor flows[n][m.did][bucket].term > m.digests[bucket].term
                              \lor flows[n][m.did][bucket].timestamp > m.digests[bucket].timestamp]
           SendMessage(m.src, [type])
                                               \mapsto BucketsRequest,
                                      did
                                               \mapsto m.did,
                                               \mapsto n,
                                      src
                                      buckets \mapsto buckets])
   \land UNCHANGED \langle states, terms, masterships, backups, flows <math>\rangle
 Handles a bucket request 'm' on node 'n'
 This implementation differs from the real-world implementation in that it handles an arbitrary number of
 buckets in the request and thus is not designed for scalability.
HandleBucketsRequest(n, m) \stackrel{\Delta}{=}
   \land LET buckets \stackrel{\triangle}{=} [bucket \in m.buckets \mapsto flows[n][m.did][bucket]]
          SendMessage(m.src, [type \mapsto BucketsResponse,
                                      did \mapsto m.did,
                                      src \mapsto n,
                                      buckets \mapsto buckets])
   ∧ UNCHANGED ⟨states, terms, masterships, backups, flows⟩
```

Handles a bucket response 'm' on node 'n'

This implementation differs from the real-world implementation in that it handles an arbitrary number of buckets in the response and thus is not designed for scalability.

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HandleBucketsResponse(n, m) \stackrel{\Delta}{=}
    \land flows' = [flows \ EXCEPT \ ![n][m.did] = [b \in 1 .. \ NumBuckets \mapsto
                      IF b \in \text{DOMAIN } m.buckets Then m.buckets[b] else flows[n][m.did][b]]
    \land UNCHANGED \langle states, terms, backups, flows \rangle
 Handles a message 'm' on node 'n'
HandleMessage(n, m) \triangleq
    \land \lor \land m.type = BackupRequest
           \land HandleBackupRequest(n, m)
       \lor \land m.type = BackupResponse
           \land HandleBackupResponse(n, m)
       \lor \land m.type = DigestsRequest
           \land HandleDigestsRequest(n, m)
       \lor \land m.type = DigestsResponse
           \land HandleDigestsResponse(n, m)
       \lor \land m.type = BucketsRequest
           \land HandleBucketsRequest(n, m)
       \lor \land m.type = BucketsResponse
           \land HandleBucketsResponse(n, m)
    \land ReceiveMessage(n, m)
vars \stackrel{\triangle}{=} \langle messages, states, backups, clocks, flows \rangle
Init \triangleq
    \land messages = [n \in Node \mapsto \langle \rangle]
    \land states = [n \in Node \mapsto [d \in Device \mapsto [term \mapsto 0, master \mapsto FALSE]]]
    \land terms = [d \in Device \mapsto 0]
    \land \ masterships = [d \in Device \mapsto [n \in Node \mapsto \langle \rangle]]
    \land backups = [n \in Node \mapsto [d \in Device \mapsto [b \in 1 .. NumBuckets \mapsto 0]]]
    \land clocks = [n \in Node \mapsto 0]
    \land flows = [n \in Node \mapsto [d \in Device \mapsto [b \in 1 .. NumBuckets \mapsto [term \mapsto 0, timestamp \mapsto 0, entries \mapsto \{\}]]
Next \triangleq
    \vee \exists n \in Node : \exists f \in Flow : AddFlow(n, f)
    \vee \exists n \in Node : \exists f \in Flow : RemoveFlow(n, f)
    \vee \exists n \in Node : \exists m \in Node : \exists d \in Device : \exists b \in 1 ... NumBuckets : Backup(n, d, b, m)
    \vee \exists n \in Node : \exists m \in Node : \exists d \in Device : RequestDigests(n, d, m)
    \vee \exists n \in Node : \exists d \in Device : \exists t \in 1 ... Nat : AddTerm(n, d, t)
    \forall \exists n \in Node : \exists d \in Device : \exists t \in masterships[d][n] : LearnTerm(n, d, t)
    \vee \exists n \in Node : \exists m \in messages[n] : HandleMessage(n, m)
Spec \stackrel{\triangle}{=} Init \wedge \Box [Next]_{vars}
```