Variable messageCount

The invariant checks that:

* No client can hold more than one lock at a time

 $messageVars \triangleq \langle requests, responses, messageCount \rangle$

* No two clients hold a lock with the same ${\it ID}$

```
* The lock is held by an active session
```

Note that more than one client may believe itself to hold the lock at the same time, e.g. if a client's session has expired but the client hasn't been notified, but lock IDs must be unique and monotonically increasing.

```
TypeInvariant \stackrel{\Delta}{=}
```

```
Returns a sequence with the head removed
Pop(q) \triangleq SubSeq(q, 2, Len(q))
Sends a request on the given client's channel
SendRequest(m, c) \triangleq
    \land requests' = [requests \ EXCEPT \ ![c] = Append(requests[c], m)]
    \land messageCount' = messageCount + 1
Sends a response on the given client's channel
SendResponse(m, c) \stackrel{\triangle}{=}
    \land responses' = [requests \ EXCEPT \ ![c] = Append(responses[c], m)]
    \land messageCount' = messageCount + 1
Removes a request from the given client's channel
AcceptRequest(m, c) \triangleq
    \land requests' = [requests \ EXCEPT \ ![c] = Pop(requests[c])]
    \land messageCount' = messageCount + 1
Removes a response from the given client's channel
AcceptResponse(m, c) \triangleq
    \land responses' = [responses \ EXCEPT \ ! [c] = Pop(responses[c])]
    \land messageCount' = messageCount + 1
```

This section models a lock state machine. The state machine supports three types of request:

- * LockRequest attempts to acquire the lock. If the lock is owned by another process, the request is enqueued until the lock is released.
- * TryLockRequest attempts to acquire the lock and fails if the lock is already owned by another process.
- * UnlockRequest attempts to release a lock that's owned by a process.

Additionally, any process's session can be expired, causing any locks held by the session to be released and lock requests enqueued for the session to be removed.

Handles a lock request. If the lock is not currently held by another process, the lock is granted to the client. If the lock is held by a process, the request is added to a queue.

 $HandleLockRequest(m, c) \triangleq$

```
 \begin{tabular}{l} $\lor$ $\land$ sessions[c].state \neq Active \\ $\land$ \begin{tabular}{l} $\land$ UNCHANGED $$\langle$ client Vars, server Vars, responses $$\rangle$ \\ $\lor$ $\land$ sessions[c].state = Active \\ $\land$ lock = Nil \\ $\land$ lock' = m \\ $\land$ id' = id + 1 \\ $\land$ SendResponse([type \mapsto LockResponse, acquired \mapsto \text{TRUE}, id \mapsto id'], c) \\ $\land$ UNCHANGED $$\langle$ queue, sessions, client Vars $$\rangle$ \\ $\lor$ $\land$ sessions[c].state = Active \\ $\land$ lock \neq Nil \\ $\land$ queue' = Append(queue, m) \\ $\land$ UNCHANGED $$\langle$ lock, id, sessions, client Vars, responses $$\rangle$ \\ \end{tabular}
```

Handles a tryLock request. If the lock is not currently held by another process, the lock is granted to the client. Otherwise, the request is rejected.

```
HandleTryLockRequest(m, c) \stackrel{\Delta}{=}
    \lor \land sessions[c].state \neq Active
        ∧ UNCHANGED ⟨client Vars, server Vars, responses⟩
    \vee \wedge sessions[c].state = Active
        \wedge lock = Nil
        \wedge lock' = m
        \wedge id' = id + 1
        \land SendResponse([type \mapsto LockResponse, acquired \mapsto TRUE, id \mapsto id'], c)
        \land UNCHANGED \langle queue, sessions, clientVars \rangle
     \lor \land sessions[c].state = Active
        \land lock \neq Nil
        \land m.timeout = 1
        \land queue' = Append(queue, m)
        ∧ UNCHANGED ⟨clientVars, lock, id, sessions, responses⟩
     \lor \land sessions[c].state = Active
        \land lock \neq Nil
        \wedge m.timeout = 0
        \land SendResponse([type \mapsto LockResponse, acquired \mapsto FALSE], c)
        ∧ UNCHANGED ⟨client Vars, server Vars⟩
```

Handles an unlock request. If the lock is currently held by the given client, it will be unlocked. If any client's requests are pending in the queue, the next lock request will be removed from the queue and the lock will be granted to the requesting client.

```
 \begin{split} Handle Unlock Request(m,\ c) &\stackrel{\triangle}{=} \\ &\lor \land sessions[c].state \neq Active \\ &\land \texttt{UNCHANGED}\ \langle client Vars,\ server Vars,\ responses \rangle \\ &\lor \land sessions[c].state = Active \\ &\land lock = Nil \\ &\land \texttt{UNCHANGED}\ \langle client Vars,\ server Vars,\ responses \rangle \\ &\lor \land sessions[c].state = Active \\ &\land lock \neq Nil \end{split}
```

Times out a pending TryLockRequest. When the request is timed out, the request will be removed from the queue and a response will be sent to the client notifying it of the failure.

```
TimeoutTryLock(c) \triangleq
```

```
 \land \exists \ i \in \text{DOMAIN} \ queue : queue[i].client = c \land queue[i].type = TryLockRequest \\ \land \text{LET} \ i \stackrel{\triangle}{=} \text{CHOOSE} \ i \in \text{DOMAIN} \ queue : queue[i].client = c \land queue[i].type = TryLockRequest condition(note that it is a substitution of the condition of th
```

Expires a client's session. If the client currently holds the lock, the lock will be released and the lock will be granted to another client if possible. Additionally, pending lock requests from the client will be removed from the queue.

∧ UNCHANGED ⟨clientVars, lock, id, sessions, requests⟩

```
ExpireSession(c) \triangleq
     \land sessions[c].state = Active
     \land sessions' = [sessions \ EXCEPT \ ![c].state = Inactive]
     \wedge LET isActive(m) \stackrel{\triangle}{=} sessions'[m.client].state = Active
            If lock \neq Nil \land lock.client = c then
                 LET q \triangleq SelectSeq(queue, isActive)
                 IN
                      \lor \land Len(q) > 0
                         \wedge lock' = Head(q)
                          \wedge id' = id + 1
                         \land queue' = Pop(q)
                         \land SendResponse([type \mapsto LockResponse, acquired \mapsto TRUE, id \mapsto id'], lock'.client)
                         ∧ UNCHANGED ⟨requests⟩
                      \lor \land Len(queue) = 0
                         \wedge lock' = Nil
                         \land queue' = \langle \rangle
                          \land UNCHANGED \langle id, message Vars \rangle
             ELSE
```

```
 \land queue' = SelectSeq(queue, isActive) \\ \land \texttt{UNCHANGED} \ \langle lock, id, messageVars \rangle \\ \land \texttt{UNCHANGED} \ \langle clientVars \rangle
```

This section models a lock client. A client can interact with a lock state machine using three types of requests:

- * Lock attempts to acquire a lock and blocks until successful or the session expires
- * TryLock attempts to acquire a lock, failing if the lock is owned by another process
- * Unlock attempts to release a lock owned by the process

Additionally, a client can assume its session has expired either before or after it actually has. This models the possibility that a client believes its session has expired when it hasn't or that the state machine can expire a client's session without the client knowing.

Sends a lock request to the cluster with a unique ID for the client.

Sends a try lock request to the cluster with a unique ID for the client.

```
TryLock(c) \triangleq
```

- $\land clients[c].state = Active$
- \land SendRequest([type \mapsto TryLockRequest, client \mapsto c, id \mapsto clients[c].next, timeout \mapsto 0], c)
- \land clients' = [clients EXCEPT ![c].next = clients[c].next + 1]
- \land UNCHANGED $\langle serverVars, responses \rangle$

Sends a try lock request to the cluster with a timeout and a unique ID for the client.

```
TryLockWithTimeout(c) \stackrel{\Delta}{=}
```

- $\land clients[c].state = Active$
- \land SendRequest([type \mapsto TryLockRequest, client \mapsto c, id \mapsto clients[c].next, timeout \mapsto 1], c)
- $\land clients' = [clients \ EXCEPT \ ![c].next = clients[c].next + 1]$
- \land UNCHANGED $\langle serverVars, responses \rangle$

Sends an unlock request to the cluster if the client is active and current holds a lock.

```
Unlock(c) \triangleq
```

- $\land clients[c].state = Active$
- $\land Cardinality(clients[c].locks) > 0$
- \land SendRequest([type \mapsto UnlockRequest, client \mapsto c, $id \mapsto$ CHOOSE $l \in clients[c].locks : TRUE], c)$
- $\land clients' = [clients \ EXCEPT \ ![c].locks = clients[c].locks \ \{CHOOSE \ l \in clients[c].locks : TRUE\}]$
- \land UNCHANGED $\langle serverVars, responses \rangle$

Handles a lock response from the cluster. If the client's session is expired, the response is ignored. If the lock was acquired successfully, it's added to the client's lock set.

 $HandleLockResponse(m, c) \triangleq$

Closes a client's expired session. This is performed in a separate step to model the time between the cluster expiring a session and the client being notified. A client can close its session either before or after it's expired by the cluster. Once the client believes its session has expired, its locks are removed, meaning a client can also believe itself to hold a lock after its session has expired in the cluster.

Receives a request 'm' from the client 'c' to the cluster.

```
Receive Request(m, c) \triangleq \\ \land \lor \land m.type = Lock Request \\ \land Handle Lock Request(m, c) \\ \lor \land m.type = TryLock Request \\ \land Handle TryLock Request(m, c) \\ \lor \land m.type = Unlock Request \\ \land Handle Unlock Request(m, c) \\ \land Accept Request(m, c)
```

Receives a response 'm' from the cluster to the client 'c'.

```
ReceiveResponse(m, c) \triangleq \\ \land \lor \land m.type = LockResponse \\ \land HandleLockResponse(m, c) \\ \land AcceptResponse(m, c)
```

```
Initial state predicate
```

```
Init \stackrel{\triangle}{=}
\land requests = [c \in Clients \mapsto \langle \rangle]
\land responses = [c \in Clients \mapsto \langle \rangle]
\land messageCount = 0
\land lock = Nil
```

```
\land queue = \langle \rangle
     \wedge id = 0
     \land clients = [c \in Clients \mapsto [state \mapsto Active, locks \mapsto \{\}, next \mapsto 1]]
     \land sessions = [c \in Clients \mapsto [state \mapsto Active]]
 Next state predicate
Next \triangleq
     \forall \exists c \in \text{DOMAIN } clients : \exists i \in \text{DOMAIN } requests[c] : ReceiveRequest(requests[c][i], c)
     \forall \exists c \in \text{DOMAIN } clients : \exists i \in \text{DOMAIN } responses[c] : ReceiveResponse(responses[c][i], c)
     \vee \exists c \in DOMAIN \ clients : Lock(c)
     \vee \exists c \in DOMAIN \ clients : TryLock(c)
     \forall \exists c \in DOMAIN \ clients : TryLockWithTimeout(c)
     \vee \exists c \in DOMAIN \ clients : Unlock(c)
     \vee \exists c \in DOMAIN \ clients : TimeoutTryLock(c)
     \vee \exists c \in DOMAIN \ clients : ExpireSession(c)
     \vee \exists c \in DOMAIN \ clients : CloseSession(c)
 The specification includes the initial state predicate and the next state
Spec \stackrel{\Delta}{=} Init \wedge \Box [Next]_{\langle server Vars, client Vars, message Vars \rangle}
```

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