The current version domain

The number of nodes must be greater than 1

Assume Cardinality(Nodes) > 1

The partition size is a number

 $CurrentVersion(v) \stackrel{\Delta}{=} CHOOSE \ i \in DOMAIN \ Versions : Versions[i] = v$

Assume $PartitionSize \in Nat \land PartitionSize \leq Cardinality(Nodes)$

Predicate indicating whether the version can be incremented $CanUpgrade(v) \stackrel{\Delta}{=} CurrentVersion(v) < Len(Versions)$

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The next version
NextVersion(v) \triangleq Versions[CurrentVersion(v) + 1]
 The previous version
Previous Version(v) \stackrel{\Delta}{=} Versions[Current Version(v) - 1]
 Return the minimum value from a set
Min(s) \stackrel{\Delta}{=} CHOOSE \ x \in s : \forall \ y \in s : x \leq y
 Whether the node has been upgraded
IsUpgraded(n) \triangleq
     \land nodes[n].version - 1 \in DOMAIN Versions
     \land Cardinality(\{i \in Nodes : nodes[i].version = PreviousVersion(nodes[n].version)\}) > 0
 Whether other nodes have been upgraded
NotUpgraded(n) \triangleq
     \land nodes[n].version + 1 \in domain Versions
     \land Cardinality(\{i \in Nodes : nodes[i].version = NextVersion(nodes[n].version)\}) > 0
The type invariant asserts that:
 * The total number of versions in the cluster at any given time is \leq 2
 * All devices have a master in the current version
TypeInvariant \triangleq
     \land Cardinality(\{nodes[n].version : n \in Nodes\}) \in \{1, 2\}
     \land \forall d \in Devices:
          LET master \triangleq devices[d].master
          IN master = Nil \lor nodes[master].version = upgradeVersion
     \land \forall n \in Nodes:
          LET active \stackrel{\triangle}{=} nodes[n].active
          IN \forall p \in \text{DOMAIN } active :
                 IF IsUpgraded(n) THEN
                      Cardinality(active[p]) = Min(\{Cardinality(\{i \in Nodes : nodes[i].version = nodes[n].version\})
                      Cardinality(active[p]) = PartitionSize
 Selects a master node for a device with the given version
ChooseMaster(v, ns) \stackrel{\Delta}{=}
    LET choices \stackrel{\triangle}{=} \{n \in ns : nodes[n].version = v\}
    IN IF Cardinality(choices) = 0 THEN Nil ELSE CHOOSE n \in choices: TRUE
 Run the command to initialize an upgrade
RunInitialize \triangleq
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 $\land upgradeState = Inactive$

 $\land \forall n \in Nodes : nodes[n].version = upgrade Version$

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\land upgradeState' = Initialized
    \land UNCHANGED \langle upgradeVersion, nodes, devices \rangle
 Run the command to perform an upgrade
RunUpgrade \triangleq
    \land upgradeState = Initialized
    \wedge CanUpgrade(upgradeVersion)
    \land Cardinality(\{n \in Nodes : nodes[n].version = NextVersion(upgradeVersion)\}) > 0
    \land upgradeState' = Upgraded
    \land upgrade Version' = NextVersion(upgrade Version)
    \land devices' = [d \in Devices \mapsto [master \mapsto ChooseMaster(NextVersion(upgradeVersion), Nodes)]]
    \land UNCHANGED \langle nodes \rangle
 Run the command to rollback an upgrade
RunRollback \triangleq
    \land upgradeState = Upgraded
    \land Cardinality(\{n \in Nodes : nodes[n].version = Previous Version(upgrade Version)\}) > 0
    \land upgradeState' = RolledBack
    \land upgrade Version' = Previous Version(upgrade Version)
    \land devices' = [d \in Devices \mapsto [master \mapsto ChooseMaster(Previous Version(upgrade Version), Nodes)]]
    \land UNCHANGED \langle nodes \rangle
Run the command to commit a version change
RunCommit \triangleq
    \land upgradeState = Upgraded
    \land \forall n \in Nodes : nodes[n].version = upgrade Version
    \land upgradeState' = Inactive
    ∧ UNCHANGED ⟨upgrade Version, nodes, devices⟩
Run the command to reset ISSU
RunReset \triangleq
    \land upgradeState = RolledBack
    \land \forall n \in Nodes : nodes[n].version = upgradeVersion
    \land upgradeState' = Inactive
    \land UNCHANGED \langle upgradeVersion, nodes, devices \rangle
Helper predicate indicating whether an upgrade can be rolled back during a node state change
CanRollback \triangleq
    \land upgradeState = Upgraded
    \land Cardinality(\{n \in Nodes : nodes[n].version = PreviousVersion(upgradeVersion)\}) > 0
Helper to roll back an upgrade during a node state change
RollbackUpgrade \triangleq
    \land upgradeState' = RolledBack
    \land upgrade Version' = Previous Version(upgrade Version)
```

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\land devices' = [d \in Devices \mapsto [master \mapsto ChooseMaster(Previous Version(upgrade Version), Nodes)]]
 Helper to rebalance masters after a node state change
RebalanceMasters(except) \triangleq
         \land devices' = [d \in Devices \mapsto [master \mapsto ChooseMaster(upgradeVersion, Nodes \setminus except)]]
         \land UNCHANGED \langle upgradeState, upgradeVersion \rangle
 Helper to determine whether no nodes are in the given version
IsFirstNode(version) \stackrel{\Delta}{=}
         \land Cardinality(\{n \in Nodes : nodes[n].version = version\}) = 0
  Upgrade a node in a single atomic step
UpgradeNode(n) \triangleq
         \land \lor \land upgradeState = Initialized
                    \land nodes[n].version = upgradeVersion
              \lor \land upgradeState = Upgraded
                    \land nodes[n].version = Previous Version(upgrade Version)
         \land CanUpgrade(nodes[n].version)
         \land RebalanceMasters(IF upgradeState = Upgraded THEN \{\} ELSE \{n\})
         \land IF IsFirstNode(NextVersion(nodes[n].version)) THEN
                      nodes' = [nodes \ EXCEPT \ ![n].version = NextVersion(nodes[n].version),
                                                                                ![n].inactive = nodes[n].active,
                                                                                ![n].active = [i \in 1..NumPartitions \mapsto \{n\}]]
               ELSE
                     LET active \triangleq nodes[CHOOSE \ x \in Nodes : nodes[x].version = NextVersion(nodes[n].version)].active
                     IN nodes' = [nodes \ EXCEPT \ ![n].version = NextVersion(nodes[n].version),
                                                                                           ![n].inactive = nodes[n].active,
                                                                                           ![n].active = [i \in 1 ... NumPartitions \mapsto IF \ Cardinality(active[i]) < Period | Pe
         \land UNCHANGED \langle upgradeState, upgradeVersion \rangle
 Downgrade a node in a single atomic step
DowngradeNode(n) \stackrel{\Delta}{=}
         \land upgradeState = RolledBack
         \land nodes[n].version = NextVersion(upgradeVersion)
         \land RebalanceMasters(\{\})
         \land nodes' = [nodes \ EXCEPT \ ![n].version = Previous Version(nodes[n].version),
                                                                       ![n].inactive = [i \in 1 ... NumPartitions \mapsto \{\}],
                                                                       ![n].active = nodes[n].inactive]
         \land UNCHANGED \langle upgradeState, upgradeVersion \rangle
 Crash a node in a single atomic step
CrashNode(n) \triangleq
         \land \lor \land CanRollback
                    \land RollbackUpgrade
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 $\lor \land \neg CanRollback$

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\land UNCHANGED \langle nodes \rangle
 Initial state predicate
Init \triangleq
     \land upgradeState = Inactive
     \land upgradeVersion = Head(Versions)
     \land nodes = [n \in Nodes \mapsto [
            version \mapsto Head(Versions),
            inactive \mapsto [i \in 1 ... NumPartitions \mapsto \{\}],
            active \mapsto [i \in 1 \ldots \textit{NumPartitions} \mapsto \textit{Choose} \ x \in \textit{Subset} \ (\textit{Nodes}) : \textit{Cardinality}(x) = \textit{PartitionSize}]
     \land \ devices = [d \in Devices \mapsto [master \mapsto ChooseMaster(upgradeVersion,\ Nodes)]]
 Next state predicate
Next \triangleq
     \vee \ RunInitialize
     \vee RunUpgrade
     \vee \ RunCommit
     \vee \ RunRollback
     \vee \ RunReset
     \vee \exists n \in Nodes :
         \vee UpgradeNode(n)
         \lor DowngradeNode(n)
         \vee CrashNode(n)
```

 $\land RebalanceMasters(\{n\})$

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