Full Adder and Ripple-Carry Adder

Based on slides by Jared Moore

Extending Half Adder

Half adder is all we need for one bit

But, when looking at inputs of more than one bit, a half adder would be sufficient only for the rightmost bit

$$\begin{array}{r} 0100_2 \\ +\ 0101_2 \\ \hline \end{array}$$

All other columns of our sum actually have a third input: the carry in

Construct truth table for 1-bit addition with a carry-in input that can also be 0 or 1

Now have 3 inputs => 8 rows of truth table

Outputs are the same – sum and carry out

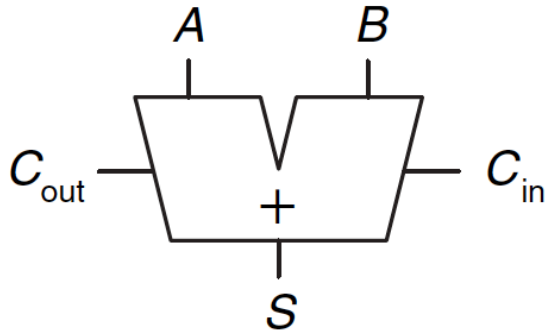
C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Full Adder

Add a carry-in input to a half adder

Circuit can be drawn in several ways

PLA-style would be rather large



C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

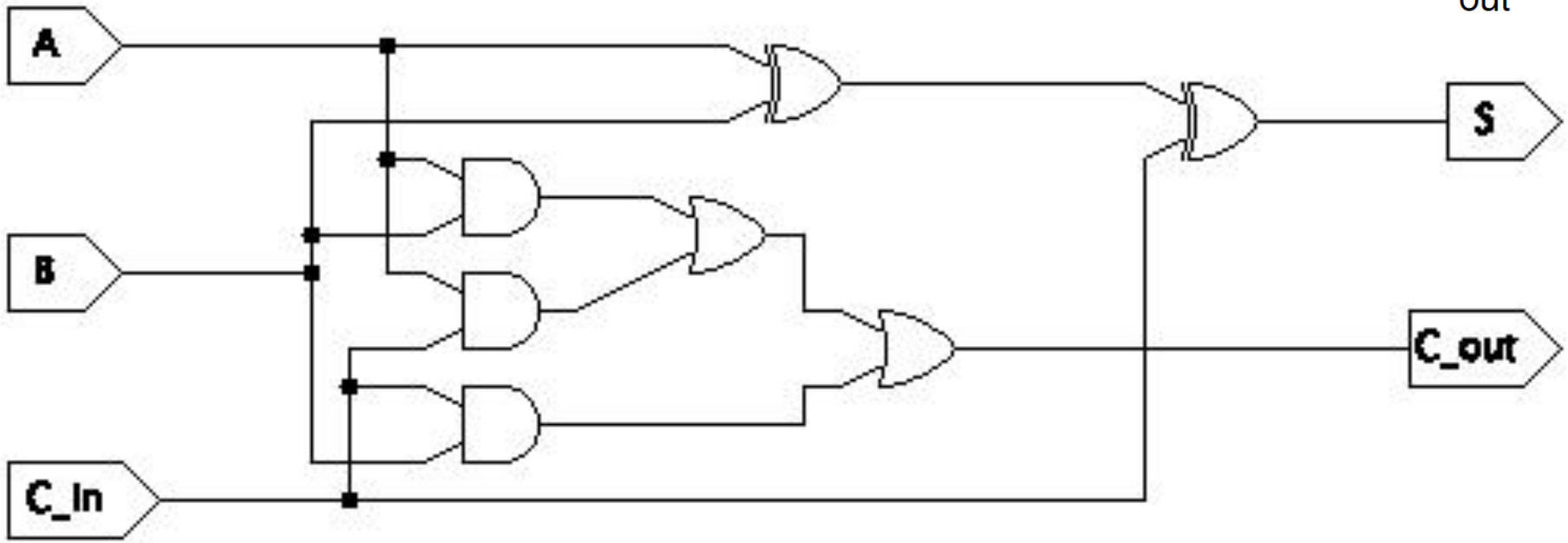
$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

Full Adder – Simple

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

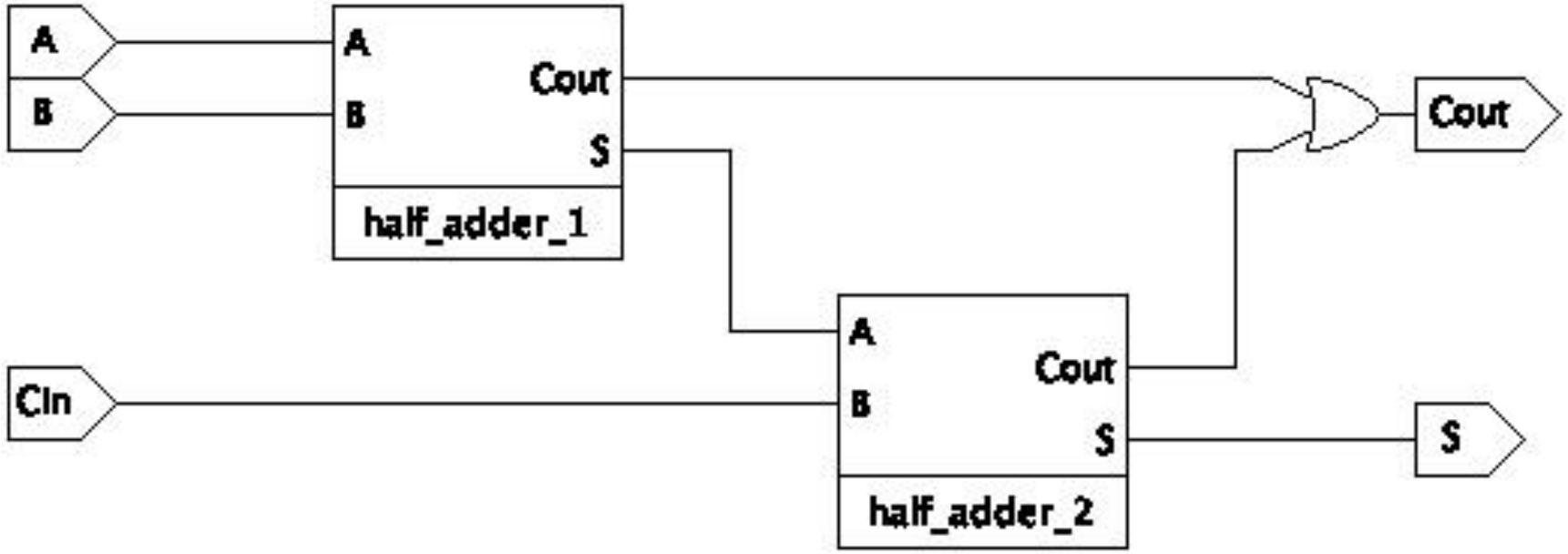


This way follows directly from the Boolean algebra, but we will see later that it is more efficient to build it another way

Full Adder – Combining Half Adders

$$S = A \oplus B \oplus C_{in}$$

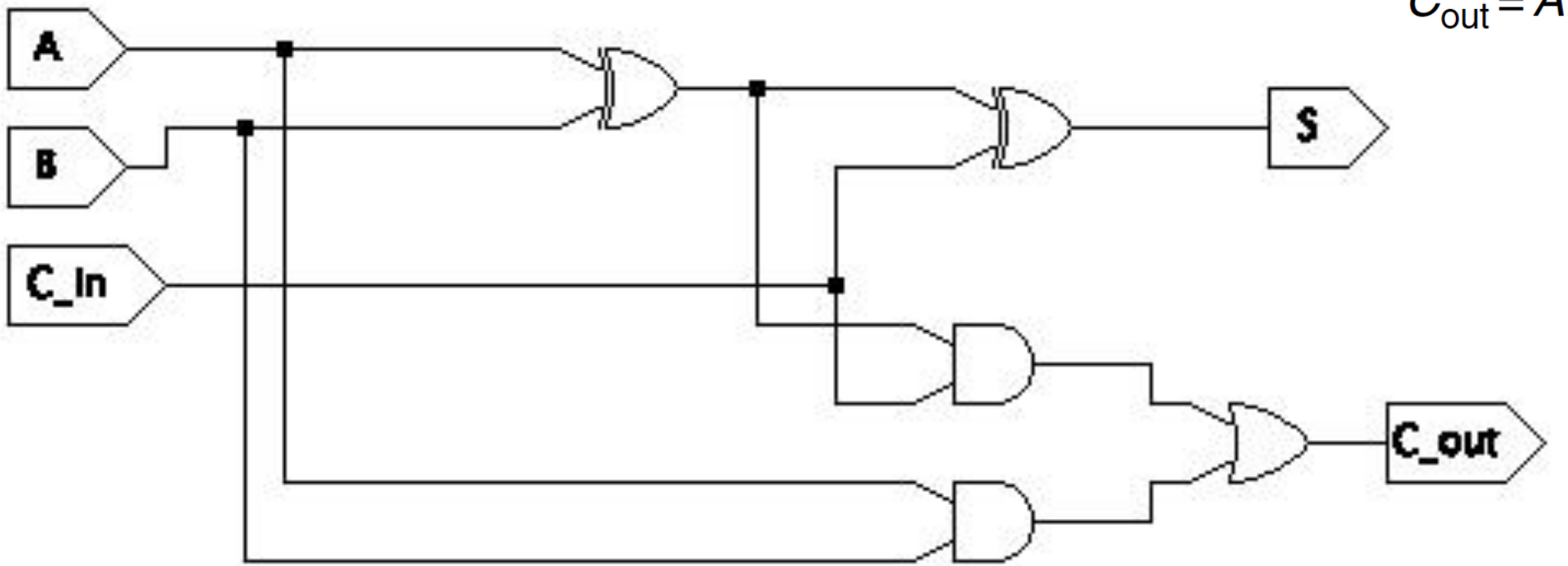
$$C_{out} = AB + AC_{in} + BC_{in}$$



Full Adder – Combining Half Adders

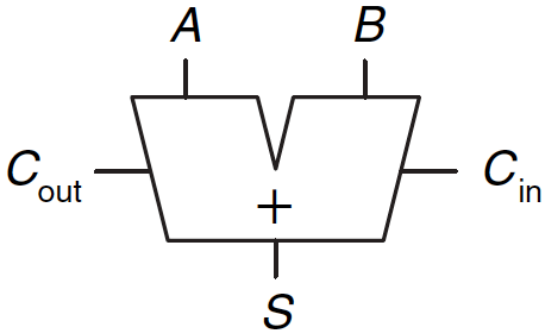
$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$



Full Adder

Now that we know we can build a full adder, we will use it as a black box for even larger circuits



C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

Carry Propagate Adder

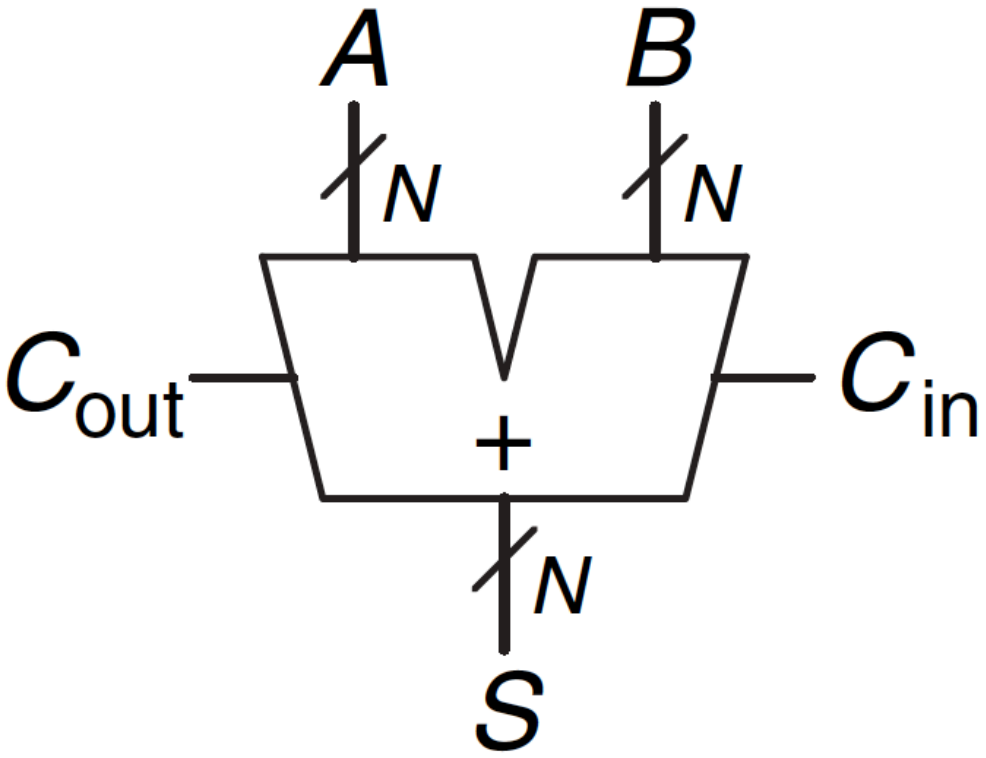
N-bit adder sums two N-bit inputs plus a C_{in}

Outputs are S and C_{out}

Carry out of one bit propagates to the next bit.

Three common types:

- Ripple Carry Adder
- Carry-Lookahead Adder
- Prefix Adder

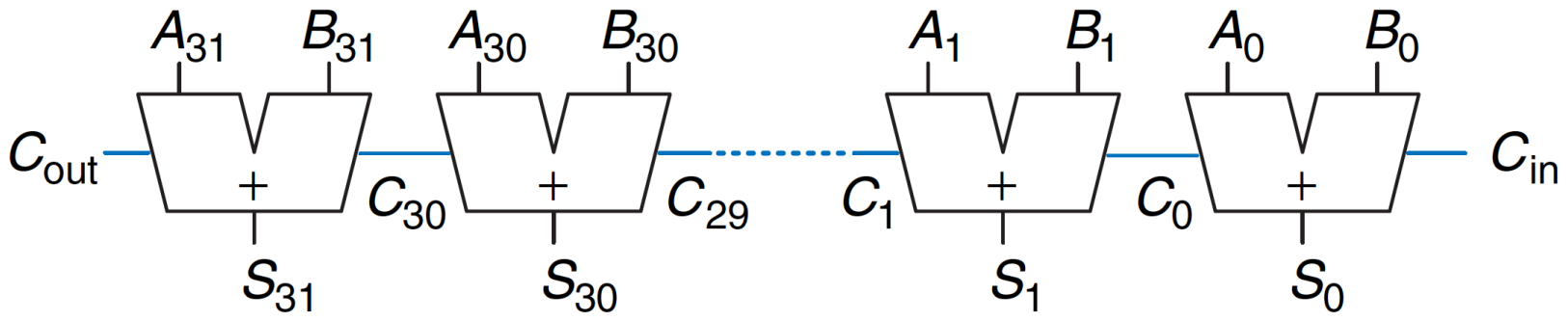


Ripple Carry Adder

Chain together full adders to get the required number of bits.

Hook C_{in} to C_{out} of previous bit

Reuse a full adder module repeatedly.



$$\begin{array}{r} 0100_2 \\ +\ 0101_2 \\ \hline \end{array}$$

Ripple Carry Adder

Ripple carry adder is just one of several types of adders we will study in this course

If it works correctly, why bother with other kinds of adders?

Circuits have not just an input and output, but also a *time* it takes to get from input to output

Ripple carry is simple, but very slow compared to other adders