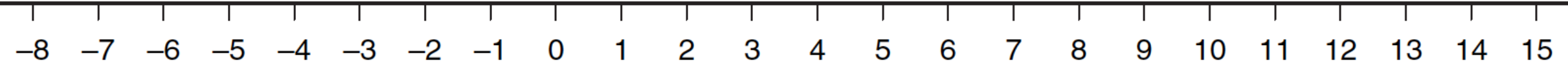


Overflow

Based on slides by Jared Moore

System	Range
Unsigned	$[0, 2^N - 1]$
Sign/Magnitude	$[-2^{N-1} + 1, 2^{N-1} - 1]$
Two's Complement	$[-2^{N-1}, 2^{N-1} - 1]$



Unsigned

0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111

1000 1001 1010 1011 1100 1101 1110 1111 0000 0001 0010 0011 0100 0101 0110 0111

Two's Complement

1111 1110 1101 1100 1011 1010 1001 0000 0001 0010 0011 0100 0101 0110 0111
1000

Sign/Magnitude

Overflow in unsigned addition

Digital systems have a fixed number of bits.

Carry bit exceeds the number of bits.

Leads to incorrect results!

$$\begin{array}{r} 1100_2 \\ + 1001_2 \\ \hline \end{array}$$

Overflow in unsigned addition

Overflow: When the result of an operation cannot be represented in the given system.

4-bit system can represent $[0, 15]$. Any operands that lead to a sum greater than 15 will cause overflow in this system.

How to detect overflow in unsigned addition in a circuit?

Check for a carry on the most significant column.

When can two's
complement addition
overflow?

How can we detect overflow in two's complement?

Overflow means the same thing for unsigned and two's complement – that the result is outside the representable range

However, we use different methods to detect overflow in unsigned and two's complement arithmetic

Two's Complement Overflow

Overflow occurs when adding two N-bit positive numbers or negative numbers if result is greater than $2^{N-1} - 1$ or less than -2^{N-1}

Adding a positive and negative will **never** cause an overflow.

Remember: Overflow detection is not the same as unsigned numbers.

What if I really want to use unsigned overflow detection for two's complement?

Don't.

$$\begin{array}{r} 0100_2 \\ + 1110_2 \\ \hline 1\ 0010_2 \end{array}$$

Unsigned
(overflows)

$$\begin{array}{r} 4 \\ + 14 \\ \hline 2 \end{array}$$

Signed
(no overflow)

$$\begin{array}{r} 4 \\ + -2 \\ \hline 2 \end{array}$$

How can we detect
overflow in two's
complement?

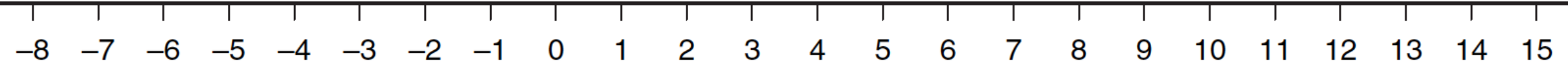
How can we detect overflow in two's complement?

Two numbers being added have the same sign

- Not possible for positive and negative to overflow!

Result has the opposite sign.

System	Range
Unsigned	$[0, 2^N - 1]$
Sign/Magnitude	$[-2^{N-1} + 1, 2^{N-1} - 1]$
Two's Complement	$[-2^{N-1}, 2^{N-1} - 1]$



Unsigned

0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111

1000 1001 1010 1011 1100 1101 1110 1111 0000 0001 0010 0011 0100 0101 0110 0111

Two's Complement

1111 1110 1101 1100 1011 1010 1001 0000 0001 0010 0011 0100 0101 0110 0111
1000

Sign/Magnitude

Important to remember the distinction between what overflow *is* vs how we *detect it*

Remember – detecting overflow in *signed* arithmetic follows different rules from detecting overflow in *unsigned* arithmetic