# Truth Tables

Truth tables, however, are not the only way to represent a combinational circuit

When limiting ourselves to combinational circuits, all three of these are equally expressive
- Truth tables
- Boolean algebra
- Logic gates

We will need to understand all three, including their pros and cons and how to switch between them

# Boolean Expressions

Expression that produces a Boolean value when evaluated

Examples:

X > 1

X

X > Y || Y == Z

(X && Y) + (Z + X)

ABC + D

# Logic Gates

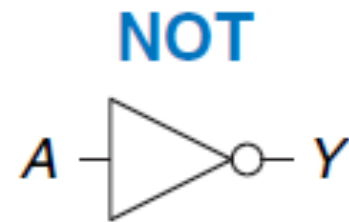Simple digital circuits that take one or more binary inputs and produce a binary output

Drawn with a symbol showing inputs and outputs

- Inputs at left or top
- Outputs at right or bottom
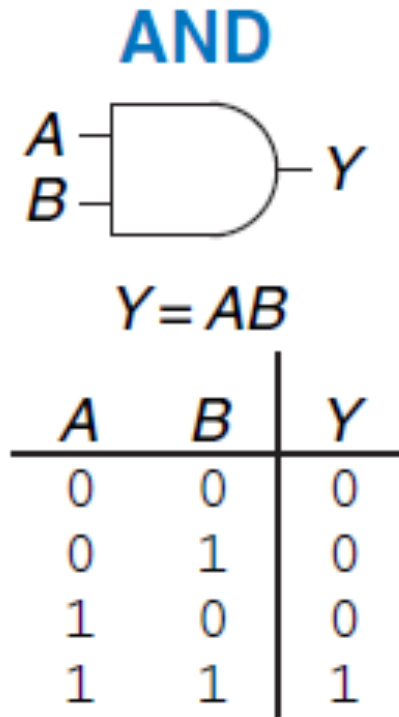
# NOT

Inverter

• Changes 1 to 0, vice versa

**NOT**

$A$ —▷o— $Y$

$Y = \overline{A}$

| A | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

# AND

**AND**



$$Y = AB$$

Output true when both inputs
are true

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

*Other Forms:*

Y = A * B,

y = AB,

y = A (intersection) B

# OR

Output true when A, B, or both are 1

*Other Forms:* Y = A + B,

Y = A (union) B

**OR**

$$Y = A + B$$

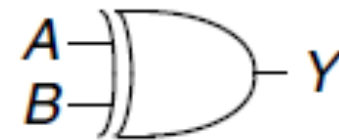| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |
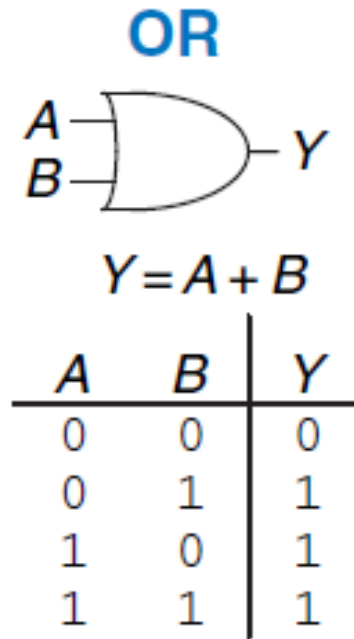
# XOR

Output true only when A is 1 and B is 0, or A is 0 and B is 1

Multiple inputs make it a parity checker, true when odd number of inputs are 1, 0 if even

## XOR



$$Y = A \oplus B$$

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# Other Gates

**NOR**

$$Y = \overline{A + B}$$

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

**NAND**

$$Y = \overline{AB}$$

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# XNOR

**XNOR**

A ─┐
   │  )o─ Y
B ─┘
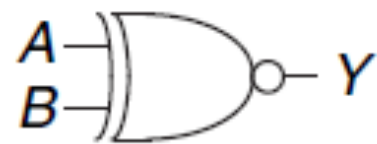
$$Y=\overline{A \oplus B}$$

| A | B | Y |
|---|---|---|
| 0 | 0 |   |
| 0 | 1 |   |
| 1 | 0 |   |
| 1 | 1 |   |

# Multiple Input Gates

Adding more than two inputs possible for most gates.

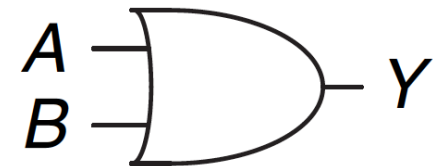For example, a 5-input AND gate would output 1 only if all 5 inputs were 1

Similarly, a 5-input OR gate would output 1 if *any* of the 5 inputs were 1

# Combinational Circuit Rules

1. Every subcircuit is combinational

2. A wire cannot be connected to the output of two different subcircuits
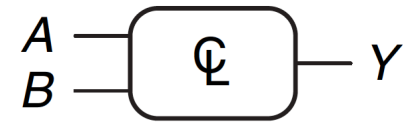
3. No cyclic paths!
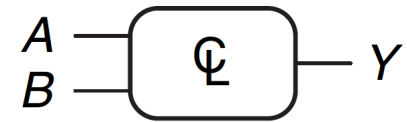
$Y = F(A, B) = A + B$

# Combinational Circuit

Restricting ourselves to combinational circuits is an example of *discipline*

We limit our own options in order to make things easier to work with and understand

$$Y = F(A, B) = A + B$$