Building an Adder

Based on slides by Jared Moore

Circuits that do things

When working with Booleans, we are accustomed to yes/no questions

How can we perform addition (or any task) using just yes/no?

We have seen that when we get enough bits together, we can treat them as numbers instead of True/False values

If we transition our thinking slightly, we can make circuits that perform calculations

Adders

An adder is a circuit that… adds

Inputs will be binary numbers, as will outputs

Assume for now that we are working with unsigned numbers, *but* you already saw that adding works the same way with signed two's complement numbers

$$\begin{array}{r} 0100_2 \\ +\ 0101_2 \\ \hline \end{array}$$

Adders

Example problem would require 8 input bits, 4 output bits for the sum, and 1 output bit to detect carry out

How many rows in truth table?

Adders

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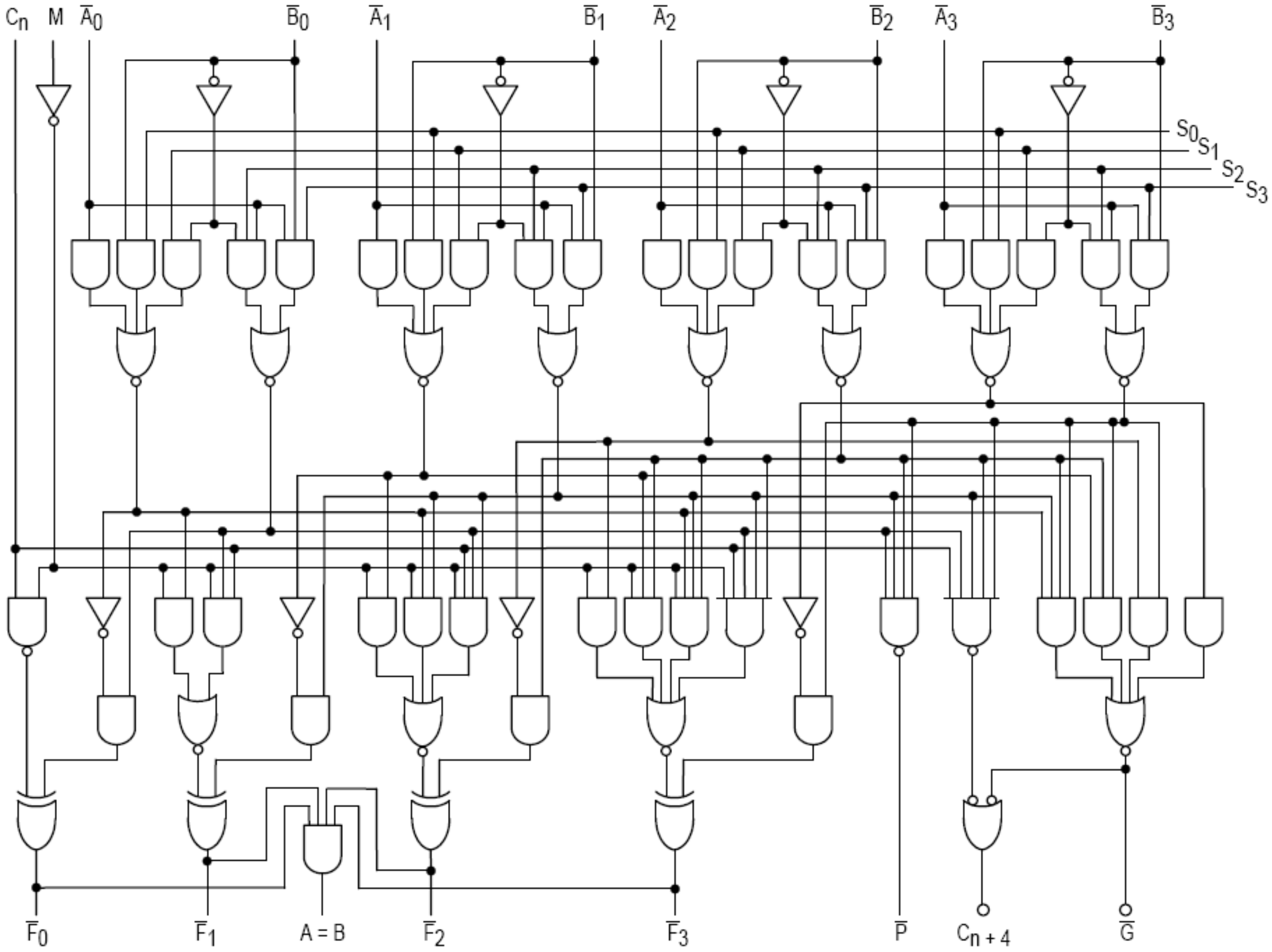
How many rows in truth table?

$2^8 = 256$ rows

How would the circuit look?



C_n
M
$\overline{A}_0$
$\overline{B}_0$
$\overline{A}_1$
$\overline{B}_1$
$\overline{A}_2$
$\overline{B}_2$
$\overline{A}_3$
$\overline{B}_3$
S_0
S_1
S_2
S_3
$\overline{F}_0$
$\overline{F}_1$
A = B
$\overline{F}_2$
$\overline{F}_3$
$\overline{P}$
C_{n+4}
$\overline{G}$

Adders

We’re going to use our design principles to make that more manageable

Start off with just one bit arithmetic, then work our way up

One Bit Addition

Construct a truth table for 1-bit addition

What are the inputs? How many are there?

How many rows in the truth table?

How many outputs?

$$\begin{array}{r} 1_2 \\ +\,1_2 \\ \hline \end{array}$$

The inputs are the bits being added, of which there are two

Two inputs => 4 rows in truth table

Two outputs: sum and carry out

$$\begin{array}{r} 1_2 \\ +\ 1_2 \\ \hline \end{array}$$

$$\begin{array}{r} 0_2 \\ +\,0_2 \\ \hline \end{array} \qquad \begin{array}{r} 0_2 \\ +\,1_2 \\ \hline \end{array} \qquad \begin{array}{r} 1_2 \\ +\,0_2 \\ \hline \end{array} \qquad \begin{array}{r} 1_2 \\ +\,1_2 \\ \hline \end{array}$$

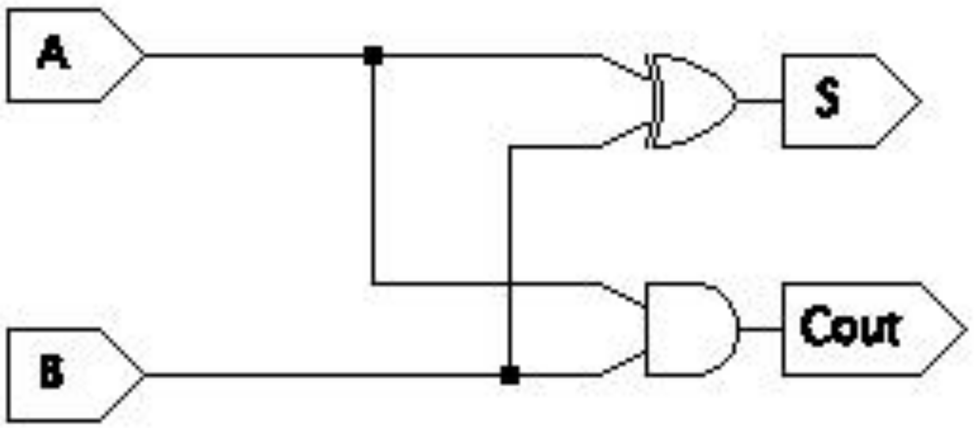
A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

From truth table to circuit

Design a circuit that implements the truth table for a one-bit adder

Remember that the two outputs are independent of one another – you can almost imagine you are designing two separate circuits

PLA-style is always an option, though you can find a more convenient way for this table



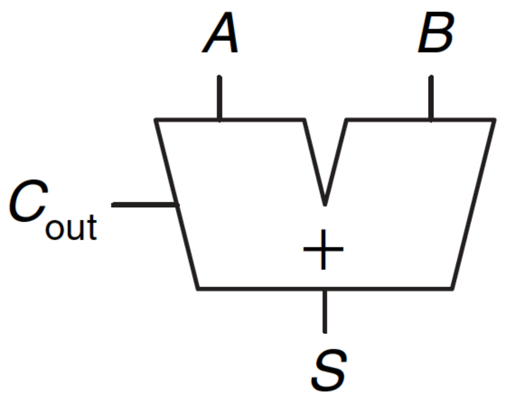
A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Half Adder

2 inputs (A and B)

2 outputs (S and C_{out})

Now that we know how this circuit is built, we use it as a "black box" to create other circuits without drawing out the inside



A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0