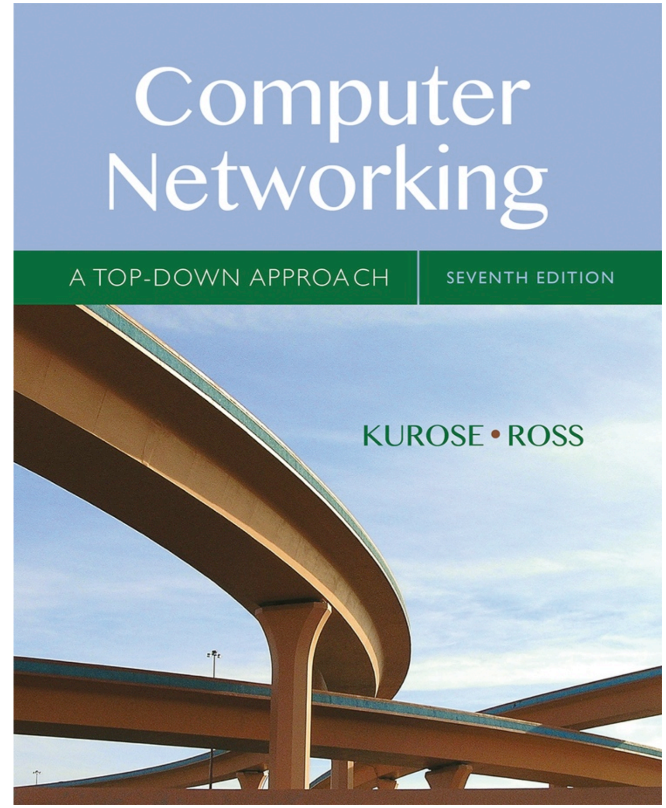
Chapter 8
Security



Computer Networking: A Top Down Approach

7th edition
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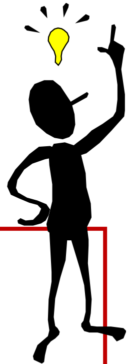
Thanks and enjoy! JFK/KWR

Public Key Cryptography

symmetric key crypto

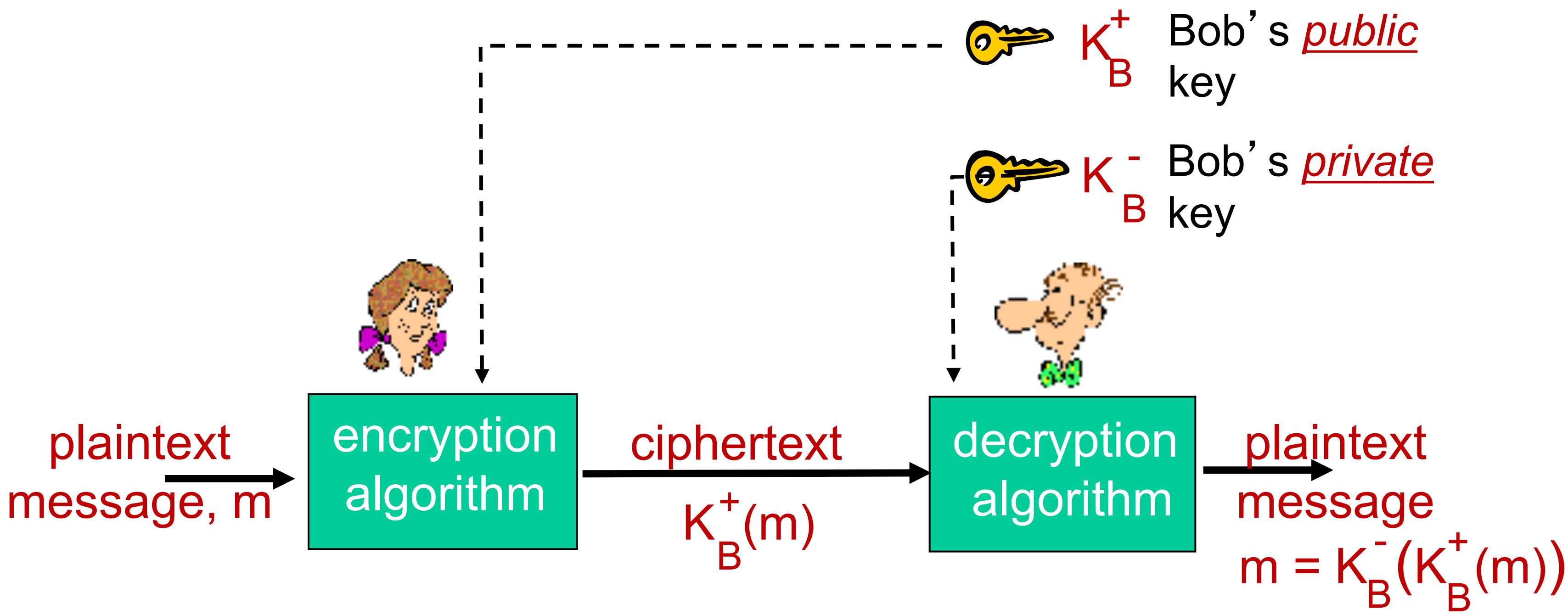
- requires sender, receiver know shared secret key
- Q: how to agree on key in first place (particularly if never "met")?

public key crypto



- radically different approach [Diffie-Hellman76, RSA78]
- sender, receiver do *not* share secret key
- *public* encryption key known to *all*
- *private* decryption key known only to receiver

Public key cryptography



Public key encryption algorithms

requirements:

1. need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$

2. given public key K_B^+, it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

Prerequisite: modular arithmetic

- x mod n = remainder of x when divide by n
- facts:

  [(a mod n) + (b mod n)] mod n = (a+b) mod n

  [(a mod n) - (b mod n)] mod n = (a-b) mod n

  [(a mod n) * (b mod n)] mod n = (a*b) mod n
- thus

  $(a \bmod n)^d \bmod n = a^d \bmod n$
- example: x=14, n=10, d=2:

  $(x \bmod n)^d \bmod n = 4^2 \bmod 10 = 6$

  $x^d = 14^2 = 196 \quad x^d \bmod 10 = 6$

RSA: getting ready

- message: just a bit pattern
- bit pattern can be uniquely represented by an integer number
- thus, encrypting a message is equivalent to encrypting a number

example:

- m= 10010001 . This message is uniquely represented by the decimal number 145.
- to encrypt m, we encrypt the corresponding number, which gives a new number (the ciphertext).

RSA: Creating public/private key pair

1. choose two large prime numbers p, q. (e.g., 1024 bits each)
2. compute $n = pq$, $z = (p-1)(q-1)$
3. choose e (with $e<n$) that has no common factors with z (e, z are "relatively prime").
4. choose d such that $ed-1$ is exactly divisible by z. (in other words: $ed \bmod z = 1$).
5. *public* key is $\underbrace{(n,e)}_{K_B^+}$. *private* key is $\underbrace{(n,d)}_{K_B^-}$.

RSA: encryption, decryption

0. given (n,e) and (n,d) as computed above

1. to encrypt message m ($<n$), compute

   $c = m^e \bmod n$

2. to decrypt received bit pattern, c, compute

   $m = c^d \bmod n$

magic happens! $m = (\underbrace{m^e \bmod n}_{c})^d \bmod n$

RSA example:

Bob chooses *p=5, q=7*. Then *n=35, z=24*.

e=5 (so *e, z* relatively prime).

d=29 (so *ed-1* exactly divisible by z).

encrypting 8-bit messages.

encrypt: $\underbrace{\text{bit pattern}}_{00001100}$ $\quad \underbrace{m}_{12}$ $\quad \underbrace{m^e}_{24832}$ $\quad \underbrace{c = m^e \bmod n}_{17}$

decrypt: $\underbrace{c}_{17}$ $\quad \underbrace{c^d}_{481968572106750915091411825223071697}$ $\quad \underbrace{m = c^d \bmod n}_{12}$

Why does RSA work?

- must show that $c^d \bmod n = m$
  where $c = m^e \bmod n$
- fact: for any x and y: $x^y \bmod n = x^{(y \bmod z)} \bmod n$
  - where $n = pq$ and $z = (p-1)(q-1)$
- thus,

$$
\begin{aligned}
c^d \bmod n &= (m^e \bmod n)^d \bmod n \\
&= m^{ed} \bmod n \\
&= m^{(ed \bmod z)} \bmod n \\
&= m^1 \bmod n \\
&= m
\end{aligned}
$$

RSA: another important property

The following property will be *very* useful later:

$$\underbrace{K_B^-(K_B^+(m))}_{\text{use public key first, followed by private key}} = m = \underbrace{K_B^+(K_B^-(m))}_{\text{use private key first, followed by public key}}$$

result is the same!

Why $K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$?

follows directly from modular arithmetic:

$$
\begin{aligned}
(m^e \bmod n)^d \bmod n &= m^{ed} \bmod n \\
&= m^{de} \bmod n \\
&= (m^d \bmod n)^e \bmod n
\end{aligned}
$$

Why is RSA secure?

- suppose you know Bob's public key (n,e). How hard is it to determine d?
- essentially need to find factors of n without knowing the two factors p and q
  - fact: factoring a big number is hard

RSA in practice: session keys

- exponentiation in RSA is computationally intensive
- DES is at least 100 times faster than RSA
- use public key crypto to establish secure connection, then establish second key – symmetric session key – for encrypting data

session key, K_S

- Bob and Alice use RSA to exchange a symmetric key K_S
- once both have K_S, they use symmetric key cryptography