Combinatorics Basics

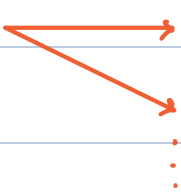
TABLE OF CONTENTS

.

< *Question* > : Given 10 Girls and 7 Boys. How many different pairs of 1:1 can be formed?

Pair → 1 Girl & 1 Boy

Boys	Girls
B_1	G_1
B_2	G_2
B_3	G_3
B_4	G_4
B_5	G_5
B_6	G_6
B_7	G_7
	G_8
	G_9
	G_{10}



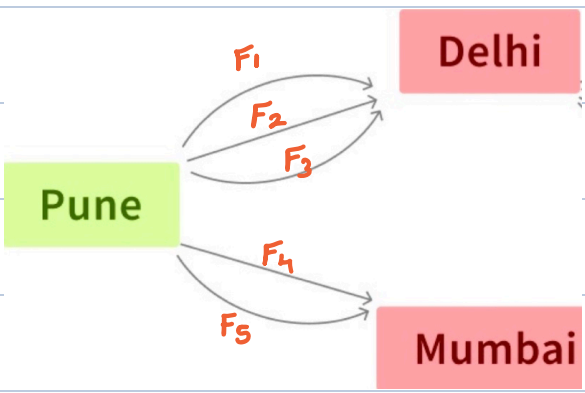
ways = 7 * 10 = 70

Task A → n ways

Task B → m ways

Task A & B → n * m ways

< *Question* > : Find number of ways to travel from Pune to Delhi or Mumbai?



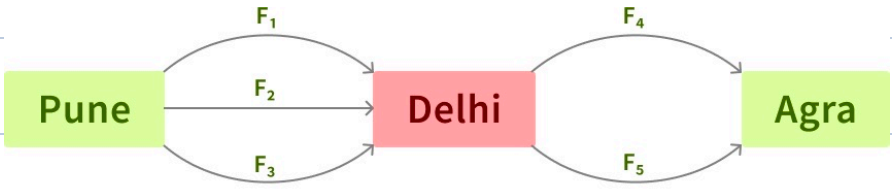
3 + 2 = 5

Task A → n ways

Task B → m ways

Task A or B → n + m ways

Example :



Number of ways to reach Agra from Pune via Delhi

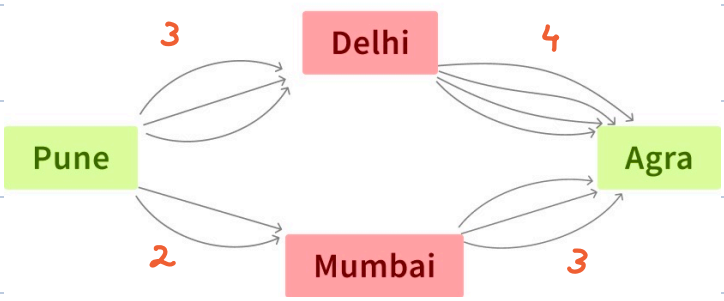
P ⟶ D ⇒ 3 ways

D ⟶ A ⇒ 2 ways

P ⟶ D & D ⟶ A ⇒ 3 * 2 = 6 ways

Number of ways of reaching Agra from Pune ?

Quiz :



P ⟶ D ⟶ A ⇒ 3 * 4 = 12

P ⟶ M ⟶ A ⇒ 2 * 3 = 6

P ⟶ D ⟶ A or P ⟶ M ⟶ A ⇒ 12 + 6 = 18

Scenerio

Zomato, features an exciting option for its users - meal combos. Each combo includes one main course, one *dessert*, and one *beverage*, offering a complete dining experience from various restaurants. Zomato believes that a greater variety of combos can significantly enhance customer satisfaction.

Problem

You're tasked with helping **Zomato** identify which restaurant offers the most variety in its meal combos. You're provided with a list, shaped like a grid or a 2D matrix **A**, where each row corresponds to a different restaurant's offerings.

Each row is divided into three parts:

1. A[i][0] tells you the number of main courses,
2. A[i][1] the number of desserts, and
3. A[i][2] the number of beverages a restaurant offers.

Your challenge is to analyze this data and pinpoint which restaurant gives its customers the most options to mix and match their meal combo.

Examples

Example 1 :

```
A = [
    [3, 2, 2],   # Restaurant 1
    [4, 3, 3],   # Restaurant 2
    [1, 1, 1]    # Restaurant 3
]
```

Ans = $\forall i$, max (A[i][0] * A[i][1] * A[i][2])

Output : 2

Explanation for input 1 :

- Restaurant 1 offers 12 combos (3 mains x 2 desserts x 2 beverages) → 12
- Restaurant 2 offers 36 combos (4 mains x 3 desserts x 3 beverages) → 36
- Restaurant 3 offers 1 combo (1 main x 1 dessert x 1 beverage) → 1

So, Restaurant 2 provides the most variety with 36 possible combinations.

Permutations → Arrangement of objects

< *Question-1* > : Given 3 distinct characters. In how many ways, we can arrange them?

S = " a b c "

a b c b a c c a b

a c b b c a c b a

Ans = 6

< Question-2 > : In how many ways, you can arrange 4 distinct characters?

s = "d a t e"

↓ ↓ ↓ ↓

4 & 3 & 2 & 1

⇒ 4 * 3 * 2 * 1 = 24

< Question-3 > : In how many ways n distinct characters can be arranged?

↓ ↓ ↓ ↓

N N-1 N-2 . . . 1

ways = N * (N-1) * (N-2) * . . . * 1

= N!

< Question-4 > : Given 4 distinct characters, in how many ways can we arrange 2 ?

s = "d a t e"

↓ ↓

4 & 3

ways = 4 * 3 = 12

< Question-5 > : In how many ways can we arrange R out of N distinct characters?

1 2 R

N N-1 . . . N-(R-1)

R = 4

N N-1 N-2 N-3

ways = N * (N-1) * (N-2) * . . . (N-(R-1))

N * (N-1) * (N-2) * . . . (N-(R-1)) * (N-R) * (N-R-1) * . . . 2 * 1

(N-R) * (N-R-1) * . . . 2 * 1

$= \frac{N!}{(N-R)!} \rightarrow {}^{N}P_{R}$

Combinations → selection of objects

< Question > : In how many ways can we select 3 players from a pool of 4 players?

[P1 P2 P3 P4]

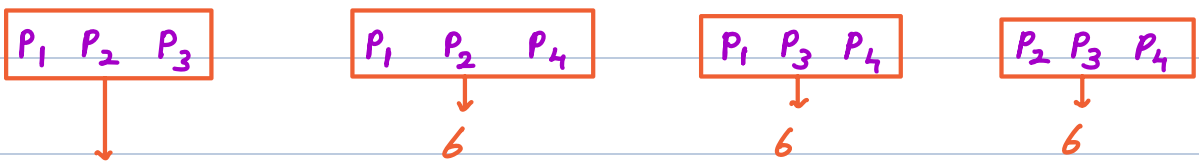
P_1 P_2 P_3 P_1 P_3 P_4 # ways = 4

P_1 P_2 P_4 P_2 P_3 P_4

< Question > : What is the number of ways to arrange the players in 3 slots ?

Given 4 players → [P1 P2 P3 P4]

$$\text{Ans} = {}^4P_3 = \frac{4!}{(4-3)!} = \frac{24}{1} = 24$$



$P_1\ P_2\ P_3$

$P_1\ P_3\ P_2$

$P_2\ P_3\ P_1$

$P_2\ P_1\ P_3$

$P_3\ P_1\ P_2$

$P_3\ P_2\ P_1$

6

ways = 4 * 6 = 24

Arrange R out of N items
= Select R out of N items &
Arrange selected R items

$$\boxed{{}^NP_R = {}^NC_R * R!}$$

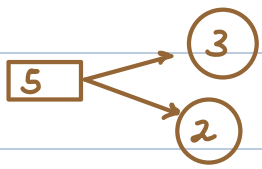
$$\Rightarrow \boxed{{}^NC_R = \frac{{}^NP_R}{R!} = \frac{N!}{(N-R)! * R!}}$$

Properties

1. ${}^{N}C_{0}$ → Select nothing from N items → 1 $\quad \frac{N!}{(N-0)! * 0!} \to 1$

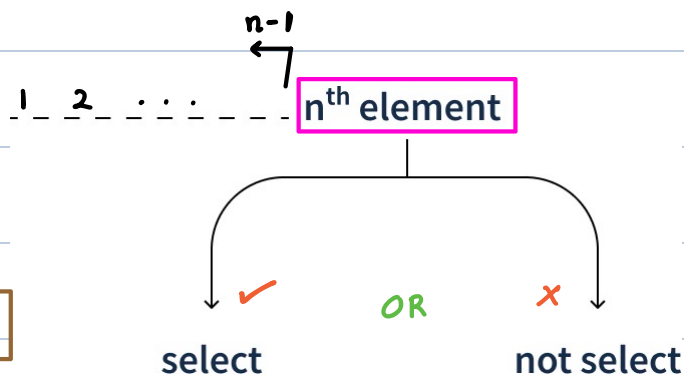
2. ${}^{N}C_{N}$ → Select everything → 1 $\quad \frac{N!}{(N-N)! * N!}$

3. ${}^{N}C_{N-r}$ → Select (N-R) items



$$\frac{N!}{(N-(N-R))! * (N-R)!} = \frac{N!}{R! * (N-R)!} = {}^{N}C_{R}$$

< *Question* > : Given N distinct elements, select r distinct elements. → ${}^{N}C_{R}$



$x! = x * (x-1)!$

$$ {}^{N-1}C_{R-1} + {}^{N-1}C_{R} = {}^{N}C_{R} $$

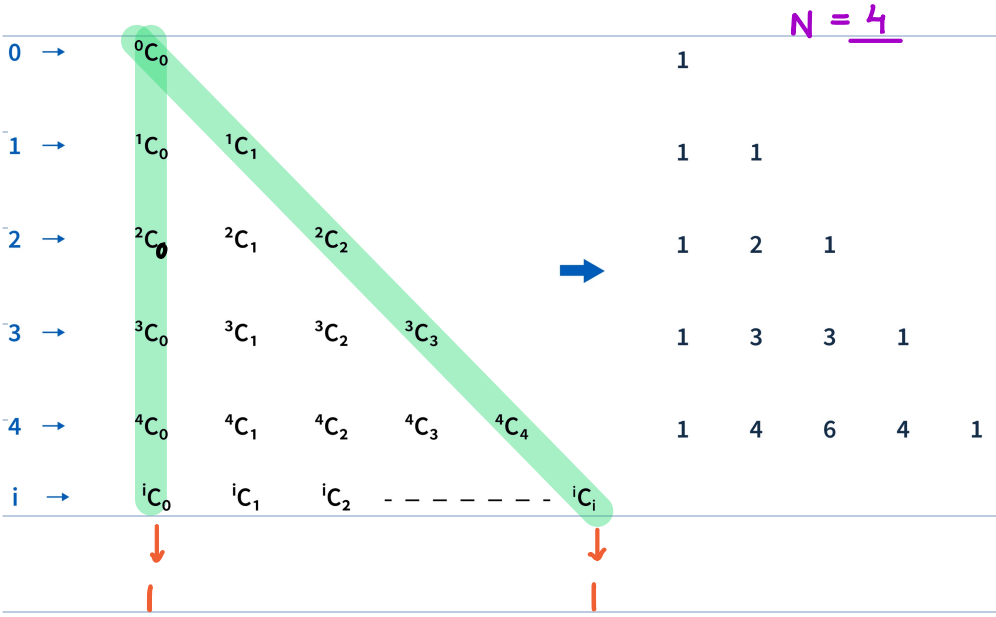
$$\frac{(N-1)!}{(N-1-(R-1))! * (R-1)!} + \frac{(N-1)!}{(N-1-R)! * R!} = \frac{(N-1)!}{(N-R-1)! * (R-1)!} \left(\frac{1}{N-R} + \frac{1}{R}\right)$$

$\frac{1}{N-R} + \frac{1}{R} = \frac{R + N - R}{(N-R) * R} = \frac{N}{(N-R) * R}$

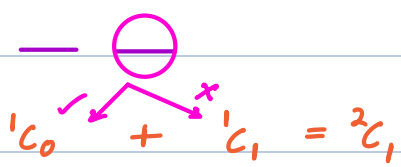
$\frac{(N-1)!}{(N-R-1)! * (R-1)!} * \frac{N}{(N-R) * R} = \frac{N!}{(N-R)! * R!} = {}^{N}C_{R}$

Pascal Triangle

- Generate the Pascal's triangle for given N



${}^{2}C_{1} = {}^{1}C_{0} + {}^{1}C_{1}$



$${}^{N-1}C_{R-1} + {}^{N-1}C_{R} = {}^{N}C_{R}$$

```
for i → 0 to N {
    c[i][0] = 1        c[i][i] = 1
    for j → 1 to (i-1) {
        c[i][j] = (c[i-1][j-1] + c[i-1][j]) % M
    }
} return c
```

TC = $O(N^2)$ SC = $O(1)$

Nth Column Title

- Find the Nth column title

N =	1	2	3		26	27	28		50	51	52	53	54		
	A	B	C	- - -	Z	AA	AB	- - - - - - - - - -	AX	AY	AZ	BA	BB	- - -	BZ

N = 30 → AD ✓

N = 50 → AX ✓

N = 100 → CV ✓

int → binary

10 → 2

10 → 26

N = 50 = 110010

A → 1 0

B → 2

C → 3

Z → 26 25

N = 500 → SF

26	500	6	F
26	19	19	S
	0		

2	50	0
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

N = 100 → CV

26	100	22	V
26	3	3	C
	0		

0 <= A % 26 <= 25

N = 26

26	26 - 1	25 → Z
	0	

N = 27

26	27 - 1 = 26	0
26	1 - 1 = 0	0
	0	

↑ AA

ans = " "

while (N > 0) {

    ch = (char) ((N-1) % 26 + 'A')

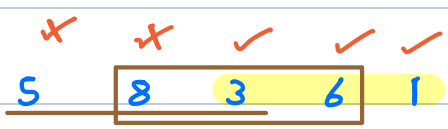
    ans = ch + ans

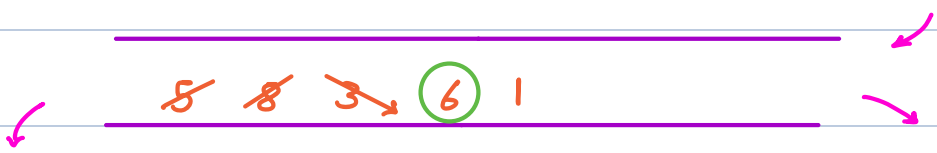
    N = (N-1) / 26

}

return ans

TC = O($\log_{26}$ (N)) SC = O(1)





Ans = q.front()