

Q → Find the smallest number that can be formed by rearranging the digits in the given array.

$$0 \leq A[i] \leq 9$$

$A = [6 \ 3 \ 4 \ 6 \ 5 \ 2]$

→ 2 3 4 5 6 6

sorting in ascending order.

$$TC = O(N \log(N))$$

$A = [4 \ 6 \ 0 \ 2 \ 5 \ 2]$

→ 0 2 2 4 5 6

$F[i] = \text{frequency of 'i'}$

$F = [1 \ 0 \ 2 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0]$

```
for i → 0 to (N-1) {
    F[A[i]]++
}
```

```
for d → 0 to 9 {
    for i → 1 to F[d] {
        print(d)
    }
}
```

Count Sort

$$\text{Total } TC = O(N + N) = O(N)$$

$$SC = O(|A[i]|) = O(10) = O(1)$$

If  $A[i] \leq 10^9 \Rightarrow$  MLE error (SC is very high)

works for range  $\sim 10^6$  to  $10^7$

sort array where  $-5 \leq A[i] \leq 5$ .

$A = [ \overset{0}{3} \ \overset{1}{-5} \ \overset{2}{2} \ \overset{3}{3} \ \overset{4}{0} \ \overset{5}{-1} \ \overset{6}{4} \ \overset{7}{-1} ]$

|                 |    |    |    |    |    |   |   |   |   |   |    |
|-----------------|----|----|----|----|----|---|---|---|---|---|----|
| $x \rightarrow$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5  |
|                 | 0  | 1  | 2  | 3  | 4  | 5 | 6 | 7 | 8 | 9 | 10 |
| $F \rightarrow$ | 1  | 0  | 0  | 0  | 2  | 1 | 0 | 1 | 2 | 1 | 0  |

$\text{index} = x - \text{min}$

$\text{o/p} \rightarrow -5 \ -1 \ -1 \ 0 \ 2 \ 3 \ 3 \ 4$

$$= x - (-5) = \underline{x + 5}$$

// find minimum of  $A[i] \rightarrow m$

for  $i \rightarrow 0$  to  $(N-1)$  {

$F[A[i] - m]++$

}

for  $i \rightarrow -5$  to  $5$  { // Range

$id = i - m$

for  $j \rightarrow 1$  to  $F[id]$  { // # times an element is present

print ( $i$ )

// for ( $i = 1$ ;  $i \leq F[id]$ ;  $i++$ )

}

}

$TC = \underline{O(N)}$

$SC = \underline{O(|A[i]|)}$

Q  $\rightarrow$  Merge two sorted arrays into a single sorted array.

$A = [ \overset{0}{2} \ \overset{1}{4} \ \overset{2}{5} \ \overset{3}{10} ]$

$B = [ \overset{0}{1} \ \overset{1}{3} \ \overset{2}{5} \ \overset{3}{9} ]$

$\text{o/p} \rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 5 \ 9 \ 10$

$A = [2 \ 4 \ 5 \ 10]$   
~~1~~ ~~2~~ ~~3~~ ~~4~~

$B = [1 \ 3 \ 5 \ 9]$   
~~1~~ ~~2~~ ~~3~~ ~~4~~

$C = [1 \ 2 \ 3 \ 4 \ 5 \ 5 \ 9 \ 10]$

int[] merge (A[], B[]) {

$N = A.length$        $M = B.length$

// C[N+M] → Merged array

$i = 0$        $j = 0$        $k = 0$

while (i < N && j < M) {

if (A[i] <= B[j]) {

$C[k] = A[i]$

$i++$        $k++$

} else {

// select from right half

$C[k] = B[j]$

//  $int += N - i$

$j++$        $k++$

}

}

while (i < N) {

$C[k] = A[i]$

$i++$        $k++$

}

while (j < M) {

$C[k] = B[j]$

$j++$        $k++$

}

return C

}

A[l — mid]

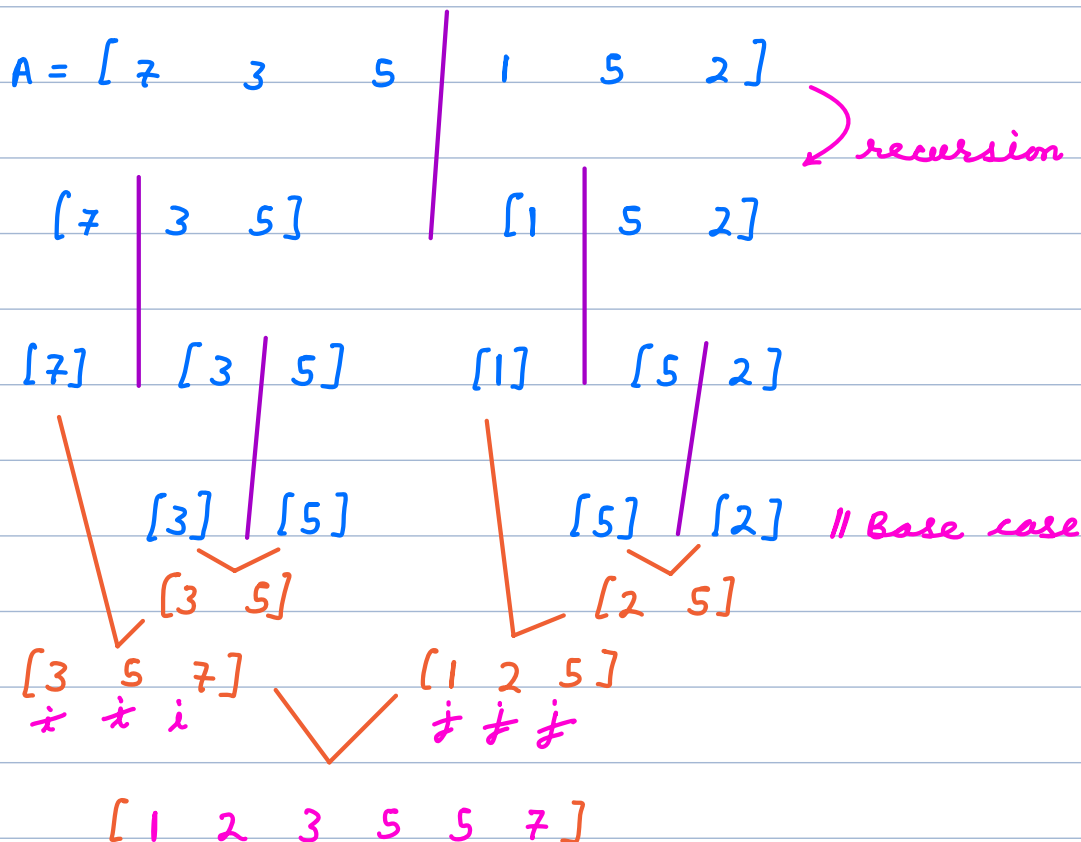
A[mid+1 — r]

↗ ↖  
 A, B → i/p  
 C → o/p

TC =  $O(N+M)$

SC =  $O(1)$

## Merge Sort (Divide & Conquer)



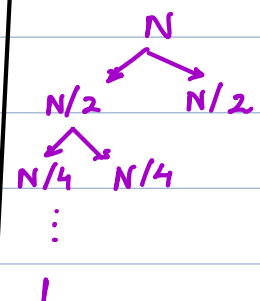
```

void sort (A[], l, r) {
    if (l >= r) return
    mid = (l+r)/2
    sort (A, l, mid)
    sort (A, mid+1, r)
    merge (A, l, mid, r) // l — mid (mid+1) — r
}
    
```

$TC = O(N) \quad SC = O(N)$

$TC = O(N * \# \text{ levels}) \Rightarrow \text{divide from mid}$   
 $\# \text{ levels} = \underline{\log(N)}$

$TC = O(N * \log(N))$   
 $SC = O(N + \log(N)) = O(N)$   
 merge recursion



Q → Given two sorted arrays  $A[]$  &  $B[]$ .

Find # pairs  $i, j$  s.t.  $A[i] > B[j]$ .

$A = [3 \quad 5 \quad 7]$   
 $B = [0 \quad 2 \quad 6]$

Ans = 7

| $i$ | $j$ |
|-----|-----|
| 0   | 0   |
| 0   | 1   |
| 1   | 0   |
| 1   | 1   |

| $i$ | $j$ |
|-----|-----|
| 2   | 0   |
| 2   | 1   |
| 2   | 2   |

Bruteforce →  $\forall i, j$  check

$A[i] > B[j]$ .  $TC = O(N * M)$

$A = [1 \quad 3 \quad 5 \quad 7]$   
 $\checkmark \quad \checkmark \quad \checkmark$   
 $\neq \quad \neq \quad \neq \quad i$

$B = [0 \quad 2 \quad 6]$   
 $\checkmark \quad \checkmark \quad \checkmark$   
 $\neq \quad \neq \quad \neq$

cnt =  $0 + 4 + 3 + 1$   
= 8

when selecting  $B[j]$

Ans += # remaining  
elements in  $A[]$   
=  $A.length - i$

$i = 0 \quad j = 0$

while ( $i < N \ \&\& \ j < M$ ) {

if ( $A[i] \leq B[j]$ )  $i++$

else {  $j++$

cnt +=  $N - i$

}

} return cnt

$TC = O(N + M)$

$SC = O(1)$

Q → Given an integer array  $A$ , find # pairs  
s.t.  $i < j$  &  $A[i] > A[j]$ .

$A = [10^0 \ 3^1 \ 8^2 \ 15^3 \ 6^4]$

Ans = 5

| i | j |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 0 | 4 |
| 2 | 4 |
| 3 | 4 |

$A = [5^0 \ 2^1 \ 6^2 \ 1^3]$

Ans = 4

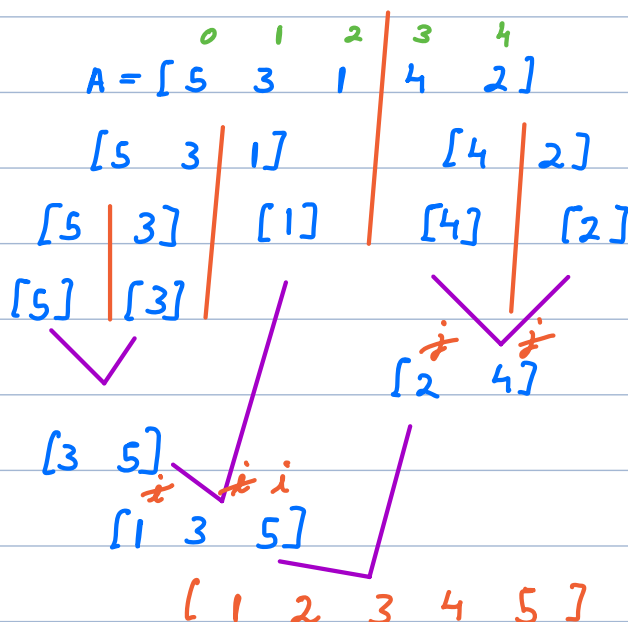
| i | j |
|---|---|
| 0 | 1 |
| 0 | 3 |
| 1 | 3 |
| 2 | 3 |

$A = [5^0 \ 3^1 \ 1^2 \ 4^3 \ 2^4]$

Ans = 7

Brute force  $\rightarrow \forall i, j$  s.t.  $i < j$  check  $A[i] > A[j]$ .

TC =  $O(N^2)$  SC =  $O(1)$



cnt += # of remaining elements in left half when selecting element from right half.

cnt =  $0 + 1 + 2 + 1 + 2 + 1 = \underline{7}$

TC =  $O(N \log(N))$

SC =  $O(N)$

Stable Sorting  $\rightarrow$  Relative order of equal elements should not change while sorting.

For equal elements  $\rightarrow$  use index to compare.

---