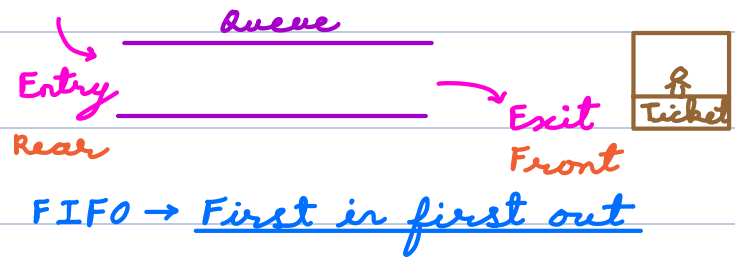
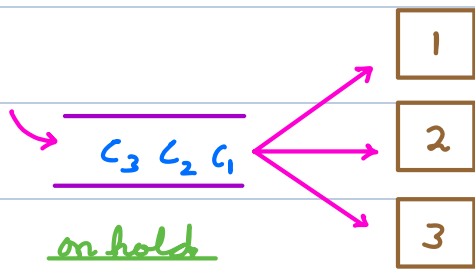






Queue



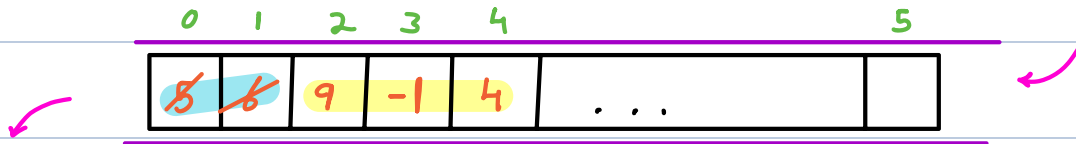
Operations in Queues

1. Enqueue(x) → Insert x at rear end.
 2. Dequeue() → Remove element from front end.
 3. front() / rear() → Get element of front/rear end.
 4. isEmpty() → Check if the queue is empty.
- } TC = O(1)



Array Implementation of Queues

Example: \checkmark Eq(5), \checkmark Eq(6), \checkmark Eq(9), \checkmark Eq(-1), \checkmark Dq(), \checkmark Dq(), \checkmark front, \checkmark Eq(4)



$$f = \cancel{0} + 2$$

$$\text{Queue} \rightarrow [f \text{ --- } r]$$

$$r = \cancel{-1} + 2 = 4$$

```
void enqueue(x) {
```

```
    r++
```

```
    A[r] = x
```

```
}
```

overflow \rightarrow \nrightarrow limit insertion.

\Rightarrow Use dynamic array. \checkmark

```
int dequeue() {
```

```
    if (isEmpty()) return -1
```

```
    f++
```

```
    return A[f-1]
```

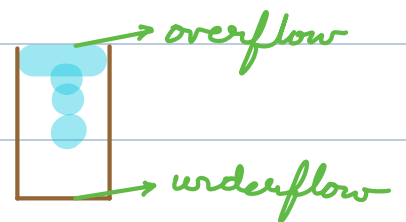
```
}
```

```
bool isEmpty() {
```

```
    len = r - f + 1
```

```
    return (len == 0) // f > r
```

```
}
```

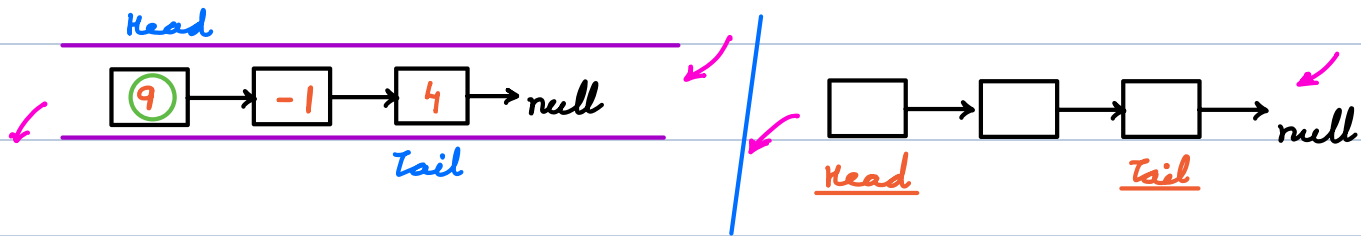


\forall operations, $TC = O(1)$



Implementation of Queue using LinkedList

Example : \checkmark Eq(5) , \checkmark Eq(6) , \checkmark Eq(9) , \checkmark Eq(-1) , \checkmark Dq() , \checkmark Dq() , \checkmark front , \checkmark Eq(4)



$Tail.next = new Node(x)$

$Head = null$

$Tail = Tail.next$

$Tail = null$

1) Enqueue(x) \rightarrow Insert at tail

2) Dequeue() \rightarrow Remove head.

underflow \checkmark

3) front() \rightarrow Head.data

4) rear() \rightarrow Tail.data

5) isEmpty() \rightarrow Head == null

TC = $O(1)$



What will be the state of the queue after these operations enqueue(3), enqueue(7), enqueue(12), dequeue(), dequeue(), enqueue(8), enqueue(3)

~~3~~ ~~7~~ 12 8 3

What will be the state of the queue after these operations enqueue(4), dequeue(), enqueue(9), enqueue(3), enqueue(7), enqueue(11), enqueue(20), dequeue()

~~4~~ ~~9~~ 3 7 11 20



Implementation of Queue using Stack² - [Directi]

behind the scene only these functions can be used

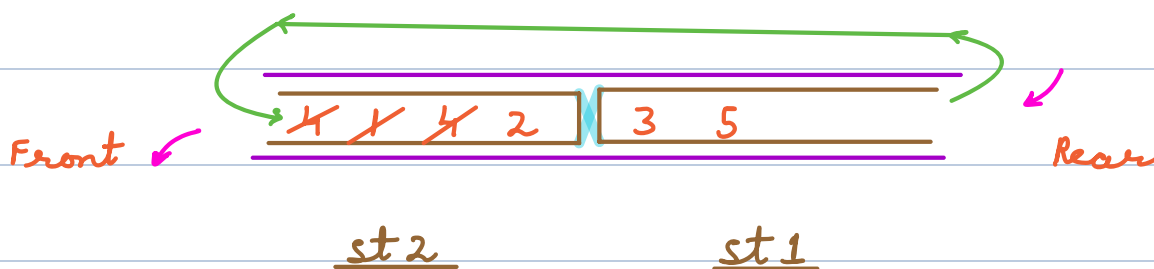
→ Dequeue

→ Enqueue

→ front()/rear()

→ isEmpty()

→ push
→ pop
→ top/peek
→ isEmpty
→ size



```
void enqueue(x) {
    st1.push(x)
}
```

```
boolean isEmpty() {
    return st1.isEmpty()
    && st2.isEmpty()
}
```

```
void move() {
    while (!st1.isEmpty()) {
        st2.push(st1.pop())
    }
}
```

TC = O(K)



```
int dequeue () {
```

```
    if (isEmpty ()) return -1
```

```
    if (st2.isEmpty ()) {
```

```
        move () // st1 → st2 K elements
```

```
    }
```

⇒ for next K dequeue

```
    return st2.pop ()
```

operations → TC = O(1)

```
}
```

⇒ 2 steps per element.

$O(2) \rightarrow \underline{O(1)}$

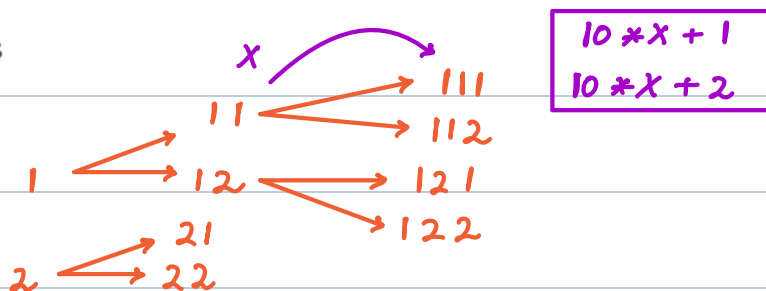
Perfect Numbers

Q → Find N^{th} number formed using digits 1 & 2.

N → 1 2 3 4 5 6 7 8

1 2 11 12 21 22 111 112

	↓	↓
0	10	20
①	11	21
②	12	22
⋮	⋮	⋮
9	19	29



x is generated before y

$\Rightarrow x$ will generate next numbers before y .

FIFO

$N = 5$

$1 \ 2 \ 11 \ 12$

$i = 3$ $x = 12$ $a = 21$ $b = 22$

if $(N \leq 2)$ return N

$q.enqueue(1)$ ✓ $q.enqueue(2)$ ✓

$i = 3$

while $(i \leq N)$ {

$x = q.dequeue()$

$a = x * 10 + 1$ ✓ $b = x * 10 + 2$ ✓

if $(i == N)$ return a ←

if $(i + 1 == N)$ return b

$q.enqueue(a)$ $q.enqueue(b)$

$i += 2$

}

$TC = O(N)$

$SC = O(N)$



Doubly Ended Queue



T.C $\rightarrow O(1)$

1. **Insertion at Front (push_front):** Add an element to the front (head) of the deque. ✓
2. **Insertion at Back (push_back):** Add an element to the back (tail) of the deque. ✓
3. **Removal from Front (pop_front):** Remove and return the element from the front of the deque. ✓
4. **Removal from Back (pop_back):** Remove and return the element from the back of the deque. ✓
5. **Front Element Access (front):** Get the element at the front of the deque without removing it. ✓
6. **Back Element Access (back):** Get the element at the back of the deque without removing it. ✓

Stack + Queue \rightarrow Doubly Ended Queue

Sliding Window Maximum

< Question > : Find max of every subarray of size K.

[3 , 2 , 3 , 4 , 5 , 5 , 4 , 5 , 6]

0 1 2 3 4 5 6 7 8

K = 4

$1 \leq N \leq 10^5$



[0 - 3] → 4

[1 - 4] → 5

[2 - 5] → 5

[3 - 6] → 5

[3, 2, 3, 4, 5, 5, 4, 5, 6]

[4 - 7] → 5

[5 - 8] → 6

Bruteforce → $TC = O(N^3) \rightarrow \underline{O(N^2)}$
 $SC = \underline{O(1)}$

H.W → Sliding Window + Deque.



3	2	9	4	-1	16	1	7	-2	5	-5
---	---	---	---	----	----	---	---	----	---	----

K = 4

0 1 2 3 4 5 6 7 8 9 10

