$A \% B \rightarrow$ Remainder when A / B.

$10 \% 3 = 1$ $\qquad 10 = \underset{q}{3} * 3 + \underset{r}{1}$

$\boxed{D = q * d + r}$

dividend = q * divisor + r $\qquad$ int $\rightarrow$ $\underline{2 * 10^9}$ $\qquad$ long $\rightarrow$ $\underline{9 * 10^{18}}$

$6 = 1 * 3 + 3$ ✗

$\;\;= 2 * 3 + 0$

$\boxed{0 <= A \% B <= B - 1}$

Properties

1) $(\underbrace{a+b}_{①}) \underset{②}{\%} m = (a \% m + b \% m) \% m$

$a = 9$ $\qquad$ (we can store <= 10)

$b = 8$ $\qquad (9 + 8) \% 5 = 17 \% 5 \longrightarrow \underline{2}$

$m = 5$ $\qquad 9 \% 5 = 4$

$8 \% 5 = 3 \qquad (4 + 3) \% 5 = 7 \% 5 = \underline{2}$

2) $(a * b) \% m = (a \% m) * (b \% m) \% m$

3) $(a - b) \% m = (\underset{(0}{\underline{a \% m}} - \underset{m-1}{\underline{b \% m}} + m) \% m$

$+ m) > 0$

$a = 7 \qquad b = 10 \qquad m = 5$

$(a - b) \% m = (7 - 10) \% 5 = \underline{-3 \% 5}$ $\xrightarrow{\text{Python}}$ $\underline{2}$ ✓

$a \% m = 7 \% 5 = 2 \qquad -3 + 5 = \underline{2}$ $\xrightarrow{\text{Java/C++}}$ $\underline{-3}$

$b \% m = 10 \% 5 = 0$

$(2 - 0) \% 5 = \underline{2}$ ✓

4) $(a^b) \% m = (a \% m)^b \% m$

$x^n \rightarrow x * x^{n/2} * x^{n/2}$ $\qquad (n \rightarrow \text{odd})$

$= x * (x^2)^{n/2}$

$6^5 \% 8 \rightarrow 6 * (36)^2 \% 8$

$= 6 * (36 \% 8)^2 \% 8 = 6 * (4^2) \% 8 = 6 * 16 \% 8$

$= (6 * 0) \% 8 = \underline{0}$

Q → $(37^{103} - 1) \% 12 = ((37^{103}) \% 12 - (1 \% 12) + 12) \% 12$

$= ((37 \% 12)^{103} - 1 + 12) \% 12$

$= (1 - 1 + 12) \% 12 = \underline{0}$ (Ans)

$$\boxed{\begin{array}{l} \text{if } (A < B) \\ \quad A \% B = A \end{array}}$$

Q → $(25 + 13) \% 7 = ((25 \% 7) + (13 \% 7)) \% 7$

$= (4 + 6) \% 7 = 10 \% 7 = \underline{3}$

Q → Given an integer array, find count of pairs (i, j) s.t $\boxed{(A[i] + A[j]) \% M = 0}$ & $\boxed{i < j}$

$$\begin{array}{lcccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ A = [& 4 & 3 & 6 & 3 & 8 & 12] \end{array} \qquad M = 6$$

i	j	
0	4	$(4 + 8) \% 6 = 0$
1	3	$(3 + 3) \% 6 = 0$
2	5	$(6 + 12) \% 6 = 0$

Ans = $\underline{3}$

Bruteforce → TC = $\underline{O(N^2)}$ SC = $\underline{O(1)}$

Solution → $(A[i] + A[j]) \% M = 0$ $[0 \quad M-1]$

$$= \underbrace{\left(\overbrace{(A[i] \% M)}^{M-1} + \overbrace{(A[j] \% M)}^{M-1}\right)}_{\to \{0,\ M\}} \% M = 0$$

$M = 6$

	0	1	2	3	4	5	6	7	8	9	10
A = [	2	3	4	8	6	15	5	12	17	7	18]
A[i] % M →	2	3	4	2	0	3	5	0	5	1	0

x	freq(x)
0 →	3
1 →	1
2 →	2
3 →	2
4 →	1
5 →	2

$0 \to 3 \longrightarrow \text{sum} = 0 \Rightarrow {}^3C_2 = \dfrac{3*(3-1)}{2} = \underline{3}$

$3 \to 2 \to {}^2C_2 = \underline{1}$

$2*1 = \underline{2}$

$f(x) * f(M-x)$

$1*2 = \underline{2}$

$\text{Ans} = 3 + 2 + 2 + 1 = \underline{8}$

$M = 6$

	0	1	2	3	4	5	6	7	8	9	10
A = [	2	3	4	8	6	15	5	12	17	7	18]
A[i] % M →	2	3	4	2	0	3	5	0	5	1	0
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓ 6-1=5	

$\underline{0}$

0 →	2
1 →	1
2 →	2
3 →	2
4 →	1
5 →	2

$\forall x$, find freq$(M-x)$

$(A[i] \% M)$

$\text{ans} = 1 + 1 + 1 + 1 + 2 + 2$

$= \underline{8}$

$\forall i,\ freq[i] = 0$

$ans = 0$

```
for i → 0 to (N-1) {
    x = A[i] % M
    y = (M - x) % M      // (M-0) % M = 0
    ans += freq(y)
    freq[x]++   ↳ [0  M-1]
}
return ans
```

$TC = \underline{O(N)}$ $\quad SC = \underline{O(M)}$

<u>GCD</u> (Greatest Common Divisor)

$gcd(x, y) = d \Rightarrow x \% d = 0 \ \& \ y \% d = 0$

$gcd(15, 25) = \underline{5}$ $\qquad gcd(12, 30) = \underline{6}$

$\{1, 2, 3, 6\} \rightarrow$ common factors

$gcd(0, 4) = \underline{4}$

0	4
1	1
2	2
3	4
4	
⋮	

$\boxed{gcd(0, x) = x}$

$gcd(0, 0) = \underline{\infty}$ $\qquad gcd(4, 7) = \underline{1}$

<u>Properties</u>

1) $gcd(0, x) = x$ ✓
2) $gcd(x, y) = gcd(y, x)$ ✓
3) $gcd(x, y, z) = gcd(x, gcd(y, z))$ ✓

$= gcd(y, gcd(x, z)) = gcd(z, gcd(x, y))$

4) $gcd(x, y) = gcd(x-y, y)$ ✓

$(x > y)$

5) $gcd(x, y) = gcd(x-y, y)$

$= gcd(x-y-y, y)$

$= gcd(x-y \ldots, y)$

$= gcd(x \% y, y)$ ✓

$gcd(100, 15) = gcd(100 \% 15, 15) = gcd(10, 15)$

gcd	(15,	21,	33,	45)
	↓	↓	↓	↓
	1	1	1	1
	3	3	3	3 ← Ans = 3
	5	7	11	5
	15	21	33	9
				15
				45

Find $gcd(x, y)$

```
g = 1
for i → 2 to min(x, y) {
    if ( x % i == 0 && y % i == 0)
        g = i
}
return g
```

$TC = O(\min(x, y))$

$gcd(100, 15) = gcd(100 \% 15, 15) = gcd(10, 15)$

$= gcd(15, 10) = gcd(15 \% 10, 10) = gcd(5, 10)$

$= gcd(10, 5) = gcd(10 \% 5, 5) = gcd(0, 5) = \underline{5}$

$gcd(x, y) = gcd(x \% y, y)$
$= gcd(y, x \% y)$

$gcd(153, 971) = gcd(971, 153 \% 971 = 153)$ // $x < y \Rightarrow$ swap
$= gcd(153, 971 \% 153 = 53)$
$= gcd(53, 153 \% 53 = 47)$
$= gcd(47, 53 \% 47 = 6)$
$= gcd(6, 47 \% 6 = 5)$
$= gcd(5, 6 \% 5 = 1) = gcd(1, 5 \% 1 = 0) = \underline{1}$

```
int gcd (x, y) {      // x = 5, y = 12
    if (y == 0) return x
    return gcd (y, x % y)      // 12, 5 % 12 = 5
}
```

$TC = \underline{O(\log(\min(x, y)))}$ ✓

$gcd(1538276, 1538275)$
$= gcd(1538275, 1538276 \% 1538275 = 1)$
$= gcd(1, 1538275 \% 1 = 0) = \underline{1}$

Q → Given an integer array,
find max gcd of the array after deleting an element.

	0	1	2	3	4	
A =	[24	16	18	30	15]	GCD
	X	16	18	30	15	1
	24	X	18	30	15	3 (Ans)
	24	16	X	30	15	1
	24	16	18	X	15	1
	24	16	18	30	X	2

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 \\ A = [\,21 & 7 & \cancel{2} & 14\,] \end{array} \qquad gcd = \underline{7}$$

Bruteforce → ∀ i, iterate & find gcd excluding A[i].

TC = $\underline{O(N * N \log(A[i]))}$

i	A =	[24	16	18	30	15]
		0	1	2	3	4
0		x	16	18	30	15
1		24	x	18	30	15
2		24	16	x	30	15
3		24	16	18	x	15
4		24	16	18	30	x

y

x = gcd (y, 18)

gcd (x, 30)

$$\boxed{\text{gcd excluding } A[i] = \text{gcd}(P[i-1],\ S[i+1])}$$

```
P[0] = A[0]
for i → 1 to (N-1) {
    P[i] = gcd (P[i-1], A[i])
}

S[N-1] = A[N-1]
for i → N-2 to 0 {
    S[i] = gcd ( S[i+1], A[i] )
}

ans = S[1]        // excluding A[0]
for i → 1 to (N-2) {
    ans = max (ans , gcd(P[i-1], S[i+1]))
}
```

ans = max(ans, P[N-2]) //excluding A[N-1]
return ans

$TC = O(N \log(A[i]))$ $SC = O(N)$

To prove $\gcd(x, y) = \gcd(x-y, y)$

Let $\gcd(x, y) = d$ $x \% d = 0$ $y \% d = 0$

$\Rightarrow (x-y) \% d = 0$

$\Rightarrow$ d is factor of x, y, $(x-y)$.

Let $\gcd(x-y, y) = t$ $(x-y) \% t = 0$ $y \% t = 0$

$\Rightarrow (x-y+y) \% t = 0$

$= x \% t = 0$

$\Rightarrow$ t is a factor of x, y, $(x-y)$.

d is common factor of $(x-y)$, y
& t is greatest common factor $\Rightarrow d <= t$

t is common factor of x, y
& d is greatest common factor $\Rightarrow t <= d$

$\boxed{d = t}$

Hence Proved!
