Binary Search - 3

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Notes

Painter's Partition Problem

Given N boards with length of each board

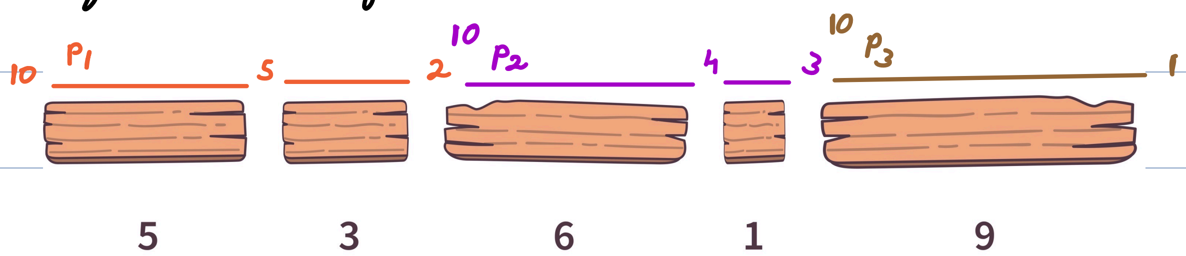
a) A painter takes 1 unit of time to paint 1 unit of length.

b) A board can only be painted by 1 painter.

c) A painter can only paint boards placed next to each other (i.e continuous segment).

A → Find min # painters required to paint all the boards in T unit of time. If not possible return -1.

N = 5



T = 7 Ans = -1

✓ T = 10 Ans = 3 { 5+3 , 6+1 , 9 }

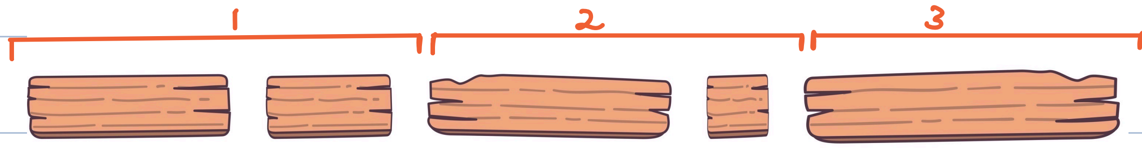
T = 20 Ans = 2 { 5 + 3 + 6 + 1 , 9 }

```
cnt = 1      t = T   → i/p

for i → 0 to (N-1) {
    if ( A[i] > T )  return -1   // INT_MAX
    if ( A[i] <= t )  t -= A[i]
    else {  cnt++
            t = T - A[i]  }
}  return cnt
```

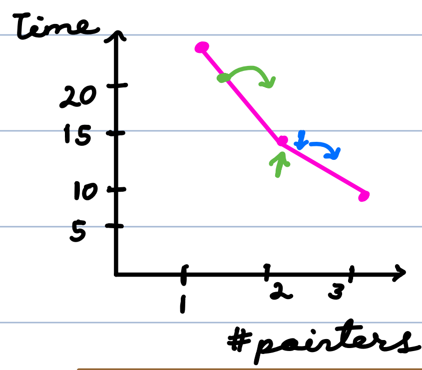
TC = O(N) SC = O(1)

B → Find min time required to paint all boards if K painters are available.



[5 3 6] [1 9]

# Painters	Minimum Time
1	$5+3+6+1+9 = \underline{24}$
2	$\max(5+3+6,\ 1+9) = \underline{14}$
3	$\max(5+3,\ 6+1,\ 9) = \underline{9}$



$\boxed{\text{Time} \propto 1/\#\text{painter}}$ → Binary Search on Answer

$A = [\underbrace{1\quad 2\quad 3\quad 4}\quad \underbrace{100}]$ $K = 2$

$\max(10,\ 100) = \underline{100}$ (Ans)

$l = \max\limits_{\forall i}(A[i])$ $r = \sum\limits_{\forall i}(A[i])$ // sum $\forall i\ (A[i])$

```
while (l <= r) {
    m = l + (r-l)/2
    cnt = countOfPainters(m, A)    // TC = O(N)
    if (cnt <= K && countOfPainters(m-1, A) > K)
        return m
```

	0	1	2	3	4
A =	10	10	10	10	10

K = 4

m = 20 → cnt = 3

m - 1 = 19 → cnt = 5

if (cnt > K) l = m+1
else r = m-1

}

$TC = \underline{O(N \log(\Sigma A[i]))}$

$N <= 10^5$

$A[i] <= 10^9$

$\Sigma A[i] <= 10^9 * 10^5 = \underline{10^{14}}$ → long

Example :

$$\begin{array}{ccccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 3 & 5 & 1 & 7 & 8 & 2 & 5 & 3 & 10 & 1 & 4 & 7 & 5 & 4 & 6 \end{array}$$

[3 5 1 7 8 2 5 3 10 1 4 7 5 4 6], K = 4

$\underbrace{3\ 5\ 1\ 7}_{P1\ 16}\ \underbrace{8\ 2\ 5\ 3}_{P2\ 18}\ \underbrace{10\ 1\ 4}_{P3\ 15}\ \underbrace{7\ 5\ 4}_{P4\ 16}\ \underbrace{6}_{P5}$

l	r	mid	Is it possible to paint all boards by P painters in time = mid?
10	71	40	cnt = 2
10	39	24	cnt = 4 count (m-1) = 4
10	23	16	cnt = 5
17	23	20	cnt = 5
21	23	22 (Ans)	cnt = 4 count (m-1) = 5

Situation:

Imagine you are tasked with developing a system for evenly distributing the workload among a team of email response handlers in a customer service department. Each email is assigned a 'complexity score' which represents the estimated time and effort required to address it. The complexity scores are represented as an array, where each element corresponds to a single email.

Task

The goal is to divide the array into K contiguous blocks (where K is the number of email handlers), such that the maximum sum of the complexity scores in any block is minimized. This approach aims to ensure that no single email handler is overwhelmed with high-complexity emails while others have a lighter load.

8   16   2   45

[3 5 1 7 8 2 5 3 10 1 4 7 5 4 6],   K = 4

16   18   22   15

Sol → Same as above

maximise the minima /

minimise the maxima ⟹ Binary Search

(observation)

Aggressive Cows

< Question > : Farmer has built a bar with N stalls.

A[i] → location of ith stall in sorted order.

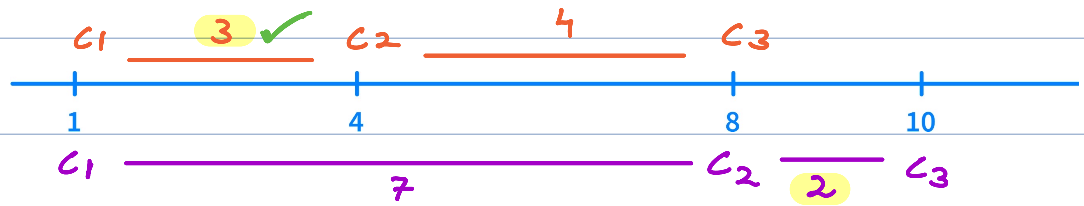
cows

$2 \le \cancel{M}\ K \le N$

Cows are aggressive towards each other. So, farmer wants to maximise the minimum distance between any pair of cows. → | A[i] - A[j] |

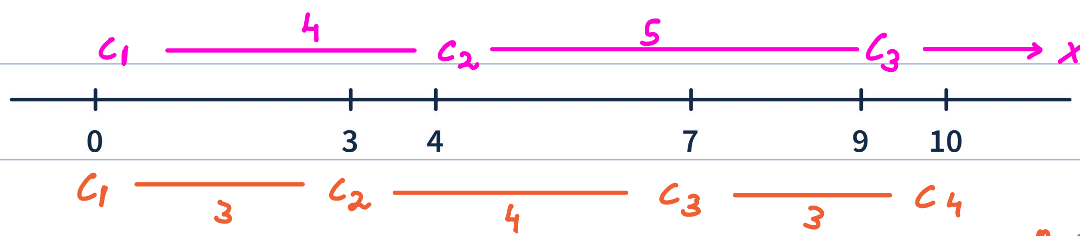
Find max possible minimum distance.

A[ ] → [1 4 8 10] , $\cancel{M}\ K = 3$



Quiz :

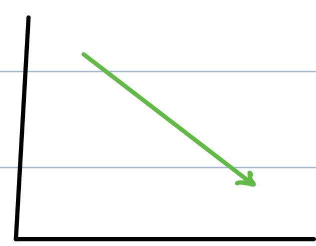
A = [0 3 4 7 9 10]



K = 4

Ans = 3

$\boxed{\#\ cows \propto 1/minDistance}$



```
int maxCows( dist, A[]) {
    cnt = 1        p = A[0]
    for i → 1 to (N-1) {
        if (A[i] - p >= dist) {
            cnt++        p = A[i]
        }
    } return cnt
}
```

$TC = \underline{O(N)}$

```
l = 1                r = A[N-1] - A[0]
while (l <= r) {
    m = l + (r-l) / 2        // distance
    cnt = maxCows(m, A)
    if ( cnt >= K  &&  maxCows(m+1, A) < K)
            return m
    if ( cnt < K )      r = m-1
    else        l = m+1
}
```

$TC = \underline{O(N \log (A[N-1] - A[0]))}$

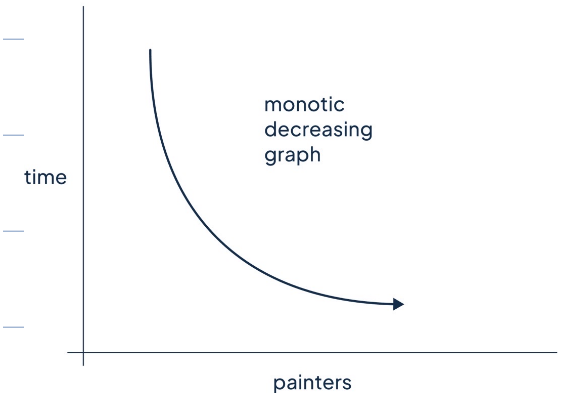
$SC = \underline{O(1)}$

$$\begin{array}{ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ A = [& 1 & 3 & 4 & 7 & 11 & 14] \\ & c_1 & \times & \times & c_2 & c_3 & \times \end{array} \qquad K = 4$$

l	r	mid	Can we place M cows with distance ≥ mid?	
1	13	7	cnt = 2	
1	6	3 (Ans)	cnt = 5 count (m+1) = 3	

- **Common observation for these two problems :**

Painters Partition



Aggressive – Cows