

One Dimensional Array

TABLE OF CONTENTS

1. Max Subarray Sum
2. Prefix Sum
3. Rain Water Trapped



Notes

stock

$A = [-20 \quad 30 \quad 40 \quad -10 \quad 50 \quad -100 \quad 70]$

Max profit in one trade $\rightarrow 30 + 40 - 10 + 50 = \underline{110}$



< Question > : Given arr[N]. Find the maximum subarray sum out of all subarrays.

$$1 \leq N \leq 10^5$$

Example 1 : arr[] \rightarrow [-3 , 2 , 4 , -1 , 3 , -4 , 3]

0 1 2 3 4 5 6

Ans = 8

Example 2 : arr[] \rightarrow [4 , 5 , 2 , 1 , 6]

0 1 2 3 4

Ans = 18

Example 3 : arr[] \rightarrow [-4 , -3 , -6 , -9 , -2]

Ans = -2

$$\begin{aligned} x &> y \\ -x &< -y \end{aligned}$$

Bruteforce \rightarrow ✓ subarray, calculate sum & take max.

$$\frac{N * (N+1)}{2}$$

$$\text{travel} \rightarrow O(N)$$

$$\text{Total TC} = O(N^3)$$

$$\text{SC} = O(1)$$

$$O(1)$$

Prefix Sum ✓

Carry Forward ✓



```

ans = 0 INT_MIN // = A[0]
for i → 0 to (N-1) {
    sum = 0
    for j → i to (N-1) { // i — j
        sum += A[j]
        ans = max(ans, sum)
    }
}
return ans

```

TC = $O(N^2)$ SC = $O(1)$

Observations

Case 1

When all elements are positive

2	4	5	1	3
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$$\text{Ans} = \sum A[i] \quad \forall i$$

Case 2

When all elements are negative

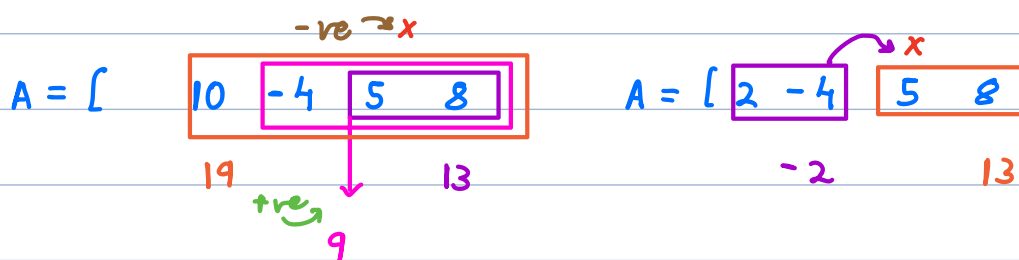
-7	-3	-11	-8	-15
----	----	-----	----	-----

$$\text{Ans} = \max(A[i]) \quad \forall i$$

Case 3

	+ve	
--	-----	--

Kadane's Algo





*Dry-Run

arr[] → [5 , 6 , 7 , -3 , 2 , -10 , -12 , 8 , 12 , -4 , 7 , -2]

0 1 2 3 4 5 6 7 8 9 10 11

sum = ~~0~~ 5 ~~11~~ 18 ~~15~~ 17 ~~7~~ -5 ~~8~~ 20 ~~16~~ 23 21

ans = 5 ~~18~~ ~~20~~ 23

</> Code

```

ans = INT_MIN
sum = 0      l = 0
for i → 0 to (N-1) {
    sum += A[i]
    if (ans < sum) {
        ans = sum
        l = l      R = i
    }
    if (sum < 0) {
        sum = 0      l = i+1
    }
}
return ans

```

Kadane's Algo

Subarray → L — R

0 1 2 i

A = [-2 -1 -5]

sum = 0 -2 0 -1 0 -5 0

ans = -2 -1

TC = O(N) SC = O(1)



Continuous Sum Query

< Question > : There are A beggars sitting in a row outside a temple. Each beggar has an empty pot initially. There are N devotees. Each devotee gives some fixed amount to beggars from indices l to r.

Given a 2-D Array $B[N][3]$, where $B[i][0]$ represents l [left index]

$B[i][1]$ represents r [right index]

$B[i][2]$ represents amount

Given an integer array, where every element is 0 initially. Update the array with multiple queries.

Query (l, r, x) $\rightarrow \forall i \rightarrow l \text{ to } r \quad A[i] += x$

Example : A = 5, $B[N][3] \rightarrow$

l	r	Amount(x)
1	2	10
2	3	20
2	5	25

0	0	0	0	0
1	2	3	4	5

+10 +10
+20 +20
+25 +25 +25 +25

10 55 45 25 25 (Ans)

Query (i, x) \rightarrow
Add x to all the numbers from i to (N-1).

A = [0 0 0 0 0]

+3 +3 +3 +3
+2 +2 +2 +2

+1
2 5 5 5 6 (Ans)

(N-1)
↓
I/p \rightarrow 1 4 3
0 4 2
4 4 1



BF Idea

✓ query, travel from l to r & add x.

length of array $\rightarrow A$

queries $\rightarrow N$

```

for i  $\rightarrow$  0 to (N-1) { // query // For 1-based do -1
    l = B[i][0]      r = B[i][1]      x = B[i][2]
    for j  $\rightarrow$  l to r {
        ans[j] += x
    }
}
return ans

```

TC = $O(N \times A)$ SC = $O(1)$

	0	1	2	3	4	
A =	[0	0	0	0	0]	I/P \rightarrow
		+3	+3	+3	+3	1 4 3
	+2	+2	+2	+2	+2	0 4 2
				+1		4 4 1
	<u>2</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>6</u>	(Ans)

$(N-1)$
 \downarrow
 Add x from i $\rightarrow (N-1)$

A = [0 0 0 0 0]

		+3			
	+2				
			+1		
	<u>2</u>	<u>3</u>	<u>0</u>	<u>0</u>	<u>1</u>
	2	5	5	5	6
	<u>2</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>6</u>
					(Ans)

Prefix Sum

$$P[i] = A[i] + P[i-1]$$



```

for i → 0 to (N-1) { // query
    l = B[i][0]      x = B[i][1]
    ans[l] += x
}

```

```

for i → 1 to (A-1) {
    ans[i] = ans[i-1] + ans[i]
}
    P[i]      P[i-1] + a[i]

```

return ans

TC = $O(N + A)$ SC = $O(1)$

$A = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$
 ⁰ ¹ ² ³ ⁴ ⁵ ⁶
 +2 -2
 -5
 +3 -3
 0 2 3 0 -7 0 -3
 ↪ 0 2 5 5 -2 -2 -5

Queries

l	r	x
1	3	2
4	6	-5
2	5	3

```

for i → 0 to (N-1) { // query

```

```

    l = B[i][0]      r = B[i][1]      x = B[i][2]

```

```

    ans[l] += x

```

```

    if (r != (N-1)) ans[r+1] -= x
}

```

```

for i → 1 to (A-1) {
    ans[i] = ans[i-1] + ans[i]
}
    P[i]      P[i-1] + a[i]

```

return ans

TC = $O(N + A)$ SC = $O(1)$

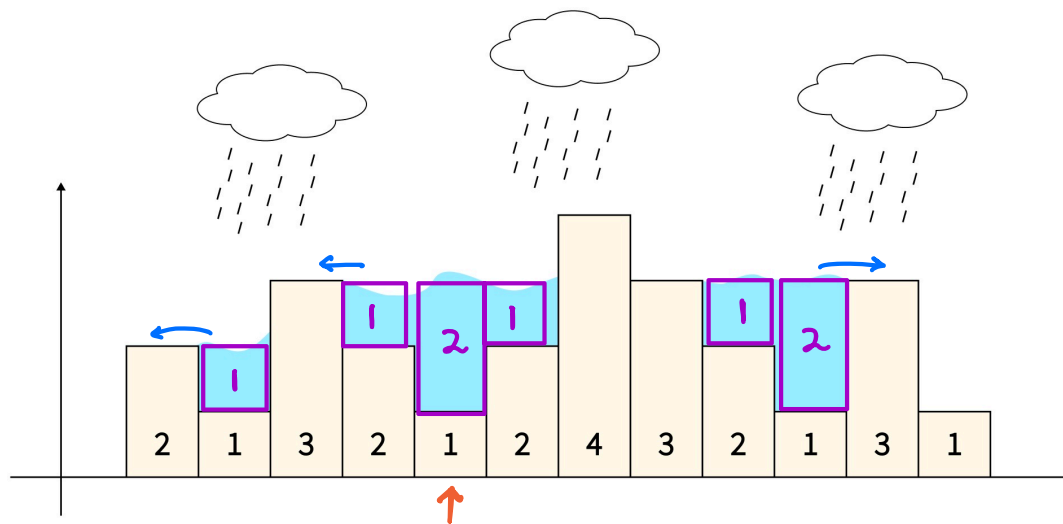


< Question > : Given $arr[N]$, where $arr[i] \rightarrow$ height of building. *Base = 1 \forall buildings*

Return amount of water trapped on all the buildings.

$\text{Area} = \underline{B * H}$

$arr[2 \ 1 \ 3 \ 2 \ 1 \ 2 \ 4 \ 3 \ 2 \ 1 \ 3 \ 1]$ $1 \leq N \leq 10^5$

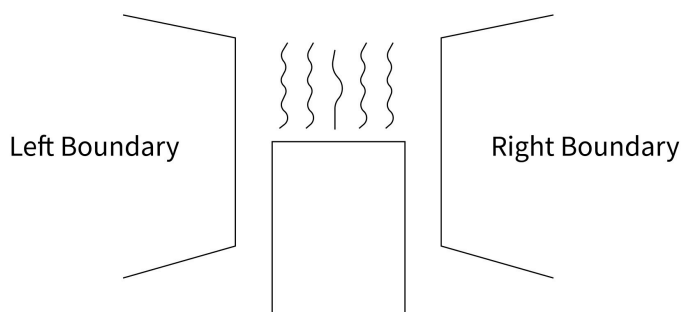


Ans = 8

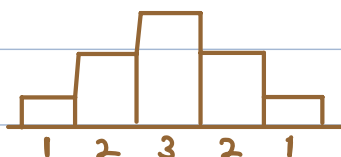


Idea -1

Find the amount of water trapped on every building.



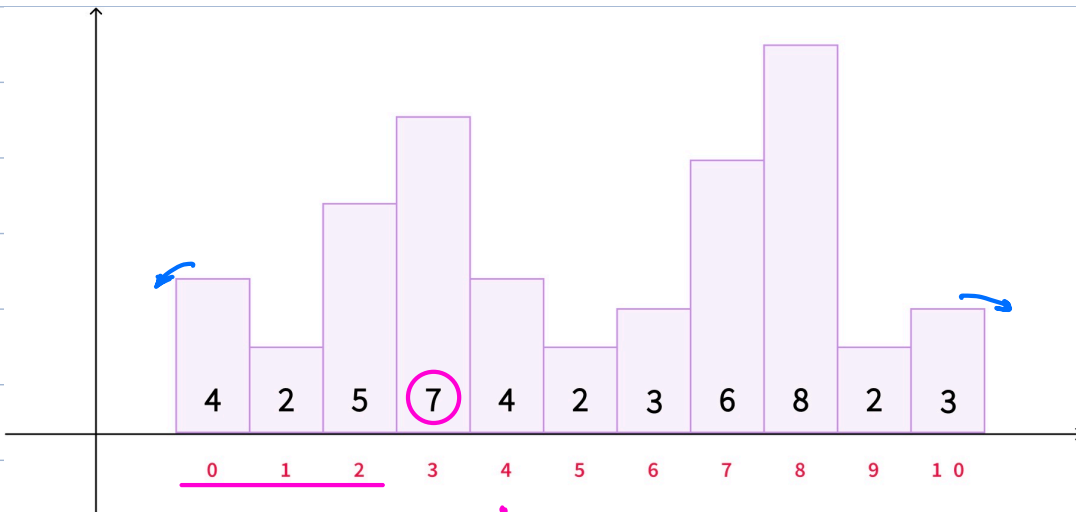
$\min(\max A[i] \text{ on left}, \max A[i] \text{ on right}) - A[i]$



Ans = 0

Bruteforce $\rightarrow TC = O(N^2)$

$SC = O(1)$



→ lmax[] → [4 4 5 7 7 7 7 7 8 8 8]

→ rmax[] → [8 8 8 8 8 8 8 8 8 3 3]

$lmax[0] = A[0]$

for $i \rightarrow 1$ to $(N-1)$ do // →

$lmax[i] = \max(lmax[i-1], A[i])$

}

$rmax[N-1] = A[N-1]$

for $i \rightarrow (N-2)$ to 0 do // ←

$rmax[i] = \max(rmax[i+1], A[i])$

}

Prefix/Suffix → sum, max, min etc

ans = 0

for $i \rightarrow 0$ to $(N-1)$ do

$ans += \min(lmax[i], rmax[i]) - A[i]$

} return ans



$$TC = \underline{O(N)} \quad SC = \underline{O(N)}$$
