

Q → Given an integer array, consider first element as pivot, rearrange the elements s.t

$\forall i, A[i] < p \rightarrow$  move left

$A[i] > p \rightarrow$  move right (distinct elements)



$A = [54, 26, 93, 17, 77, 31, 44, 55, 20]$



$A = [10, 13, 7, 8, 25, 20, 23, 5]$



$A = [54, 26, 93, 17, 77, 31, 44, 55, 20]$

$\begin{matrix} p & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} & \cancel{x} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix}$

$\begin{matrix} 31 & & 20 & & 44 & & 54 & & 77 & & 93 \end{matrix}$

$p = A[0] \quad // L$

$l = 1 \quad // L+1$

$r = N-1 \quad // R$

while ( $l \leq r$ ) {

if ( $A[l] < p$ )  $l++$

else if ( $A[r] > p$ )  $r--$

else { swap( $A, l, r$ )

$l++ \quad r--$

compare via comparator

```

    }
}

```

$$TC = O(N)$$

$$SC = O(1)$$

swap(A, r, 0) // pi = r

## Quick Sort (Divide & Conquer)

A = [54, 26, 93, 17, 77, 31, 44, 55, 20]

[26, 17, 31, 44, 20] [54] [93, 77, 55]

subproblem

right position  
in sorted order

[17, 20] 26 [31, 44] [77, 55] 93

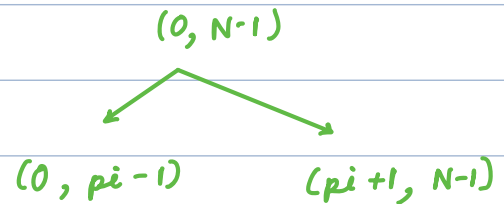
```

void quickSort(A, L, R) {
    if (L >= R) return
    pi = partition(A, L, R)
    quickSort(A, L, pi-1)
    quickSort(A, pi+1, R)
}

```

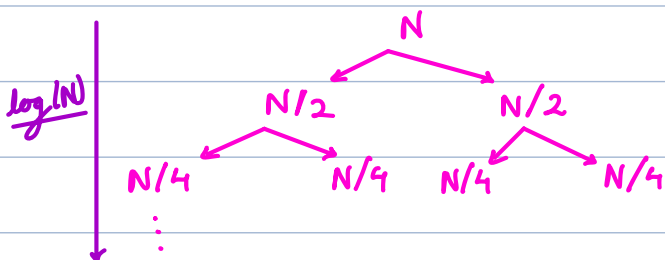
$$TC = O(N)$$

$$SC = O(1)$$



## Time Complexity

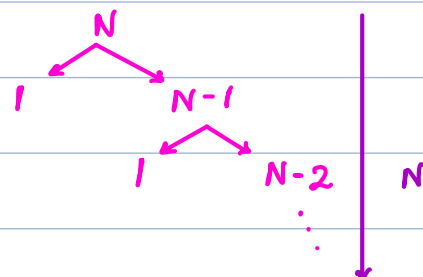
### Best Case



$$TC = O(N \log(N))$$

$$SC = O(\log N)$$

### Worst Case



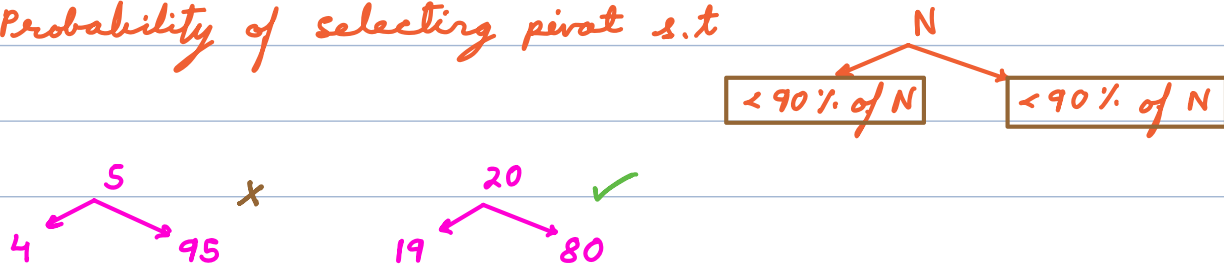
$$TC = O(N^2)$$

$$SC = O(N)$$

## Random Pivot

1 2 3 ... 9 10 11 ... 98 99 100

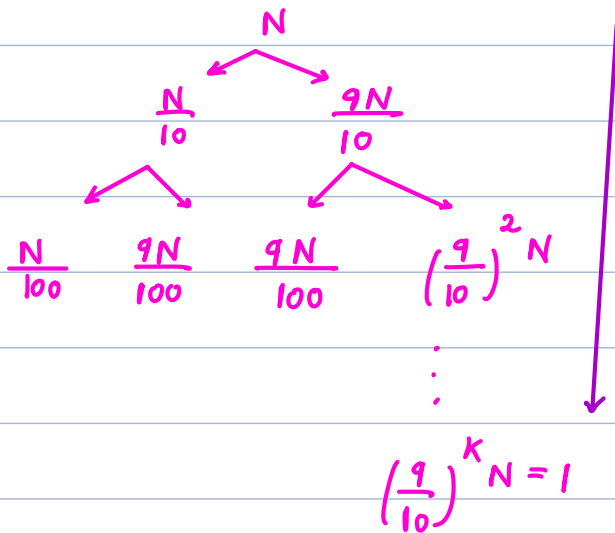
Probability of selecting pivot s.t



valid numbers  $\rightarrow [11 \quad 90] \rightarrow 90 - 11 + 1 = 80$

$\Rightarrow \text{probability} = \frac{80}{100} = 0.8$  (80% cases)

Worst scenario with 80% probability



# levels  $\rightarrow$

$$(9/10)^K N = 1 \Rightarrow N = \left(\frac{10}{9}\right)^K$$

$$\Rightarrow K = \underline{\underline{\log_{10/9}(N)}}$$

$$N = 10^5 \Rightarrow \log_{10/9}(N) \approx \underline{10^2}$$

$$TC = O(N \log_{10/9}(N)) \rightarrow \underline{10^7}$$

$$SC = O(\log_{10/9}(N)) \rightarrow \underline{10^2}$$

(better SC wrt Merge Sort)

## comparators

It defines the parameter wrt which we organise data.

```
int compare (object u, object v) {  
    if 'u' should be on left wrt v → return -ve  
    if 'u' should be on right wrt v → return +ve  
    if 'u' & 'v' are same → return 0  
}
```

Q → Sort the given integer array wrt #factors in ascending order. If count of factors are same use their actual values to compare.

A = [ 9   3   10   6   4 ]

#factors → 3   2   4   4   3

o/p → [ 3   4   9   6   10 ]

A = [ 10   4   5   13   1 ]

#factors → 4   3   2   2   1

o/p → [ 1   5   13   4   10 ]

```
int compare (Integer u, Integer v) {  
    uf = factors(u)      vf = factors(v)  
    if (uf < vf) return -1  
  
    else if (uf > vf) return 1  
    else {
```

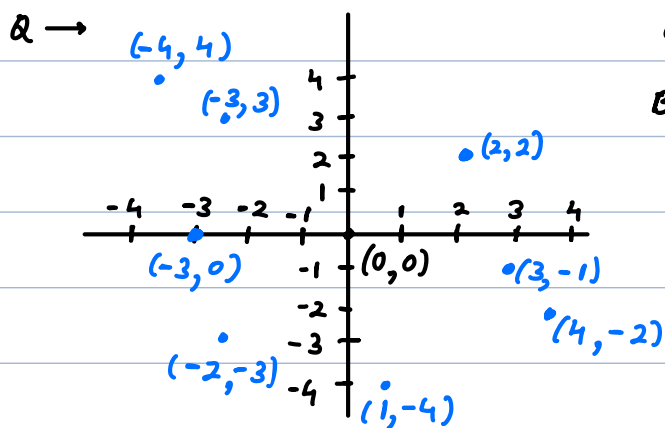
```

    if (u < v) return -1
    else if (u > v) return 1
    else return 0
}
}

```

H.W → check comparator syntax  
in your programming language.

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Given array of points find  
B closest points to origin.

$$\text{Distance from origin} = \sqrt{x^2 + y^2}$$

eg → B = 3

o/p → (2, 2) (-3, 0) (3, 1)

$-4, 4 \rightarrow \sqrt{32}$	$2, 2 \rightarrow \sqrt{8} \checkmark$
$-3, 3 \rightarrow \sqrt{18}$	$3, 1 \rightarrow \sqrt{10} \checkmark$
$-3, 0 \rightarrow \sqrt{9} \checkmark$	$4, -2 \rightarrow \sqrt{20}$
$-2, -3 \rightarrow \sqrt{13}$	$1, -4 \rightarrow \sqrt{17}$

$$\begin{aligned}
 &x < y \\
 \Rightarrow &\sqrt{x} < \sqrt{y}
 \end{aligned}$$

int compare (Point u, Point v) {

$$sdu = u.x * u.x + u.y * u.y$$

$$sdv = v.x * v.x + v.y * v.y$$

return sdu - sdv

}

---

Q → Given an integer array with the numbers

Answer an integer array with the numbers.  
 Arrange the array s.t it forms largest number & return it as a string.

$A = [10 \quad 2 \quad 55 \quad 100]$

↳ "55 2 10 100" (Ans)  
 ↑

(large)

MSD

LSD

$A = [10 \quad 5 \quad 2 \quad 8 \quad 200]$

↳ "8 5 2 200 10" (Ans)  
 ↑

$\downarrow \quad \downarrow$   
 $53 > 402$   
 $53 < 532 \rightarrow$ 

532	53
53	532

 $\times \Rightarrow$  (string comparison)  
 $\uparrow \uparrow \quad \uparrow \uparrow$

$53 < 536 \rightarrow 536 \quad 53$

// int  $\rightarrow$  strings

int compare (string u, string v) {

$x = u + v$  // concatenation

$y = v + u$

    if ( $x < y$ ) return 1

    else if ( $x > y$ ) return -1

    else return 0

}

$u = 53$  (left of 534)  
 $v = 534$   
 $x = u + v = 53534 \checkmark$   
 $y = v + u = 53453$