

Q → Given a rotated sorted array, find index of element K . If not present return -1 . (unique elements)

l m r
 $A = [32 \quad 38 \quad 40 \quad 2 \quad 5 \quad 10 \quad 25 \quad 26]$

$K = 5$

Ans = 4

Linear Search → $TC = O(N)$

Search space → $l = 0$ $r = N - 1$

check if mid is ans → if $(A[m] == K)$ return m

decide left / right →

P_1 P_2
 $A = [32 \quad 38 \quad 40 \quad 2 \quad 5 \quad 10 \quad 25 \quad 26]$ $K = 5$

l m r
 $A = [10 \quad 12 \quad 14 \quad 15 \quad 22 \quad 28 \quad 1 \quad 3]$ $K = 3$

check if x is in part 1/2 →

if $(x \geq A[0]) \Rightarrow$ Part 1

else \Rightarrow Part 2

$l = 0$ $r = N - 1$

while $(l \leq r)$ {

$m = (l + r) / 2$

 if $(A[m] == K)$ return m

 if $(K \geq A[0])$ { // $K \rightarrow$ Part 1

 if $(A[m] \geq A[0])$ { // $A[m] \rightarrow$ Part 1

 if $(A[m] < K)$ $l = m + 1$

l	r	m
0	7	3
4	7	5
6	7	6
7	7	7

```

    else r = m - 1
  } else {
    // A[m] → Part 2
    r = m - 1
  }
} else {
  // K → Part 2
  if (A[m] < A[0]) { // A[m] → Part 2
    if (A[m] < K) l = m + 1
    else r = m - 1
  } else {
    // A[m] → Part 1
    l = m + 1
  }
}
}
}

TC =  $O(\log(N))$    SC =  $O(1)$ 

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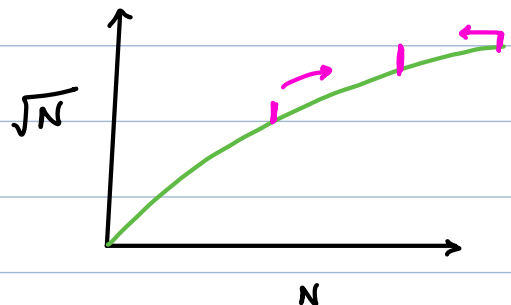
Q → Find $\text{sqrt}(N)$ → only integer part $N \geq 1$

$N = 10$ Ans = 3

$N = 49$ Ans = 7

$$1 \leq \sqrt{N} \leq N$$

$N = 49 \rightarrow 5 < \sqrt{N}$
 $5^2 < N$



$l = 1$ $r = N$

while ($l \leq r$) {

$m = l + (r - l) / 2$

if ($m * m \leq N$ && $(m + 1) * (m + 1) > N$) // $m^2 \leq N < (m + 1)^2$

return m

$$m \leq \sqrt{N} < (m + 1)$$

Binary Search on Answer

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    if (m * m > N)    l = m - 1
    else              l = m + 1
}

```

$$TC = \underline{O(\log(N))} \quad SC = \underline{O(1)}$$

Nth Magical Number

1) $\text{gcd}(x, y)$ ✓ $TC = \underline{O(\log(x))}$

2) $\text{lcm}(x, y) = \frac{x * y}{\text{gcd}(x, y)}$ ✓

3) Count of numbers $\leq K$, divisible by $x \rightarrow \underline{\frac{K}{x}}$

$K=20 \quad x=3 \quad \{3, 6, 9, 12, 15, 18\} \quad \text{Ans} = \underline{6}$

$K=50 \quad x=10 \quad \text{Ans} = \underline{5}$

4) Count of numbers $\leq K$, divisible by x or y or both

$$\boxed{\frac{K}{x} + \frac{K}{y} - \frac{K}{\text{lcm}(x, y)}}$$

$K=20$

$x=3 \quad \{3, 6, 9, 12, 15, 18\} \quad \text{Ans} = 20/3 + 20/5$

$y=5 \quad \{5, 10, 15, 20\} \quad = 6 + 4 = \underline{10} \rightarrow \underline{9}$

$K=60$

$x=6 \quad \{6, 12, 18, 24, 30, 36, 42, 48, 54, 60\}$

$y=5 \quad \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60\}$

$\text{Ans} = \frac{60}{6} + \frac{60}{5} - \textcircled{2} = 10 + 12 - 2 = \underline{20}$

\underline{K}
 $\text{lcm}(x, y)$

Q → Find N^{th} number which is divisible by x or y or both.

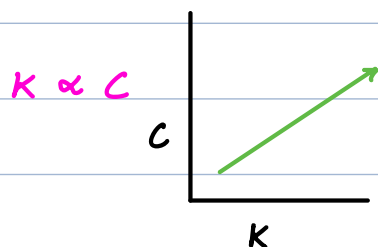
$$N = 5$$

$$x = 6 \quad \left\{ \begin{array}{l} 6 \quad 10 \quad 12 \quad 18 \quad \underline{20} \text{ (Ans)} \end{array} \right.$$

$$y = 10$$

$$(i \% x == 0 \parallel i \% y == 0)$$

Count of numbers $\leq K$, divisible by x or y or both = C



$$N = 5$$

$$x = 6 \quad \{ \textcircled{6} \quad 12 \quad 18 \quad 24 \quad \textcircled{30} \}$$

$$y = 10 \quad \{ 10 \quad 20 \quad 30 \quad 40 \quad 50 \}$$

$$\text{Range} \rightarrow [\min(x, y) \quad N * \min(x, y)]$$

check if T is ans →

$$\text{count}(T) = T/x + T/y - T/\text{lcm}(x, y) = N$$

$$\Rightarrow \text{Ans} = \underline{T} \quad \times$$

$$l = \min(x, y) \quad r = N * l$$

$$\text{lcm} = x * y / \text{gcd}(x, y)$$

while ($l \leq r$) {

$$m = l + (r - l) / 2$$

$$\text{cnt} = m/x + m/y - m/\text{lcm}$$

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    if (cnt == N && (m % x == 0 || m % y == 0))
        return m
    if (cnt < N)    l = m + 1
    else            r = m - 1
}

```

$$TC = O(\log(N * \min(x, y)))$$

$$SC = O(\log(\min(x, y))) \quad (ACD)$$

$$N = 6 \quad lcm = 4 * 6 / 2 = 12$$

$$x = 4 \quad l = 4 \quad 15 \quad 17 \quad 18$$

$$y = 6 \quad r = 24 \quad 18$$

$$m = 14 \quad 19 \quad 16 \quad 17 \quad (18) \checkmark$$

$$cnt = 18/4 + 18/6 - 18/12 = 4 + 3 - 1 = 6$$

Q → Find median of given array.

↘ middle element in sorted order

$$[1 \quad 3 \quad 5] \quad Ans = 3$$

$$[3 \quad 5 \quad 1] \quad Ans = 3$$

even → Average of 2 mid

$$[1 \quad (2) \quad 6 \quad 10]$$

↘ smaller mid

$$[18 \quad 4 \quad 5 \quad 2 \quad 1]$$

$$1 \quad 2 \quad (4) \quad 5 \quad 18$$

(Ans)

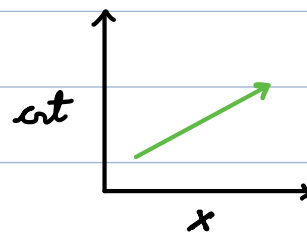
sort & find middle

$$TC = O(N \log(N))$$

Find cnt elements $\leq x$.

$$N/2$$

$$\Rightarrow Ans = x$$



$$half = (N+1)/2$$

$$l = \min(A[i])$$

$$r = \max(A[i])$$

while ($l \leq r$) {

$$m = (l + r) / 2$$

```

    cnt = count(m) // # elements <= m in A[]
    if (cnt == half &&
        count(m-1) < half)
        return m
    if (cnt < half)
        l = m+1
    else r = m-1
}

```

TC = $O(N)$

1 2 5 10 15

half = $(5+1)/2 = \underline{3}$

l = 1 r = 15

m = 8 cnt = 3

m-1 = 7 count(7) = 3

⇒ 8 is not in A[]

A = [2 10 3 5 1 12]

half = $(6+1)/2 = \underline{3}$

count(2) = 2

Ans = 3

l	r	m	cnt
1	12	6	4
1	5	3	3

TC = $O(N \log(A[i]))$ SC = $O(1)$

HW. → Find median of 2 sorted array. ↘

TC < $O(N)$

A = [1 3 5 6]

B = [2 8 10]

1 2 3 5 6 8 10

(Ans)

// # elements <= m in A[]

TC = $O(N)$ → $O(\log(N))$ (sorted array)