Many-Body Localization in Optical Lattice

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Overview

定义

孤立系统:

定义

孤立系统:

与环境无粒子和能量交换。

定义

孤立系统:

与环境无粒子和能量交换。

粒子数与能量守恒。

系综理论

系综理论的基本观点：

宏观量的系统时间平均等于系综平均

$$\bar{u} = < u >_t$$

各态遍历假说

系统的运动方程由哈密顿正则方程给出：

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

系统状态随时间的变化可以用代表点在相空间中的运动轨道来表示。

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上述运动方程表明，从不同初态出发系统在相空间内的运动轨道不相交。

各态遍历假说

玻尔兹曼提出各态遍历假说:

各态遍历假说

对于孤立的力学系统，只要时间足够长，系统从任意初态出发，都将经过能量曲面上的一切微观态。

Thermalization

热力学平衡

- 系统的宏观量不再变换，经历的所有可能微观状态。
- 系统的热力学平衡时的统计性质与初态无关。

Thermalization

系统能够进行各态遍历，平衡时宏观量的统计性质与初态无关。

微正则系综宏观量的平衡统计

各态遍历下，对于微正则系综所有可能微观状态出现的概率相同，状态分布在等能面上

$$\rho_n = \begin{cases} \frac{1}{\Omega}, & E \le E_n \le E + \Delta E \\ 0, & \text{other} \end{cases}$$

由此可以推导宏观量的热力学平衡期望值

$$\bar{u} = \sum_n \rho_n u_n$$

为什么要提出 ETH?

量子系统的态在 Hamiltonian 作用下的含时变化遵循薛定谔方程

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$$

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ETH 的目标就是解释对于幺正变化的量子系统，它们是如何各态遍历，并被平衡统计理论所描述。

从宏观量的平衡统计出发

在能量本征态基矢下，初态 $|\Psi>$ 可以表示为能量本征态的叠加：

$$|\Psi>=\sum_{\alpha} c_{\alpha}|\alpha>$$

在 H 作用下随时间变化：

$$|\Psi(t)>=\sum_{\alpha} c_{\alpha} e^{-iE_{\alpha}t/\hbar}|\alpha>$$

宏观量在 H 作用下随时间变化：

$$A(t)=\sum_{\alpha,\beta} c_{\alpha}c_{\beta}^{*}e^{-i(E_{\alpha}-E_{\beta})t/\hbar}A_{\alpha\beta}$$

$$=\sum_{\alpha}|c_{\alpha}|^{2}A_{\alpha\alpha}+\sum_{\alpha\neq\beta}c_{\alpha}c_{\beta}^{*}e^{-i(E_{\alpha}-E_{\beta})t/\hbar}A_{\alpha\beta}$$

宏观量的时间平均

$$< A >_t = \frac{1}{\tau} \lim_{\tau \to \infty} \int_0^{\tau} A(t) dt$$

$$= \sum_{\alpha} |c_\alpha|^2 A_{\alpha\alpha} + i\hbar \lim_{\tau \to \infty} [\sum_{\alpha \neq \beta} \frac{c_\alpha c_\beta^* A_{\alpha\beta}}{E_\alpha - E_\beta} (\frac{e^{-i(E_\alpha - E_\beta)\tau/\hbar - 1}}{\tau})]$$

平衡统计理论下微正则系综的守恒量的时间平均等于系综平均:

$$< A >_{mc} = \sum_{\alpha} \rho_\alpha A_{\alpha\alpha} = \frac{1}{N} \sum_{\alpha} A_{\alpha\alpha}$$

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如何将上述两个公式等价呢？

ETH 的表述

For an arbitrary initial state, $<\hat{A}>$ will ultimately evolve to its value predicted by a microcanonical ensemble, only small fluctuations around that value, provided that the following two conditions are met:

- The diagonal matrix elements $A_{\alpha\alpha}$ vary smoothly as a function of energy.
- The off-diagonal matrix elements $A_{\alpha\beta}$, with $\alpha \neq \beta$, are much smaller than the diagonal matrix elements.

在两个约束条件下，非对角项非常小，为随时间变化的涨落项。

$$< A >_t = \sum_{\alpha} |C_\alpha|^2 A_{\alpha\alpha} \approx A \sum_{\alpha} |C_\alpha|^2 = A$$

微正则系综的结果:

$$< A >_{mc} \approx \frac{1}{N} * N * A = A$$

So,

$$< A >_t = < A >_{mc}$$

违背 ETH 的系统：

- Intergrate System, 有着自己的 ETH 表述, 可以在一定条件下热化。
- Localized System, 无法热化, non-ergodic, Anderson localization, Many-body localization.
- ...

Introduction

Proposed by P.W.Anderson

- First introduced in 1958（Anderson, P. W. (1958).PhysRev.109.1492）
- suggest the possibility of eletron localization in semiconductor which contains random impurities or defects.
- Consider single particle without interaction but can hop from one site to another.

Quantum Model

Tight-binding model hamiltonian

$$H = -\sum_{i,j} J_{ij} \hat{a}_i^+ \hat{a}_j + \sum_i U_i \hat{a}_i^+ \hat{a}_i$$

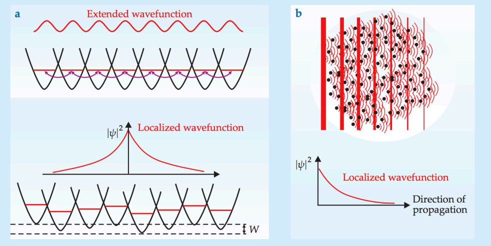
uniform U_i: extended wavefunction(Bloch wave)

random U_i: single particle localized wavefunction

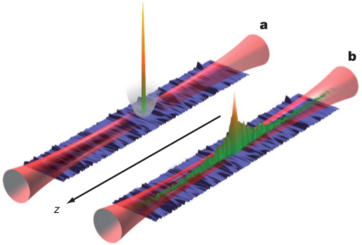
Explanation:matter wave interference

For single particle in lattice,

- uniform potential: matter wave freely propagation through lattice
- random potential: destructive interference between initial wave and wave scattered by impurities

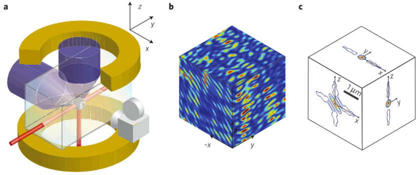


Experiment of Anderson localization in ultracold atoms



Anderson localization in non-intereacting 1d BEC

Juliette Billy et.al.Nature 453, 891-894

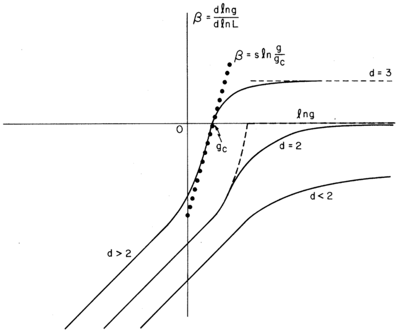


Anderson Localization in 3D ultracold atoms

F. Jendrzejewski et.al.Nature Physics 8, 398–403 (2012)

scaling-law theory of anderson localization

According to scaling-law theory, for models of different dimension system:



g is conductance,

$$\lim_{g\to\infty} \beta(g) = d - 2$$

$$\lim_{g\to 0} \beta(g) = \log(g)$$

So,

- d $\leqslant 2$, system is localized for arbitary weak disorder
- d $\geqslant 3$, system is localized for some critical disorder strength.

Thermalization in anderson localization.

- Localized states fails to thermalize,violate ETH, so some memory of initial conditiion is preserved in local observables at long times.

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- Local observables differ from eigenstate to eigenstate in same energy shell.
- Mott insulator—> ground state localization,but Anderson localization is not limited in ground state.

Many-body localization

Interaction leads eigenstates to be many-body states due to degree coupling,

Localization with interaction, so called ' Many-body localization' .

Bloch group:Localization of interacting fermions in a quasirandom optical lattice

Theoritical Model

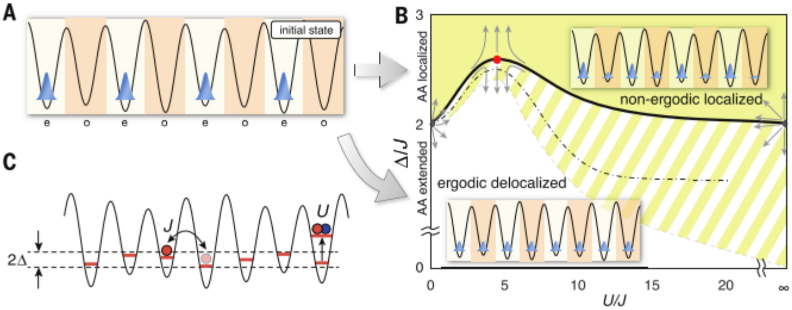
$$\hat{H} = -J \sum_{i,\sigma} (\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{i+1,\sigma} + h.c.) +$$

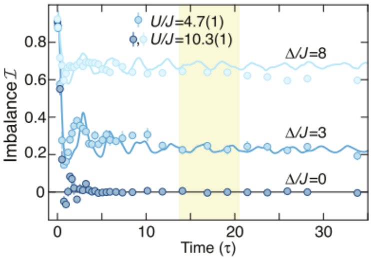
$$\triangle \sum_{i,\sigma} \cos(2\pi\beta i + \phi) \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{i,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

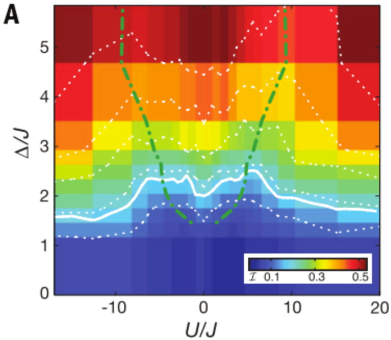
System Setup and Phase diagram

$$\beta = \frac{\lambda_\mathrm{s}}{\lambda_\mathrm{d}} = 0.721, \lambda_\mathrm{d} = 738\mathrm{nm}, \text{Qusirandom!}$$

$$\text{Initial State : CDW, imbanlance } \mathrm{I} = \frac{\mathrm{N_e} - \mathrm{N_o}}{\mathrm{N_e} + \mathrm{N_o}} \approx 1$$







Schreiber, M. et.al(2015). Science, 349(6250), 842–845.

Coupling identical one-dimensional many-body localized system ,$J_\perp$: coupling between tubes

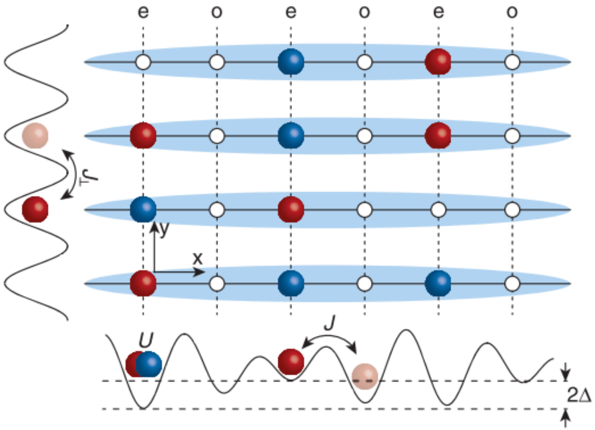
Theoritical Model

$$\hat{H} = -J\sum_{i,j,\sigma}(\hat{c}^\dagger_{i+1,j,\sigma}\hat{c}_{i,j,\sigma} + h.c.) - J_\perp\sum_{i,j,\sigma}(\hat{c}^\dagger_{i,j+1,\sigma}\hat{c}_{i,j,\sigma} + h.c.) +$$

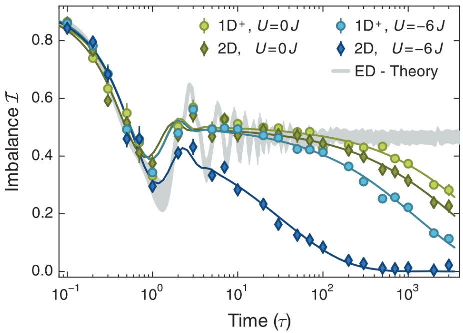
$$\triangle\sum_{i,j,\sigma}\cos(2\pi\beta i + \phi)\hat{n}_{i,j,\sigma} + U\sum_{i,j}\hat{n}_{i,j,\uparrow}\hat{n}_{i,j,\downarrow}$$

Bordia, P. et.al(2016). Physical Review Letters, 116(14), 1–6.

System Setup



Results



$$\Delta = 5\mathrm{J}$$

$$2\mathrm{D} : \mathrm{J}_{\perp} = \mathrm{J}$$

$$1\mathrm{D}^{+} : \mathrm{J}_{\perp} \lesssim 10^{-3}\mathrm{J}$$

non-zero $\mathrm{J}_{\perp}$ makes atom can move between tubes free, with interaction, tubes act as a thermal bath for each other

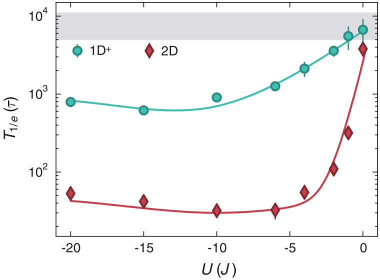
Extract Lifetime

Fitting function:

$$I = (Ae^{-t/t_1}\cos(\nu t) + I_{st})e^{-(\Gamma t)^\beta}$$

Imbanlance lifetime:

$$T_{1/e} = 1/\Gamma$$



Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass – energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End