Many-Body Localization in Optical Lattice

Bo Xiao

University of Science and Technology of China

xbustc@gmail.com

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Overview

1. Quantum Thermalization
   - Microcanonical Ensemble
   - Eigenstate Thermalization Hypothesis (ETH)
   - Anderson localization
   - Many-body localization
2. Experiments
   - one-dimensional quasirandom system
   - 2-dimension many-body localized system

Definition

Isolated System:

Definition

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No Particle and energy exchange with environment

Definition

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No Particle and energy exchange with environment

Conserved energy and particle number

系综理论

系综理论的基本观点:

宏观量的系统时间平均等于系综平均

$$\bar{u} = \langle u \rangle_t$$

各态遍历假说

玻尔兹曼提出各态遍历假说：

各态遍历假说

对于孤立的力学系统，只要时间足够长，系统从任意初态出发，都将经过能量曲面上的一切微观态。

Thermalization

热力学平衡

- 系统的宏观量不再变换，经历所有可能微观状态。
- 系统的热力学平衡时的统计性质与初态无关。

Thermalization

系统能够进行各态遍历，平衡时宏观量的统计性质与初态无关。

微正则系综宏观量的平衡统计

各态遍历下，对于微正则系综所有可能微观状态出现的概率相同，状态分布在等能面上

$$\rho_n = \begin{cases} \frac{1}{\Omega}, & E \le E_n \le E + \Delta E \\ 0, & \text{other} \end{cases}$$

由此可以推导宏观量的热力学平衡期望值

$$\bar{u} = \sum_n \rho_n u_n$$

为什么要提出 ETH?

量子系统的态在 Hamiltonian 作用下的含时变化遵循薛定谔方程

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$$

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但是有一类量子系统可以被平衡统计理论表述，平衡态的宏观量大小与初态无关，有着经典统计理论的各态遍历性质，这个被称为 Quantum ergodic.

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为了解释 Quantum ergodic，人们提出了 Eigenstate Thermalization Hypothesis。

从宏观量的平衡统计出发

在能量本征态基矢下，初态 $|\Psi>$ 可以表示为能量本征态的叠加：

$$|\Psi>=\sum_\alpha c_\alpha|\alpha>$$

在 H 作用下随时间变化：

$$|\Psi(t)>=\sum_\alpha c_\alpha e^{-iE_\alpha t/\hbar}|\alpha>$$

宏观量在 H 作用下随时间变化：

$$A(t)=\sum_{\alpha,\beta} c_\alpha c_\beta^* e^{-i(E_\alpha-E_\beta)t/\hbar}A_{\alpha\beta}$$

$$=\sum_\alpha |c_\alpha|^2 A_{\alpha\alpha}+\sum_{\alpha\neq\beta} c_\alpha c_\beta^* e^{-i(E_\alpha-E_\beta)t/\hbar}A_{\alpha\beta}$$

宏观量的时间平均

$$< A >_t = \frac{1}{\tau} \lim_{\tau \to \infty} \int_0^{\tau} A(t) dt$$

$$= \sum_{\alpha} |c_\alpha|^2 A_{\alpha\alpha} + i\hbar \lim_{\tau \to \infty} [\sum_{\alpha \neq \beta} \frac{c_\alpha c_\beta^* A_{\alpha\beta}}{E_\alpha - E_\beta} (\frac{e^{-i(E_\alpha - E_\beta)\tau/\hbar - 1}}{\tau})]$$

平衡统计理论下微正则系综的守恒量的时间平均等于系综平均:

$$< A >_{mc} = \sum_{\alpha} \rho_\alpha A_{\alpha\alpha} = \frac{1}{N} \sum_{\alpha} A_{\alpha\alpha}$$

宏观量的时间平均

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$$< A >_{mc} = \sum_\alpha \rho_\alpha A_{\alpha\alpha} = \frac{1}{N}\sum_\alpha A_{\alpha\alpha}$$

如何将上述两个公式等价呢?

ETH 的表述

For an arbitrary initial state, $<\hat{A}>$ will ultimately evolve to its value predicted by a microcanonical ensemble, only small fluctuations around that value, provided that the following two conditions are met:

- The diagonal matrix elements $A_{\alpha\alpha}$vary smoothly as a function of energy.
- The off-diagonal matrix elements $A_{\alpha\beta}$, with $\alpha \neq \beta$, are much smaller than the diagonal matrix elements.

在两个约束条件下，非对角项非常小，为随时间变化的涨落项。

$$< A >_t = \sum_\alpha |c_\alpha|^2 A_{\alpha\alpha} \approx A \sum_\alpha |c_\alpha|^2 = A$$

微正则系综的结果：

$$< A >_{mc} \approx \frac{1}{N} * N * A = A$$

So,

$$< A >_t = < A >_{mc}$$

违背 ETH 的系统:

- Intergrate System, 有着自己的 ETH 表述, 可以在一定条件下热化。
- Localized System, 无法热化, non-ergodic, Anderson localization, Many-body localization.
- ...

Introduction

Proposed by P.W.Anderson

- First introduced in 1958 (Anderson, P. W. (1958).PhysRev.109.1492)
- suggest the possibility of eletron localization in semiconductor which contains random impurities or defects.
- Consider single particle without interaction but can hop from one site to another.

Quantum Model

Tight-binding model hamiltonian

$$H = -\sum_{i,j} J_{ij}\hat{a}_i^{+}\hat{a}_j + \sum_i U_i\hat{a}_i^{+}\hat{a}_i$$

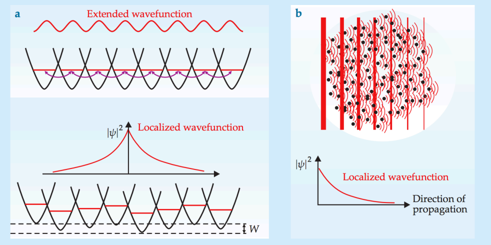
uniform U_i: extended wavefunction(Bloch wave)

random U_i: single particle localized wavefunction

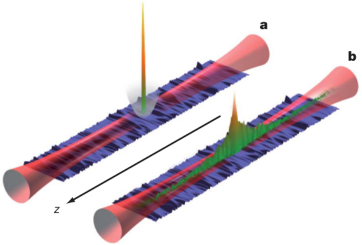
Explanation:matter wave interference

For single particle in lattice,

- uniform potential: matter wave freely propagation through lattice
- random potential: destructive interference between initial wave and wave scattered by impurities

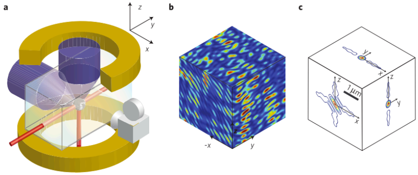


Experiment of Anderson localization in ultracold atoms



Anderson localization in non-intereacting 1d BEC

Juliette Billy et.al.Nature 453, 891-894

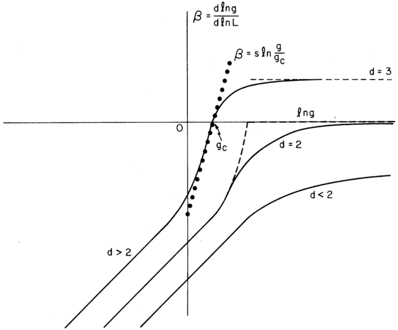


Anderson Localization in 3D ultracold atoms

F. Jendrzejewski et.al.Nature Physics 8, 398–403 (2012)

scaling-law theory of anderson localization

According to scaling-law theory, for models of different dimension system:



g is conductance,

$$\lim_{g\to\infty} \beta(g) = d - 2$$

$$\lim_{g\to 0} \beta(g) = \log(g)$$

So,

- d $\leqslant 2$, system is localized for arbitary weak disorder
- d $\geqslant 3$, system is localized for some critical disorder strength.

Thermalization in anderson localization.

- Localized states fails to thermalize,violate ETH, so some memory of initial condition is preserved in local observables at long times.

Thermalization in anderson localization.

- Localized states fails to thermalize,violate ETH, so some memory of initial condition is preserved in local observables at long times.
- Local observables differ from eigenstate to eigenstate in same energy shell.

Many-body localization

Interaction leads eigenstates to be many-body states due to degree coupling,

Localization with interaction, so called 'Many-body localization'.

Bloch group:Localization of interacting fermions in a quasirandom optical lattice

Theoritical Model

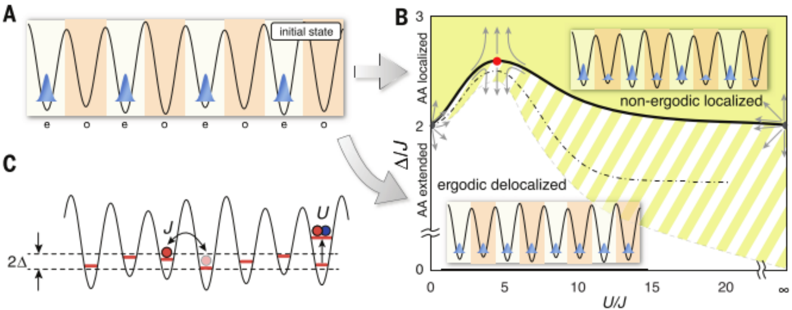
$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.) +$$

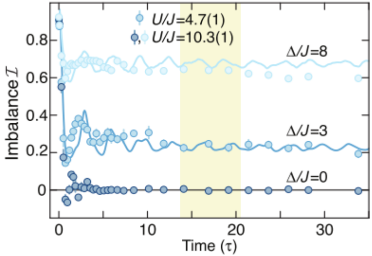
$$\triangle\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

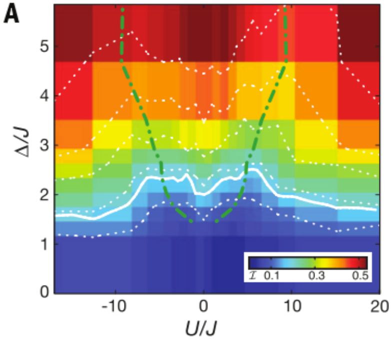
System Setup and Phase diagram

$$\beta = \frac{\lambda_s}{\lambda_d} = 0.721, \lambda_d = 738\text{nm}, \text{Qusirandom!}$$

$$\text{Initial State} : \text{CDW}, \text{imbanlance } I = \frac{N_e - N_o}{N_e + N_o} \approx 1$$







Schreiber, M. et.al(2015). Science, 349(6250), 842–845.

Coupling identical one-dimensional many-body localized system ,$J_\perp$: coupling between tubes

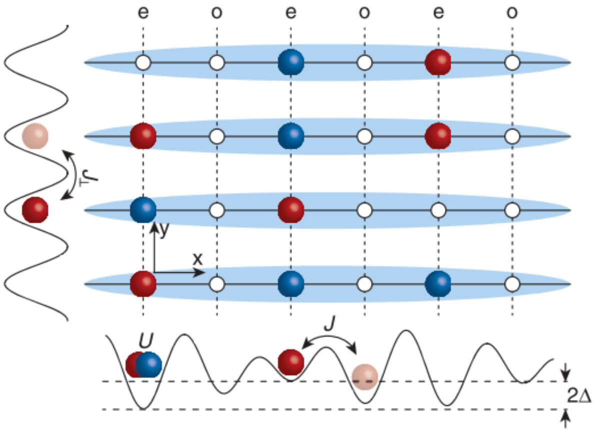
Theoritical Model

$$\hat{H} = -J\sum_{i,j,\sigma}(\hat{c}^\dagger_{i+1,j,\sigma}\hat{c}_{i,j,\sigma}+h.c.)-J_\perp\sum_{i,j,\sigma}(\hat{c}^\dagger_{i,j+1,\sigma}\hat{c}_{i,j,\sigma}+h.c.)+$$

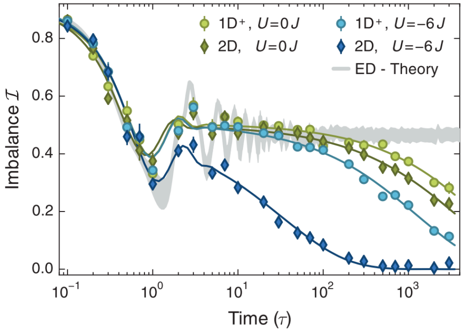
$$\triangle\sum_{i,j,\sigma}\cos(2\pi\beta i+\phi)\hat{n}_{i,j,\sigma}+U\sum_{i,j}\hat{n}_{i,j,\uparrow}\hat{n}_{i,j,\downarrow}$$

Bordia, P. et.al(2016). Physical Review Letters, 116(14), 1–6.

System Setup



Results



$$\Delta = 5\mathrm{J}$$

$$2\mathrm{D} : \mathrm{J}_{\perp} = \mathrm{J}$$

$$1\mathrm{D}^{+} : \mathrm{J}_{\perp} \lesssim 10^{-3}\mathrm{J}$$

non-zero $\mathrm{J}_{\perp}$ makes atom can move between tubes free, with interaction, tubes act as a thermal bath for each other

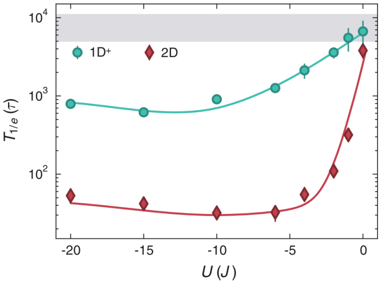
Extract Lifetime

Fitting function:

$$I = (Ae^{-t/t_1}\cos(\nu t) + I_{st})e^{-(\Gamma t)^{\beta}}$$

Imbanlance lifetime:

$$T_{1/e} = 1/\Gamma$$



Periodically Driving a Many-Body Localized Quantum System, ν : modulation frequency

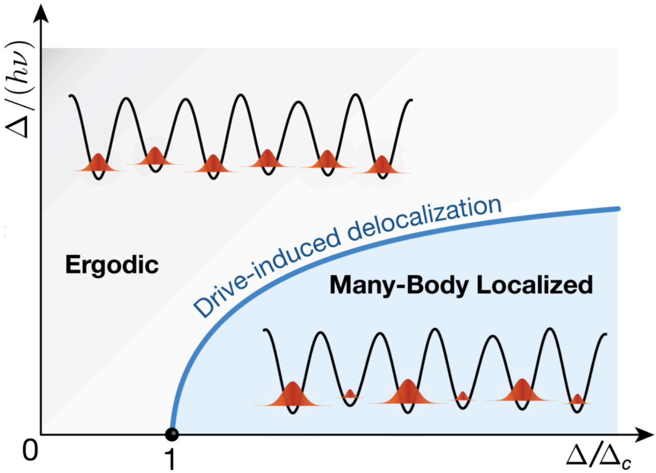
Theoritical Model

$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.) +$$

$$(\triangle + A\sin(2\pi\nu t))\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

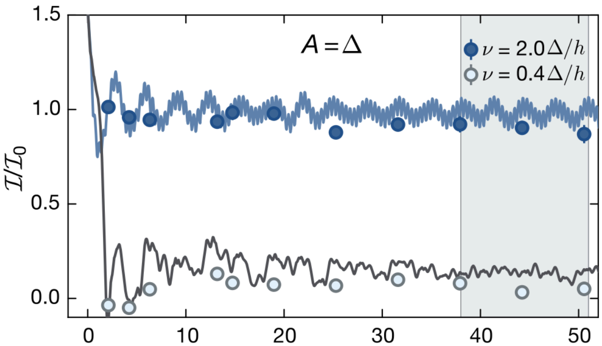
Pranjal, B. et.al(2016). arxiv:1607.07868

Phase Diagram

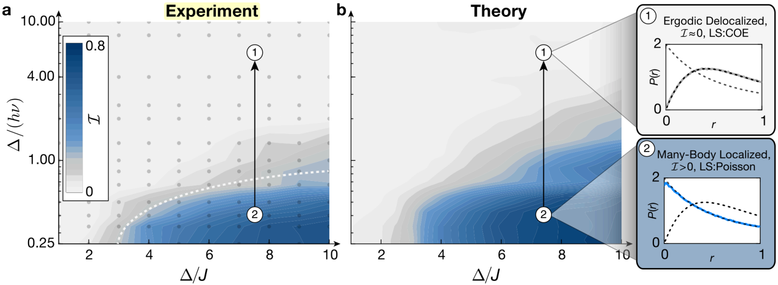


Dynamical Response

- When ν is small, system can response to the time-dependent Hamilatonian.
- When ν is large, the dynamics is effectively governed by the time-averaged non-ergodic Hamilatonian.



Dynamical Phase Diagram



Level Statistic

Define level statistic parameter:

$$\mathrm{r} = \min\left[\frac{\epsilon_{\alpha+1} - \epsilon_\alpha}{\epsilon_\alpha - \epsilon_{\alpha-1}}, \frac{\epsilon_\alpha - \epsilon_{\alpha-1}}{\epsilon_{\alpha+1} - \epsilon_\alpha}\right]$$

Typical distributions of r:

- Circular orthogonal ensemble(COE):

$$\mathrm{P}_{\mathrm{COE}} = \frac{2}{3}\left(\frac{1}{(1+\mathrm{r})^2} + \frac{\sin(\frac{2\pi \mathrm{r}}{\mathrm{r}+1})}{2\pi \mathrm{r}^2} + \frac{\sin(\frac{2\pi}{\mathrm{r}+1})}{2\pi} - \frac{\cos(\frac{2\pi}{\mathrm{r}+1})}{\mathrm{r}+1} - \frac{\cos(\frac{2\pi \mathrm{r}}{\mathrm{r}+1})}{\mathrm{r}(\mathrm{r}+1)}\right)$$

- Possion:

$$\mathrm{P}_{\mathrm{POI}} = \frac{2}{(1+\mathrm{r})^2}$$

Level Statistic

Use the distribution to determine different phases:

- For MBL phase, there is no level repulsion, eigenstates has large distance and little overlap in Fock space,so Possion Statistic.
- For ergodic phase, coupling leads to level repulsion,so COE.

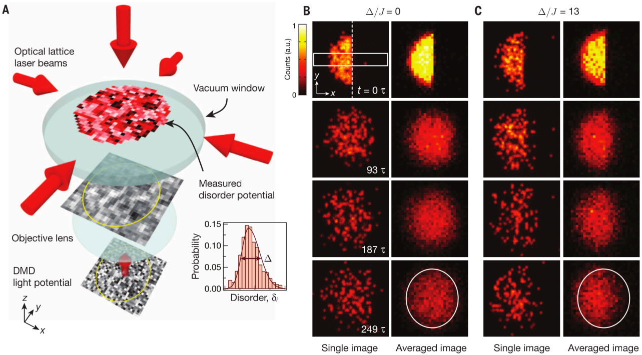
Exploring the many-body localization transition in two dimensions

Theoritical Model

$$\hat{H} = -J \sum_{<i,j>} (\hat{a}_i^\dagger \hat{a}_j) - \frac{U}{2} \sum_i (\hat{n}_i^\dagger (\hat{n}_i - 1) + \sum_i (\delta_i + V_i) \hat{n}_i^\dagger$$

$\delta :$ onsite random potential
disorder strength $\Delta :$ full $-$ width half $-$ maximum of disorder distribution

Experimental Setup



Technique:High resolution Imaging, DMD with 787.5nm laser

Interaction $U = 24.4J, J/h = 24.8Hz$

Choi, J. et.al(2016). Science, 352(6293), 1547-1552.

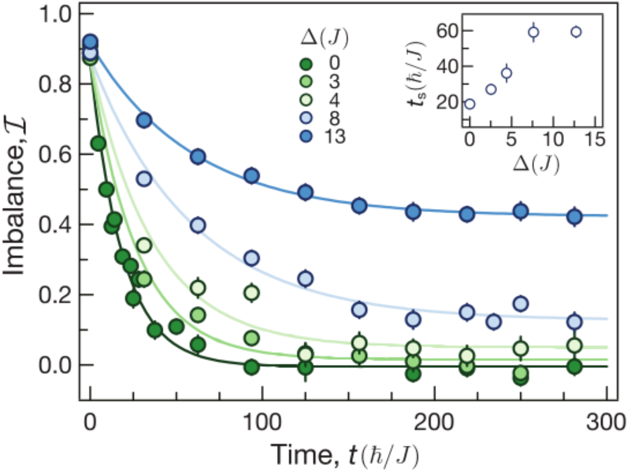
Number Imbanlance

Use Number Imbalance to identify diffrent phases:

$$I = \frac{N_L - N_R}{N_L + N_R}$$

In this experiment, I is meausured in the center region over 5 sites in the y direction.

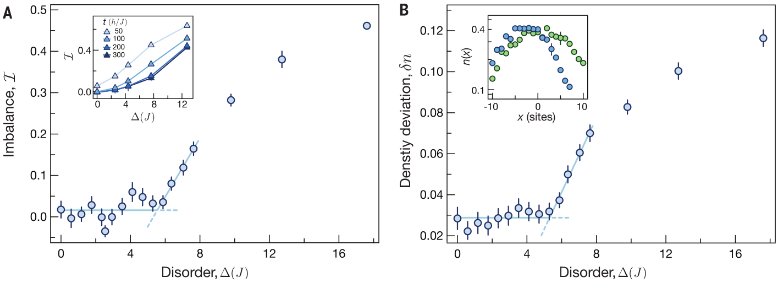
Dynamics


Imbalance, $\mathcal{I}$
Time, $t\,(\hbar/J)$
$\Delta(J)$
0
3
4
8
13
$t_s(\hbar/J)$
$\Delta(J)$

Identify MBL Transition

$$\delta n = \left(\sum_i [n_i(0) - n_i(\Delta)]^2\right)$$

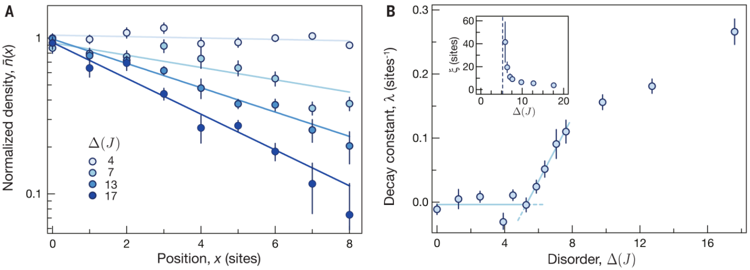
$$n_i(\Delta) = \frac{1}{5}\sum_{j=-2}^{2} n_{i,j}(0)$$



Identify MBL Transition

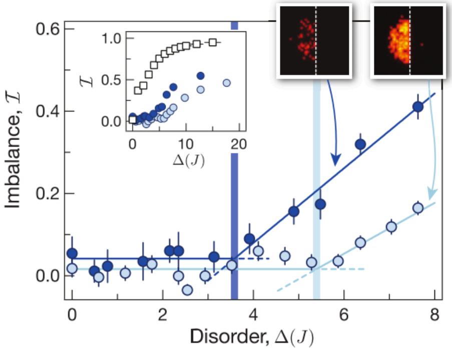
Normalized Density:

$$\bar{n}_i(\Delta) = n_i(\Delta)/n_i(0)^{-\lambda(\Delta)i}$$



Effect of Interaction

Reduce the density, therefore interaction effect.
Critical point shift can be observed, $5.3J \to 3.6J$



The End