

# Many-Body Localization in Optical Lattice

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# Overview

- 1 Quantum Thermalization
  - Microcanonical Ensemble
  - Eigenstate Thermalization Hypothesis (ETH)
  - Anderson localization

- 2 Second Section

# 定义

孤立系统:

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与环境无粒子和能量交换。

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与环境无粒子和能量交换。

粒子数与能量守恒。

# 系综理论

系综理论的基本观点：

宏观量的系统时间平均等于系综平均

$$\bar{u} = \langle u \rangle_t$$

# 各态遍历假说

系统的运动方程由哈密顿正则方程给出：

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

系统状态随时间的变化可以用代表点在相空间中的运动轨道来表示。

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上述运动方程表明，从不同初态出发系统在相空间内的运动轨道不相交。



# 各态遍历假说

玻尔兹曼提出各态遍历假说：

## 各态遍历假说

对于孤立的力学系统，只要时间足够长，系统从任意初态出发，都将经过能量曲面上的一切微观态。

# Thermalization

## 热力学平衡

- 系统的宏观量不再变换，经历的所有可能微观状态。
- 系统的热力学平衡时的统计性质与初态无关。

## Thermalization

系统能够进行各态遍历，平衡时宏观量的统计性质与初态无关。

# 微正则系综宏观量的平衡统计

各态遍历下，对于微正则系综所有可能微观状态出现的概率相同，状态分布在等能面上

$$\rho_n = \begin{cases} \frac{1}{\Omega}, & E \leq E_n \leq E + \Delta E \\ 0, & \text{other} \end{cases}$$

由此可以推导宏观量的热力学平衡期望值

$$\bar{u} = \sum_n \rho_n u_n$$

# 为什么要提出 ETH?

量子系统的态在 Hamiltonian 作用下的含时变化遵循薛定谔方程

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

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ETH 的目标就是解释对于幺正变化的量子系统，它们是如何各态遍历，并被平衡统计理论所描述。

# 从宏观量的平衡统计出发

在能量本征态基矢下，初态  $|\Psi\rangle$  可以表示为能量本征态的叠加：

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\alpha\rangle$$

在  $H$  作用下随时间变化：

$$|\Psi(t)\rangle = \sum_{\alpha} c_{\alpha} e^{-iE_{\alpha}t/\hbar} |\alpha\rangle$$

宏观量在  $H$  作用下随时间变化：

$$\begin{aligned} A(t) &= \sum_{\alpha, \beta} c_{\alpha} c_{\beta}^* e^{-i(E_{\alpha} - E_{\beta})t/\hbar} A_{\alpha\beta} \\ &= \sum_{\alpha} |c_{\alpha}|^2 A_{\alpha\alpha} + \sum_{\alpha \neq \beta} c_{\alpha} c_{\beta}^* e^{-i(E_{\alpha} - E_{\beta})t/\hbar} A_{\alpha\beta} \end{aligned}$$



宏观量的时间平均

$$\langle A \rangle_t = \frac{1}{\tau} \lim_{\tau \rightarrow \infty} \int_0^\tau A(t) dt$$

$$= \sum_{\alpha} |c_{\alpha}|^2 A_{\alpha\alpha} + i\hbar \lim_{\tau \rightarrow \infty} \left[ \sum_{\alpha \neq \beta} \frac{c_{\alpha} c_{\beta}^* A_{\alpha\beta}}{E_{\alpha} - E_{\beta}} \left( \frac{e^{-i(E_{\alpha} - E_{\beta})\tau/\hbar} - 1}{\tau} \right) \right]$$

平衡统计理论下微正则系综的守恒量的时间平均等于系综平均：

$$\langle A \rangle_{m.c.} = \sum_{\alpha} \rho_{\alpha} A_{\alpha\alpha} = \frac{1}{N} \sum_{\alpha} A_{\alpha\alpha}$$

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如何将上述两个公式等价呢？

# ETH 的表述

For an arbitrary initial state,  $\langle \hat{A} \rangle$  will ultimately evolve to its value predicted by a microcanonical ensemble, only small fluctuations around that value, provided that the following two conditions are met:

- The diagonal matrix elements  $A_{\alpha\alpha}$  vary smoothly as a function of energy.
- The off-diagonal matrix elements  $A_{\alpha\beta}$ , with  $\alpha \neq \beta$ , are much smaller than the diagonal matrix elements.

在两个约束条件下，非对角项非常小，为随时间变化的涨落项。

$$\langle A \rangle_t = \sum_{\alpha} |c_{\alpha}|^2 A_{\alpha\alpha} \approx A \sum_{\alpha} |c_{\alpha}|^2 = A$$

微正则系综的结果：

$$\langle A \rangle_{mc} \approx \frac{1}{N} * N * A = A$$

So,

$$\langle A \rangle_t = \langle A \rangle_{mc}$$

违背 ETH 的系统:

- Intergrate System, 有着自己的 ETH 表述, 可以在一定条件下热化。
- Localized System, 无法热化, non-ergodic, Anderson localization, Many-body localization.
- ...

# Introduction

Proposed by P.W.Anderson

- First introduced in 1958 (Anderson, P. W. (1958).PhysRev.109.1492)
- suggest the possibility of electron localization in semiconductor which contains random impurities or defects.
- Consider single particle without interaction but can hop from one site to another.

# Quantum Model

Tight-binding model hamiltonian

$$H = - \sum_{i,j} J_{ij} \hat{a}_i^\dagger \hat{a}_j + \sum_i U_i \hat{a}_i^\dagger \hat{a}_i$$

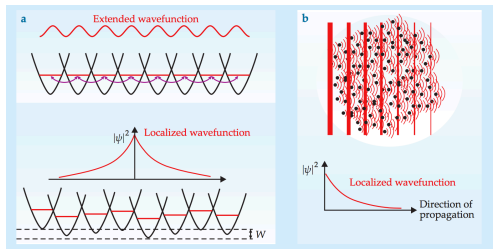
uniform  $U_i$ : extended wavefunction (Bloch wave)

random  $U_i$ : single particle localized wavefunction

# Explanation: matter wave interference

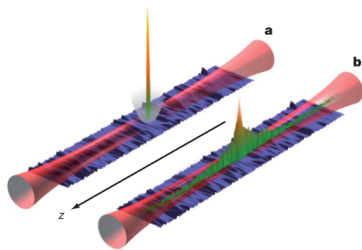
For single particle in lattice,

- uniform potential: matter wave freely propagation through lattice
- random potential: destructive interference between initial wave and wave scattered by impurities



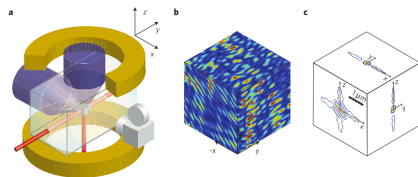


# Experiment of Anderson localization in ultracold atoms



Anderson localization in non-interacting 1d BEC

Juliette Billy et.al.Nature 453, 891-894

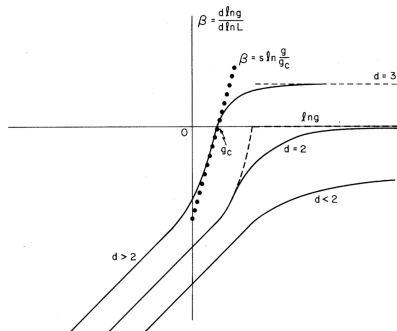


Anderson Localization in 3D ultracold atoms

F. Jendrzejewski et.al.Nature Physics 8, 398-403 (2012)

# scaling-law theory of anderson localization

According to scaling-law theory, for models of different dimension system:



$g$  is conductance,

$$\lim_{g \rightarrow \infty} \beta(g) = d - 2$$

$$\lim_{g \rightarrow 0} \beta(g) = \log(g)$$

So,

- $d \leq 2$ , system is localized for arbitrary weak disorder
- $d \geq 3$ , system is localized for some critical disorder strength.

Thermalization in anderson localization.

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- Localized states fails to thermalize, violate ETH, so some memory of initial condition is preserved in local observables at long times.
- Local observables differ from eigenstate to eigenstate in same energy shell.
- Mott insulator—> ground state localization, but Anderson localization is not limited in ground state.

# Many-body localization

Interaction leads eigenstates to be many-body states due to degree coupling,

Localization with interaction, so called ' Many-body localization' .

# Experiments

Bloch group: Localization of interacting fermions in a quasirandom optical lattice

## Theoretical Model

$$\hat{H} = -J \sum_{i,\sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{h.c.}) + \\ \Delta \sum_{i,\sigma} \cos(2\pi\beta i + \phi) \hat{c}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$



# Multiple Columns

## Heading

- 1 Statement
- 2 Explanation
- 3 Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

# Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

# Theorem

Theorem (Mass – energy equivalence)

$$E = mc^2$$

# Verbatim

## Example (Theorem Slide Code)

```
\begin{frame}  
\frametitle{Theorem}  
\begin{theorem}[Mass--energy equivalence]  
$E = mc^2$  
\end{theorem}  
\end{frame}
```

# Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

# Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

# References



John Smith (2012)

Title of the publication

Journal Name 12(3), 45 – 678.

# The End