清华大学 数学作业纸

TSINGHUA UNIVERSITY

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Problem A.

1. Sol. Length $= \int_{-\frac{2}{3}}^{1} \sqrt{\left(\frac{d(1-t)}{dt}\right)^2 + \left(\frac{d(2+3t)}{dt}\right)^2}\, dt = \int_{-\frac{2}{3}}^{1} \sqrt{10}\, dt = \sqrt{10}\, t \Big|_{-\frac{2}{3}}^{1} = \frac{5\sqrt{10}}{3}$.

2. Sol. Length $= \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\pi} \sqrt{(-\sin t)^2 + (1+\cos t)^2}\, dt = \int_0^{\pi} \sqrt{2+2\cos t}\, dt$

$= \sqrt{2}\int_0^{\pi} \sqrt{\frac{1-\cos t}{1-\cos t}\cdot(1+\cos t)}\, dt = \sqrt{2}\int_0^{\pi} \frac{\sin t}{\sqrt{1-\cos t}}\, dt = \sqrt{2}\int_0^{2} \frac{1}{\sqrt{u}}\, du \quad (u = 1-\cos t)$

$= 2\sqrt{2}\sqrt{u}\Big|_0^2 = 4$

3. Sol. Length $= \int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\sqrt{3}} \sqrt{9t^4+9t^2}\, dt = \int_0^{\sqrt{3}} 3t\sqrt{t^2+1}\, dt$

$= \frac{3}{2}\int_1^4 \sqrt{u}\cdot du \quad (u = t^2+1) = \frac{3}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_1^4 = 7.$

Problem B.

9. Sol. $S = \int_0^4 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx = \int_0^4 2\pi\cdot\frac{x}{2}\sqrt{1+\frac{1}{4}}\, dx = \frac{\sqrt{5}\pi}{4}\cdot x^2\Big|_0^4 = 4\sqrt{5}\pi$

By geometry formula: $S = \frac{1}{2}\times 2\pi r \times$ slant height $= \frac{1}{2}\times 2\pi\times 2\times\sqrt{4^2+2^2} = 4\sqrt{5}\pi$.

11. Sol. $S = \int_1^3 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_1^3 \pi(x+1)\cdot\sqrt{1+\frac{1}{4}}\, dx = \frac{\sqrt{5}\pi}{2}\left(\frac{1}{2}x^2+x\right)\Big|_1^3 = 3\sqrt{5}\pi$.

By geometry formula: $S = \frac{1}{2}\times 2\pi(r_1+r_2)$ slant height $= \pi\times 3\times\sqrt{5} = 3\sqrt{5}\pi$

Problem C

26. Sol. $S = 2\int_0^1 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = 2\int_0^1 2\pi\cdot(1-x^{\frac{2}{3}})^{\frac{3}{2}}\cdot\sqrt{1+x^{-\frac{2}{3}}\cdot(1-x^{\frac{2}{3}})}\, dx$

$= 2\int_0^1 2\pi(1-x^{\frac{2}{3}})^{\frac{3}{2}}\cdot x^{-\frac{1}{3}}\, dx = 2\int_1^0 2\pi\cdot u^{\frac{3}{2}}\cdot x^{-\frac{1}{3}}\frac{du}{-\frac{2}{3}\cdot x^{-\frac{1}{3}}} = -\frac{12}{5}\pi\cdot u^{\frac{5}{2}}\Big|_1^0 = \frac{12}{5}\pi.$

Problem D.

17. Sol. $y = x^5 \Rightarrow x = \sqrt[5]{y}$, $f^{-1}(x) = \sqrt[5]{x}$, domain: $\mathbb{R}$ range: $\mathbb{R}$

$f(f^{-1}(x)) = (\sqrt[5]{x})^5 = f^{-1}(f(x)) = \sqrt[5]{x^5} = x$.

20. Sol. $y = x^4 \Rightarrow x = \sqrt[4]{y}$ so $f^{-1}(x) = \sqrt[4]{x}$ domain: $x \geq 0$ range: $y \geq 0$.

$f(f^{-1}(x)) = (\sqrt[4]{x})^4 = f^{-1}(f(x)) = \sqrt[4]{x^4} = x$.

23. Sol. $y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$, $f^{-1}(x) = \frac{1}{\sqrt{x}}$, domain $x > 0$, range $y > 0$

$f(f^{-1}(x)) = \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^2} = f^{-1}(f(x)) = \frac{1}{\sqrt{\frac{1}{x^2}}} = x$.

24. Sol. $y=\frac{1}{x^3} \Rightarrow x=\frac{1}{\sqrt[3]{y}}$. $f^{-1}(x)=\frac{1}{\sqrt[3]{x}}$. domain $x\neq 0$, range $y\neq 0$.

$f(f^{-1}(x))=\frac{1}{(\frac{1}{\sqrt[3]{x}})^3}=f^{-1}(f(x))=\frac{1}{\sqrt[3]{\frac{1}{x^3}}}=x$

Problem E.

39. Sol. $\int\frac{2y\,dy}{y^2-25}=\int\frac{1}{y^2-25}\cdot d(y^2-25)=\ln|y^2-25|+C$.

44. Sol. $\int_2^4\frac{dx}{x\ln x}=\int_2^4\frac{1}{x\ln x}\cdot\frac{d(\ln x)}{\frac{1}{x}}=\ln(\ln x)\Big|_2^4=\ln 2$.

45. Sol. $\int_2^4\frac{dx}{x(\ln x)^2}=\int_2^4\frac{1}{x(\ln x)^2}\cdot\frac{d(\ln x)}{\frac{1}{x}}=-\frac{1}{\ln x}\Big|_2^4=\frac{1}{2\ln 2}=\frac{1}{\ln 4}$

Problem F.

42. Sol. $\int(2e^x-3e^{-2x})dx=2e^x+\frac{3}{2}e^{-2x}+C$.

49. Sol $\int\frac{e^{\sqrt{r}}}{\sqrt{r}}dr=\int\frac{e^{\sqrt{r}}}{\sqrt{r}}\cdot\frac{d(\sqrt{r})}{\frac{1}{2}r^{-\frac{1}{2}}}=\int 2e^{\sqrt{r}}\,d(\sqrt{r})=2e^{\sqrt{r}}+C$.

55 Sol. $\int_0^{\frac{\pi}{4}}(1+e^{\tan\theta})\sec^2\theta\cdot d\theta=\int_0^{\frac{\pi}{4}}(1+e^{\tan\theta})\cdot\sec^2\theta\,\frac{d(\tan\theta)}{\frac{1}{\cos^2\theta}}=\int_0^{\frac{\pi}{4}}1+e^{\tan\theta}\cdot d(\tan\theta)$

$=1+e^{\tan\theta}\Big|_0^{\frac{\pi}{4}}=e$.

Problem G.

68. Sol. The function $f(x)=2e^{\sin\frac{x}{2}}$ has maximum value when $\sin\frac{x}{2}=1\Rightarrow x=\pi+4k\pi$,

has a minimum when $\sin\frac{x}{2}=-1\Rightarrow x=3\pi+4k\pi$

Maximum Value : $f(x)=2e$ $(x=4k\pi+\pi)$

Minimum Value : $f(x)=2e^{-1}$ $(x=4k\pi+3\pi)$