



班级: 计01 姓名: 谷逸朗 编号: 2020010869 科目: 离散 第 1 页

1. (1) 不是函数 (2) 不是函数 (3) 是函数

2. (1) 是, $\text{dom}(f) = \{1, 2, 3\}$, $\text{ran}(f) = \{\langle 2, 3 \rangle, \langle 3, 2 \rangle, \langle 4, 1 \rangle\}$

(2) 不是

(3) 是, $\text{dom}(f) = \{1, 2, 3\}$, $\text{ran}(f) = \{\langle 2, 3 \rangle\}$

3. (1) 不是函数

令 $A = \{1, 2, 3\}$, $B = \{1, 2\}$

$f = \{\langle 1, 1 \rangle, \langle 2, 1 \rangle, \langle 3, 1 \rangle\}$, $g = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 2 \rangle\}$

$f \cap g = \{\langle 1, 1 \rangle\}$ 不是函数

(2) 不是函数

令 $A = \{1, 2, 3\}$, $B = \{1, 2\}$

$f = \{\langle 1, 1 \rangle, \langle 2, 1 \rangle, \langle 3, 1 \rangle\}$, $g = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 2 \rangle\}$

$f \cup g = \{\langle 1, 1 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle\}$ 不是函数

4. $f(0) = 0$

$f[\{0\}] = \{0\}$

$f[\{0, 2, 4, 6, \dots\}] = \{0, 1, 2, 3, \dots\}$

$f[\{1, 3, 5, \dots\}] = \{1\}$

$f^{-1}[\{2\}] = \{4\}$

$f^{-1}[\{3, 4\}] = \{6, 8\}$

6. (1) 非满射, 非单射, 非双射

(2) 非满射, 单射, 非双射

(3) 非满射, 非单射, 非双射

(4) 非满射, 非单射, 非双射

7. 要使 g 为双射, 则 $|A| = |A/R|$. 又对 $\forall x \in A$, 必有 $x \in [x]_R$, 故 $[x]_R = \{x\}$, 即 R 为恒等关系 I_A

当 R 为恒等关系时 $g(x) = [x]_R$. 若 $x_1, x_2 \in A$, 使得 $g(x_1) = g(x_2)$, 则

$$[x_1]_R = [x_2]_R = g(x_1) = g(x_2) = [x_2]_R = \{x_2\}$$

故 $x_1 = x_2$, 即 g 是单射的.

当 R 为恒等关系时, $[x]_R = \{x\}$, 此时 $g(x) = [x]_R$.

由 $A/R = \{y \mid (\exists x)(x \in A \wedge y = [x]_R)\}$ 知, 对 $\forall y \in A/R$, 必 $\exists x \in A$, 使得 $g(x) = y$.

故 g 是满射的.

所以, 当且仅当 R 为恒等关系时, g 是双射的.



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第 2 页

9 (1) $m \leq n$ (2) 若 $n=0$, 则 $m=0$, 若 $n \neq 0$, 则 $m \geq n$.(3) $m=n$ 10. (2) $f: A \rightarrow B$, $f(x) = 2x+1$ (3) $P(X) = \{ \emptyset, \{a, b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$

$$f_1 = \{ \langle a, 0 \rangle, \langle b, 0 \rangle, \langle c, 0 \rangle \}$$

$$f_2 = \{ \langle a, 0 \rangle, \langle b, 0 \rangle, \langle c, 1 \rangle \}$$

$$f_3 = \{ \langle a, 0 \rangle, \langle b, 1 \rangle, \langle c, 0 \rangle \}$$

$$f_4 = \{ \langle a, 0 \rangle, \langle b, 1 \rangle, \langle c, 1 \rangle \}$$

$$f_5 = \{ \langle a, 1 \rangle, \langle b, 0 \rangle, \langle c, 0 \rangle \}$$

$$f_6 = \{ \langle a, 1 \rangle, \langle b, 0 \rangle, \langle c, 1 \rangle \}$$

$$f_7 = \{ \langle a, 1 \rangle, \langle b, 1 \rangle, \langle c, 0 \rangle \}$$

$$f_8 = \{ \langle a, 1 \rangle, \langle b, 1 \rangle, \langle c, 1 \rangle \}$$

$$X_Y = \{ f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8 \}$$

$$f = \{ \langle \emptyset, f_1 \rangle, \langle \{a\}, f_2 \rangle, \langle \{b\}, f_3 \rangle, \langle \{c\}, f_4 \rangle, \langle \{a, b\}, f_5 \rangle, \langle \{b, c\}, f_6 \rangle, \langle \{a, c\}, f_7 \rangle, \langle \{a, b, c\}, f_8 \rangle \}.$$