数 学 作 业 纸

(科目: Calculus)

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Problem A.

Sol. 11. $h(x) = -x^3+2x^2$, $h'(x) = -3x^2+4x$

x	$(-\infty,0)$	0	$(0,\frac{4}{3})$	$\frac{4}{3}$	$(\frac{4}{3},+\infty)$
$h'(x)$	$-$	0	$+$	0	$-$
$h(x)$	↘	local min	↗	local max	↘

(a) increasing on $(0,\frac{4}{3})$

decreasing on $(-\infty,0)$, $(\frac{4}{3},+\infty)$

(b) local maximum value $h(\frac{4}{3}) = \frac{32}{27}$ (at $x=\frac{4}{3}$), local minimum value $h(0)=0$ (at $x=0$)

(c) no absolute extrema.

17. $f(x) = x^4-8x^2+16$, $f'(x) = 4x^3-16x$

x	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,+\infty)$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	↘	local min	↗	local max	↘	local min	↗

(a) increasing on $(-2,0)$, $(2,+\infty)$

decreasing on $(-\infty,-2)$, $(0,2)$

(b) local maximum is $f(0)=16$ at $x=0$, local minimum is $f(\pm 2) = 0$ at $x=\pm 2$.

(c) No absolute maximum, absolute minimum is 0 at $x=\pm 2$.

21. $g(x) = x\sqrt{8-x^2}$　$g'(x) = \frac{8-2x^2}{\sqrt{8-x^2}} = \frac{2(x+2)(2-x)}{\sqrt{(2\sqrt{2}-x)(2\sqrt{2}+x)}}$

x	$-2\sqrt{2}$	$(-2\sqrt{2},-2)$	-2	$(-2,2)$	2	$(2,2\sqrt{2})$	$2\sqrt{2}$
$g'(x)$	0	$-$	0	$+$	0	$-$	0
$g(x)$	local max, ↘		local min	↗	local max	↘	local min

(a) increasing on $(-2,2)$

decreasing on $(-2\sqrt{2},-2)$, $(2,2\sqrt{2})$

(b) local maxima are $\begin{cases} g(-2\sqrt{2})=0 & \text{at } x=-2\sqrt{2} \\ g(2)=4 & \text{at } x=2 \end{cases}$

local minima are $g(-2)=-4$ at $x=-2$ and $g(2\sqrt{2})=0$ at $x=2\sqrt{2}$

(c) absolute maximum is 4 at $x=2$

absolute minimum is -4 at $x=-2$.

Problem B.

Sol. 1. $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$, $y' = x^2-x-2$　$y'' = 2x-1$

x	$(-\infty,-1)$	-1	$(-1,2)$	2	$(2,+\infty)$
y'	$+$	0	$-$	0	$+$
y	↗	local max	↘	local min	↗

x	$(-\infty,\frac{1}{2})$	$\frac{1}{2}$	$(\frac{1}{2},+\infty)$
y''	$-$	0	$+$

inflection point: $(\frac{1}{2}, -\frac{3}{4})$

local maximum: $\frac{3}{2}$ at $x=-1$

local minimum: -3 at $x=2$

concave up on $(\frac{1}{2},+\infty)$

concave down on $(-\infty,-\frac{1}{2})$

2. $y = \frac{x^4}{4} - 2x^2 + 4$, $y' = x^3-4x$　$y'' = 3x^2-4$

x	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,+\infty)$
y'	$-$	0	$+$	0	$-$	0	$+$
y	↘	local min	↗	local max	↘	local min	↗

x	$(-\infty,-\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}},+\infty)$
y''	$+$	$-$	$+$

inflection point: $(-\frac{2}{\sqrt{3}}, \frac{16}{9})$, $(\frac{2}{\sqrt{3}}, \frac{16}{9})$

local maximum: 4 at $x=0$

local minimum: 0 at $x=\pm 2$

concave up on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}},+\infty)$

concave down on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$.

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Problem C.

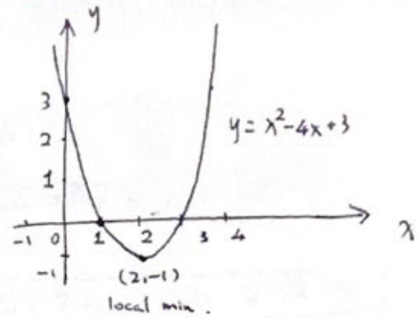
Sol. 9. $y=x^2-4x+3 \Rightarrow y'=2x-4 \Rightarrow y''=2$.

a. no local maximum, local minimum is $(2,-1)$

b. increasing on $(2,+\infty)$, decreasing on $(-\infty,2)$

c. concave up on $(-\infty,+\infty)$ as $y''>0$.

d. no inflection point.



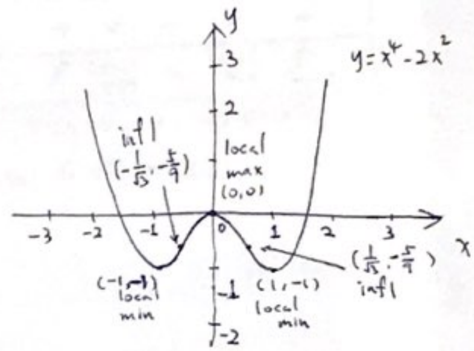
17. $y=x^4-2x^2 \Rightarrow y'=4x^3-4x \quad y''=12x^2-4$

a. local maximum is $(0,0)$,
local minima are $(-1,-1)$ and $(1,-1)$

b. increasing on $(-1,0)$, $(1,+\infty)$
decreasing on $(-\infty,-1)$, $(0,1)$

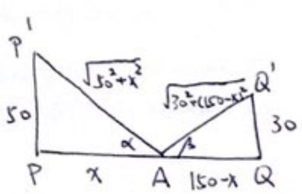
c. concave up on $(-\infty,-\frac{1}{\sqrt3})$ and $(\frac{1}{\sqrt3},+\infty)$
concave down on $(-\frac{1}{\sqrt3},\frac{1}{\sqrt3})$

d. inflection point are $(-\frac{1}{\sqrt3},-\frac{5}{9})$ and $(\frac{1}{\sqrt3},-\frac{5}{9})$



Problem D.

58. Sol.



a. The length $f(x)=\sqrt{50^2+x^2}+\sqrt{30^2+(150-x)^2}$, $x\in[0,150]$

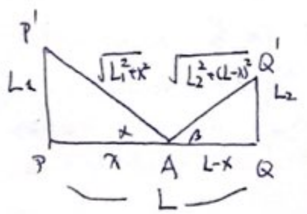
Let $f'(x)=\dfrac{2x}{\sqrt{50^2+x^2}}+\dfrac{2x-300}{\sqrt{30^2+(150-x)^2}}$

$= 2\cos\alpha - 2\cos\beta = 0$

then $\cos\alpha=\cos\beta$, so $\alpha=\beta$ and $\triangle APP' \sim \triangle AQQ'$

which means $\dfrac{x}{50}=\dfrac{150-x}{30} \Rightarrow x=\dfrac{375}{4}=93.75$ ft.

(from the left tower)



b. length $f(x)=\sqrt{L_1^2+x^2}+\sqrt{L_2^2+(L-x)^2}$

Let $f'(x)=\dfrac{2x}{\sqrt{L_1^2+x^2}}-\dfrac{2L-2x}{\sqrt{L_2^2+(L-x)^2}}$

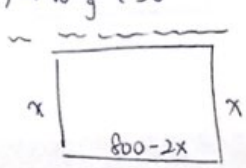
$=2\cos\alpha-2\cos\beta$. so $\alpha=\beta$ gives the minimum.

Problem E.

63. Sol. $s'=-gt+V_0$, Let $s'=0$, we get $t=\frac{V_0}{g}$, thus, $S=-\frac{1}{2}g(\frac{V_0}{g})^2+V_0\frac{V_0}{g}+S_0$

$=\dfrac{V_0^2}{2g}+S_0$ is the body's maximum height.

Problem F



7. Sol. Let $S(x)=x(800-2x) \Rightarrow S'(x)=800-4x$; let $S'(x)=0$ we get $x=200$.

So maximum area is $S(200)=80000\,m^2$, dimensions are 200m and 400m.