（科目：Calculus）

数 学 作 业 纸

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Problem A.

3. Sol. a. $\lim_{x\to 2^+} f(x) = 2$, $\lim_{x\to 2^-} f(x) = 1$

b. $\lim_{x\to 2} f(x)$ do not exist because the right- and left-hand limits are not equal.

c. $\lim_{x\to 4^+} f(x) = \frac{4}{2}+1 = 3$　$\lim_{x\to 4^-} f(x) = \frac{4}{2}+1 = 3$

d. Yes $\lim_{x\to 4} f(x) = 3$ because $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = 3$.

Problem B.

5. Sol. a. Yes. $f(-1) = (-1)^2 - 1 = 0$

b. Yes. $\lim_{x\to -1^+} f(x) = 0$

c. Yes, because $\lim_{x\to -1^+} f(x) = f(-1) = 0$.

d. Yes, because $\lim_{x\to -1^+} f(x) = f(-1) = 0$.

6. Sol. a. Yes, $f(1) = 1$

b. Yes, $\lim_{x\to 1^+} f(x) = 2$, $\lim_{x\to 1^-} f(x) = 2$, so $\lim_{x\to 1} f(x) = 2$

c. No

d. No

7. Sol. a. No

b. No

8. Sol. $[-1,0)\cup(0,1)\cup(1,2)\cup(2,3)$

Problem C.

37. Sol. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(\frac{2}{x}-3\right) = \lim_{x\to\infty}\frac{2}{x} + \lim_{x\to\infty}(-3) = 0-3 = -3$

39. Sol $\lim_{x\to\infty} g(x) = \lim_{x\to\infty}\frac{1}{2+\frac{1}{x}} = \frac{\lim_{x\to\infty} 1}{\lim_{x\to\infty}(2+\frac{1}{x})} = \frac{1}{2+0} = \frac{1}{2}$

43 Sol. As $\frac{-1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x}$, with sandwich theoren we have $\lim_{x\to\infty}\frac{\sin 2x}{x} = 0$.

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Problem D.

35. Sol. $g(x) = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x+3 \quad (x \neq 3) \Rightarrow g(3) = \lim_{x \to 3}(x+3) = 3+3 = 6$

37. Sol. $f(s) = \frac{s^3-1}{s^2-1} = \frac{(s-1)(s^2+s+1)}{(s+1)(s-1)} = \frac{s^2+s+1}{s+1} \quad (s \neq 1) \Rightarrow f(1) = \lim_{s \to 1} \frac{s^2+s+1}{s+1} = \frac{1+1+1}{1+1} = \frac{3}{2}$

Problem E.

11. Sol. $m = \lim_{h \to 0} \frac{[(h+2)^2+1]-(2^2+1)}{h} = \lim_{h \to 0} \frac{h^2+4h}{h} = \lim_{h \to 0}(h+4) = 4$

At $(2,5)$, the tangent line is $y-5 = 4(x-2)$

12 Sol $m = \lim_{h \to 0} \frac{[h+1-2(h+1)^2]-(-1)}{h} = \lim_{h \to 0} \frac{-2h^2-3h}{h} = \lim_{h \to 0}(-2h-3) = -3$

At $(1,-1)$, the tangent line is $y+1 = -3(x-1)$

Problem F.

13. Sol. (a) $y' = (3-x^2) \cdot \frac{d}{dx}(x^3-x+1) + (x^3-x+1)\frac{d}{dx}(3-x^2)$

$= (3-x^2)(3x^2-1) + (x^3-x+1)(-2x)$

$= -3x^4+10x^2-3 - 2x^4+2x^2-2x$

$= -5x^4+12x^2-2x-3$

(b) $y = -x^5+4x^3-x^2-3x+3 \Rightarrow y' = -5x^4+12x^2-2x-3$

14. Sol (a) $y' = (x-1)\frac{d}{dx}(x^2+x+1) + (x^2+x+1)\frac{d}{dx}(x-1)$

$= (x-1)(2x+1) + (x^2+x+1) \cdot 1$

$= 2x^2-x-1+x^2+x+1$

$= 3x^2$

(b) $y = x^3-1 \Rightarrow y' = 3x^2$