

Calculus A(1): Homework 8

Each assigned exercise is worth 20 points. The bonus exercises are optional. We may (or may not) decide to grade the bonus exercises and use them to replace one assigned exercise (if it improves your total grade). We refer to Thomas' Calculus book (whose PDF is available on the weblearn) for the exercises given by a paragraph and number. If you are using your own Thomas' Calculus book, make sure that the numbering of exercises is identical with the PDF.

Routine exercises (do not hand-in)

§6.1 Exercises 7, 16, 55

§6.2 Exercises 14, 25

§6.3 Exercises 29, 30

§6.5 Exercises 9, 26

§7.1 Exercises 7, 16, 31, 48, 50

§7.2 Exercises 23, 42, 68

§7.3 Exercises 31, 55, 68

§7.6 Exercises 16

§7.7 Exercises 71, 80, 94, 140

Assigned exercises (hand-in)

A1. Compute the volume of the solid of revolution obtained by rotating the disk $x^2 + y^2 \leq a^2$ about the line $x = b$, where $0 < a < b$.

A2. §6.5 Exercise 29

A3. §7.3 Exercise 82

A4. Compute $\lim_{x \rightarrow 0^+} x^x$ (if it exists). You should justify your answer.

A5. Prove that for all $x \in \mathbb{R}$ with $|x| \geq 1$, we have $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ (with the convention taken in class/in the book for the range of \sec^{-1}).

Bonus exercises

B1. Consider a smooth parametric curve C given by parametric equations $x = f(t)$ and $y = g(t)$ for $t \in [a, b]$ ($a < b$). Let $c < d$ and $\varphi : [a, b] \rightarrow [c, d]$ be C^1 and increasing, with $\varphi(a) = c$ and $\varphi(b) = d$. We then get a new parametrization of C by considering the curve C' whose parametric equations are $x = (f \circ \varphi)(u)$ and $y = (g \circ \varphi)(u)$ for $u \in [c, d]$ (here, u is the new parameter). Prove that the length of C' is equal to the length of C . We say that the length of a smooth parametric curve is independent of the parametrization.

B2. Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function which is one-to-one. Show that f is monotonic.