班级：CST01 姓名：客逸朗 编号：2020010869 科目：Calculus 第 1 页

4.(a) $\frac{\partial w}{\partial x} = \frac{2x}{x^2+y^2+z^2}$, $\frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2}$, $\frac{\partial w}{\partial z} = \frac{2z}{x^2+y^2+z^2}$, $\frac{dx}{dt} = -\sin t$, $\frac{dy}{dt} = \cos t$, $\frac{dz}{dt} = 2t^{-1/2}$

So $\frac{dw}{dt} = -\frac{2x\sin t}{x^2+y^2+z^2} + \frac{2y\cos t}{x^2+y^2+z^2} + \frac{4zt^{-1/2}}{x^2+y^2+z^2} = \frac{4\cdot 4\cdot t^{\frac{1}{2}}\cdot t^{-\frac{1}{2}}}{1+16t} = \frac{16}{1+16t}$.

another way: $w = \ln(\cos^2 t + \sin^2 t + 16t) = \ln(16t+1)$, $\frac{dw}{dt} = \frac{16}{16t+1}$

(b) $\frac{dw}{dt}(3) = \frac{16}{49}$

8.(a) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u} = \frac{\frac{1}{y}}{(\frac{x}{y})^2+1}\cdot\cos v + \frac{-\frac{x}{y^2}}{(\frac{x}{y})^2+1}\cdot\sin v = \frac{y\cos v - x\sin v}{x^2+y^2} = \frac{u\sin v\cos v - u\cos v\sin v}{x^2+y^2} = 0$

$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v} = \frac{\frac{1}{y}}{(\frac{x}{y})^2+1}\cdot(-u\sin v) + \frac{-\frac{x}{y^2}}{(\frac{x}{y})^2+1}\cdot(u\cos v) = \frac{-u^2\sin^2 v - u^2\cos^2 v}{x^2+y^2} = \frac{-u^2}{u^2(\cos^2 v+\sin^2 v)} = -1$

by expressing z: $z = \tan^{-1}(\frac{x}{y}) = \tan^{-1}(\cot v)$

so $\frac{\partial z}{\partial u} = 0$, $\frac{\partial z}{\partial v} = \frac{-\csc^2 v}{(\cot v)^2+1} = \frac{-\csc^2 v}{\csc^2 v} = -1$.

(b) $(u,v) = (1.3, \frac{\pi}{6}) \Rightarrow \frac{\partial z}{\partial u} = 0$, $\frac{\partial z}{\partial v} = -1$

28. Let $F(x,y) = xe^y + \sin xy + y - \ln 2 = 0$, then $F_x(x,y) = e^y + y\cos xy$, $F_y(x,y) = xe^y + x\cos xy + 1$

$\Rightarrow \frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{e^y + y\cos xy}{xe^y + x\cos xy + 1} \Rightarrow \frac{dy}{dx}(0, \ln 2) = -\frac{e^{\ln 2} + \ln 2\cdot 1}{1} = -2-\ln 2$.

42. $V = abc$, so $\frac{\partial V}{\partial t} = bc\cdot\frac{\partial a}{\partial t} + ac\cdot\frac{\partial b}{\partial t} + ab\frac{\partial c}{\partial t} \Rightarrow \left.\frac{\partial V}{\partial t}\right|_{a=1,b=2,c=3} = 2\times3\times1 + 1\times3\times1 + 1\times2\times(-3) = 3\ m^3/s$

$S = 2(ab+bc+ca)$, $\frac{\partial S}{\partial t} = 2\left((b+c)\frac{\partial a}{\partial t} + (a+c)\frac{\partial b}{\partial t} + (a+b)\frac{\partial c}{\partial t}\right) \Rightarrow \left.\frac{\partial S}{\partial t}\right|_{a=1,b=2,c=3} = 2\times(5\times1 + 4\times1 + 3\times(-3)) = 0\ m^2/s$

$D = \sqrt{a^2+b^2+c^2} \Rightarrow \frac{\partial D}{\partial t} = \frac{1}{2\sqrt{a^2+b^2+c^2}}\left(2a\cdot\frac{\partial a}{\partial t} + 2b\frac{\partial b}{\partial t} + 2c\cdot\frac{\partial c}{\partial t}\right) \Rightarrow \left.\frac{\partial D}{\partial t}\right|_{a=1,b=2,c=3} = \frac{2\times1 + 4\times1 + 6\times(-3)}{2\sqrt{1+4+9}} = -\frac{6}{\sqrt{14}}\ m/s$

16. $\vec{u} = \frac{i+j+k}{\sqrt{1+1+1}} = \frac{1}{\sqrt3}\vec{i} + \frac{1}{\sqrt3}\vec{j} + \frac{1}{\sqrt3}\vec{k}$, $f_x(x,y,z) = 2x$, $f_y(x,y,z) = 4y$, $f_z(x,y,z) = -6z \Rightarrow f_x(1,1,1) = 2$,

$f_y(1,1,1) = 4$, $f_z(1,1,1) = -6 \Rightarrow \nabla f = 2\vec{i} + 4\vec{j} - 6\vec{k} \Rightarrow (D_u f)_{P_0} = \frac{2}{\sqrt3} + \frac{4}{\sqrt3} - \frac{6}{\sqrt3} = 0$

34 $\nabla T = 2y\vec{i} + (2x-z)\vec{j} - y\vec{k} \Rightarrow |\nabla T(1,-1,1)| = |-2\vec{i} + \vec{j} + \vec{k}| = \sqrt6$, as $-\sqrt6 > -3$, the answer is no.

4.(a) $\nabla f = (2x+2y)\vec{i} + (2x-2y)\vec{j} + 2z\cdot\vec{k}$, $\nabla f(1,-1,3) = 4\vec{j} + 6\vec{k}$

so Tangent plane: $0 + 4(y+1) + 6(z-3) = 0 \Rightarrow 2y + 3z = 7$

(b) normal line: $x = 1$, $y = -1+4t$, $z = 3+6t$