Calculus A(1): Homework 5

Each assigned exercise is worth 20 points. The bonus exercise is optional. We may (or may not) decide to grade the bonus and use it to replace one assigned exercise (if it improves your total grade). We refer to Thomas' Calculus book (whose PDF is available on the weblearn) for the exercises given by a paragraph and number. If you are using your own Thomas' Calculus book, make sure that the numbering of exercises is identical with the PDF.

Routine exercises (do not hand-in)

§3.5, Exercises 1, 6, 14, 18, 34, 52, 64
§3.6, Exercises 59, 68, 76
§3.7, Exercises 1, 7, 13
§3.8, Exercises 4, 15, 17, 31, 35, 6
§4.1, Exercises 4, 7, 10, 13, 23, 25, 65, 66, 67, 68

Assigned exercises (hand-in)

A1. Consider the curve $\mathcal{C}$ in the plane given by the equation $x^2 + xy + y^2 = 12$. Find the points where $\mathcal{C}$ has a horizontal tangent or a vertical tangent. In the case of a vertical tangent, $\frac{dy}{dx}$ is not well-defined; in this case what is the value of $\frac{dx}{dy}$?

A2. Recall that the linear approximation of a differentiable function f at a is

$$L(x) = f'(a)(x - a) + f(a)$$

Prove that the linear function L has the following two properties

1. $L(a) = f(a)$
2. $L'(a) = f'(a)$

Prove that any other linear function satisfying these two properties must be equal to L.

A3. Let $a \in \mathbb{R}$. Prove that any degree 2 polynomial $P(x)$ is equal to $A(x - a)^2 + B(x - a) + C$ for some constants A, B, and C in $\mathbb{R}$.
Let f be a function twice differentiable at a (which means that $f''(a)$ exists). Prove that there is a unique polynomial $P(x)$ of degree 2 satisfying the following three conditions:

1. $P(a) = f(a)$
2. $P'(a) = f'(a)$
3. $P''(a) = f''(a)$

Such a P is called the *quadratic approximation* of f at a.

A4. Find the global extrema of $f(x) = | x^3 - 9x |$ in $\mathbb{R}$, and the points where these extrema are attained by f.

A5. Among all the rectangles whose diagonal has length equal to 1cm, which rectangle has the largest area? (You need to justify your answer.)

Bonus exercises (optional)

B1. Is it true that any continuous function $f : [a, b] \to \mathbb{R}$ necessarily has a local extremum at $x = a$? (If the answer is yes, you need to prove it. Otherwise, you need to give a counter example.) Same question if f is assumed to be differentiable?