清華大學 TSINGHUA UNIVERSITY 数学作业纸

班级：CST01 姓名：[illegible] 编号：2020010869 科目：Calculus 第 1 页

18. By Ratio Test, we have $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}\cdot(n+1)^2\cdot e^{-(n+1)}}{(-1)^n\, n^2\cdot e^{-n}}\right| = \lim_{n\to\infty}\frac{(n+1)^2}{e n^2} = \lim_{n\to\infty}\frac{1}{e}\cdot\left(1+\frac{2}{n}+\frac{1}{n^2}\right)$

$= \frac{1}{e} < 1$, so it is converges.

28. By Root Test, $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\sqrt[n]{\frac{(\ln n)^n}{n^n}} = \lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\frac{\frac{1}{n}}{1} = 0 < 1$, converges

38 By Ratio Test, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!} = \lim_{n\to\infty}\frac{n^n}{(n+1)^n} = \lim_{n\to\infty}\frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e} < 1$, converges

42. By Ratio Test, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{3^{n+1}}{(n+1)^3\cdot 2^{n+1}}\cdot\frac{n^3\cdot 2^n}{3^n} = \lim_{n\to\infty}\frac{3}{2}\cdot\frac{1}{(1+\frac{1}{n})^3} = \frac{3}{2} > 1$, diverges

18. We have $\frac{1}{1+\sqrt{n}} > \frac{1}{1+\sqrt{n+1}} > 0$ and $\lim_{n\to\infty}\frac{1}{1+\sqrt{n}} = 0$ gives converges, but

$\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty}\frac{1}{1+\sqrt{n}} \geq \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}}$, and $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is a p-series with $p=\frac{1}{2}<1$ is divergent.

so it is converges conditionally.

48. Let $1+\frac{1}{4}-\frac{1}{9}-\frac{1}{16}+\frac{1}{25}+\frac{1}{36}-\cdots = \sum_{n=1}^{\infty}a_n$

by absolute convergence test, $\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty}\frac{1}{n^2}$ is a p-series with $p=2>1$, so it is converges.

8. $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x+2)^{n+1}}{n+1}\cdot\frac{n}{(-1)^n(x+2)^n}\right| < 1 \Rightarrow |x+2|\cdot\lim_{n\to\infty}\frac{n}{n+1} < 1 \Rightarrow |x+2|<1 \Rightarrow -3<x<-1.$

For $x=-1$, the series become $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is converges,

For $x=-3$, $\sum_{n=1}^{\infty}\frac{1}{n}$ is a diverge series.

(a) The radius is 1, interval of convergence is $x\in(-3,-1]$

(b) The interval of absolute converge is $x\in(-3,-1)$

(c) The series converges conditionally at $x=-1$.

34. $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{3\cdot5\cdot7\cdots(2n+3)\cdot x^{n+1}\cdot n^2\cdot 2^n}{(n+1)^2\cdot 2^{n+1}\cdot 3\cdot5\cdot7\cdots(2n+1)}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{x\cdot(2n+3)\cdot n^2}{2\,(n+1)^2}\right| < 1 \Rightarrow |x|\lim_{n\to\infty}\left|\frac{2n+3}{2(1+\frac{1}{n})^2}\right| < 1$

gives $x=0$

(a) The radius is 0, interval of converge is $x=0$

(b) The interval of absolute converge is $x=0$

(c) There is no value for conditionally converges series.