4. (a) $v\cdot u = 2\times2 + 10\times2 - 11\times1 = 13$

$|v| = \sqrt{2\times2 + 10\times10 + (-11)\times(-11)} = 15$

$|u| = \sqrt{2^2 + 2^2 + 1^2} = 3$

(b) $\cos\theta = \dfrac{u\cdot v}{|u|\cdot|v|} = \dfrac{13}{3\times15} = \dfrac{13}{45}$

(c) scalar component u in direction v: $u\cdot\dfrac{v}{|v|} = \dfrac{13}{15}$

(d) $\text{proj}_v u = \left(\dfrac{u\cdot v}{|v|^2}\right)\cdot v = \dfrac{13}{225}(2i + 10j - 11k)$

18. $\vec{CA} = -\vec{v}-\vec{u}$, $\vec{CB} = \vec{u}-\vec{v}$

With $\vec{CA}\cdot\vec{CB} = (-\vec{v}-\vec{u})(\vec{u}-\vec{v}) = \vec{v}\cdot\vec{v} - \vec{u}\cdot\vec{u} = |\vec{v}|^2 - |\vec{u}|^2 = 0$, as $|\vec{v}| = |\vec{u}| = \text{radius} = \dfrac{|AB|}{2}$.

we can say $\vec{CA}$ and $\vec{CB}$ are orthogonal.

8. $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3/2 & -1/2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -2\vec{i} - 2\vec{j} + 2\vec{k}$, direction: $-\frac{1}{\sqrt3}\vec{i} - \frac{1}{\sqrt3}\vec{j} + \frac{1}{\sqrt3}\vec{k}$

$\vec{v}\times\vec{u} = -(\vec{u}\times\vec{v}) = 2\vec{i} + 2\vec{j} - 2\vec{k}$. direction: $\frac{1}{\sqrt3}\vec{i} + \frac{1}{\sqrt3}\vec{j} - \frac{1}{\sqrt3}\vec{k}$.

24. (a) $\vec{u}\cdot\vec{v} = 1\times(-1) + 2\times1 + (-1)\times1 = 0$

$\vec{u}\cdot\vec{w} = 1\times1 + 2\times0 + (-1)\times1 = 0$

$\vec{u}\cdot\vec{r} = 1\times(-\frac{\pi}{2}) + 2\times(-\pi) + (-1)\times\frac{\pi}{2} = -3\pi$

$\vec{v}\cdot\vec{w} = -1\times1 + 1\times0 + 1\times1 = 0$

$\vec{v}\cdot\vec{r} = (-1)\times(-\frac{\pi}{2}) + 1\times(-\pi) + 1\times(\frac{\pi}{2}) = 0$

$\vec{w}\cdot\vec{r} = 1\times(-\frac{\pi}{2}) + 0\times(-\pi) + 1\times(\frac{\pi}{2}) = 0$

So $\vec{u}\perp\vec{v}$, $\vec{u}\perp\vec{w}$, $\vec{v}\perp\vec{w}$, $\vec{v}\perp\vec{r}$, $\vec{w}\perp\vec{r}$

(b) $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 3\vec{i} + 3\vec{k} \neq \vec{0}$

$\vec{u}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 2\vec{i} - 2\vec{j} - 2\vec{k} \neq \vec{0}$

$\vec{u}\times\vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -\pi/2 & -\pi & \frac{\pi}{2} \end{vmatrix} = \vec{0}$

$\vec{v}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \vec{i} + 2\vec{j} - \vec{k} \neq \vec{0}$

$\vec{v}\times\vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ -\pi/2 & -\pi & \frac{\pi}{2} \end{vmatrix} = \frac{3\pi}{2}\vec{i} + \frac{3}{2}\pi\vec{k} \neq \vec{0}$

$\vec{w}\times\vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ -\pi/2 & -\pi & \pi/2 \end{vmatrix} = \pi\vec{i} - \pi\vec{j} - \pi\vec{k} \neq 0$

only $\vec{u}\,/\!/\,\vec{r}$.

30. $\vec{i}\times\vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k}$, $\vec{k}\times\vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\vec{i}$. so $(\vec{i}\times\vec{j})\times\vec{j} = -\vec{i}$.

$\vec{j}\times\vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{0}$, $\vec{i}\times\vec{0} = \vec{0}$, so $\vec{i}\times(\vec{j}\times\vec{j}) = \vec{0}$.

which means cross product is not associative.

68 take vector $\vec{V_1} = A_1\vec{i} + B_1\vec{j} + C_1\vec{k}$ in plane $A_1x + B_1y + C_1z = D_1$

and vector $\vec{V_2} = A_2\vec{i} + B_2\vec{j} + C_2\vec{k}$ in plane $A_2x + B_2y + C_2z = D_2$

If two planes parallel, we have $\vec{V_1} \times \vec{V_2} = 0$, which means $(A_1\vec{i} + B_1\vec{j} + C_1\vec{k}) \times (A_2\vec{i} + B_2\vec{j} + C_2\vec{k}) = 0$

$\Rightarrow (B_1C_2 - B_2C_1)\vec{i} + (A_2C_1 - C_2A_1)\vec{j} + (A_1B_2 - A_2B_1)\vec{k} = 0$

$\Rightarrow B_1C_2 = B_2C_1$, $A_2C_1 = C_2A_1$, $A_1B_2 = A_2B_1$

If two planes are perpendicular, we have $\vec{V_1} \cdot \vec{V_2} = 0$, which is $A_1A_2 + B_1B_2 + C_1C_2 = 0$.

As $|\vec{V_1}|, |\vec{V_2}| \neq 0$,

$(\vec{V_1} \times \vec{V_2} = 0 \Leftrightarrow |\vec{V_1}||\vec{V_2}| \sin\theta = 0 \Rightarrow \theta = 0 \Rightarrow$ parallel, $\vec{V_1} \cdot \vec{V_2} = 0 \Leftrightarrow |\vec{V_1}||\vec{V_2}| \cos\theta = 0 \Rightarrow \theta = 90^\circ \Rightarrow \perp)$.

22. $V(t) = (-\sin t)\vec{i} + (\cos t)\vec{j} + 2\cos 2t \cdot \vec{k}$

$V(t_0) = V(\frac{\pi}{2}) = -\vec{i} - 2\vec{k}$

$r(t_0) = P_0 = (\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, \sin 2\cdot\frac{\pi}{2}) = (0, 1, 0)$

$x = x_0 - t = 0 - t = -t$

$y = y_0 \cdot \quad = 1$

$z = z_0 - 2t = 0 - 2t = -2t$

so $x = -t$, $y = 1$, $z = -2t$ are the parametric equations of the tangent line.