Sample Problems for Calculus A(2) Midterm Exam

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April 5, 2021

Below are some problems I made but decided to not use for the midterm exam. Solving these problems may serve as good preparation.

For the midterm, I will also include multiple choice problems that assess your understanding of basic concepts.

1. Do the following infinite series converge or diverge? Explain.
   (a) $\sum_{n=1}^{\infty} \frac{n^3}{n^{2.9}+1}$
   (b) $\sum_{n=1}^{\infty} \frac{n^3}{n^{3.1}+1}$
   (c) $\sum_{n=1}^{\infty} \left(\frac{n+2}{3n+4}\right)^n$
   (d) $\sum_{n=1}^{\infty} (-1)^n(\sqrt{n^2+1}-n)$
   (e) $\sum_{n=1}^{\infty} ((n+1)/n)^{n^2}/n$
   (f) $\sum_{n=1}^{\infty} (\sqrt{n^2+1}-n)$
   (g) $\sum_{n=1}^{\infty} 3^{-\ln n}$
2. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} x^{2n+1}/n(2^n+1)$.
3. Calculate the sum of the geometric series $\sum_{n=2}^{\infty} 3/2^n$.
4. Write down the Taylor series generated by $\cos x$ at $x = \pi/4$.
5. Show $\sum_{k=0}^{\infty} (-1)^k(\pi/4)^{2k+1}/(2k+1)! = \sum_{k=0}^{\infty} (-1)^k(\pi/4)^{2k}/(2k)!$.
6. Consider a triangle whose sides have length $6\,\mathrm{cm}$, $x\,\mathrm{cm}$ and $(10-x)\,\mathrm{cm}$. As function of x, for what value of x does the area become largest? Explain.
7. Consider the hyperbola described by the equation $x^2/a^2 - y^2/b^2 = 1,\ a > 0,\ b > 0$. Which of the following statements are true?

   A. The eccentricity e is always greater than 1.

   B. If P is a point on the hyperbola, and PF_1 and PF_2 are the distances from P to the two foci F_1 and F_2, respectively, then $|PF_1 - PF_2|$ is independent of the choice of P.

   C. The foci are located on the x-axis if $a > b$, and they are located on the y-axis if $a < b$.

   D. The graph of the hyperbola is symmetric under both a reflection about the x-axis and a reflection about the y-axis.
8. Let $\mathbf{u}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + \mathbf{k}$ and $\mathbf{v}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + 2\mathbf{k}$.
   (a) Calculate the dot product $\mathbf{u} \cdot \mathbf{v}$.
   (b) The cross product $\mathbf{u} \times \mathbf{v}$.
   (c) Calculate the length of the trajectory whose velocity is equal to $\mathbf{v}(t)$, where the time t varies from $t = 0$ to $t = 2\pi$.