数 学 作 业 纸

（科目：Linear Algebra）

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Problem 2.7.5.

Sol. (a) $x^T A y = [0\ 1]\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = [5]$

(b) This is the row $x^T A = [4\ 5\ 6]$ times y.

(c) This is the row x^T times the column $Ay = \begin{bmatrix}2\\5\end{bmatrix}$

Problem 2.7.11.

Sol.

As $PA = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&6\\1&2&3\\0&4&5\end{bmatrix} = \begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$ is the upper triangular.

P_1 exchanges the row 2, 3 of A, P_2 exchange columns of A.

$P_1 A P_2 = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} A \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix} = \begin{bmatrix}6&0&0\\5&4&0\\3&2&1\end{bmatrix}$

Problem 2.7.16

Sol. A^2-B^2 and ABA are symmetric but $(A+B)(A-B)$ and $ABAB$ are not.

Problem 2.7.22

Sol. As $\begin{bmatrix}0&1&1\\1&0&1\\2&3&4\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&1\\2&3&4\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&1\\0&3&2\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&1\\0&0&-1\end{bmatrix}$

So. $PA = LU = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix} A = \begin{bmatrix}1&0&0\\0&1&0\\2&3&1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\\0&0&-1\end{bmatrix}$

As $\begin{bmatrix}1&2&0\\2&4&1\\1&1&1\end{bmatrix} \to \begin{bmatrix}1&2&0\\0&0&1\\0&-1&1\end{bmatrix} \to \begin{bmatrix}1&2&0\\0&-1&1\\0&0&1\end{bmatrix}$

So $PA = LU = \begin{bmatrix}1&&\\&&1\\&1&\end{bmatrix} A = \begin{bmatrix}1&&\\1&1&\\2&0&1\end{bmatrix}\begin{bmatrix}1&2&0\\&-1&1\\&&1\end{bmatrix}$

Problem 2.7.32.

Sol. $P\cdot\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}&&1\\1&&\\&1&\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}z\\x\\y\end{bmatrix}$, $P^2 = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$, $P^3 = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

so P^3 rotations for $360°$ and P rotations for $120°$

thus, $v=(2,3,-5)$ to $Pv=(-5,2,3)$ has rotate $120°$.

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Problem 2.7.39

Sol. As $Q^TQ = I$, so $\begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_n^T \end{bmatrix}[q_1\ q_2 \cdots q_n] = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$

(a) The diagonal entries gives $q_i^T \cdot q_i = 1$ $(i=1,2\dots n)$, so q_i $(i=1,2\dots n)$ are unit vectors.

(b) The off-diagonal entries $q_1^T q_2 = 0$

(c) $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ is one possible example.

Problem 3.1.4

Sol. Zero vector: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\frac{1}{2}A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ $-A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$, cA in the smallest subspace of M that contains A.

Problem 3.1.10.

Sol. (a) (d) (e) are subspaces.

for (b), As $u=(1,b_2,b_3)$ is in the subspace, but $cu=(c,cb_2,cb_3)$ do not.

for (c), $u=(0,1,1)$ and $v=(1,0,1)$ are in the subspace, but $(1,1,2)$ do not.

for (f), $u=(1,2,3)$ and $v=(4,6,8)$ are contain by the subspace, but $u-v=(3,-4,-5)$ do not.

Problem 3.1.14

Sol. (a) subspaces of $\mathbb{R}^2$ are $\mathbb{R}^2$ itself, lines (through (0,0)) and Z contains only (0,0)

(b) subspaces of D are D itself, zero metrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $c\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$ for all c.

Problem 3.1.15

Sol (a) Two matrix through (0,0,0) is intersect in a line through (0,0,0) but it could be a space.

(b) The plane and line intersect in a point (0,0,0) but it could be a line.

(c) For every $x,y \in S\cap T$, we have $x,y \in S$ and $x,y \in T$, so $x+y$, $cx \in S$ and $x+y$, $cx \in T$, so $x+y$, $cx \in S\cap T$, which means $S \cap T$ is a subspase.

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Problem 3.1.19

Sol. The column space of A is $\begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}$ for all c. is a line.

The column space of B is $\begin{bmatrix} c \\ d \\ 0 \end{bmatrix}$ for all c and d.

The column space of c is $\begin{bmatrix} c \\ 2c \\ 0 \end{bmatrix}$ for all c.

Problem 3.1.20.

Sol. (a) The column space of $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix}$ is $c\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ for all c, if the system is solvable,

we have $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = c\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, which means $b_2 = 2b_1$ and $b_3 = -b_1$.

(b) Do elimination: $\begin{bmatrix} 1 & 4 & b_1 \\ 2 & 9 & b_2 \\ -1 & -4 & b_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & b_1 \\ 0 & 1 & b_2-2b_1 \\ 0 & 0 & b_1+b_3 \end{bmatrix}$, which means $b_1+b_3=0$ makes the system solvable.

Problem 3.1.25.

Sol. $Az = b+b^* = Ax+Ay = A(x+y)$ so $z = x+y$.

As $Ax=b$ and $Ay=b^*$ solvable, b and b^* both in C(A), by definition we have $b+b^*$ in C(A), which means $Az=b+b^*$ solvable.

Problem 1.

Sol. (a) Let V be the set of all symmetric matrix (where V is a subspace of $M(n \times n)$)

(1) First, we have $O_{n\times n} = (a_{ij})$, where $a_{ij}=0$ for all i and j.

so $a_{ij} = a_{ji} = 0$, and $O_{n\times n} \in V$.

Then, let $A = (a_{ij})$ and $B = (b_{ij})$, where $A \in V$ and $B \in V$.

(2) Let $C = (c_{ij})$ where $C = A+B$, we have. $c_{ij} = a_{ij}+b_{ij} = a_{ji}+b_{ji} = c_{ji}$ for all i, j, so $C \in V$.

(3) Lastly, let $A = (a_{ij})$ $(A \in V)$ and $c \in R$.

we have $a_{ij} = a_{ji} \Rightarrow c \cdot a_{ij} = c \cdot a_{ji}$, so $cA \in V$.

Thus, the subset of symmetric matrix is a subspace of the vector space M of n×n metrixs.

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(b) No. let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. we have $A \cdot B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ which is not invertible.

Problem 2.

Sol.

(1) First, the zero vector $p(x) = 0$ degree $= 0 \leq n$ so it is in P_n.

(2) Next, let $f(x), g(x) \in P_n$

where $f(x) = a_0 + a_1 x + \cdots + a_n x^n$, $g(x) = b_0 + b_1 x + \cdots + b_n x^n$.

let $h(x) = f(x) + g(x)$

We have $h(x) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)x^n$ is a polynomial which degree $\leq n$, so $h(x) \in P_n$.

(3) Then, let $f(x) \in P_n$, where $f(x) = a_0 + a_1 x + \cdots + a_n x^n$

let $k(x) = c f(x)$ $(c \in \mathbb{R}.)$

We have $k(x) = (ca_0) + (ca_1)x + \cdots + (c \cdot a_n)x^n$ is a polynomial which degree $\leq n$, so $k(x) \in P_n$.

Thus, P_n is a subspace of F.