清華大學 TSINGHUA UNIVERSITY 数学作业纸

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Problem A.

Sol. 1 (a) x^2　(b) $\frac{1}{3}x^3$　(c) $\frac{1}{3}x^3 - x^2 + x$

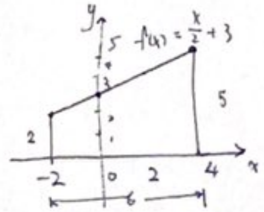
5 (a) $-x^{-1}$　(b) $-5\cdot x^{-1}$　(c) $2x + 5x^{-1}$

9. (a) $x^{\frac{2}{3}}$　(b) $x^{\frac{1}{3}}$　(c) $x^{-\frac{1}{3}}$

Problem B.

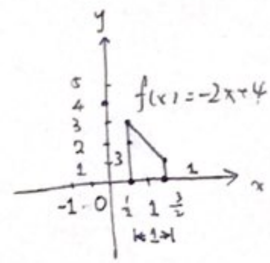
Sol. 15. $S = \frac{1}{2}(2+5)\times 6$
$= 21$

So $\int_{-2}^{4}(\frac{x}{2}+3)dx = 21.$



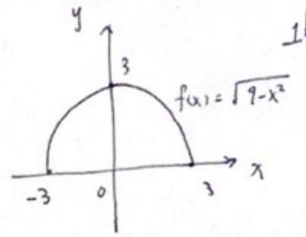
16. $S = \frac{1}{2}\times(1+3)\times 1$
$= 2$

So $\int_{\frac{1}{2}}^{\frac{3}{2}}(-2x+4)dx = 2$



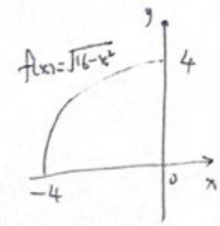
17. $S = \frac{1}{2}\pi\times 3^2$
$= \frac{9}{2}\pi$

So $\int_{-3}^{3}\sqrt{9-x^2}\,dx = \frac{9}{2}\pi$



18. $S = \frac{1}{4}\pi\times 4^2 = 4\pi.$

So $\int_{-4}^{0}\sqrt{16-x^2}\,dx = 4\pi$



Problem C.

Sol. 3. $\int_0^4 (3x - \frac{x^3}{4})dx = \frac{3}{2}x^2 - \frac{1}{16}x^4 \Big|_0^4 = 8$

8. $\int_{-2}^{-1}\frac{2}{x^2}dx = -2x^{-1}\Big|_{-2}^{-1} = \frac{-2}{-1} - \frac{-2}{-2} = 1$

10. $\int_0^{\pi}(1+\cos x)dx = x + \sin x\Big|_0^{\pi} = \pi + \sin\pi - (0+\sin 0) = \pi.$

26. $\int_0^{\pi}\frac{1}{2}(\cos x + |\cos x|)dx = \int_0^{\frac{\pi}{2}}\cos x\,dx + \int_{\frac{\pi}{2}}^{\pi} 0\,dx = \sin x\Big|_0^{\frac{\pi}{2}} = 1.$

Problem D.

1. $\int \sin 3x\,dx = \int \sin u\cdot\frac{du}{3} = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos 3x + C$

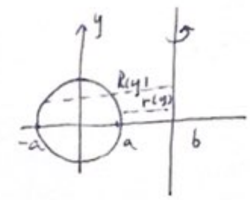
6. $\int x^3(x^4-1)^2dx = \int x^3\cdot u^2\cdot\frac{du}{4x^3} = \frac{1}{4}\int u^2\cdot du = \frac{1}{12}u^3 + C = \frac{1}{12}(x^4-1)^3 + C$

12. (a) $\int\frac{dx}{\sqrt{5x+8}} = \int\frac{1}{\sqrt{u}}\cdot\frac{du}{5} = \frac{2}{5}u^{\frac{1}{2}} + C = \frac{2}{5}\sqrt{5x+8} + C.$

(b) $\int\frac{dx}{\sqrt{5x+8}} = \int\frac{1}{\sqrt{5x+8}}\cdot\frac{du}{\frac{5}{2\sqrt{5x+8}}} = \int\frac{2}{5}du = \frac{2}{5}u + C = \frac{2}{5}\sqrt{5x+8} + C$

Problem E.

49. Sol. $R(y) = b + \sqrt{a^2-y^2}$, $r(y) = b - \sqrt{a^2-y^2}$.



$$V = \int_{-a}^{a} \pi \left([R(y)]^2 - [r(y)]^2\right) dy$$

$$= \int_{-a}^{a} \pi \left[\left(b^2 + 2b\sqrt{a^2-y^2} + a^2 - y^2\right) - \left(b^2 - 2b\sqrt{a^2-y^2} + a^2 - y^2\right)\right] dy$$

$$= \int_{-a}^{a} \pi \cdot 4b\sqrt{a^2-y^2} \cdot dy.$$

$$= 4b\pi \int_{-a}^{a} \sqrt{a^2-y^2}\, dy$$

$= 4b\pi \cdot$ area of semicircle of radius a.

$$= 4b\pi \cdot \frac{1}{2}\pi a^2 = 2a^2 b \pi^2$$

Problem F.

56. Sol. Let $h(x) = f(x) - g(x)$ (so $h(x)$ is also differentiable on $[a,b]$)

We have $h(a) = f(a) - g(a) = 0$, $h(b) = f(b) - g(b) = 0$

(with mean value theorem)

there must have one point c (in (a,b)) where $\frac{h(a)-h(b)}{a-b} = h'(c) = 0$.

as $h(c) = f(c) - g(c) \Rightarrow h'(c) = f'(c) - g'(c) = 0 \Rightarrow f'(c) = g'(c)$

So there must exist at least one point between a and b where tangents to the graphs of f and g are parallel or the same line.