Problem 4.1.6

Sol. $y = (1, 1, -1)$ at left nullspace

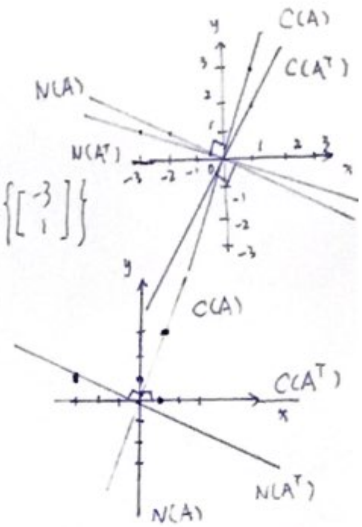
Problem 4.1.11.

Sol. $[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 6 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{array}\right] = [R|E]$

so $C(A) = \left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right\}$, $N(A) = \left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$, $C(A^T) = \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$, $N(A^T) = \left\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right\}$

$[B|I] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{array}\right] = [R|E]$

$C(A) = \left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right\}$ $N(A) = \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ $C(A^T) = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ $N(A^T) = \left\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right\}$



Problem 4.1.17.

Sol. If S contains only zero vector, $S^\perp$ is all of $\mathbb{R}^3$.

If S is spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $S^\perp$ spanned by $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

If S is spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $S^\perp$ spanned by $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Problem 4.1.20.

Sol. zero vector ; all of $\mathbb{R}^4$; $(V^\perp)^\perp = V$.

Problem 4.1.26

Sol

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 2 \\ 1 & -4 & -1 \end{bmatrix}$ $A^TA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 14 \end{bmatrix}$ is diagonal matrix.

As we can see $A^TA = \begin{bmatrix} c_1^T \\ c_2^T \\ c_3^T \end{bmatrix} \cdot [c_1 \cdot c_2 \cdot c_3] = \begin{bmatrix} c_1^T c_1 & c_1^T c_2 & c_1^T c_3 \\ c_2^T c_1 & c_2^T c_2 & c_2^T c_3 \\ c_3^T c_1 & c_3^T c_2 & c_3^T c_3 \end{bmatrix} = \begin{bmatrix} c_1^T c_1 & 0 & 0 \\ 0 & c_2^T c_2 & 0 \\ 0 & 0 & c_3^T c_3 \end{bmatrix}$ is diagonal.

(because col 1, col2, col3 are perpendicular to each other)

Problem 4.2.5

Sol.

$P_1 = \frac{a_1 a_1^T}{a_1^T a_1} = \frac{1}{9}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}[-1\ 2\ 2] = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$ $P_2 = \frac{1}{9}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}[2\ 2\ -1] = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$

$P_1 P_2 = 0$ becance. a_1 and a_2 are perpendicular.

Problem 4.2.6.

Sol $\vec{p}_1 = P_1\vec{b} = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ $\vec{p}_2 = P_2\vec{b} = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}$ $\vec{p}_3 = P_3\vec{b} = \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$

so $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \frac{1}{9}\begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = b$. Δ

Problem. 4.2.7.

Sol. $P_3 = \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$ and $P_1+P_2+P_3 = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$

Problem. 4.2.13

Sol

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $P = A(A^TA)^{-1}A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot (I)^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$ $\vec{p} = P\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$

P is a square matrix.

Problem 4.2.19.

Sol.

Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $P = A(A^TA)^{-1}A^T = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}^T \cdot \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 1 & 2 \\ 5 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$

Problem 4.2.20.

Sol

$\vec{e} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ $Q = \frac{\vec{e}\vec{e}^T}{\vec{e}^T\vec{e}} = \frac{1}{6}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}[1\ -1\ -2] = \frac{1}{6}\begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix}$ $P = I - Q = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$

Problem 4.2.21

Sol. $P^2 = A(A^TA)^{-1}A^T \cdot A(A^TA)^{-1} \cdot A^T = A \cdot I \cdot (A^TA)^{-1} \cdot A^T = P$

$P\vec{b}$ is in the column space of A so its projection onto that column space is $P\vec{b}$.
$(P(P\vec{b}))$

Problem 4.2.25.

Sol. S is the column space of P.

Problem 1.

Sol. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 6 \\ 1 & 2 & 1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R.$

$x_n = c\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + e\begin{bmatrix} -2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

hence, $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms a basis for the orthogonal complement $V^\perp$.

Problem 2.

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 2 \\ 0 & -1 \end{bmatrix}$, then $P = A(A^TA)^{-1}A^T = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 2 \\ 0 & -1 \end{bmatrix} \left(\begin{bmatrix} 2 & 4 \\ 4 & 10 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & -1 & 2 & -1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 2 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & -1 & 2 & -1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & -1 & 2 & -1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

So. $\vec{p} = P \cdot \vec{x} = P \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \end{bmatrix}$

$\vec{e} = \vec{x} - \vec{p} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$

So $\|\vec{e}\| = 2$.