

NAME :

STUDENT ID:

NOTE: There are 3 Parts to this FINAL EXAM (total of 6 pages). For Part 1 (multiple choice) be sure to indicate your answer clearly as no partial credit will be awarded. Each question has a unique right answer. Part 3 consists of a bonus exercise, which is not compulsory, and can give you some extra points. It is advised to finish all the compulsory questions before attempting the bonus one. If your total is > 100 points, your assigned score for the exam will be 100.

In order to receive full credit for Parts 2 and 3, you must show work to explain your reasoning. If you require additional sheets for Parts 2 or 3, be sure to put your name and ID on each additional page that you turn in with this exam. Use of calculators will NOT be permitted. You have 120 minutes to complete this test.

Part 1 (40 pts)

1. Let $f : (0, \frac{\pi}{2}) \rightarrow \mathbb{R}$ be given by $f(x) = (\tan(x))^2$. Denote as usual by f^{-1} the inverse function of f . What is the value of $(f^{-1})'(2)$?
 - (A) $\frac{1}{2(1+\tan(2)^2)\tan(2)}$
 - (B) 1
 - (C) $\frac{1}{6\sqrt{2}}$
 - (D) $\arctan(2)$
 - (E) None of the above.
2. What is the value of $\int_1^{\exp(\frac{\pi}{4})} \frac{\tan(\log(x))}{x} dx$?
 - (A) $\frac{\log(2)}{2}$
 - (B) 0
 - (C) $\log(\pi)$
 - (D) $\frac{\sqrt{2}}{2}$
 - (E) None of the above.
3. Consider a differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfying $f'(x) = x^3(x-1)^2(x+1)$ for all $x \in \mathbb{R}$. How many local extrema does f have?
 - (A) 3
 - (B) 2
 - (C) 1
 - (D) 0
 - (E) None of the above.
4. What is $\lim_{x \rightarrow 1} \frac{x^4+3x^3-3x^2-7x+6}{x^3-x^2-x+1}$?
 - (A) 0
 - (B) 3
 - (C) 6
 - (D) $+\infty$
 - (E) None of the above.
5. What is the equation of the tangent line to the curve $x^3 + xy + y^2 = 3$ at the point $(1, 1)$?
 - (A) $y = -\frac{4}{3}x + \frac{7}{3}$
 - (B) $y = -x + 2$
 - (C) $y = -\frac{4}{3}x + 2$
 - (D) $x = 1$
 - (E) None of the above.

Part 2a. (20 pts) Find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = \frac{1}{(4-x^2)^{3/4}}$ for $x \in [1, \sqrt{2}]$. Show ALL of your working.

Part 2b. (20pts) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x(8 - x)^{1/3}$. Sketch the graph of f (in particular, find the possible local extrema, the possible inflection points of f and give the possible intervals on which f is convex or concave). You need to justify your answer.

Part 2c. (20 pts) Compute

$$\int_0^{\pi/2} \frac{\sin(x)(\cos(x) - 1)}{(\cos(x))^2 - 5\cos(x) + 6} dx .$$

Part 3 (BONUS QUESTION). (10pts)

Consider a differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that for all $x \in \mathbb{R}$ we have $f'(x) = f(x)$. Prove that there exists a constant $C \in \mathbb{R}$ such that for all $x \in \mathbb{R}$ we have $f(x) = C \cdot \exp(x)$.