Linear Algebra – Homework 4

14 Oct 2020
Due: 22 Oct 2020

Textbook Problems. These problems will not be graded, but you must submit solutions to receive full credit for the homework.

Problem 2.3.9.

(a) E_{21} subtracts row 1 from row 2 and then P_{23} exchanges rows 2 and 3. What matrix $M = P_{23}E_{21}$ does both steps at once?

(b) P_{23} exchanges rows 2 and 3 and then E_{31} subtracts row 1 from row 3. What matrix $M = E_{31}P_{23}$ does both steps at once? Explain why the M's are the same but the E's are different.

Problem 2.3.12. Multiply these matrices:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \end{bmatrix}.$$

Problem 2.3.17. The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

Problem 2.3.26. The equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x}^* = \mathbf{b}^*$ have the same matrix A. What double augmented matrix should you use in elimination to solve both equations at once?

Solve both of these equations by working on a 2×4 matrix:

$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Problem 2.3.28. If $AB = I$ and $BC = I$, use the associative law to prove $A = C$.

Problem 2.4.6. Show that $(A+B)^2$ is different from $A^2 + 2AB + B^2$, when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule for $(A+B)(A+B) = A^2 + \underline{\hspace{2cm}} + B^2$.

Problem 2.4.15. True or false:

(a) If A^2 is defined, then A is necessarily square.

(b) If AB and BA are both defined, then A and B have to be square.

(c) If AB and BA are both defined, then AB and BA are both square.

(d) If $AB = B$, then A has to be the identity matrix I.

Problem 2.4.18. Write down the 3×3 matrices whose entries are:

(a) $a_{ij} =$ minimum of i and j.

(b) $a_{ij} = (-1)^{i+j}$.

(c) $a_{ij} = i/j$.

Problem 2.4.21. Compute A^2, A^3, A^4, and also $A\mathbf{v}$, $A^2\mathbf{v}$, $A^3\mathbf{v}$, $A^4\mathbf{v}$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}.$$

Problem 2.4.24. By experiment with $n = 2$ and $n = 3$, predict A^n for these matrices:

$$A_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}.$$

Problem 2.4.26. Multiply AB using columns times rows:

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} + \underline{\qquad\qquad} = \underline{\qquad\qquad}.$$

Problem 2.4.32. Suppose you solve $A\mathbf{x} = \mathbf{b}$ for three special right sides $\mathbf{b}$:

$$A\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad A\mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad A\mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

If the three solutions $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ are the columns of a matrix X, what is A times X?

Graded Problems.

Problem 1.

(a) Show that if r is any scalar, then the matrix $rI = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$ commutes with every 2×2 matrix. That is, show $(rI)\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}(rI)$.

(b) Find all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that commute with the matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Also find all 2×2 matrices that commute with $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. What matrices commute with *both* $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$?

Problem 2. Multiply the matrices:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & 1 \\ -1 & 0 \end{bmatrix}.$$