清華大學 TSINGHUA UNIVERSITY 数学作业纸

班级：计01　姓名：宋逸朗　编号：2020010869　科目：概统　第 1 页

20：$P(|X|>c) = \int_{|X|>c} f(x)\,dx$

又 $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx \geq \int_{|x|\geq c} x^2 f(x)\,dx \geq c^2 \int_{|x|\geq c} f(x)\,dx = c^2 \cdot P(|X|>c)$

故 $P(|X|>c) \leq \frac{E(X^2)}{c^2}$

21. $P(|X-m|>c) = \int_{|X-m|>c} f(x)\,dx \leq \int_{|X-m|>c} \frac{|X-m|}{c} \cdot f(x)\,dx$.（积分域上 $\frac{|X-m|}{c} \geq 1$）.

$= \frac{1}{c}\int_{|X-m|>c} |X-m| f(x)\,dx \leq \frac{1}{c}\int_{-\infty}^{\infty} |X-m| f(x)\,dx = \frac{E(|X-m|)}{c}$

23. 对 $\forall x \geq 0$，$\Psi(x) = P(|X|\leq x) = \int_{-x}^{x} e^{-\frac{u^2}{2}}\,du = \int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,du + \int_{-x}^{\infty} e^{-\frac{u^2}{2}}\,du - \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}}\,du$

$= \Phi(x) + [1-\Phi(-x)] - 1$　（又 $\Phi(x)+\Phi(-x)=1$）

$= 2\Phi(x) - 1$　，故　$\Psi = 2\Phi - 1$.（$x \geq 0$）

25. 由 $P(X^2<x) = P(-\sqrt{x}<X<\sqrt{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\sqrt{x}}^{\sqrt{x}} e^{-u^2/2}\,du = \frac{2}{\sqrt{2\pi}}\int_0^{\sqrt{x}} e^{-u^2/2}\,du$.

当 $x\to\infty$ 时有　$1 = \frac{2}{\sqrt{2\pi}}\int_0^{\infty} e^{-u^2/2}\,du$　$(x = u^2/2) \longrightarrow 1 = \frac{2}{\sqrt{2\pi}}\int_0^{\infty} \frac{e^{-x}}{\sqrt{2x}}\,dx = \frac{1}{\sqrt{\pi}} \cdot \int_0^{\infty} \frac{e^{-x}}{\sqrt{x}}\,dx$

由此 $\int_0^{\infty} x^{-\frac{1}{2}} e^{-x}\,dx = \sqrt{\pi}$

29. 由 Chebyshev's inequality 知　$P(|X-Y|>\varepsilon) \leq \frac{E[(X-Y)^2]}{\varepsilon^2} = 0$　对 $\forall \varepsilon>0$ 成立，故 $P(|X-Y|>0)=0$

又 $P(|X-Y|<0)=0$，故 $P(|X-Y|=0) = 1-P(|X-Y|<0) - P(|X-Y|>0) = 1$，即 $P(X=Y)=1$

30. $(\hat{X}\pm\hat{Y})^2 = (\hat{X})^2 \pm 2\hat{X}\cdot\hat{Y} + (\hat{Y})^2$

$= \frac{X^2 - 2X\cdot E(X) + E(X)^2}{\sigma^2(X)} \pm \frac{2(X-E(X))(Y-E(Y))}{\sigma(X)\sigma(Y)} + \frac{Y^2 - 2Y\cdot E(Y) + E(Y)^2}{\sigma^2(Y)}$

则 $E(\hat{X}\pm\hat{Y})^2 = \frac{E(X^2)-E(X)^2}{\sigma^2(X)} \pm \frac{2E[(X-E(X))(Y-E(Y))]}{\sigma(X)\sigma(Y)} + \frac{E(Y^2)-E(Y)^2}{\sigma^2(Y)}$

$= 2 \pm 2\rho(X,Y)$

当 $\rho(X,Y)=1$ 时，$E(\hat{X}-\hat{Y})^2=0$．由 29 的讨论知 $\hat{X},\hat{Y}$ 是 almost surely identical 的

当 $\rho(X,Y)=-1$ 时　$E(\hat{X}+\hat{Y})^2=0$．将 29 中 Y 换为 $-Y$，有　$P(|X+Y|=0)=1$，即 $\hat{X}=-\hat{Y}$ 是 almost surely identical 的.

4.75 (a) $\Phi(-1.78) = 1-\Phi(1.78) = 1-0.9625 = 0.0375$

(b) $\Phi(0.56) = 0.7123$

(c) $1-\Phi(-1.45) = \Phi(1.45) = 0.9265$

(d) $1-\Phi(2.16) = 1-0.9846 = 0.0154$

(e) $\Phi(1.53)-\Phi(-0.8) = \Phi(1.53)-(1-\Phi(0.8)) = 0.9370-(1-0.7881) = 0.7251$

(f) $1-(\Phi(1.83)-\Phi(2.52)) = 1-\Phi(1.83)+1-\Phi(2.52) = 2-0.9664-0.9941 = 0.0395$

4.78. $P(Z\geq z_1) = 1-\Phi(z_1) = \Phi(-z_1) = 0.84$, $\Rightarrow -z_1 = 0.995 \Rightarrow z_1 = -0.995$.

4.79. $P(X>8) = P(X-5>3) = P\left(\frac{X-5}{2}>\frac{3}{2}\right) = 1-\Phi(1.5) = 1\ 0.9332 = 0.0668$

4.91. X: 次品数量 则 $P(X=k) = \binom{100}{k}\left(\frac{3}{100}\right)^k\left(1-\frac{3}{100}\right)^{100-k} \approx \pi_k(3) = \frac{e^{-3}\cdot 3^k}{k!}$

(a) $1-\sum_{i=0}^{5}\pi_i(3) = 1-e^{-3}\cdot\sum_{i=0}^{5}\frac{3^k}{k!} = 1-e^{-3}\left(\frac{1}{1}+\frac{3}{1}+\frac{9}{2}+\frac{27}{6}+\frac{81}{24}+\frac{243}{120}\right) = 0.0839$

(b). $\sum_{i=1}^{3}\pi_i(3) = e^{-3}\left(\frac{3}{1}+\frac{9}{2}+\frac{27}{6}\right) = 0.5974$

(c) $\sum_{i=0}^{2}\pi_i(2) = e^{-3}\left(\frac{1}{1}+\frac{3}{1}+\frac{9}{2}\right) = 0.4232$