数 学 作 业 纸

（科目：linear Algebra）

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Problem 3.3.18.

Sol. $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 11 & 5 \\ -1 & 2 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & 3 & 5 \\ 0 & 6 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$, rank A = rank $A^T = 2$.

$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & q \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & q-1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & q-2 \end{bmatrix}$ if $q=2$, rank$A$ = rank $A^T = 2$

when $q \neq 2$, rank$A$ = rank$A^T = 3$

Problem 3.3.24

Sol. (a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}[x] = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. (b) $\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [b_1]$ (c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}[x] = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Problem 3.4.2.

Sol. As $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, so the largest possible number is 3.

(v_1, v_2, v_3 is one possible solution)

Problem 3.4.8

Sol. $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0 \Leftrightarrow c_1(w_3 + w_2) + c_2(w_1 + w_3) + c_3(w_1 + w_2) = 0$

$\Leftrightarrow (c_2 + c_3)w_1 + (c_1 + c_3)w_2 + (c_1 + c_2)w_3 = 0$

Because w_1, w_2, w_3 are independent, $c_2 + c_3 = c_1 + c_3 = c_1 + c_2 = 0$, so

$c_1 = c_2 = c_3 = 0$, that means v_1, v_2, v_3 are independent.

Problem 3.4.11

Sol. (a) line in $\mathbb{R}^3$ (b) plane in $\mathbb{R}^3$ (c) all of $\mathbb{R}^3$ (d) all of $\mathbb{R}^3$

Problem 3.4.20

Sol. $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ is one basis of the plane. the intersection with xy-plane means $z=0$, so

$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is a basis, $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ is a basis for all vectors perpendicar to the plane.

Problem 3.4.23

Sol. column 1 and column 2 are bases of column spaces. (different)

row 1 and row 2 are bases of the row spaces (same)

$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is the nullspace for both A and U (same)

Problem 3.4.38

Sol. (a) No, 2 vectors can't span $\mathbb{R}^3$

(b) No, 4 vectors are dependent in $\mathbb{R}^3$.

(c) Yes

(d) No, these vectors are dependent, as $2\begin{bmatrix}1\\2\\2\end{bmatrix}+2\begin{bmatrix}-1\\2\\1\end{bmatrix}=\begin{bmatrix}0\\8\\6\end{bmatrix}$.

Problem 3.5.2

Sol. $C(A^T)$. basis: $\begin{bmatrix}1\\2\\4\end{bmatrix}$ dim=1　$C(A)$: basis: $\begin{bmatrix}1\\2\end{bmatrix}$ dim=1

$N(A^T)$. basis: $\begin{bmatrix}-2\\1\end{bmatrix}$ dim=1　$N(A)$: basis: $\begin{bmatrix}-2\\1\\0\end{bmatrix},\begin{bmatrix}-4\\0\\1\end{bmatrix}$ dim=2

$C(B^T)$. basis: $\begin{bmatrix}1\\2\\4\end{bmatrix},\begin{bmatrix}2\\5\\8\end{bmatrix}$ dim=2　$C(B)$: basis: $\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix}$ dim=2

$N(B^T)$. basis: empty ($\begin{bmatrix}0\\0\end{bmatrix}$) dim=0　$N(B)$: basis: $\begin{bmatrix}-4\\0\\1\end{bmatrix}$ dim=1.

Problem 3.5.11

Sol. (a) No solution means $r<m$ (of course $r\le n$)

(b) As $m-r>0$, $N(A^T)$ should have nonzero vector.

Problem 3.5.18

Sol. row3 − 2row2 + row1 produced the zero row. so $\begin{bmatrix}1\\-2\\1\end{bmatrix}$ in the nullspace of A^T

and $\begin{bmatrix}1\\-2\\1\end{bmatrix}$ is also in the nullspace of A.

Problem 3.5.24

Sol. If solvable, d is in row space and the leftnullspace gives unique y.

Problem 1.

Sol. $\left[\begin{array}{cccc|c}2&3&-1&2&-1\\4&6&2&2&1\\6&9&1&2&-1\end{array}\right]\to\left[\begin{array}{cccc|c}2&3&-1&2&-1\\0&0&4&-2&3\\0&0&4&-4&2\end{array}\right]\to\left[\begin{array}{cccc|c}2&3&-1&2&-1\\0&0&4&-2&3\\0&0&0&-2&-1\end{array}\right]\to\left[\begin{array}{cccc|c}2&3&-1&0&-2\\0&0&4&0&4\\0&0&0&2&1\end{array}\right]$

$\to\left[\begin{array}{cccc|c}1&3/2&0&0&-1/2\\0&0&1&0&1\\0&0&0&1&1/2\end{array}\right]$, so the solution $x=x_p+x_n=\begin{bmatrix}-\frac{1}{2}\\0\\1\\\frac{1}{2}\end{bmatrix}+x_2\begin{bmatrix}-\frac{3}{2}\\1\\0\\0\end{bmatrix}$

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Problem 2.

Sol. (a) Put them into a matrix.

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1 & -2/3 & -2/3 \\ 0 & 0 & 4/3 & -2/3 \\ 0 & 0 & -2/3 & 4/3 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1 & -2/3 & -2/3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As the matrix have 4 pivots, these vectors form bases for $\mathbb{R}^4$.

(b) $$\begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 3/2 & -1 & -1/2 \\ 0 & -1 & 2 & -1 \\ 0 & -\frac{1}{2} & -1 & \frac{3}{2} \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 3/2 & -1 & -1/2 \\ 0 & 0 & 4/3 & -4/3 \\ 0 & 0 & -4/3 & 4/3 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 3/2 & -1 & -1/2 \\ 0 & 0 & 4/3 & -4/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then $\begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} -1 \\ 2 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, so they do not form bases for $\mathbb{R}^4$.

Problem 3.

Sol. $$A = \begin{bmatrix} -1 & 2 & -3 & 4 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} -1 & 2 & -3 & 4 \\ 0 & 10 & -10 & 12 \\ 0 & 5 & -5 & 6 \end{bmatrix} \to \begin{bmatrix} -1 & 2 & -3 & 4 \\ 0 & 1 & -1 & \frac{6}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & -\frac{8}{5} \\ 0 & 1 & -1 & \frac{6}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so $C(A)$: basis is the first two column. $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

$C(A^T)$: basis : $\begin{bmatrix} 1 \\ 0 \\ 1 \\ -\frac{8}{5} \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ -1 \\ \frac{6}{5} \end{bmatrix}$

$N(A)$: basis : $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 8/5 \\ -4/5 \\ 0 \\ 1 \end{bmatrix}$

$N(A^T)$: $$\begin{bmatrix} -1 & 3 & 2 \\ 2 & 4 & 1 \\ -3 & -1 & 1 \\ 4 & 0 & -2 \end{bmatrix} \to \begin{bmatrix} -1 & 3 & 2 \\ 0 & 10 & 5 \\ 0 & -10 & -5 \\ 0 & 12 & 6 \end{bmatrix} \to \begin{bmatrix} -1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

basis : $\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$

rank $A = 2$