数 学 作 业 纸 （科目：Calculus）

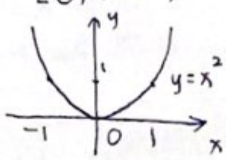
班级：CST01 姓名：容逸朗 编号：2020010869 第 1 页

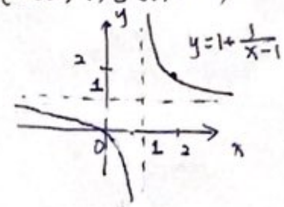
Problem A.

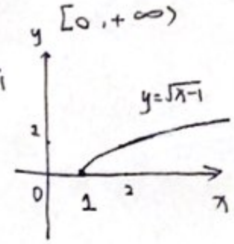
Sol.

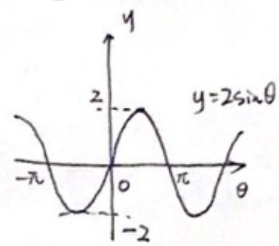
Function	(i) $y=x^2$	(ii) $y=1+\frac{1}{x-1}$	(iii) $y=\sqrt{x-1}$	(iv) $y=2\sin\theta$
(a) Domain	$(-\infty,+\infty)$	$(-\infty,1)\cup(1,+\infty)$	$[1,+\infty)$	$(-\infty,+\infty)$
(b) Range	$[0,+\infty)$	$(-\infty,1)\cup(1,+\infty)$	$[0,+\infty)$	$[-2,2]$

(c) Graph



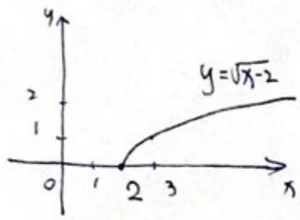


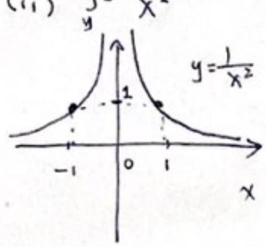


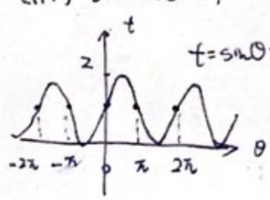


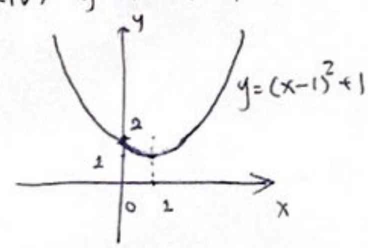
Problem B

Sol. (i) $y=\sqrt{x-2}$ (ii) $y=\frac{1}{x^2}$ (iii) $t=\sin\theta+1$ (iv) $y=(x-1)^2+1$









Problem C.

Sol. (i) $\lim\limits_{x\to 2} x^2-2 = 2^2-2=2$

(ii) $\lim\limits_{x\to 0}\frac{1}{\sqrt{x+3}-1} = \frac{1}{\sqrt{0+3}-1} = \frac{\sqrt{3}+1}{2}$

(iii) $\lim\limits_{x\to 1}\frac{x^2+x-2}{x^2-1} = \lim\limits_{x\to 1}\frac{(x-1)(x+2)}{(x+1)(x-1)} = \lim\limits_{x\to 1}\frac{x+2}{x+1} = \frac{1+2}{1+1} = \frac{3}{2}$

(iv) $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

(v) $\lim\limits_{\theta\to\infty}\frac{\cos\theta}{\theta} = 0$

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Problem D

Sol. Function (i) $f(x)=\frac{1}{x}$, $x=1$.　　(ii) $g(x)=\sin x$, $x=\pi$　　(iii) $h(x)=\frac{1}{1+x^2}$, $x=1$

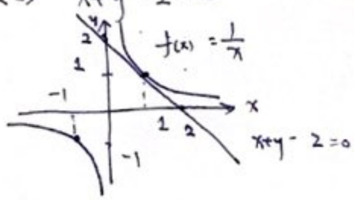
Derivative: $f'(x)=-x^{-2}=-\frac{1}{x^2}$　　$g'(x)=\cos x$　　$h'(x)=-(1+x^2)^{-2}\cdot 2x = \frac{-2x}{(x^2+1)^2}$

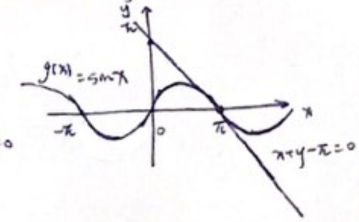
Tangent line: $y-\frac{1}{1}=-\frac{1}{1^2}(x-1)$ $\Leftrightarrow$ $x+y-2=0$

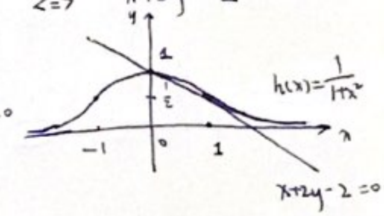
$y-\sin\pi=\cos\pi(x-\pi)$ $\Leftrightarrow$ $x+y-\pi=0$

$y-\frac{1}{1+1}=\frac{-2}{(1+1)^2}(x-1)$ $\Leftrightarrow$ $x+2y-2=0$

Graph:







Problem E.

Sol. (i) $y'=2x$

(ii) $y'=-2\sin x$

(iii) $y=\sqrt{x}=(x)^{\frac{1}{2}}$　so $y'=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

(iv) $y=(1-x^2)(1+x^2)=-x^4+1$, so $y'=-4x^3$

(v) $y=\frac{1-x}{1+x}=(1-x)(1+x)^{-1}$, so $y'=-(1+x)^{-1}+-(1+x)^{-2}(1-x)=-\frac{2}{(1+x)^2}$

Problem F.

Sol. (i) $y'=(\sin 2x)'=\cos 2x\cdot(2x)'=2\cos 2x$

(ii) $y'=(\cos x^2)'=-\sin x^2\cdot(x^2)'=-2x\sin x^2$

(iii) $y'=(\sqrt{1-x^2})'=\frac{1}{2}(1-x^2)^{\frac{1}{2}}\cdot(1-x^2)'=-\frac{x}{\sqrt{1-x^2}}$