数 学 作 业 纸

（科目：Calculus）

班级：CS 01　　姓名：宋逸朗　　编号：2020010869　　第 1 页

Problem A.

1. Sol. $\lim_{x\to -7}(2x+5) = 2\times(-7)+5 = -9$

3. Sol. $\lim_{x\to 2}(-x^2+5x-2) = -2^2+5\times 2-2 = 4$

15. Sol. $\lim_{h\to 0}\frac{3}{\sqrt{3h+1}+1} = \frac{3}{\sqrt{3\times 0+1}+1} = \frac{3}{2}$

Problem B.

42. Sol. a. $\lim_{x\to -2}(p(x)+r(x)+s(x)) = \lim_{x\to -2}p(x)+\lim_{x\to -2}r(x)+\lim_{x\to -2}s(x) = 4+0-3 = 1$

b. $\lim_{x\to -2}p(x)\cdot r(x)\cdot s(x) = \lim_{x\to -2}p(x)\cdot\lim_{x\to -2}r(x)\cdot\lim_{x\to -2}s(x) = 4\times 0\times(-3) = 0$

c. $\lim_{x\to -2}\frac{-4p(x)+5r(x)}{s(x)} = \frac{-4\times 4+5\times 0}{-3} = \frac{16}{3}$

Problem C.

49. Sol. By sandwich theorem, we have $\lim_{x\to 0}\sqrt{5-2x^2} = \sqrt{5} = \lim_{x\to 0}\sqrt{5-x^2}$,

so $\lim_{x\to 0}f(x) = \sqrt{5}$.

50. Sol. By sandwich theorem, we have $\lim_{x\to 0}2-x^2 = 2 = \lim_{x\to 0}2\cos\theta$

so $\lim_{x\to 0}g(x) = 2$.

Problem D.

53. Sol. If $\lim_{x\to c}f(x)$ exists, $\lim_{x\to c}x^4 = \lim_{x\to c}x^2$, then $c^4 = c^2 \Rightarrow c^2(c^2-1)=0$,

which means $c = 0, 1, -1$, thus. $\lim_{x\to 0}f(x) = \lim_{x\to 0}x^4 = 0$, $\lim_{x\to 1}f(x) = \lim_{x\to 0}x^4 = 1$

$\lim_{x\to -1}f(x) = \lim_{x\to -1}x^4 = 1$.

Problem E.

7. Sol. $|x-5|<\delta \Rightarrow -\delta<x-5<\delta \Rightarrow 5-\delta<x<5+\delta$

From the graph we have $5-\delta = 4.9 \Rightarrow \delta = 0.1$ or $5+\delta = 5.1 \Rightarrow \delta = 0.1$, thus $\delta = 0.1$

9. Sol. $|x-1|<\delta \Rightarrow -\delta<x-1<\delta \Rightarrow 1-\delta<x<1+\delta$

From the graph we have $1-\delta = \frac{9}{16} \Rightarrow \delta = \frac{7}{16}$ or $1+\delta = \frac{25}{16} \Rightarrow \delta = \frac{9}{16}$, thus $\delta = \frac{7}{16}$

11. Sol. $|x-2|<\delta \Rightarrow -\delta<x-2<\delta \Rightarrow 2-\delta<x<2+\delta$

From the graph we have $2-\delta = \sqrt{3} \Rightarrow \delta = 2-\sqrt{3}$ or $2+\delta = \sqrt{5} \Rightarrow \delta = \sqrt{5}-2$, thus $\delta = \sqrt{5}-2$.

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Problem F.

51. As x approaches 0, the value $g(x)$ approaches k.
    For any number $\varepsilon > 0$, there exists a $\delta > 0$ that $0 < |x-0| < \delta \Rightarrow |g(x)-k| < \varepsilon$

52. We have $x = h+c$ and $0 < |x-c| < \delta \Leftrightarrow -\delta < x-c < \delta,\ x \neq c \Leftrightarrow -\delta < (h+c)-c < \delta,\ h+c \neq c$
    $\Leftrightarrow -\delta < h < \delta,\ h \neq 0 \Leftrightarrow 0 < |h-0| < \delta$
    Thus $\lim_{x\to c} f(x) = L \Leftrightarrow$ for any $\varepsilon > 0$, there have $\delta > 0$ such that $|f(x)-L| < \varepsilon$ where $0 < |x-c| < \delta$
    $\Leftrightarrow |f(h+c)-L| < \varepsilon$ while $0 < |h-0| < \delta \Leftrightarrow \lim_{h\to 0} f(h+c) = L$