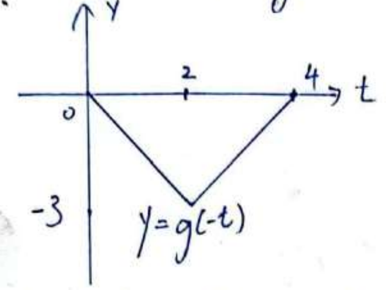
清華大學

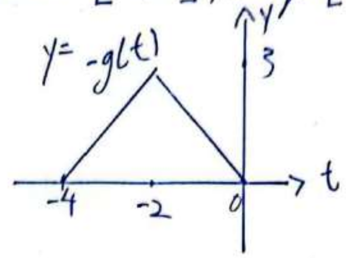
Tsinghua University

§1.5 Exercise 50

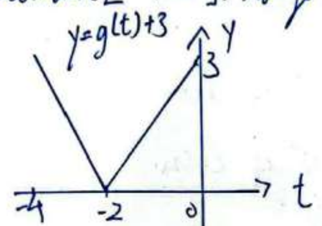
a. domain [0, 4], range [-3, 0]



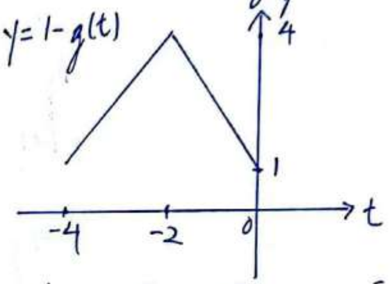
b. domain [-4, 0], range [0, 3]



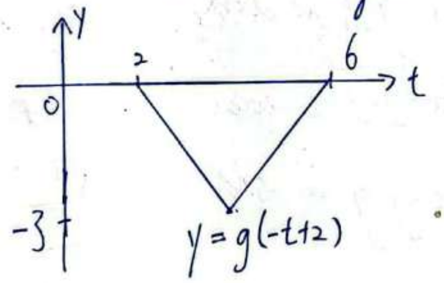
c. domain [-4, 0], range [0, 3]



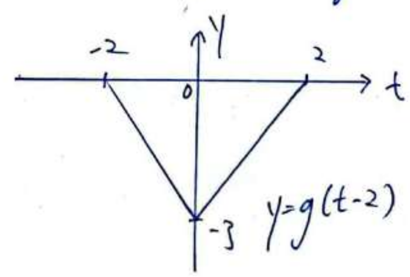
d. domain [-4, 0], range [1, 4]



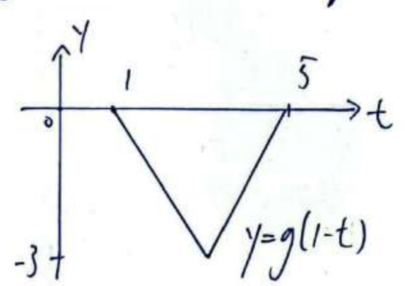
e. domain [2, 6], range [-3, 0]



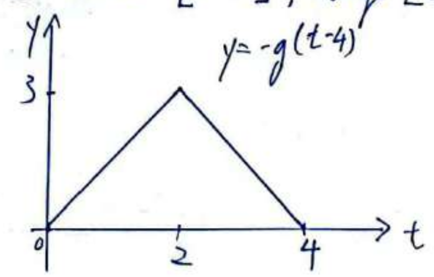
f. domain [-2, 2], range [-3, 0]



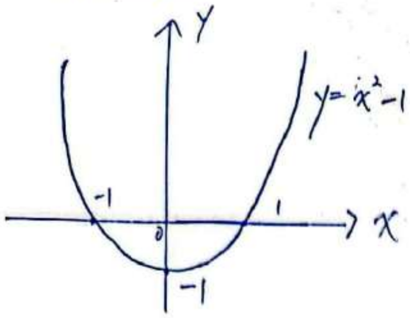
g. domain [1, 5], range [-3, 0]



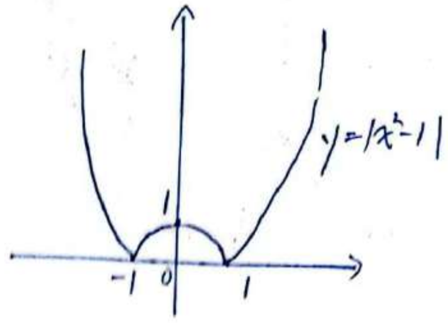
h. domain [0, 4], range [0, 3]



§ 1.5 Exercise 67:



$\longrightarrow$



this is the graph of $y=|x^2-1|$

§ 1.5 Exercise 79

a. $f(-x)\cdot g(-x) = f(x)\cdot(-g(x)) = -(f(x)\cdot g(x))$ so fg is odd

b. $f(-x)/g(-x) = f(x)/(-g(x)) = -\left(\frac{f(x)}{g(x)}\right)$ so f/g is odd

c. $g(-x)/f(-x) = (-g(x))/f(x) = -\left(\frac{g(x)}{f(x)}\right)$ so g/f is odd

d. $f(-x)\cdot f(-x) = f(x)\cdot f(x)$ so f^2 is even

e. $g(-x)\cdot g(-x) = -g(x)\cdot(-g(x)) = g(x)\cdot g(x)$ so g^2 is even

f. $f(g(-x)) = f(-g(x)) = f(g(x))$ so $f\circ g$ is even

g. $g(f(-x)) = g(f(x))$ so $g\circ f$ is even

h. $f(f(-x)) = f(f(x))$ so $f\circ f$ is even

i. $g(g(-x)) = g(-g(x)) = -g(g(x))$ so $g\circ g$ is odd

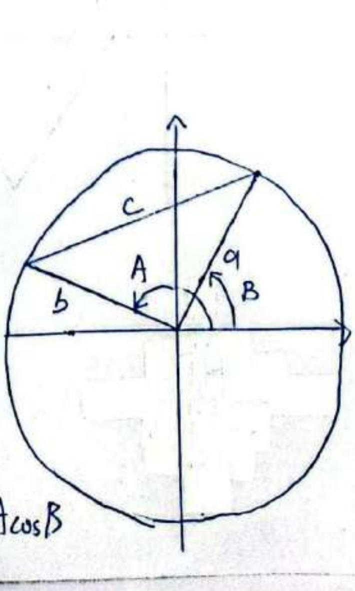
§ 1.5 Exercise 80

$f(x)=0$ if both even and odd. cause $f(x)=-f(-x)=0$, and $f(x)=f(-x)=0$

§ 1.6 Exercise 49

$$\sin^2\frac{\pi}{12} = \frac{1-\cos(2\cdot\frac{\pi}{12})}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$$

§ 1.6 Exercise 53



the law of cosines: $c^2 = a^2+b^2-2ab\cos(A-B) = 2-2\cos(A-B)$

and at the same time, according to the distance between two point, we get:

$c^2 = (\sin B - \sin A)^2 + (\cos B - \cos A)^2 = 2 - 2\sin A\sin B - 2\cos B\cos A$

so $-2\cos(A-B) = -2(\sin A\sin B + \cos A\cos B) \Leftrightarrow \cos(A-B) = \sin A\sin B + \cos A\cos B$

清華大學
Tsinghua University

A1. $\cos(3x) = \sin(2x)$
$\Downarrow$
$\sin(\frac{\pi}{2} - 3x) = \sin(2x)$

$\frac{\pi}{2} - 3x + 2k\pi = 2x$ or $\pi - (\frac{\pi}{2} - 3x) + 2k\pi = 2x$ $(k \in \mathbb{Z})$

$x = \frac{\pi}{10} + \frac{2}{5}k\pi$ or $x = \frac{\pi}{2} + 2k\pi$ $(k \in \mathbb{Z})$

A2. according to condition 1 and 2 we get:

$\forall x \in [1, 2]$, $f(x) = f(x-1) + 2$ ①

$\forall x \in [1, 2]$, $f(x-1) = 2x - 2$ ②

we substitute ② into ①:

$\forall x \in [1, 2]$, $f(x) = 2x - 2 + 2 = 2x$

and $\forall x \in [0, 1]$, $f(x) = 2x$ (condition) 2

because $[0, 2] = [0, 1] \cup [1, 2]$, so $f(x) = 2x$

Bonus exercise

§1.6 exercise 57.

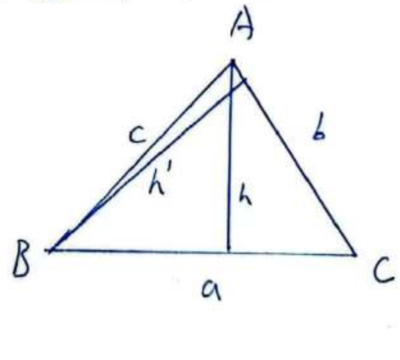
first $c \cdot \sin B = h$, and if $C \leq 90°$, $b \cdot \sin C = h$, if $C > 90°$, $b \cdot \sin(\pi - C) = h \Leftrightarrow b \cdot \sin C = h$

so $h = c \cdot \sin B = b \cdot \sin C \Leftrightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$

if we draw other height of the triangle h', we get:

$h' = c \cdot \sin A = a \cdot \sin C \Leftrightarrow \frac{\sin A}{a} = \frac{\sin C}{c}$

so $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



B1. $\cos(\frac{\pi}{10}\cdot 3) = \sin(\frac{\pi}{10}\cdot 2)$

$\cos(3x) = 4\cos^3 x - 3\cos x \Leftrightarrow \cos(\frac{\pi}{15}\cdot 3) = 4\cdot\cos^3(\frac{\pi}{15}) - 3\cos(\frac{\pi}{15})$

$\sin(2x) = 2\sin x\cos x \Leftrightarrow \sin(\frac{\pi}{15}\cdot 2) = 2\sin\frac{\pi}{15}\cos\frac{\pi}{15}$

$4\cos^3(\frac{\pi}{10}) - 3\cos(\frac{\pi}{10}) = 2\sin(\frac{\pi}{1})\cos(\frac{\pi}{10})$

$4\cos^2(\frac{\pi}{10}) - 3 = 2\sin(\frac{\pi}{10})$

$4\sin^2(\frac{\pi}{10}) + 2\sin(\frac{\pi}{10}) - 1 = 0$

$\sin(\frac{\pi}{10}) = \frac{\sqrt{5}-1}{4}$ or $\sin(\frac{\pi}{10}) = \frac{-\sqrt{5}-1}{4}$

because $\sin(\frac{\pi}{10}) \geq -1$, so $\sin(\frac{\pi}{10}) = \frac{\sqrt{5}-1}{4}$

$\cos(\frac{\pi}{5}) = 1 - 2\sin^2(\frac{\pi}{10}) = 1 - (\frac{\sqrt{5}-1}{4})^2 = \frac{\sqrt{5}+1}{4}$