清华大学 TSINGHUA UNIVERSITY 数学作业纸

班级：CST01 姓名：寇逸朗 编号：2020010869 科目：Calculus 第 1 页

Problem set A.

(i) sol. $y(x) = \int y'(x)\,dx = \int 3x^2\,dx = x^3 + C$

(ii) sol. $y(x) = \int y'(x)\,dx = \int \cos x\,dx = \sin x + C$

(iii) sol. $y(x) = \int y'(x)\,dx = \int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$

(iv) sol. $y(x) = \int 2x\sqrt{x^2+1}\,dx = \int \sqrt{u}\,du \ (u = x^2+1) = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$.

Problem set B

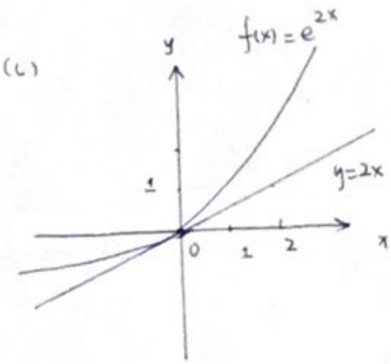
(i) sol. (a). $f'(x) = 2e^{2x}$

(b) $f(0) = e^0 - 1 = 0$, $f'(0) = 2\cdot e^{2\cdot 0} = 2$.

so equation we want is $y = f(0) + f'(0)(x-0)$

$= 2x$.

(c)




$f'(x) = 2e^{2x} > 0$, $f''(x) = 4e^{2x} > 0$

(ii) sol. (a) $f'(x) = \frac{1}{2(x+1)}$

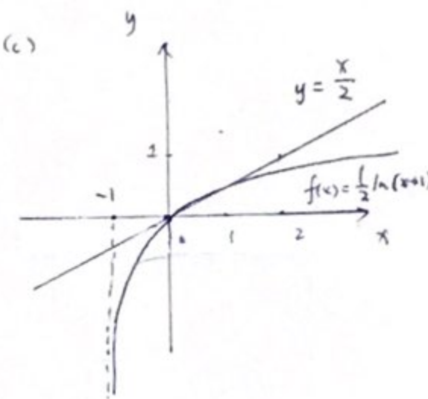
(b) $f(0) = \frac{\ln(0+1)}{2} = 0$, $f'(0) = \frac{1}{2(0+1)} = \frac{1}{2}$

tangent line of $x=0$: $y = f(0) + f'(0)(x-0)$

$= 0 + \frac{1}{2}x$

$= \frac{1}{2}x$

(c)



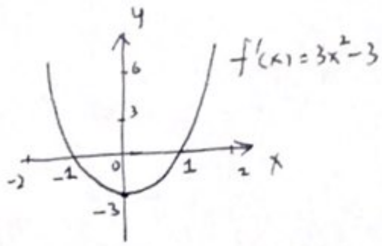

$f''(x) = -\frac{1}{2(x+1)^2} < 0$

| x | $(-\infty,-1)$ | $(-1,+\infty)$ |
|---|---|---|
| y' | − | + |
| y | ↘ | ↗ |

Problem set C.

Sol. (a) $f'(x) = 3x^2 - 3$

(b)




(c)

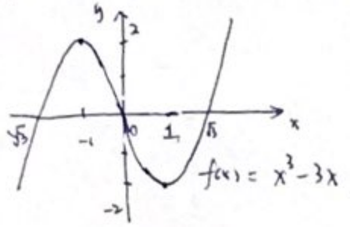
| x | $(-\infty, -1)$ | $-1$ | $(-1, 1)$ | $1$ | $(1, +\infty)$ |
|---|---|---|---|---|---|
| y' | + | 0 | − | 0 | + |
| y | ↗ | | ↘ | | ↗ |

for $x \in (-1, 1)$, $f'(x) < 0$.

(d) $(-1, 1)$ is the largest interval where $f(x)$ is decreasing

(e) $f(x) = x^3 - 3x = x(x^2-3) = x(x+\sqrt{3})(x-\sqrt{3})$, so roots of $f(x)$ is $0, -\sqrt{3}, \sqrt{3}$.

(f)

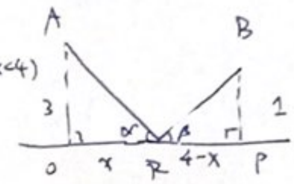



| x | $(-\infty, 0)$ | $0$ | $(0, +\infty)$ |
|---|---|---|---|
| $f''(x)$ | − | 0 | + |
| $f(x)$ | concave down | | concave up |

$f''(x) = 6x$

Problem set D.

Sol. (a) $f(x) = \sqrt{x^2+3^2} + \sqrt{(4-x)^2+1^2} = \sqrt{x^2+9} + \sqrt{x^2-8x+17}$ $(0<x<4)$




(b) $f'(x) = \dfrac{x}{\sqrt{x^2+9}} + \dfrac{x-4}{\sqrt{x^2-8x+17}}$ $(0<x<4)$

(c) Let $\alpha$ be the angle between AR and OR, so $\cos\alpha = \dfrac{x}{\sqrt{x^2+9}}$,

Let $\beta$ be the angle between BR and PR, so $\cos\beta = \dfrac{4-x}{\sqrt{x^2-8x+17}}$

and $f'(x) = \dfrac{x}{\sqrt{x^2+9}} - \dfrac{4-x}{\sqrt{x^2-8x+17}} = 2\cos\alpha - 2\cos\beta$

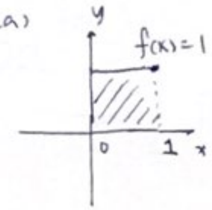
let $f'(x) = 0$, which is $\cos\alpha = \cos\beta$, (as $0<x<4$, $\alpha, \beta < \frac{\pi}{2}$), because $\alpha, \beta < \frac{\pi}{2}$.

so $\alpha = \beta$, then $\triangle RAO \sim \triangle RBP$, with $\frac{3}{x} = \frac{1}{4-x}$ gives $x = 3$, where $f(x) = 4\sqrt{2}$

is the local minimum of $f(x)$.

Problem set E.

(i) sol. (a)



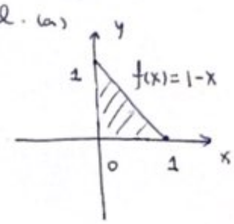

(b) $\int_0^1 f(x)\,dx$

$= \int_0^1 1\,dx$

$= x \Big|_0^1$

$= 1-0 = 1$

(ii). sol. (a)



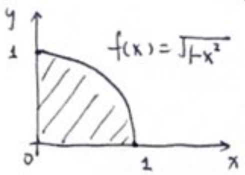

(b) $\int_0^1 f(x)\,dx$

$= \int_0^1 1-x \;dx$

$= x - \frac{1}{2}x^2 \Big|_0^1$

$= 1 - \frac{1}{2} - (0-0) = \frac{1}{2}$

(iii) sol (a)

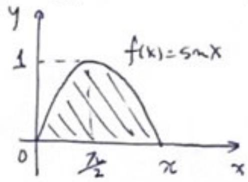



(b) $\int_0^1 f(x)\,dx$

$= \frac{1}{4}\pi \cdot 1^2$

$= \frac{\pi}{4}$

(iv) sol.



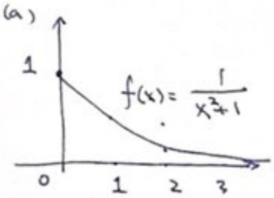

(b) $\int_0^{\pi} f(x)\,dx$

$= \int_0^{\pi} \sin x\,dx$

$= -\cos x \Big|_0^{\pi}$

$= 1-(-1) = 2$

(v) sol. (a)




(b) $\int_0^{\infty} f(x)\,dx$

$= \lim_{b\to\infty} \int_0^b \frac{1}{x^2+1}\,dx$

$= \lim_{b\to\infty} \tan^{-1} x \Big|_0^b$

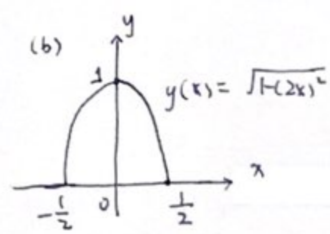
$= \lim_{b\to\infty} \tan^{-1} b - \tan^{-1} 0$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Problem set F.

sol. (a) we should have $1-(2x)^2 \geq 0$, so $x \in [-\frac{1}{2}, \frac{1}{2}]$ (is the domain)

(b)




(c) range of $y(x)$: $[0, 1]$

(d) $V = \int_{-1/2}^{1/2} \pi\, y^2(x)\,dx = \int_{-1/2}^{1/2} \pi\,[1-4x^2]\,dx$

(e) $V = \int_{-1/2}^{1/2} \pi\,(1-4x^2)\;dx = \pi\left(x - \frac{4}{3}x^3\right)\Big|_{-1/2}^{1/2} = \pi\left[\frac{1}{2} - \frac{1}{6} - \left(-\frac{1}{2}+\frac{1}{6}\right)\right] = \frac{2\pi}{3}$

(f) $V = \frac{2\pi}{3}$.

Problem set G.

(i) sol. $\int x\cos(x^2)\,dx = \int x\cos x^2 \frac{dx^2}{2x} = \frac{1}{2}\int \cos x^2 dx^2 = \frac{1}{2}\sin x^2 + C$

(ii) sol. $\int \frac{2x}{x^2+16}dx = \int \frac{2x}{x^2+16}\cdot\frac{d(x^2+16)}{2x} = \int \frac{d(x^2+16)}{x^2+16} = \ln|x^2+16| + C = \ln(x^2+16) + C.$

(iii) sol. $\int \sec^2\theta\, d\theta = \tan\theta + C$

Problem set H.

Sol. (a) $e^x + \frac{dy}{dx} = e^x + y^2e^x \Rightarrow \frac{dy}{dx} = y^2\cdot e^x$ (where $a(x)=e^x$ and $b(y)=y^2$)

(b) $\frac{dy}{y^2} = e^x dx$

$\int \frac{dy}{y^2} = \int e^x dx$

$-\frac{1}{y} = e^x + C.$

$y = -\frac{1}{e^x+C}$