Problem 5.1.2.

Sol. As $|A|=3$, $\det(\frac{1}{2}A)=(\frac{1}{2})^3\cdot\det A=-\frac{1}{8}$

$\det(-A)=(-1)^3\det A = 1$

$\det(A^2)=(\det A)^2 = 1$

$\det(A^{-1})=\frac{1}{\det A} = -1$

Problem 5.1.7

Sol. $\det Q = \cos^2\theta - (-\sin^2\theta) = 1$

$\det Q = (1-2\cos^2\theta)(1-2\sin^2\theta) - (-2\cos\theta\sin\theta)^2 = 1+4\cos^2\theta\sin^2\theta - 2(\sin^2\theta+\cos^2\theta) - 4\cos^2\theta\sin\theta^2$

$= -1.$

Problem 5.1.13

Sol $A=\begin{bmatrix}1&1&1\\1&2&2\\1&2&3\end{bmatrix}\to\begin{bmatrix}1&1&1\\0&1&1\\0&1&2\end{bmatrix}\to\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}=U, \det A = 1$

$A=\begin{bmatrix}1&2&3\\2&2&3\\3&3&3\end{bmatrix}\to\begin{bmatrix}1&2&3\\0&-2&-3\\0&-3&-6\end{bmatrix}\to\begin{bmatrix}1&2&3\\0&-2&-3\\0&0&-3/2\end{bmatrix}=U$, $\det A = 3$

Problem 5.1.18

Sol. $\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix} = \begin{vmatrix}1&a&a^2\\0&b-a&b^2-a^2\\0&c-a&c^2-a^2\end{vmatrix} = (b-a)(c-a)\begin{vmatrix}1&a&a^2\\0&1&b+a\\0&1&c+a\end{vmatrix} = (b-a)(c-a)\begin{vmatrix}1&a&a^2\\0&1&b+a\\0&0&c-b\end{vmatrix}$

$= (b-a)(c-a)(c-b).$

Problem 5.1.22

Sol. $\det A = 2\times2 - 1\times1 = 3$

$\det A^{-1} = (\frac{1}{3})^2(2\times2-(-1)\times(-1)) = 1/3$

Let $(2-\lambda)^2-1=0$, then $\lambda=1$ or $\lambda=3$, which is

$A-I=\begin{bmatrix}1&1\\1&1\end{bmatrix}$, $A-3I=\begin{bmatrix}-1&1\\1&-1\end{bmatrix}$

Problem 5.1.30

Sol $\begin{bmatrix}\frac{df}{da}&\frac{df}{dc}\\\frac{df}{db}&\frac{df}{dd}\end{bmatrix} = \begin{bmatrix}\frac{d}{ad-bc}&\frac{-b}{ad-bc}\\\frac{-c}{ad-bc}&\frac{a}{ad-bc}\end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix} = A^{-1}$

Problem 5.2.1

Sol. $\det A = 1+12+18-9-6-4 = 12$, the rows of A are independent

$\det B = 28+40+72-60-24-56 = 0$. rows of B are dependent (As row1+row2=row3)

$\det C = 0+0+0-1-0-0=-1$, the rows of C are independent.

Problem 5.2.4

Sol. For A, we can see $a_{11}a_{23}a_{32}a_{44} = -1$ and $a_{14}a_{23}a_{32}a_{41} = 1$, $\det A = -1+1 = 0$

For B, with same entry of A, $\det B = -b_{11}b_{23}b_{32}b_{44} + b_{14}b_{23}b_{32}b_{41} = -16+64 = 48$.

Problem 5.2.15

Sol. (a) the cofactor of entry 1.1 is E_{n-1}

the cofactor of entry 1.2 has single 1 on the first column, which cofactor is E_{n-2}.

As $1+2=3$ is odd, the sign should be $-E_{n-2}$

so $E_n = E_{n-1} - E_{n-2}$.

(b) $E_1 = 1$, $E_2 = 0$, $E_3 = -1$, $E_4 = -1$, $E_5 = 0$, $E_6 = 1$, $E_7 = 1$, $E_8 = 0$

(c) We can see the period is 6, so $E_{100} = E_4 = -1$.

Problem 5.2.19.

Sol. (a) This is because $x \cdot x^2$ and x^3 is in the same row of V_4, they can't multiple each other.

(b) Let $r_1=a, r_2=b, r_3=c$, then V_4 have same rows so $V_4=0$.

(c) From (b). we can see V_4 have vector $(x-a)(x-b)(x-c)$, so $V_4 = A(x-a)(x-b)(x-c)$.

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & x & x^2 & x^3 \end{vmatrix} = \begin{vmatrix} 0 & a-x & a^2-x^2 & a^3-x^3 \\ 0 & b-x & b^2-x^2 & b^3-x^3 \\ 0 & c-x & c^2-x^2 & c^3-x^3 \\ 1 & x & x^2 & x^3 \end{vmatrix} = -(a-x)(b-x)(c-x)\begin{vmatrix} 1 & a+x & a^2+ax+x^2 \\ 1 & b+x & b^2+bx+x^2 \\ 1 & c+x & c^2+cx+x^2 \end{vmatrix}$$

As $$\begin{vmatrix} 1 & a+x & a^2+ax+x^2 \\ 1 & b+x & b^2+bx+x^2 \\ 1 & c+x & c^2+cx+x^2 \end{vmatrix} = \begin{vmatrix} 1 & a+x & a^2+ax+x^2 \\ 0 & b-a & b^2-a^2+x(b-a) \\ 0 & c-a & c^2-a^2-x(c-a) \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & b+a+x \\ 1 & c+a+x \end{vmatrix} = (b-a)(c-a)(c-b)$$

(d) So that $V_4 = (x-a)(x-b)(x-c)\cdot(b-a)(c-a)(c-b)$, where $A = (b-a)(c-a)(c-b)$,

Problem 5.2.31

Sol. $\det P = -1$, as the cofactor of P_{14} is I_3. which determinant is 1, and sign of P_{14} is minus.

and it takes $2n+1$ $(n\in Z^+)$ times to change 4, 1, 2 3 into 1, 2, 3, 4.

Problem 5.2.34.

Sol. (a) Because there exists same rows.

(b) Choose the last three rows first, and the third choise will be zero.

Problem 1.

Sol. $A=\begin{bmatrix}1&1&1&1&1\\1&2&3&4&5\\1&3&6&10&15\\1&4&10&20&35\\1&5&15&35&70\end{bmatrix}\rightarrow\begin{bmatrix}1&1&1&1&1\\0&1&2&3&4\\0&2&5&9&14\\0&3&9&19&34\\0&4&14&34&69\end{bmatrix}\rightarrow\begin{bmatrix}1&1&1&1&1\\0&1&2&3&4\\0&0&1&3&6\\0&0&3&10&22\\0&0&6&22&53\end{bmatrix}\rightarrow\begin{bmatrix}1&1&1&1&1\\0&1&2&3&4\\0&0&1&3&6\\0&0&0&1&4\\0&0&0&4&17\end{bmatrix}\rightarrow\begin{bmatrix}1&1&1&1&1\\0&1&2&3&4\\0&0&1&3&6\\0&0&0&1&4\\0&0&0&0&1\end{bmatrix}$

$\det A = 1.$

Problem 2.

Sol.

$$\det A=\begin{vmatrix}1&-1&1&-1&1\\1&2&3&4&5\\0&-1&0&0&0\\4&0&-2&0&1\\0&-2&0&2&0\end{vmatrix}=(-1)^{(3+2)}\cdot(-1)\begin{vmatrix}1&1&-1&1\\1&3&4&5\\4&-2&0&1\\0&0&2&0\end{vmatrix}$$

$$=(-1)^{(4+3)}\cdot 2\cdot\begin{vmatrix}1&1&1\\1&3&5\\4&-2&1\end{vmatrix}$$

$$=-2\times(3+20-2-12+10-1)$$

$$=-36$$