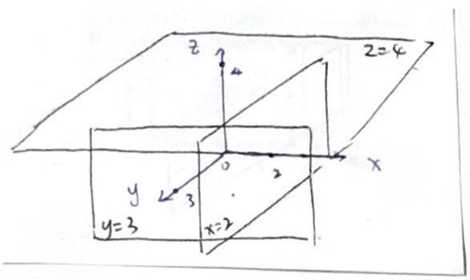
数 学 作 业 纸

（科目：Linear Algebra）

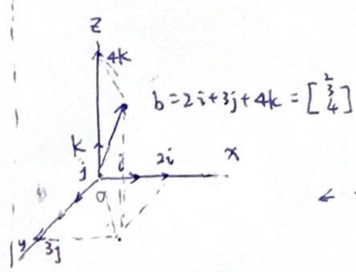
班级：CS 01　　姓名：容逸朗　　编号：2020010869　　第 1 页

Problem 2.1.1

Sol. We have $\begin{cases} x=2 \\ y=3 \\ z=4 \end{cases}$, and the row picture is:



The colmun vectors are $i=[1\ 0\ 0]^T$, $j=[0\ 1\ 0]^T$, $k=[0\ 0\ 1]^T$



← the colmun picture.

Problem 2.1.4

Sol If $z=2$, then $\begin{cases} x+y=0 \\ x-y=2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=-1 \end{cases}$, and $(1,-1,2)$ is the point we want.

If $z=0$, then $\begin{cases} x+y=6 \\ x-y=4 \end{cases} \Rightarrow \begin{cases} x=5 \\ y=1 \end{cases}$, $(5,1,0)$ is the point.

The third point halfway between is $\left(\frac{5+1}{2}, \frac{1-1}{2}, \frac{2+0}{2}\right) = (3,0,1)$

Problem 2.1.17

Sol We can see $\begin{bmatrix} 0&1&0 \\ 0&0&1 \\ 1&0&0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}$ and $\begin{bmatrix} 0&0&1 \\ 1&0&0 \\ 0&1&0 \end{bmatrix}\begin{bmatrix} y \\ z \\ x \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

so, $P = \begin{bmatrix} 0&1&0 \\ 0&0&1 \\ 1&0&0 \end{bmatrix}$ and $Q = \begin{bmatrix} 0&0&1 \\ 1&0&0 \\ 0&1&0 \end{bmatrix}$ is the answer.

Problem 2.1.22.

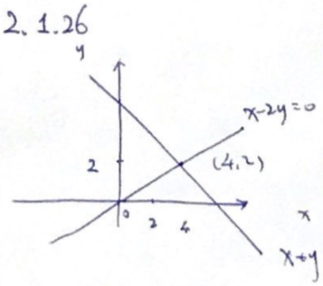
Sol. dot product $Ax = [1\ 4\ 5]\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

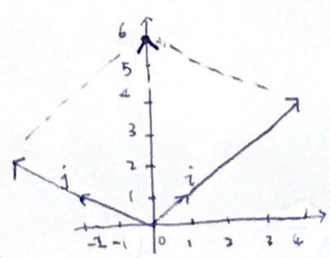
The solution of $Ax=0$ lie on a plane in $\mathbb{R}^3$ perpendicular to the vector $\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$.

and the colmuns of A are vectors in one-dimensional space.

Problem 2.1.26

Sol.





Two colmun vector are $i=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $j=\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

and the combination $4i+2j = 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

数 学 作 业 纸

（科目：Linear Algebra）

班级： CS 01　　姓名： 岑逸朗　　编号： 2020010869　　第 2 页

Problem 2.1.29

Sol.

$$u_2 = Au_1 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .64 + .06 \\ .16 + .14 \end{bmatrix} = \begin{bmatrix} .7 \\ .3 \end{bmatrix}$$

$$u_3 = Au_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .56 + .09 \\ .14 + .21 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$$

we can mention that the sum of the components alway equal to 1.

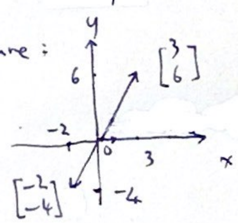
Problem 2.2.6

Sol. We can make it singular by $b=4$ because $4x+8y$ is two times $2x+4y$, then we can make it solvable by $g=32$ so that two equation become the same, and the possible solution could be $(8,0)$ and $(6,1)$.

Problem 2.2.9

Sol. We can mention that $3x-2y=b_1 \iff 6x-4y=2b_1$, with equation $6x-4y=b_2$ we can see $2b_1=b_2$ make the equations have solutions. There would have infinitely many solutions as two equation become the same.

Column picture:



($b=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $b=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ share the same column picture).

Problem 2.2.13

Sol.

$$\begin{matrix} 2x-3y = 3 \\ 4x-5y+z=7 \\ 2x-y-3z=5 \end{matrix} \xrightarrow[\text{subtract 1 time row1 from row3}]{\text{subtract 2times row1 from row 2}} \begin{matrix} 2x-3y = 3 \\ y+z = 1 \\ 2y-3z = 2 \end{matrix} \xrightarrow{\text{subtract 2 times row2 from row 3}} \begin{matrix} 2x-3y=3 \\ y+z=1 \\ -5z=0 \end{matrix} \Rightarrow \begin{matrix} x=3 \\ y=1 \\ z=0 \end{matrix}$$

Problem 2.2.18

Sol. $\begin{bmatrix} 1 & 4 & 13 \\ 2 & 8 & 26 \\ 3 & 12 & 39 \end{bmatrix}$ is one possible matrix, and there has infinitely many solutions for $b=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ because three equation are the same, and no solution for $b=\begin{bmatrix} 1 \\ 10 \\ 100 \end{bmatrix}$.

数 学 作 业 纸

（科目：Linear Algebra）

班级：CS 01　　姓名：容逸朗　　编号：2020010869　　第 3 页

Problem 2.2.21

Sol.

$$\begin{cases}2x+y=0\\ x+2y+z=0\\ y+2z+t=0\\ z+2t=5\end{cases}\xrightarrow{r_2-\frac{1}{2}r_1}\begin{cases}2x+y=0\\ \frac{3}{2}y+z=0\\ y+2z+t=0\\ z+2t=5\end{cases}\xrightarrow{r_3-\frac{2}{3}r_2}\begin{cases}2x+y=0\\ \frac{3}{2}y+z=0\\ \frac{4}{3}z+t=0\\ z+2t=5\end{cases}\xrightarrow{r_4-\frac{3}{4}r_3}\begin{cases}2x+y=0\\ \frac{3}{2}y+z=0\\ \frac{4}{3}z+t=0\\ \frac{5}{4}t=5\end{cases}$$

Pivots of A are $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$, by back substitution we have $t=4$, $z=-3$, $y=2$, $x=-1$.

$$\begin{cases}2x-y=0\\ -x+2y-z=0\\ -y+2z-t=0\\ -z+2t=5\end{cases}\xrightarrow{r_2+\frac{1}{2}r_1}\begin{cases}2x-y=0\\ \frac{3}{2}y-z=0\\ -y+2z-t=0\\ -z+2t=5\end{cases}\xrightarrow{r_3+\frac{2}{3}r_2}\begin{cases}2x-y=0\\ \frac{3}{2}y-z=0\\ \frac{4}{3}z-t=0\\ -z+2t=5\end{cases}\xrightarrow{r_4+\frac{3}{4}r_3}\begin{cases}2x-y=0\\ \frac{3}{2}y-z=0\\ \frac{4}{3}z-t=0\\ \frac{5}{4}t=5\end{cases}$$

Pivots of K are $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$, by back substitution we have $t=4$, $z=3$, $y=2$, $x=1$.

Problem 2.2.27

Sol.

$$\begin{cases}3x=3\\ 6x+2y=8\\ 9x-2y+z=9\end{cases}\xrightarrow[r_2-2r_1]{r_3-3r_1}\begin{cases}3x=3\\ 2y=2\\ -2y+z=0\end{cases}\xrightarrow{r_3+r_2}\begin{cases}3x=3\\ 2y=2\\ z=2\end{cases},\quad U=\begin{bmatrix}3&0&0\\0&2&0\\0&0&1\end{bmatrix}$$

So, U is a diagonal matrix and the solution is $\begin{cases}x=1\\y=1\\z=2\end{cases}$

Problem 1.

Sol. Matrix-vector equation: $\begin{bmatrix}2&-2\\-1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}$

vector equation: $x\cdot\begin{bmatrix}2\\-1\end{bmatrix}+y\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}$

$$\begin{cases}2x-2y=b_1\\ -x+2y=b_2\end{cases}\xrightarrow{r_2+\frac{1}{2}r_1}\begin{cases}2x-2y=b_1\\ y=b_2+\frac{1}{2}b_1\end{cases}\xrightarrow{r_1+2r_2}\begin{cases}2x=2b_1+2b_2\\ y=b_2+\frac{1}{2}b_1\end{cases}\Rightarrow\begin{cases}x=b_1+b_2\\ y=\frac{1}{2}b_1+b_2\end{cases}$$

If $b=\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&1\\\frac{1}{2}&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\\frac{1}{2}\end{bmatrix}$, which means $1\cdot\begin{bmatrix}2\\-1\end{bmatrix}+\frac{1}{2}\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$.

If $b=\begin{bmatrix}0\\1\end{bmatrix}$, $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&1\\\frac{1}{2}&1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$, so $1\cdot\begin{bmatrix}2\\-1\end{bmatrix}+1\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$

Problem 2

Sol.

$$\begin{cases}ax+by=f\\ cx+dy=g\end{cases}\xrightarrow[(a\neq0)]{r_2-\frac{c}{a}r_1}\begin{cases}ax+by=f\\ \frac{ad-bc}{a}\cdot y=\frac{ag-cf}{a}\end{cases}\xrightarrow[(ad-bc\neq0)]{r_1-\frac{ab}{ad-bc}r_2}\begin{cases}ax=\frac{-abg+adf}{ad-bc}\\ \frac{ad-bc}{a}\cdot y=\frac{ag-cf}{a}\end{cases}\Rightarrow\begin{cases}x=\frac{df-bg}{ad-bc}\\ y=\frac{ag-cf}{ad-bc}\end{cases}$$

If there is a unique solution, $ad-bc\neq0$ is the condition we want.

(If $a=0$ and $c\neq0$, we can change two equation so that the new coefficient $a'=c\neq0$ and we can do the elimination as above.

If a and c both zero, the equation can't have unique solution as x could be anything no matter what b is.