数 学 作 业 纸

（科目：Calculus）

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Problem A.

9\. Sol. Let $u = 2x+1$, then $y = u^5$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4 \cdot 2 = 10(2x+1)^4$

10\. Sol. Let $u = 4-3x$, then $y = u^9$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 9u^8 \cdot (-3) = -27(4-3x)^8$

14\. Sol. Let $u = \frac{x}{5} + \frac{1}{5x}$, then $y = u^5$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4\left(\frac{1}{5} - \frac{1}{5x^2}\right) = \cdot\left(\frac{x}{5} + \frac{1}{5x}\right)^4 \cdot \left(1 - \frac{1}{x^2}\right)$

Problem B

81\. Sol. With $(-1,-3)$ we can give two equations $x = -1+at$ and $y = -3+bt$. Let this two equations go through $(4,1)$ when $t=1$, then $4 = -1+a$, $1 = -3+b$, so $a=5$, $b=4$

one possible parameterization is $x = -1+5t$ and $y = -3-4t$ $(0 \le t \le 1)$

82\. Sol. With $(-1,3)$, we can create two equations $x = -1+at$ and $y = 3+bt$, (when $t=0$, they pass through $(-1,3)$). Let they pass through $(3,-2)$ at $t=1$, then $3 = -1+a \Rightarrow a=4$ and $-2 = 3+b \Rightarrow b=-5$, one possible parameterization is $x = -1+4t$, $y = 3-5t$, $t \in [0,1]$

84\. Sol. The vertex of the parabola is $(-1,-1)$, so the left half of parabola is $y = x^2+2x$ $(x \le -1)$, Let $x=t$, then $y = t^2+2t$ $(t \le -1)$ is one possible parameterization.

Problem C.

19\. Sol. $x^2y + xy^2 = 6 \Rightarrow 2xy + x^2 \cdot y' + y^2 + 2y \cdot y' \cdot x = 0 \Rightarrow (x^2+2xy)y' = -y^2-2xy \Rightarrow y' = \frac{-y^2-2xy}{x^2+2xy}$

20\. Sol. $x^3+y^3 = 18xy \Rightarrow 3x^2 + 3y^2 \cdot y' = 18y + 18x \cdot y' \Rightarrow (3y^2-18x)y' = 18y - 3x^2 \Rightarrow y' = \frac{6y-x^2}{y^2-6x}$

25\. Sol. $y^2 = \frac{x-1}{x+1} \Rightarrow 2y \cdot y' = \frac{x+1-(x-1)}{(x+1)^2} \Rightarrow y' = \frac{1}{y(x+1)^2}$

Problem D.

4\. Sol. (a.) $V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt}$

(b.) $V = \frac{1}{3}\pi h \cdot r^2 \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi h \cdot 2r \cdot \frac{dr}{dt} = \frac{2}{3}\pi r h \cdot \frac{dr}{dt}$

(c) $V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi \cdot (2r) \cdot \frac{dr}{dt} \cdot h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$= \frac{2}{3}\pi r h \cdot \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$