14. $f_x = 3x^2+3y=0$, $f_y = 3x+3y^2=0$, gives $x=0, y=0$ and $x=-1, y=-1$

So critical point for $f(x,y)$ are $(0,0)$ and $(-1,-1)$

for $(0,0)$, $f_{xx}(0,0) = 6x|_{(0,0)} = 0$, $f_{xy}(0,0)=3$, $f_{yy}(0,0) = 6y|_{(0,0)} = 0$

So $f_{xx}f_{yy} - f_{xy}^2 = 0-9<0$, gives $(0,0)$ a saddle point.

for $(-1,-1)$, $f_{xx}(-1,-1) = 6x|_{(-1,-1)} = -6$, $f_{xy}(-1,-1) = 3$, $f_{yy}(-1,-1) = 6y|_{(-1,-1)} = -6$

So $f_{xx}\cdot f_{yy} - f_{xy}^2 = 36-0>0$, which means $(-1,-1)$ is a local maximum (as $f_{xx}<0$)

28. $f_x = e^x(x^2-y^2) + 2xe^x = e^x(x^2+2x-y^2) = 0$ $\quad f_y = -2ye^x = 0$

because $e^x>0$, so $x^2+2x-y^2=0$ and $-2y=0$, which means $(0,0)$ and $(-2,0)$ are critical points.

$f_{xx} = e^x(x^2+2x-y^2) + (2x+2)\cdot e^x = e^x(x^2+4x-y^2+2)$, $f_{xy} = -2ye^x$, $f_{yy} = -2e^x$

For $(0,0)$ $f_{xx}(0,0) = e^0(0+0-0+2) = 2$, $f_{xy}(0,0)=0$, $f_{yy}(0,0) = -2e^0 = -2$

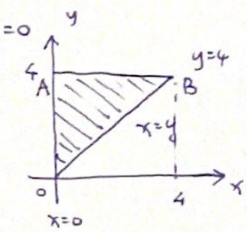
$\Rightarrow f_{xx}f_{yy} - f_{xy}^2 = 2\times(-2)-0 = -4<0 \Rightarrow (0,0)$ is a saddle point.

For $(-2,0)$, $f_{xx}(-2,0) = e^{-2}(4-8-0+2) = -2\cdot e^{-2}$, $f_{xy}(-2,0)=0$, $f_{yy}(-2,0) = -2e^{-2}$

$\Rightarrow f_{xx}f_{yy} - f_{xy}^2 = 4e^{-4}-0 = 4e^{-4}>0 \Rightarrow$ with $f_{xx}(-2,0)<0$, $(-2,0)$ is a local maximum.

32. (i) interior points; we have $f_x = 2x-y=0$ and $f_y = -x+2y=0 \Rightarrow x=0$ and $y=0$

but $(0,0)$ is not an interior point of the region.



(ii) on OA: $D(x,y) = D(0,y) = y^2+1$ $(0\le y\le 4)$

let $D'(0,y) = 2y=0 \Rightarrow y=0$

gives $D_{min}(0,y) = D(0,0) = 1$, $D_{max}(0,y) = D(0,4) = 17$.

(iii) on OB, $D(x,y) = D(x,x) = x^2+1$ $(0\le x\le 4)$

let $D'(x,x) = 2x = 0 \Rightarrow x=0$

gives $D_{min}(x,x) = D(0,0) = 1$, $D_{max}(x,x) = D(4,4) = 17$.

(iv) on AB, $D(x,y) = D(x,4) = x^2-4x+17$ $(0\le x\le 4)$

let $D'(x,4) = 2x-4=0 \Rightarrow x=2$, and $D_{min}(x,4) = D(2,4) = 13$.

$D_{max}(x,4) = D(0,4) = D(4,4) = 17$.

Hence, the absolute maximum is 17 at $(0,4)$ and $(4,4)$

the absolute minimum is 1 at $(0,0)$

12. As $f(x,y)=e^x\sin y$, $f(0,0)=0$, $f_x(0,0)=e^x\sin y=0$, $f_y(0,0)=e^x\cos y=1$, $f_{xx}(0,0)=e^x\sin y=0$

$f_{xy}(0,0)=e^x\cos y=1$, $f_{yy}(0,0)=-e^x\sin y=0$

So $f(x,y)\approx f(0,0)+xf_x(0,0)+yf_y(0,0)+\frac{1}{2}\left[x^2f_{xx}(0,0)+2xy\,f_{xy}(0,0)+y^2f_{yy}(0,0)\right]$

$= 0+0+y+\frac{1}{2}(0+2xy+0)=y+xy$ is the quadratic approximation.

For the error, $f_{xxx}=e^x\sin y$ $f_{xxy}=e^x\cos y$ $f_{xyy}=-e^x\sin y$ $f_{yyy}=-e^x\cos y$

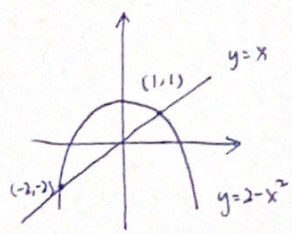
now we have $|x|\leq 0.1$ and $|y|\leq 0.1$, so

$|E(x,y)|\leq\frac{1}{6}\left(|x^3f_{xxx}|+|3x^2y\,f_{xxy}|+|3xy^2f_{xyy}|+|y^3f_{yyy}|\right)$

$\leq\frac{1}{6}\left(|(0.1)^3\cdot e^{0.1}\cdot\sin 0.1|+|3\cdot(0.1)^2\cdot(0.1)\cdot e^{0.1}\cdot\cos 0.1|+|3\cdot(0.1)\cdot(0.1)^2(-e^{0.1}\cdot\sin 0.1)|+|(0.1)^3(-e^{0.1}\cos 0.1)|\right)$

≤ 0.000807

58. $\begin{cases}y=x\\ y=2-x^2\end{cases}\Rightarrow x=-2,\ x=1$



So $V=\int_{-2}^{1}\int_{x}^{2-x^2}x^2\,dy\,dx$

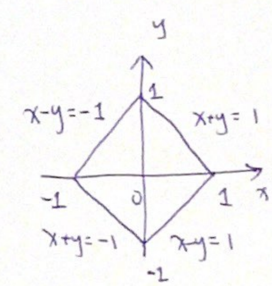
$=\int_{-2}^{1}\left[x^2y\right]_{x}^{2-x^2}dx$

$=\int_{-2}^{1}2x^2-x^3-x^4\,dx$

$=\frac{2}{3}x^3-\frac{1}{4}x^4-\frac{1}{5}x^5\Big|_{-2}^{1}$

$=\left(\frac{2}{3}-\frac{1}{4}-\frac{1}{5}\right)-\left(\frac{2}{3}\cdot(-2)^3-\frac{1}{4}(-2)^4-\frac{1}{5}(-2)^5\right)=\frac{63}{20}$

64. $V=\int_{-1}^{0}\int_{-x-1}^{x+1}3-3x\,dy\,dx+\int_{0}^{1}\int_{x-1}^{1-x}3-3x\,dy\,dx$



$=\int_{-1}^{0}\left[3y-3xy\right]_{-x-1}^{x+1}dx+\int_{0}^{1}\left[3y-3xy\right]_{x-1}^{1-x}dx$

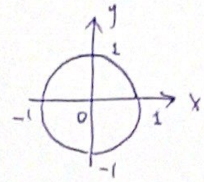
$=6\int_{-1}^{0}1-x^2\,dx+6\int_{0}^{1}1-2x+x^2\,dx$

$=6\left[x-\frac{1}{3}x^3\right]_{-1}^{0}+6\left[x-x^2+\frac{1}{3}x^3\right]_{0}^{1}$

$=0-(-4)+2-0$

$=6$

18. $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$



$= 4\int_0^{\frac{\pi}{2}}\int_0^1 \frac{2r}{(1+r^2)^2}\,dr\,d\theta$

$= 4\int_0^{\frac{\pi}{2}} \left[-\frac{1}{1+r^2}\right]_0^1 d\theta$

$= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}\,d\theta$

$= 2\theta \Big|_0^{\pi/2} = \pi$

34. average height $= \frac{1}{\pi a^2}\cdot 4\cdot \int_0^{\frac{\pi}{2}}\int_0^a r^2\,dr\,d\theta$

$= \frac{1}{\pi a^2}\; 4\cdot \int_0^{\frac{\pi}{2}} \frac{1}{3}a^3\,d\theta$

$= \frac{1}{\pi a^2}\cdot \frac{4}{3}a^3\cdot\frac{\pi}{2}$

$= \frac{2}{3}a$