Calculus A(1): Homework 1

Each assigned exercise is worth 20 points. When we refer to a paragraph number (e.g. §1.3), we refer to the PDF of the textbook *Thomas Calculus* that you can find on the weblearn. The bonus exercises are optional and more difficult. We may (or may not) decide to grade one of your bonus exercises and use it to replace one assigned exercise (if it improves your total grade).

Routine exercises (do not hand-in)

1. §1.1 Exercises 15, 27, 39, 41, 46, 48
2. §1.2 Exercises 9, 12, 18, 37
3. §1.3 Exercises 2, 6, 7, 10, 19, 23, 34
4. Sketch the graphs of the following power functions: $x^{1/3}$, $x^{2/3}$, $x^{1/4}$.
5. Find the least upper-bound (if it exists) of the following sets: $(0,1)$, $\mathbb{Z}$, $\{x \in \mathbb{R}, x^2 \leq 8\}$.

Assigned exercises (hand-in)

1. §1.3 Exercises 22, 27.
2. Find all $x \in \mathbb{R}$ such that $\sqrt{x(x-3)} = \sqrt{3x-5}$.
3. Prove that for any $a, b \in \mathbb{R}$, we have $\mid a - b \mid \geq \Big|\mid a \mid - \mid b \mid\Big|$. Give an example (of values of a and b) such that the inequality is strict.
4. Find (if it exists) the least upper-bound of the set $S = \{1 - \frac{1}{\sqrt{n}}, n \in \mathbb{N}\}$. You can use the *Archimedean property of* $\mathbb{R}$: for all $y \in \mathbb{R}$, there exists $n \in \mathbb{N}$ such that $n > y$.

Bonus exercises (optional)

1. Prove that for any $a, b \in \mathbb{R}$ with $a, b \geq 0$, we have $\frac{a+b}{2} \geq \sqrt{ab}$.
2. Prove that $\sqrt{2}$ is not in $\mathbb{Q}$, that is there does not exist $x \in \mathbb{Q}$ such that $x^2 = 2$.
3. Prove that for any $a, b \in \mathbb{R}$ with $a < b$, there exists $x \in \mathbb{Q}$ such that $a < x < b$. You can use the *Archimedean property of* $\mathbb{R}$: for all $y \in \mathbb{R}$, there exists $n \in \mathbb{N}$ such that $n > y$.
4. (Hard) Find the functions $f : \mathbb{R} \to \mathbb{R}$, such that

$$\forall x \in \mathbb{R}, \quad f(x) + xf(1-x) = 1 + x$$