数 学 作 业 纸

（科目：LA ）

班级：cs 01　　姓名：[illegible]　　编号：2020010869　　第 1 页

Problem. 1.2.7.

Sol. (a) $\cos\theta = \dfrac{v\cdot w}{\|v\|\cdot\|w\|} = \dfrac{1}{2\times 1} = \dfrac{1}{2}$ $\therefore\ \theta = \dfrac{\pi}{3}$.

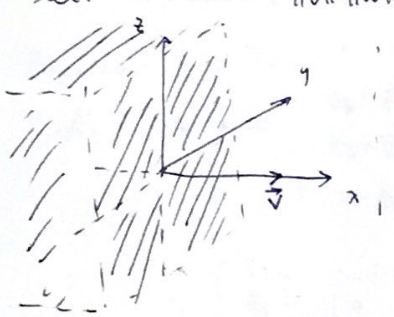
(b) $\cos\theta = \dfrac{v\cdot w}{\|v\|\cdot\|w\|} = \dfrac{0}{3\times 3} = 0$ $\therefore\ \theta = \dfrac{\pi}{2}$

(c) $\cos\theta = \dfrac{v\cdot w}{\|v\|\cdot\|w\|} = \dfrac{2}{2\times 2} = \dfrac{1}{2}$ $\therefore\ \theta = \dfrac{\pi}{3}$

(d) $\cos\theta = \dfrac{v\cdot w}{\|v\|\cdot\|w\|} = \dfrac{-5}{\sqrt{10}\cdot\sqrt{5}} = -\dfrac{\sqrt{2}}{2}$ $\therefore\ \theta = \dfrac{3}{4}\pi$.

Problem 1.2.11

Sol. As $\cos\theta = \dfrac{v\cdot w}{\|v\|\cdot\|w\|} < 0$, the angle $\theta > 90^\circ$, we can see w fill half of 3-dimensional space.



Problem 1.2.16

Sol. $\|v\| = \sqrt{1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2} = 3$

$u = \dfrac{v}{3} = \left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}, \ldots, \dfrac{1}{3}\right)$

$w = \left(\dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{1}{2}, -\dfrac{1}{2}, 0, 0, 0, 0, 0\right)$ is a vector perpendicular to v.

Problem 1.2.22.

Sol. $\|v\|^2\|w\|^2 - |v\cdot w|^2 = (v_1^2w_1^2 + v_2^2w_1^2 + v_1^2w_2^2 + v_2^2w_2^2) - (v_1^2w_1^2 + v_2^2w_2^2 - 2v_1v_2w_1w_2)$

$= v_1^2\cdot w_2^2 - 2v_1w_2\cdot v_2w_1 + v_2^2w_1^2$

$= (v_1w_2 - v_2w_1)^2 \geq 0$

Problem 1.2.27

Sol. $\|v+w\|^2 + \|v-w\|^2 = (v+w)(v+w) + (v-w)(v-w) = v\cdot v + 2v\cdot w + w\cdot w + v\cdot v - 2v\cdot w + w\cdot w$

$= 2\|v\|^2 + 2\|w\|^2$.

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（科目：linear A.）

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Problem 1.2.33

Sol. $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$ $(\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2})$

Problem 1.3.4

Sol. As. $w_1-2w_2+w_3=0$, these vectors are dependent and they lie in a plane.

Problem 1.3.5.

Sol. From Problem 1.3.4, we can see $(y_1,y_2,y_3)=(1,-2,1)$ is a solution, also, $(2,-4,2)$ is another solution.

Problem 1.3.10.

Sol. We have
$$\begin{cases} z_2-z_1=b_1 \\ z_3-z_2=b_2 \\ 0-z_3=b_3 \end{cases} \Rightarrow \begin{cases} z_1=-b_1-b_2-b_3 \\ z_2=-b_2-b_3 \\ z_3=-b_3 \end{cases}$$

$$\therefore \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \therefore \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \text{ is the inverse matrix.}$$

Problem 1.3.12

Sol.
$$C\cdot x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3-x_1 \\ x_4-x_2 \\ -x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -b_2-b_4 \\ b_1 \\ -b_4 \\ b_1+b_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

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Problem 2.

Sol. (a) By the cosine formula, $u\cdot v = |u|\cdot|v|\cdot\cos\theta = 4\cos\theta$, where $\cos\theta\in[-1,1]$

When $\cos\theta=1$, $u\cdot v$ have largest possible value is 4. where u and v lie on same line and with same direction.

If u and v lie on ame line with opposite direction, which means $\cos\theta=-1$, and $u\cdot v=-4$ is the smallest possible value.

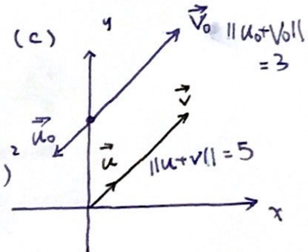
(b) By the triangle inequality, $\|u+v\| \le \|u\|+\|v\| = 1+4 = 5$

Also, we have $(\|u\|+\|v\|)^2 = \|u\|^2 - 2\|u\|\|v\| + \|v\|^2$
$\le \|u\|^2 + 2\, u\cdot v + \|v\|^2 = (\|u+v\|)^2$

which means $\|u+v\| \ge |\,\|u\|-\|v\|\,| = |1-4| = 3$

(When u and v have same direction, $\|u+v\|$ have largest value 5;
when u and v have opposite direction, $\|u+v\|$ have smallest value 3.

(c)



Problem 3.

Sol (a) $A\cdot x = \begin{bmatrix}4&7\\1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}4x_1+7x_2\\x_1+2x_2\end{bmatrix} = \begin{bmatrix}b_1\\b_2\end{bmatrix} \Rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}2b_1-7b_2\\4b_2-b_1\end{bmatrix} = \begin{bmatrix}2&-7\\-1&4\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}$

(b) So that the inverse matrix of A is $\begin{bmatrix}2&-7\\-1&4\end{bmatrix}$.

Problem 1

Sol. (a) As $\|u\| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$, so $u_0 = \frac{u}{\sqrt{6}} = [\frac{1}{\sqrt{6}}\ \ \frac{2}{\sqrt{6}}\ \ \frac{1}{\sqrt{6}}]^T$ is the unit vector.

(b) All vector perpendicular to u can be written as $[a\ \ b\ \ -a-2b]^T$ $(a,b\in\mathbb{R})$

With cross product we can find $w = \begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&2&1\\a&b&-a-2b\end{vmatrix} = (-2a-5b)\vec{i} + (2a+2b)\vec{j} + (b-2a)\vec{k}$

so, $v=[a\ \ b\ \ -a-2b]^T$ and $w=[-2a-5b\ \ 2a+2b\ \ b-2a]^T$ is the answer, $(a,b\in\mathbb{R})$

as $u\cdot v=0$, $v\cdot w=0$ and $u\cdot w=0$,