Calculus A(1): Homework 3

Each assigned exercise is worth 12.5 points. The bonus exercise is optional. We may (or may not) decide to grade your bonus exercise and use it to replace one assigned exercise (if it improves your total grade). We refer to Thomas' Calculus book (whose PDF is available on the weblearn) for the exercises given by a paragraph and number. If you are using your own Thomas' Calculus book, make sure that the numbering of exercises is identical with the PDF.

Routine exercises (do not hand-in)

§2.1 Exercise 1
§2.2 Exercises 16, 48, 53, 59
§2.3 Exercises 3, 9, 20, 45, 48, 58
§2.4 Exercises 1, 6, 10, 16, 17, 22, 25, 30, 33, 43, 46, 50, 51, 63, 68, 76, 78
§2.5 22, 32, 36, 49, 51

Assigned exercises (hand-in)

§2.3 Exercises 52, 54
§2.4 Exercises 5, 69
A1. Determine the limit of the following functions when x goes to 0: $g : x \mapsto x^2 \lfloor \frac{1}{x} \rfloor$ (recall that $\lfloor x \rfloor$ is the largest integer $\leq x$) and $h : x \mapsto x \cos(\frac{1}{x})$.
A2. Let $f : \mathbb{R} \to \mathbb{R}$ be an even function. Show that if $\lim_{x \to +\infty} f(x) = L \in \mathbb{R}$, then $\lim_{x \to -\infty} f(x) = L$.
A3. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $\lim_{x \to +\infty} f(x) = L \in \mathbb{R}$. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = f(x)^2 - f(x+1) \cdot f(x-1)$ for all $x \in \mathbb{R}$. Prove that $\lim_{x \to +\infty} g(x)$ exists and find it.
A4. Prove that $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$ exists and find it. (Hint: you can use $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$).