Linear Algebra – Week 1 Homework

16 Sept 2020
Due: 24 Sept 2020

Textbook Problems. These problems will not be graded, but you must submit solutions to receive full credit for the homework.

Problem 1.1.1. Describe geometrically (line, plane, or all 3-dimensional space) all linear combinations of:

$$(a)\quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \qquad (b)\quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \qquad (c)\quad \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}.$$

Problem 1.1.3. If $\mathbf{v} + \mathbf{w} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\mathbf{v} - \mathbf{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw the vectors $\mathbf{v}$ and $\mathbf{w}$.

Problem 1.1.6. Every linear combination of

$$\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

has components that add up to what number? Find c and d so that $c\mathbf{v} + d\mathbf{w} = (3, 3, -6)$. Why is $(3, 3, 6)$ impossible?

Problem 1.1.7. In the xy-plane, mark all nine of these linear combinations:

$$c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \text{with} \qquad c = 0, 1, 2 \quad \text{and} \quad d = 0, 1, 2.$$

Problem 1.1.9. If three corners of a parallelogram are $(1, 1)$, $(4, 2)$, and $(1, 3)$, what are all three of the possible fourth corners? Draw two of them.

Problem 1.1.25. Draw vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ so that their combinations $c\mathbf{u} + d\mathbf{v} + e\mathbf{w}$ fill only a line. Find vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ (in 3-dimensional space) so that their combinations $c\mathbf{u} + d\mathbf{v} + e\mathbf{w}$ fill only a plane.

Problem 1.1.26. What combination of $c\begin{bmatrix} 1 \\ 2 \end{bmatrix} + d\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ produces $\begin{bmatrix} 14 \\ 8 \end{bmatrix}$? Express this question as two equations for the coefficients c and d in the linear combination.

Problem 1.1.29. Find two different combinations of the three vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ that produce $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. If you take any three vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in the plane, will there always be two different combinations that produce $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

Graded Problems.

Problem 1. Consider the two vectors in 3-dimensional space.

$$\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}.$$

(a) Describe geometrically the set of all linear combinations of $\mathbf{u}$ and $\mathbf{v}$. Also, find the components of a general linear combination of $\mathbf{u}$ and $\mathbf{v}$.

(b) Find a third vector $\mathbf{w}$ in 3-dimensional space that is *not* a linear combination of $\mathbf{u}$ and $\mathbf{v}$. Describe geometrically the set of all linear combinations of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

Problem 2. Consider the three vectors from Problem 1.1.1(c):

$$\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}.$$

(a) Describe geometrically the set of all linear combinations of $\mathbf{u}$ and $\mathbf{v}$. Also, find the components of a general linear combination of $\mathbf{u}$ and $\mathbf{v}$.

(b) Describe geometrically the set of all linear combinations of $\mathbf{v}$ and $\mathbf{w}$. Also, find the components of a general linear combination of $\mathbf{v}$ and $\mathbf{w}$.

(c) Which vectors in 3-dimensional space are linear combinations of $\mathbf{u}$ and $\mathbf{v}$ and *also* are linear combinations of $\mathbf{v}$ and $\mathbf{w}$? (In other words, the intersection of two distinct _____ in 3-dimensional space is a _____.)