4\. Sol. $a_1 = 1 \quad a_2 = 3 \quad a_3 = 1 \quad a_4 = 3$

6\. Sol. $a_1 = \frac{1}{2} \quad a_2 = \frac{3}{4} \quad a_3 = \frac{7}{8} \quad a_4 = \frac{15}{16}$

10\. Sol. $a_1 = -2 \quad a_2 = -1 \quad a_3 = -\frac{2}{3} \quad a_4 = -\frac{1}{2} \quad a_5 = -\frac{2}{5} \quad a_6 = -\frac{1}{3} \quad a_7 = -\frac{2}{7}$

$a_8 = -\frac{1}{4} \quad a_9 = -\frac{2}{9} \quad a_{10} = -\frac{1}{5}$

12\. Sol. $a_1 = 2 \quad a_2 = -1 \quad a_3 = -\frac{1}{2} \quad a_4 = \frac{1}{2} \quad a_5 = -1 \quad a_6 = -2 \quad a_7 = 2$

$a_8 = -1 \quad a_9 = -\frac{1}{2} \quad a_{10} = \frac{1}{2}$

16\. Sol. $a_n = (-1)^{n+1} \cdot \frac{1}{n^2}$ , $n = 1, 2, \ldots$

18\. Sol. $a_n = \frac{2n-5}{n \cdot (n+1)}$ , $n = 1, 2, \ldots$

28\. Sol. $\lim\limits_{n\to\infty} \frac{n+(-1)^n}{n} = \lim\limits_{n\to\infty} 1 + \frac{(-1)^n}{n} = 1$ , $\{a_n\}$ is converges.

34\. Sol. $\lim\limits_{n\to\infty} \frac{1-n^3}{70-4n^2} = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2} - n}{\frac{70}{n^2} - 4} = \infty$ , $\{a_n\}$ is diverges.

36\. Sol $\lim\limits_{n\to\infty} (-1)^n (1-\frac{1}{n})$ do not exist, which means $\{a_n\}$ is diverges.

48 Sol. $\lim\limits_{n\to\infty} \frac{3^n}{n^3} = \lim\limits_{n\to\infty} \frac{3^n \ln 3}{3n^2} = \lim\limits_{n\to\infty} \frac{3^n (\ln 3)^2}{6n} = \lim\limits_{n\to\infty} \frac{3^n (\ln 3)^3}{6} = \infty$ , $\{a_n\}$ is diverges

56\. Sol $\lim\limits_{n\to\infty} \sqrt[n]{n^2} = \lim\limits_{n\to\infty} \sqrt[n]{n} \cdot \lim\limits_{n\to\infty} \sqrt[n]{n} = 1 \cdot 1 = 1$ , $\{a_n\}$ is converges

62\. Sol $\lim\limits_{n\to\infty} 3^{\frac{2n+1}{n}} = \lim\limits_{n\to\infty} 3^2 \cdot 3^{\frac{1}{n}} = 9 \times 1 = 9$ , $\{a_n\}$ is converges.

78\. Sol. $\lim\limits_{n\to\infty} \frac{1-\cos\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{\sin\frac{1}{n} \cdot (-\frac{1}{n^2})}{-\frac{1}{n^2}} = \lim\limits_{n\to\infty} \sin\frac{1}{n} = 0$ , $\{a_n\}$ is converges.

80 Sol $\ln a_n = \frac{1}{n} \ln(3^n + 5^n)$

$\lim\limits_{n\to\infty} \ln a_n = \lim\limits_{n\to\infty} \frac{\ln(3^n+5^n)}{n} = \lim\limits_{n\to\infty} \frac{3^n \ln 3 + 5^n \ln 5}{3^n + 5^n} = \lim\limits_{n\to\infty} \frac{(\frac{3}{5})^n \ln 3 + \ln 5}{(\frac{3}{5})^n + 1} = \frac{0 + \ln 5}{0 + 1} = \ln 5$

so $\lim\limits_{n\to\infty} a_n = e^{\ln 5} = 5$ , $\{a_n\}$ is converges

94. Sol. As sequence is converges, we can let $\lim\limits_{n\to\infty} a_n = L$

then $\lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} \sqrt{8+2a_n} \Rightarrow L = \sqrt{8+2\lim\limits_{n\to\infty} a_n} \Rightarrow L = \sqrt{8+2L} \Rightarrow L^2-2L-8=0$

$\Rightarrow L=-2$ or $L=4$, as $a_n>0$, we have $\lim\limits_{n\to\infty} a_n = L = 4$

98. Sol. Let $a_1 = 1$, $a_{n+1} = \sqrt{1+a_n}$ $(n=1,2,\cdots)$ and $\lim\limits_{n\to\infty} a_n = L$.

$\lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} \sqrt{1+a_n} \Rightarrow L = \sqrt{1+\lim\limits_{n\to\infty} a_n} \Rightarrow L = \sqrt{1+L} \Rightarrow L^2-L-1=0$

$\Rightarrow L = \frac{1-\sqrt{5}}{2}$ or $L = \frac{1+\sqrt{5}}{2}$, as $a_n>0$, we have $\lim\limits_{n\to\infty} a_n = L = \frac{1+\sqrt{5}}{2}$