清華大學 TSINGHUA UNIVERSITY 数学作业纸

班级：CST01 姓名：[illegible] 编号：2020010869 科目：Calculus 第 1 页

2. $A = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\cdot(2\sin\theta)^2\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1-\cos 2\theta \, d\theta = \theta - \frac{1}{2}\sin 2\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = (\frac{\pi}{2}-0) - (\frac{\pi}{4}-\frac{1}{2}) = \frac{1}{2}+\frac{\pi}{4}$.

6. $A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}\cdot(\cos 3\theta)^2\, d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1+\cos 6\theta}{4}\, d\theta = \frac{\theta}{4} + \frac{\sin 6\theta}{24}\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = (\frac{\pi}{24}+0) - (-\frac{\pi}{24}+0) = \frac{\pi}{12}$

12. $\overrightarrow{AB} = \langle 2-1,\ 0-(-1)\rangle = \langle 1, 1\rangle$, $\overrightarrow{CD} = \langle -2-(-1),\ 2-3\rangle = \langle -1, -1\rangle$

$\overrightarrow{AB} + \overrightarrow{CD} = \langle 0, 0\rangle$

14. take $\langle \cos(-\frac{3}{4}\pi), \sin(-\frac{3}{4}\pi)\rangle = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\rangle$.

26. length : $|9i-2j+6k| = \sqrt{9^2+2^2+6^2} = 11$

direction $= \frac{9i-2j+6k}{|9i-2j+6k|} = \frac{9}{11}i - \frac{2}{11}j + \frac{6}{11}k$.

so $9i-2j+6k = 11\left(\frac{9}{11}i - \frac{2}{11}j + \frac{6}{11}k\right)$

·(1) $r = \frac{ke}{1+e\cos\theta} \Rightarrow ke = r + er\cos\theta \Rightarrow ke = r + e\cdot x \Rightarrow ke = \sqrt{x^2+y^2} + ex$

$\Rightarrow x^2+y^2 = e^2(k-x)^2 \Rightarrow x^2+y^2 = e^2x^2 - 2e^2kx + e^2k^2 \Rightarrow (1-e^2)x^2 + y^2 + 2e^2kx - e^2k^2 = 0$

where $A = 1-e^2$, $B=0$ and $C=1$ ($D = 2e^2k$, $E=0$, $F = -e^2k^2$)

check: $B^2-4AC = 0 - 4\cdot(1-e^2) = 4(e^2-1)$

if $e=1$, $B^2-4AC = 4\times(1^2-1) = 0$, is a parabola

if $e<1$, $B^2-4AC = 4(e^2-1) < 0$, is an ellipse.

if $e>1$, $B^2-4AC = 4(e^2-1) > 0$, is a hyperbola.

·(2) First, we can rotate the system α degree by x-axis, then:

$x' = x$, $y' = y\cos\alpha + z\sin\alpha$, $z' = -y\sin\alpha + z\cos\alpha$

then $u'\cdot v' = u_1'\cdot v_1' + u_2'\cdot v_2' + u_3'\cdot v_3'$ (where $u = \langle u_1, u_2, u_3\rangle$, $v = \langle v_1, v_2, v_3\rangle$
$u' = \langle u_1', u_2', u_3'\rangle$, $v' = \langle v_1', v_2', v_3'\rangle$)

$= u_1\cdot v_1 + (u_2\cos\alpha + u_3\sin\alpha)(v_2\cos\alpha + v_3\sin\alpha) + (-u_2\sin\alpha + u_3\cos\alpha)(-v_2\sin\alpha + u_3\sin\alpha)$

$= u_1v_1 + u_2\cdot\cos^2\alpha + u_3\sin^2\alpha + 2\sin\alpha\cos\alpha(u_2v_3+u_3v_2) + u_2v_2\sin^2\alpha + u_3v_3\cos^2\alpha - 2\sin\alpha\cos\alpha(u_2v_3 + u_3v_2)$

$= u_1v_1 + u_2v_2 + u_3v_3 = u\cdot v$.

write the formula of y-axis rotate and z-axis rotate:

for y-axis: $y' = y$, $x' = x\cos\beta + z\sin\beta$, $z' = -x\sin\beta + z\cos\beta$.

for z-axis: $z' = z$, $x' = x\cos\theta + y\sin\theta$, $y' = -x\sin\theta + y\cos\theta$, these two equation have the same form of x-axis rotate, so the result of $u\cdot v$ doesn't change after each rotation.