24. $1.\overline{414} = 1 + \frac{414}{1000} + \frac{414}{1000^2} + \cdots$

$= 1 + \frac{414}{1000}\left(1 + \frac{1}{1000} + \frac{1}{1000^2} + \cdots\right)$

$= 1 + \frac{414}{1000}\left(\frac{1000}{999}\right)$

$= \frac{1413}{999}$

78. As $a = 1$, $r = \ln x$

If the series converges, $|r| < 1$, which means $|\ln x| < 1 \iff e^{-1} < x < e$

and the sum of series is $\frac{a}{1-r} = \frac{1}{1-\ln x}$

20. by Integral test, we have $\int_2^\infty \frac{\ln x}{\sqrt{x}}\,dx = \int_2^\infty u \cdot e^{\frac{1}{2}u}\,du$ ($u = \ln x$, $x = e^u$, $dx = e^u du$

$= \lim_{b\to\infty} 2(u-2)e^{\frac{1}{2}u}\Big|_2^b = \lim_{b\to 0}\left(2(b-2)e^{\frac{1}{2}b} - 0\right) = \infty$

as $\int_2^\infty \frac{\ln x}{\sqrt{x}}\,dx$ diverges, we can tell $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ is also diverges.

34 by nth-Term Test, we have $\lim_{n\to\infty} n\tan\frac{1}{n} = \lim_{n\to\infty} \frac{\tan\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{-\frac{1}{n^2}}{-\frac{1}{n^2}\cdot\cos\frac{1}{n}}$

$= \lim_{n\to\infty} \sec\frac{1}{n} = \sec 0 = 1 \neq 0$, so $\sum_{n=1}^{\infty} n\tan\frac{1}{n}$ is diverges

36. by Integral Test, we have $\int_1^\infty \frac{2}{1+e^x}\,dx = \int_e^\infty \frac{2}{u(u+1)}\,du$ ($u = e^x$, $x = \ln u$, $dx = \frac{du}{u}$)

$= \int_e^\infty \frac{2}{u} - \frac{2}{u+1}\,du = \lim_{b\to\infty} 2\ln u - 2\ln(u+1)\Big|_e^b = \lim_{b\to\infty} 2\ln\frac{u}{u+1}\Big|_e^b = \lim_{b\to\infty} 2\ln\frac{b}{b+1} - 2\ln\frac{e}{e+1}$

$= 2\ln 1 - 2\ln\frac{e}{e+1} = -2\ln\frac{e}{e+1}$, so $\sum_{n=1}^{\infty} \frac{2}{1+e^n}$ converges.

44. We have $\sum_{n=1}^{\infty} \frac{1}{nx} = \frac{1}{x}\sum_{n=1}^{\infty} \frac{1}{n}$, as $\sum_{n=1}^{\infty} \frac{1}{n}$ is a geometric series with $r = 1$.

so it always diverges no matter what x we choose.

28. by comparison Test, let $d_n = \frac{1}{n}$, as $n + \sqrt{n} \le n + n \le 3n$, we have $\frac{1}{n} \le \frac{3}{n+\sqrt{n}}$

with series $\sum_{n=1}^{\infty} \frac{1}{n}$ is diverges, $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ is also diverges.

22. by limit comparison Test, let $b_n = \frac{1}{n^{\frac{3}{2}}}$, then

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+1}{n^2\sqrt{n}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{5/2} + n^{3/2}}{n^{5/2}} = \lim_{n\to\infty} \frac{1+n^{-1}}{1} = \frac{1+0}{1} = 1$

because $\sum b_n = \sum_{n=0}^{\infty} \frac{1}{n^{3/2}}$ is coverges ($r = \frac{3}{2} > 1$), so $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ is also coverges

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28. by limit comparison test. let $b_n = \frac{1}{n^2}$. then

$$\lim_{n\to\infty} \frac{\frac{\ln n^2}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\ln n^2}{n} = 2\lim_{n\to\infty} \frac{\ln n}{n} = 2\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 2\lim_{n\to\infty} \frac{1}{n} = 0$$

because $\sum \frac{1}{n^2}$ is converge (p-series), so that $\sum_{n=1}^{\infty} \frac{\ln n^2}{n^3}$ is also converge.

56. by limit comparison test,

$$\lim_{n\to\infty} \frac{\frac{a_n}{n}}{a_n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

because $\sum_{n=1}^{\infty} a_n$ is converges, $\sum_{n=1}^{\infty} \frac{a_n}{n}$ is also converges.