1 (1)　证明：$\neg(\exists x)(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y))$

$= (\forall x)(\forall y)\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y))$

$= (\forall x)(\forall y)(\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \vee \neg R(x,y))$

$= (\forall x)(\forall y)((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \rightarrow \neg R(x,y))$

(4)　证明：$(\forall y)(\exists x)((P(x) \rightarrow q) \vee S(y))$

$= (\exists x)(P(x) \rightarrow q) \vee (\forall y) S(y)$

$= ((\forall x) P(x) \rightarrow q) \vee (\forall y) S(y)$

(5)　证明：$(\forall x) P(x) \rightarrow q$

$= \neg(\forall x) P(x) \vee q$

$= (\exists x) \neg P(x) \vee q$

$= (\exists x)(\neg P(x) \vee q)$

$= (\exists x)(P(x) \rightarrow q)$

(6)　证明：$(\exists x)(P(x) \rightarrow Q(x))$

$= (\exists x)(\neg P(x) \vee Q(x))$

$= \neg(\forall x) P(x) \vee (\exists x) Q(x)$

$= (\forall x) P(x) \rightarrow (\exists x) Q(x)$

(7)　证明：$(\exists x) P(x) \rightarrow (\forall x) Q(x)$

$= \neg(\exists x) P(x) \vee (\forall x) Q(x)$

$= (\forall x) \neg P(x) \vee (\forall x) Q(x)$

$\Rightarrow (\forall x)(\neg P(x) \vee Q(x))$

$= (\forall x)(P(x) \rightarrow Q(x))$

(8)　证明：$(\exists x) P(x) \vee (\forall x) Q(x)$

$= (\exists x) P(x) \vee (\forall y) Q(y)$

$= (\exists x)(P(x) \vee (\forall y) Q(y))$

$\Leftarrow (\exists x)(P(x) \vee Q(x))$

(9)　证明：$((\forall x) P(x) \wedge \forall(x) Q(x) \wedge (\exists x) R(x)) \vee ((\forall x) P(x) \wedge (\forall x) Q(x) \vee (\exists x) S(x))$

$= ((\forall x)(P(x) \vee Q(x)) \wedge (\exists x) R(x)) \vee ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists x) S(x))$

$= (\forall x)(P(x) \wedge Q(x)) \vee ((\exists x) R(x) \vee (\exists x) S(x))$

$= (\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)(R(x) \vee S(x))$

2. (1) 不是普遍有效，在 $\{1,2\}$ 域上分析，令 $P(1)=P(2)=Q(1)=F$，$Q(2)=T$，该式为假

(2) 不是普遍有效。

(3) 普遍有效，$((\exists x)P(x) \to (\forall x)Q(x)) \to (\forall x)(P(x) \to Q(x))$

$= \{\neg(\exists x)P(x) \lor (\forall x)Q(x)\} \to (\forall x)(\neg P(x) \lor Q(x))$

$= ((\forall x)\neg P(x) \lor (\forall x)Q(x)) \to (\forall x)(\neg P(x) \lor Q(x))$

$= (\forall x)(\neg P(x) \lor Q(x)) \to (\forall x)(\neg P(x) \lor Q(x))$

$= T$

(4) 不是普遍有效。

(5) 普遍有效

(6) 不是普遍有效

(7) 不是普遍有效

(8) 不是普遍有效。

4 (1) $(\forall x)(P(x) \to (\exists y)Q(x,y))$

$= (\forall x)(\neg P(x) \lor (\exists y)Q(x,y))$

$= (\forall x)(\exists y)(\neg P(x) \lor Q(x,y))$

(2) $(\forall x)(\forall y)(\forall z)(P(x,y,z) \lor ((\exists u)Q(x,u) \to (\exists w)Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(P(x,y,z) \lor (\neg(\exists u)Q(x,u) \lor (\exists w)Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(P(x,y,z) \lor ((\forall u)\neg Q(x,u) \lor (\exists w)Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(\forall u)(\exists w)(P(x,y,z) \lor (\neg Q(x,u) \lor Q(y,w)))$

(3) $(\exists x)P(x,y) \leftrightarrow (\forall z)Q(z)$

$= ((\exists x)P(x,y) \lor (\forall z)Q(z)) \land (\neg(\exists x)P(x,y) \lor \neg(\forall z)Q(z))$

$= ((\exists x)P(x,y) \lor (\forall z)Q(z)) \land ((\forall x)\neg P(x,y) \lor (\exists z)\neg Q(z))$

$= ((\exists x)P(x,y) \lor (\forall z)Q(z)) \land ((\forall u)\neg P(u,y) \lor (\exists v)\neg Q(v))$

$= ((\forall u)\neg P(u,y) \land (\exists z)Q(z)) \lor (P(x,y) \land Q(z)) \lor (\neg P(u,y) \lor \neg Q(v))$

$(4)\ ((\exists x)P(x) \lor (\forall y)Q(y)) \to (\exists z)R(z)$

$= \neg((\exists x)P(x) \lor (\forall y)Q(y)) \lor (\exists z)R(z)$

$= ((\exists x)\neg P(x) \land (\exists y)\neg Q(y)) \lor (\exists z)R(z)$

$= ((\forall x)\neg P(x) \land (\exists y)\neg Q(y)) \lor (\exists z)R(z)$

$= (\exists x)(\exists y)(\exists z)((\neg P(x) \lor \neg Q(y)) \lor R(z))$

(5) $(\forall x)(P(x) \to (\forall y)((P(y) \to (Q(x) \to Q(y))) \lor (\exists z) P(z)))$

$= (\forall x)(\exists z)(\forall y)((\neg P(x) \lor ((\neg P(y) \lor (\neg Q(x) \lor Q(y))) \land (\exists z) P(z)))) =$

$= (\forall x)(\forall y)(\exists z)((\neg P(x) \lor ((\neg P(y) \lor \neg Q(x) \lor Q(y)) \lor P(z))) =$

(9) $(\forall x)(P(x) \to (\exists y) Q(x,y)) \lor (\forall z) R(z)$

$= (\forall x)(\neg P(x) \lor (\exists y) Q(x,y)) \lor (\forall z) R(z) =$

$= (\forall x)(\exists y)(\forall z)(\neg P(x) \lor Q(x,y) \lor R(z)) =$

Skolem范式：$(\forall x)(\forall z)((\neg P(x) \lor Q(x, f(x)) \lor R(z))$

(10) $(\exists y)(\forall x)(\forall z)(\exists u)(\forall v) P(x,y,z,u,v)$

Skolem范式：$(\forall x)(\forall z)(\forall v) P(x, a, z, f(x,z), v)$