

数学作业纸

(科目: 高数)

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$$\begin{aligned}
 \text{1 (1) 证明: } & \neg (\exists x)(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) \\
 & = (\forall x)(\forall y) \neg (P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) \\
 & = (\forall x)(\forall y) (\neg (P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \vee \neg R(x, y)) \\
 & = (\forall x)(\forall y) ((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \rightarrow \neg R(x, y))
 \end{aligned}$$

$$\begin{aligned}
 \text{(4) 证明: } & (\forall y)(\exists x)((P(x) \rightarrow q) \vee S(y)) \\
 & = (\exists x)(P(x) \rightarrow q) \vee (\forall y)S(y) \\
 & = ((\forall x)P(x) \rightarrow q) \vee (\forall y)S(y)
 \end{aligned}$$

$$\begin{aligned}
 \text{(5) 证明: } & (\forall x)P(x) \rightarrow q \\
 & = \neg (\forall x)P(x) \vee q \\
 & = (\exists x)\neg P(x) \vee q \\
 & = (\exists x)(\neg P(x) \vee q) \\
 & = (\exists x)(P(x) \rightarrow q)
 \end{aligned}$$

$$\begin{aligned}
 \text{(6) 证明: } & (\exists x)(P(x) \rightarrow Q(x)) \\
 & = (\exists x)(\neg P(x) \vee Q(x)) \\
 & = \neg (\forall x)P(x) \vee (\exists x)Q(x) \\
 & = (\forall x)P(x) \rightarrow (\exists x)Q(x)
 \end{aligned}$$

$$\begin{aligned}
 \text{(7) 证明: } & (\exists x)P(x) \rightarrow (\forall x)Q(x) \\
 & = \neg (\exists x)P(x) \vee (\forall x)Q(x) \\
 & = (\forall x)\neg P(x) \vee (\forall x)Q(x) \\
 & \Rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\
 & = (\forall x)(P(x) \rightarrow Q(x))
 \end{aligned}$$

$$\begin{aligned}
 \text{(8) 证明: } & (\exists x)P(x) \wedge (\forall x)Q(x) \\
 & = (\exists x)P(x) \wedge (\forall y)Q(y) \\
 & = (\exists x)(P(x) \wedge (\forall y)Q(y)) \\
 & \Rightarrow (\exists x)(P(x) \wedge Q(x))
 \end{aligned}$$

$$\begin{aligned}
 \text{(9) 证明: } & ((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)R(x)) \vee ((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)S(x)) \\
 & = ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)R(x)) \vee ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)S(x)) \\
 & = (\forall x)(P(x) \wedge Q(x)) \wedge ((\exists x)R(x) \vee (\exists x)S(x)) \\
 & = (\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)(R(x) \vee S(x))
 \end{aligned}$$

数学作业纸

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2. (1) 不是普遍有效, 在 $\{1, 2\}$ 域上分析, 令 $p(1) = p(2) = Q(1) = F, Q(2) = T$, 该式为假
(2) 不是普遍有效.

$$\begin{aligned} (3) \text{ 普遍有效, } & ((\exists x)P(x) \rightarrow (\forall x)Q(x)) \rightarrow (\forall x)(P(x) \rightarrow Q(x)) \\ & = \neg(\neg(\exists x)P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\ & = ((\forall x)\neg P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\ & = (\forall x)(\neg P(x) \vee Q(x)) \rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\ & = T \end{aligned}$$

(4) 不是普遍有效.

(5) 普遍有效

(6) 不是普遍有效

(7) 不是普遍有效

(8) 不是普遍有效.

$$\begin{aligned} 4 (1) & (\forall x)(P(x) \rightarrow (\exists y)Q(x, y)) \\ & = (\forall x)(\neg P(x) \vee (\exists y)Q(x, y)) \\ & = (\forall x)(\exists y)(\neg P(x) \vee Q(x, y)) \end{aligned}$$

$$\begin{aligned} (2) & (\forall x)(\forall y)(\forall z)(P(x, y, z) \wedge ((\exists u)Q(x, u) \rightarrow (\exists w)Q(y, w))) \\ & = (\forall x)(\forall y)(\forall z)(P(x, y, z) \wedge (\neg(\exists u)Q(x, u) \vee (\exists w)Q(y, w))) \\ & = (\forall x)(\forall y)(\forall z)(P(x, y, z) \wedge ((\forall u)\neg Q(x, u) \vee (\exists w)Q(y, w))) \\ & = (\forall x)(\forall y)(\forall z)(\forall u)(\exists w)(P(x, y, z) \wedge (\neg Q(x, u) \vee Q(y, w))) \end{aligned}$$

$$\begin{aligned} (3) & (\exists x)P(x, y) \leftrightarrow (\forall z)Q(z) \\ & = ((\exists x)P(x, y) \wedge (\forall z)Q(z)) \vee (\neg(\exists x)P(x, y) \wedge \neg(\forall z)Q(z)) \\ & = ((\exists x)P(x, y) \wedge (\forall z)Q(z)) \vee ((\forall x)\neg P(x, y) \wedge (\exists z)\neg Q(z)) \\ & = ((\exists x)P(x, y) \wedge (\forall z)Q(z)) \vee ((\forall u)\neg P(u, y) \wedge (\exists v)\neg Q(v)) \\ & = (\exists x)(\forall z)(\forall u)(\exists v)(P(x, y) \wedge Q(z)) \vee (\neg P(u, y) \wedge \neg Q(v)) \end{aligned}$$

$$\begin{aligned} (4) & (\neg(\exists x)P(x) \vee (\forall y)Q(y)) \rightarrow (\forall z)R(z) \\ & = \neg(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \vee (\forall z)R(z) \\ & = ((\exists x)P(x) \wedge \neg(\forall y)Q(y)) \vee (\forall z)R(z) \\ & = ((\exists x)P(x) \wedge (\exists y)\neg Q(y)) \vee (\forall z)R(z) \\ & = (\exists x)(\exists y)(\forall z)((P(x) \wedge \neg Q(y)) \vee R(z)) \end{aligned}$$

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$$\begin{aligned}
 (5) & (\forall x)(P(x) \rightarrow (\forall y)((P(y) \rightarrow (Q(x) \rightarrow Q(y))) \vee (\forall z)P(z))) \\
 &= (\forall x)(\neg P(x) \vee (\forall y)((\neg P(y) \vee (\neg Q(x) \vee Q(y))) \vee (\forall z)P(z))) \\
 &= (\forall x)(\forall y)(\forall z)(\neg P(x) \vee (\neg P(y) \vee \neg Q(x) \vee Q(y)) \vee P(z))
 \end{aligned}$$

$$\begin{aligned}
 (9) & (\forall x)(P(x) \rightarrow (\exists y)Q(x,y)) \vee (\forall z)R(z) \\
 &= (\forall x)(\neg P(x) \vee (\exists y)Q(x,y)) \vee (\forall z)R(z) \\
 &= (\forall x)(\exists y)(\forall z)(\neg P(x) \vee Q(x,y) \vee R(z))
 \end{aligned}$$

Skolem范式: $(\forall x)(\forall z)(\neg P(x) \vee Q(x, f(x)) \vee R(z))$

$$(10) (\exists y)(\forall x)(\forall z)(\exists u)(\forall v)P(x,y,z,u,v)$$

Skolem范式: $(\forall x)(\forall z)(\forall v)P(x, a, z, f(x,z), v)$