



班级: CST01 姓名: 容逸朗 编号: 2020010869 科目: Calculus

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$$\begin{aligned}
 24. \quad V &= \int_0^1 \int_0^{1-x} \int_0^{2-2z} dy \, dz \, dx \\
 &= \int_0^1 \int_0^{1-x} (2-2z) \, dz \, dx \\
 &= \int_0^1 [2z - z^2]_0^{1-x} \, dx \\
 &= \int_0^1 (1-x^2) \, dx \\
 &= \left[x - \frac{1}{3}x^3 \right]_0^1 = \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad V &= \int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{3}} \int_0^a \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \\
 &= \int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{3}} \frac{1}{3} a^3 \sin \phi \, d\phi \, d\theta \\
 &= \int_0^{\frac{\pi}{6}} \left(-\frac{1}{3} a^3 \cos \frac{\pi}{2} + \frac{1}{3} a^3 \cos 0 \right) d\theta \\
 &= \frac{1}{3} a^3 \cdot \frac{\pi}{6} = \frac{\pi a^3}{18}
 \end{aligned}$$

62. first solve $x^2+y^2+z^2=2$ and $z=x^2+y^2$, which gives $z^2+z=2$ or $(z+2)(z-1)=0 \Rightarrow z=-2$ or $z=1$ but $z=x^2+y^2 \geq 0$, so $z=1$, gives $r=\sqrt{x^2+y^2}=1$

$$\begin{aligned}
 \text{then } V &= 4 \int_0^{\frac{\pi}{2}} \int_0^1 \int_{r^2}^{\sqrt{2-r^2}} dz \, r \, dr \, d\theta \\
 &= 4 \int_0^{\frac{\pi}{2}} \int_0^1 (\sqrt{2-r^2} - r^2) \cdot r \, dr \, d\theta \\
 &= 4 \int_0^{\frac{\pi}{2}} \left[-\frac{1}{4}r^4 - \frac{1}{3}(2-r^2)^{\frac{3}{2}} \right]_0^1 d\theta \\
 &= 4 \int_0^{\frac{\pi}{2}} \left(\frac{2\sqrt{2}}{3} - \frac{7}{12} \right) d\theta \\
 &= \frac{(8\sqrt{2}-7)\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad V &= \int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y} dz \, dy \, dx \\
 &= \int_0^2 \int_0^{4-x^2} (4-x^2-y) \, dy \, dx \\
 &= \int_0^2 \left[4y - x^2y - \frac{1}{2}y^2 \right]_0^{4-x^2} dx \\
 &= \int_0^2 \left(8-4x^2 + \frac{1}{2}x^4 \right) dx \\
 &= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5 \right]_0^2 = \frac{128}{15}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad V &= 4 \int_0^{\frac{\pi}{2}} \int_0^1 \int_{r^2}^{r^2+1} dz \, r \, dr \, d\theta \\
 &= 4 \int_0^{\frac{\pi}{2}} \int_0^1 1 \, r \, dr \, d\theta \\
 &= 4 \int_0^{\frac{\pi}{2}} \frac{1}{2} d\theta \\
 &= \pi
 \end{aligned}$$

$$\begin{aligned}
 66. \quad \text{average} &= \frac{1}{\frac{2}{3}\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^3 \sin \phi \cos \phi \, d\rho \, d\phi \, d\theta \\
 &= \frac{1}{\frac{2}{3}\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{1}{4} \sin 2\phi \cos \phi \, d\phi \, d\theta \\
 &= \frac{3}{16\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin 2\phi \, d\phi \, d\theta \\
 &= \frac{3}{16\pi} \int_0^{2\pi} \left[-\frac{\cos 2\phi}{2} \right]_0^{\frac{\pi}{2}} d\theta \\
 &= \frac{3}{16\pi} \int_0^{2\pi} d\theta \\
 &= \frac{3}{16\pi} \cdot 2\pi \\
 &= \frac{3}{8}
 \end{aligned}$$



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14. $x = u + \frac{v}{2}, y = v \Rightarrow J(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 1/2 \\ 0 & 1 \end{vmatrix} = 1$

| boundary of R | boundary of G | simplified |
|---------------------|-------------------------------------|------------|
| $x = \frac{y}{2}$ | $u + \frac{v}{2} = \frac{v}{2}$ | $u = 0$ |
| $x = \frac{y+4}{2}$ | $u + \frac{v}{2} = \frac{v}{2} + 2$ | $u = 2$ |
| $y = 0$ | $v = 0$ | $v = 0$ |
| $y = 2$ | $v = 2$ | $v = 2$ |

$$\begin{aligned}
 \text{So } & \int_0^2 \int_{y/2}^{(y+4)/2} y^3 (2x-y) e^{(2x-y)^2} dx dy \\
 &= \int_0^2 \int_0^2 v^3 \cdot 2u \cdot e^{4u^2} J(u, v) du dv \\
 &= \int_0^2 \int_0^2 v^3 \cdot 2u \cdot e^{4u^2} du dv \\
 &= \int_0^2 \left[\frac{1}{4} v^3 \cdot e^{4u^2} \right]_0^2 dv \\
 &= \int_0^2 \frac{1}{4} v^3 (e^{16} - 1) dv \\
 &= \frac{1}{16} \left[v^4 (e^{16} - 1) \right]_0^2 \\
 &= e^{16} - 1
 \end{aligned}$$

*. as $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$

we have $J(\rho, \phi, \theta) = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \sin \phi \cos \theta & \rho \cos \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \sin \phi \sin \theta & \rho \cos \phi \sin \theta & \rho \sin \phi \cos \theta \\ \cos \phi & -\rho \sin \phi & 0 \end{vmatrix}$

$$\begin{aligned}
 &= \cos \phi \begin{vmatrix} \rho \cos \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \rho \cos \phi \sin \theta & \rho \sin \phi \cos \theta \end{vmatrix} - (-\rho \sin \phi) \begin{vmatrix} \sin \phi \cos \theta & -\rho \sin \phi \sin \theta \\ \sin \phi \sin \theta & \rho \sin \phi \cos \theta \end{vmatrix} \\
 &= \rho^2 \sin \phi \cos^2 \phi + \rho^2 \sin^3 \phi \\
 &= \rho^2 \sin \phi
 \end{aligned}$$