7. by $f(x)=\sin x$, $f'(x)=\cos x$, $f''(x)=-\sin x$, $f'''(x)=-\cos x$ and $f(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$, $f'(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$, $f''(\frac{\pi}{4})=-\frac{\sqrt{2}}{2}$,

$f'''(\frac{\pi}{4})=-\frac{\sqrt{2}}{2}$, then: $P_0(x)=\frac{\sqrt{2}}{2}$, $P_1(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})$, $P_2(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})-\frac{\sqrt{2}}{4}(x-\frac{\pi}{4})^2$

$P_3(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})-\frac{\sqrt{2}}{4}(x-\frac{\pi}{4})^2-\frac{\sqrt{2}}{12}(x-\frac{\pi}{4})^3$

28. with $f'(x)=3(1-x)^{-4}$, $f''(x)=12(1-x)^{-5}$, we have $f^{(n)}(x)=\frac{(n+2)!}{2}(1-x)^{-n-3}$.

So $f(0)=1$, $f'(0)=3$, $f''(0)=12$ ... $f^{(n)}(0)=\frac{(n+2)!}{2}$

and Taylor series $\frac{1}{(1-x)^3}=1+3x+6x^2+\cdots=\sum_{n=0}^{\infty}\left(\frac{(n+2)(n+1)}{2}\cdot x^n\right)$

30. With $f(x)=2^x$, $f'(x)=2^x\ln 2$, $f''(x)=2^x(\ln 2)^2$ ... $f^{(n)}(x)=2^x(\ln 2)^n$

we got $f(1)=2$, $f'(1)=2\ln 2$, $f''(1)=2(\ln 2)^2$ ... $f^{(n)}(1)=2(\ln 2)^n$

So $2^x=2+\ln 2(x-1)+\frac{1}{3}(\ln 2)^2\cdot(x-1)^2+\cdots=\sum_{n=0}^{\infty}\frac{2(\ln 2)^n\cdot(x-1)^n}{n!}$

52 If f is even, then $f(-x)=f(x)\Rightarrow -f'(-x)=f'(x)$ gives f' odd.

If f is odd, then $-f(-x)=f(x)\Rightarrow f'(-x)=f'(x)$ gives f' even, and let $x=0$,

then $-f(-0)=f(0)\Rightarrow 2f(0)=0\Rightarrow f(0)=0$.

So, If $f(x)$ is even, then odd-order derivative is odd and equal to zero at $x=0$, which gives $a_1=a_3=a_5=\cdots=0$, and by Maclaurin series, we have only even power of x.

If $f(x)$ is odd, then even-order derivative is odd and is zero at $x=0$, which gives $a_0=a_2=a_4=\cdots=0$, and Maclaurin series gives there is only odd power of x.

2. $(1+x)^{1/3}=1+\sum_{k=1}^{\infty}\binom{\frac{1}{3}}{k}\cdot x^k=1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{81}x^3-\cdots$

34. $\tan^{-1}x=\int\frac{1}{1+x^2}dx=\int(1-x^2+x^4-x^6+\cdots)dx=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$

$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

So $\lim_{y\to 0}\frac{\tan^{-1}y-\sin y}{y^3\cos y}=\lim_{y\to 0}\frac{\left(y-\frac{y^3}{3}+\frac{y^5}{5}-\frac{y^7}{7}+\cdots\right)-\left(y-\frac{y^3}{3!}+\frac{y^5}{5!}-\frac{y^7}{7!}+\cdots\right)}{y^3\cdot\cos y}=\lim_{y\to 0}\frac{\left(-\frac{y^3}{6}+\frac{23y^5}{125}-\cdots\right)}{y^3\cos y}=\lim_{y\to 0}\frac{\left(-\frac{1}{6}+\frac{23y^2}{125}+\cdots\right)}{\cos y}$

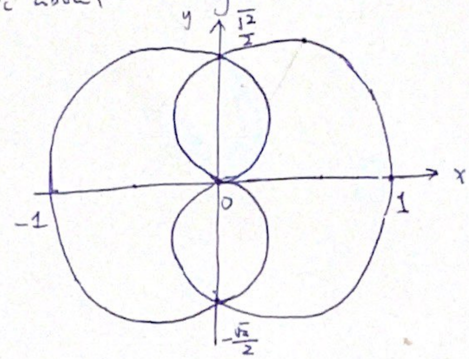
$=\frac{-\frac{1}{6}}{1}=-\frac{1}{6}$

8. As $\cos(-\frac{\theta}{2}) = \cos(\frac{\theta}{2}) = r$, the curve is symmetric about x-axis

By $\cos(\frac{2\pi-\theta}{2}) = \cos\frac{\theta}{2} = r$, the curve is symmetric about y-axis

So the curve is symmetric about the origin.

θ	$r=\cos\frac{\theta}{2}$
0	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{2\pi}{3}$	$\frac{1}{2}$
π	0



24 $r = a\sin^2\frac{\theta}{2}$, $0\le\theta\le\pi$, $a>0$

gives $\frac{dr}{d\theta} = a\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

then $L = \int_0^{\pi} \sqrt{r^2+(\frac{dr}{d\theta})^2}\, d\theta$

$= \int_0^{\pi} \sqrt{a^2\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\, d\theta$

$= \int_0^{\pi} \sqrt{a^2\sin^2\frac{\theta}{2}}\, d\theta$

$= \int_0^{\pi} a\sin\frac{\theta}{2}\, d\theta$

$= -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$

$= 0-(-2a)$

$= 2a.$