数 学 作 业 纸　　　　（科目：LA　）

班级：CS 01　　姓名：宗逸朗　　编号：2020010869　　第 1 页

Problem 1.1.1.

Sol. (a) line in $\mathbb{R}^3$ (b) plane in $\mathbb{R}^3$ (c) 3-dimensional space in $\mathbb{R}^3$

Problem 1.1.3

Sol. As $2v = (v+w) + (v-w) = \begin{bmatrix}5\\1\end{bmatrix} + \begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}6\\6\end{bmatrix}$, we have $v = \begin{bmatrix}3\\3\end{bmatrix}$

and $w = (v+w) - v = \begin{bmatrix}5\\1\end{bmatrix} - \begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$.

Problem 1.1.6.

Sol. (i) A linear combination of v and w is

$$cv + dw = c\begin{bmatrix}1\\-2\\1\end{bmatrix} + d\begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}c\\-2c+d\\c-d\end{bmatrix}$$

The sum of the components is

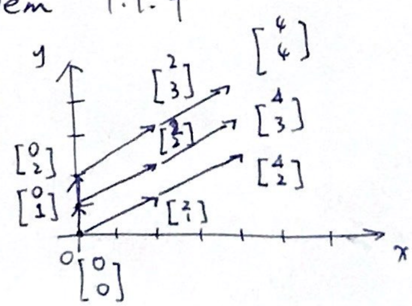
$$c + (-2c+d) + (c-d) = 0$$

(ii) We have $\begin{cases} c = 3 \\ -2c+d = 3 \\ c-d = -6 \end{cases}$ and $\begin{cases} c = 3 \\ d = 9 \end{cases}$ is the solution

(iii) We can mention that $3+3+6 = 12 \neq 0$, but from (i) we know the combination of v and w should have components which sum is zero.

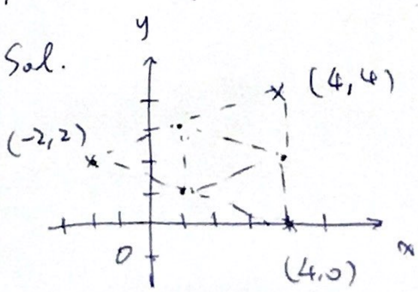
Problem 1.1.7

Sol.



Problem 1.1.9

Sol.

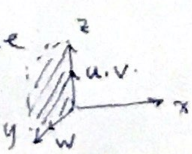


The fourth corner can be (-2,2), (4,0), (4,4)

Problem 1.1.25

Sol. Let $u = v = w = \begin{bmatrix}0\\0\\1\end{bmatrix}$, and their combination fill only a line.

Let $u = v = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $w = \begin{bmatrix}0\\1\\0\end{bmatrix}$, and their combination fill a plane.



数 学 作 业 纸

（科目：LA ）

班级：CS01 姓名：[name] 编号：2020010869 第 2 页

Problem 1.1.26

Sol. $c\begin{bmatrix}1\\2\end{bmatrix}+d\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}c+3d\\2c+d\end{bmatrix}=\begin{bmatrix}14\\8\end{bmatrix}$, so that $\begin{cases}c+3d=14\\2c+d=8\end{cases}$ and the solution is $c=2, d=4$.

Problem 1.1.29.

Sol. $1u+(-1)v+1w=\begin{bmatrix}1\\3\end{bmatrix}-\begin{bmatrix}2\\7\end{bmatrix}+\begin{bmatrix}1\\5\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$ and

$-2u+1v+0w=-2\begin{bmatrix}1\\3\end{bmatrix}+\begin{bmatrix}2\\7\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$ is two possible solution.

No, if $u=v=w=\begin{bmatrix}1\\1\end{bmatrix}$, the combinations of them would be $\begin{bmatrix}c\\c\end{bmatrix}\neq\begin{bmatrix}0\\1\end{bmatrix}$.

Problem 1.

Sol. (a) u and v span a plane in $\mathbb{R}^3$; $c\cdot u+dv=c\begin{bmatrix}2\\-1\\0\end{bmatrix}+d\begin{bmatrix}-1\\2\\-1\end{bmatrix}=\begin{bmatrix}2c-d\\2d-c\\-d\end{bmatrix}$.

(b) $w=\begin{bmatrix}1\\1\\0\end{bmatrix}$ is not a combination of u and v, and u, v, w span all of $\mathbb{R}^3$.

Problem 2.

Sol. (a) u and v span a plane in $\mathbb{R}^3$, $cu+dv=c\begin{bmatrix}2\\0\\0\end{bmatrix}+d\begin{bmatrix}0\\2\\2\end{bmatrix}=\begin{bmatrix}2c\\2d\\2d\end{bmatrix}$.

(b) v and w span a plane in $\mathbb{R}^3$, $ev+fw=e\begin{bmatrix}0\\2\\2\end{bmatrix}+f\begin{bmatrix}2\\2\\3\end{bmatrix}=\begin{bmatrix}2f\\2e+2f\\2e+3f\end{bmatrix}$

(c) Let $\begin{bmatrix}2c\\2d\\2d\end{bmatrix}=\begin{bmatrix}2f\\2e+2f\\2e+3f\end{bmatrix}$, we have $\begin{cases}2c=2f\\2d=2e+2f\\2d=2e+3f\end{cases}$ and the solution is

$\begin{cases}c=f=0\\d=e.\end{cases}$, the intersection of two distinct is $d\begin{bmatrix}0\\2\\2\end{bmatrix}$ and it is a line in $\mathbb{R}^3$.