Exercise 1. 9 Puzzle - UVa 11513

You are given a 3x3 grid of numbers, and you'd like to perform a series of operations so the grid ends up in the finishing configuration. For example,

start $\Longrightarrow$ | 3 | 8 | 7 |
1	6	2
4	5	9	$\Longrightarrow$ ... $\Longrightarrow$	1	2	3
4	5	6
7	8	9	$\Longrightarrow$ finish

The operations you are allowed to make are (1) circular shift a row horizontally (left or right) by one square and (2) circular shift a column vertically (up or down) by one square.

1. How many possible states in the game are there?
2. For a single state, how many other states can be reached?
3. Is it possible to reach the final state in the following start position?

1	2	3
4	5	6
7	**9**	**8**

4. What is the runtime of searching over the entire search space?
5. Suppose you are given N initial states that you are asked to compute the minimum number of moves to reach the finishing configuration. How do you speed up each search using an answer you've previously computed?

Exercise 2. Editing a Book - UVa 11212

You have n equal-length paragraphs numbered 1 to n. Now you want to arrange them in the order of `1, 2, ..., n`. With the help of a clipboard, you can easily do this: Ctrl-X (cut) and Ctrl-V (paste) several times. You cannot cut twice before pasting, but you can cut several contiguous paragraphs at the same time - they'll be pasted in order.

For example, in order to arrange `2, 4, 1, 5, 3, 6` in order, you can cut `1` and paste before `2`, then cut `3` and paste before `4`. As another example, one copy and paste is enough for `3, 4, 5, 1, 2`. There are two ways to do so: cut `3, 4, 5` and paste after `1, 2`, or cut `1, 2` and paste before `3, 4, 5`.

1. What is a simple cut-and-paste strategy with paragraphs one-by-one to place them in order?
2. Using this strategy, what is a loose upper-bound on the minimum number of cut-and-paste operations?

3. How many possible states are there in the search space?
4. For a single state, how many other states can be reached?
5. What is the runtime of searching over the entire search space?
6. If $n = 9$, about how many edges and nodes are there in the search space?
7. How can the search space be pruned and reduced?

Exercise 3. Clickomania - Rocky Mountain Regional 2009

You are asked to determine whether or not a string S such that $|S| \leq 150$ is valid using the following rules.

- The empty string is valid
- If `x` and `y` are valid, so are `xy`, `AxA`, and `AxAyA` for any character `A` $\in$ `[A-Z]`
- All other strings are invalid

For example, `ABBAABBAAB` is valid but `ABBAAAAB` is not.

1. How can you split the problem into subproblems?
2. How many possible states are there in the search space?
3. For a single state, how many other states can be reached?
4. With a DP technique, how many states will be reached in the search space?
5. What is the runtime of this algorithm?