Midterm Part 1A (35 Points)

1. Determine the following constants up to 1 significant figure (that is, the number of digits, and the value of the most significant digit should be correct after rounding).

   (a) $n \lg n$ where $n = 2^{30}$.

   (b) The number of times f is called in the following code (give the numerical answer):

```
int[] arr = {1,2,3,4,5,6,7,8,9,10};
do
{
  f();
} while ( nextPermutation(arr) );
```

   where nextPermutation behaves as discussed in class.

   (c) The largest **signed** 32-bit integer.

   (d) The number of times f is called in the following code:

```
for (int i = 0; i < (1<<20); ++i) f();
```

   (e) The number of times f is called in the following code:

```
int count(int pos, int val)
{
  if (pos == 0) return f();
  return count(pos-1,val) + count(pos-1,val-1);
}
```

   when count is called as count(20,20).

   (f) ($\star$) The number of times f is called in the following code:

```
for (int i = 0; i < (1<<20); ++i)
  for (int j = 0; j < 20; ++j)
    if ( (i & (1<<j)) != 0) f();
```

   (g) ($\star$) The number of times f is called in the following code:

```
int[][] cache = new int[21][21];
int count(int pos, int val)
{
  if (cache[pos][val] != 0) return cache[pos][val];
  if (pos == 0) return cache[pos][val] = f();
  return cache[pos][val] =
              count(pos-1,val) + count(pos-1,val-1);
}
```

   when count is called as count(20,20), and f always returns 1.

2. What comes immediately after $[4, 7, 23, 9, 12, 19, 15, 14, 5, 1]$ in lexicographic order?

3. Give, as **strings of bits**, the values of the following. You may omit leading (higher order) zeroes.

   (a) The subset $\{1, 3, 4\}$ of $\{0, 1, 2, 3, 4, 5\}$ expressed as a bitmask.

   (b) The Java or C++ expression: `6 ^ (1<<3)^ (1<<1)`

4. In each of the following assume the data structure has n elements. Use big-Oh notation.

   (a) What is the worst case runtime for removing the first (in the ordering) element from a Java PriorityQueue or C++ priority_queue (both have the same implementation)?

   (b) What is the **worst case** runtime of adding an element to an ArrayList or vector?

   (c) What is the worst case runtime for querying a Fenwick tree?

Midterm Part 2A (65 Points)

1. Suppose you have ants walking on a log of length 20 (the left edge at x-coordinate 0, the right edge at 20). Currently three ants are facing rightward at x-coordinates 4, 7, and 14, and three ants are facing leftward at 8, 10, and 17. In how much time will all of the ants have fallen off the log?

2. You are maintaining a fixed-length list of signed integers whose entries are changing. In addition to a large number of update operations, there will be frequent queries as to whether the product of a contiguous sub-range is positive, negative, or zero. Explain clearly but tersely:

   (a) What data structure(s) you will use.

   (b) What you do on an update operation that changes a positive entry to zero?

   (c) What you do on a query operation.

   You do **not** have to explain how to implement the data structure you have chosen.

3. You are given a 5-d cube with opposing corners at $(0, 0, 0, 0, 0)$ and $(1, 1, 1, 1, 1)$. If the point (a, b, c, d, e) is represented by the string of bits $abcde$, what are the bitstrings for the neighboring corners of 11010?

4. You are living in a world whose currencies are all powers-of-two between 2^0 and 2^{13}, inclusive. Write Java or C++ code that computes the number of ways to make exact change for the amount n, where $1 \leq n \leq 10000$. You may declare functions and allocate memory as necessary.

5. Assuming you are given a function **boolean** canComplete(**int** strength) that returns whether a given strength (in $[0, 10^8]$) is large enough to complete a task, write a function **int** minStrength() that returns the minimum strength needed. Your code should be Java or C++ code, but you will not be penalized for benign compiler errors as long as your code makes sense. You may write helper functions if necessary.