Deliver Time Optimization Project

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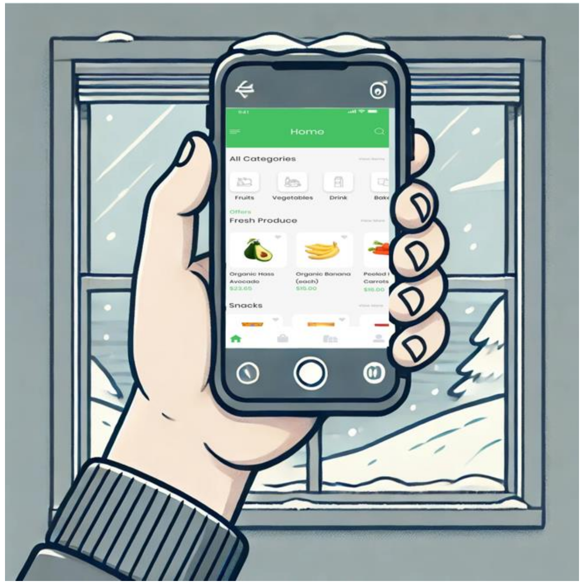
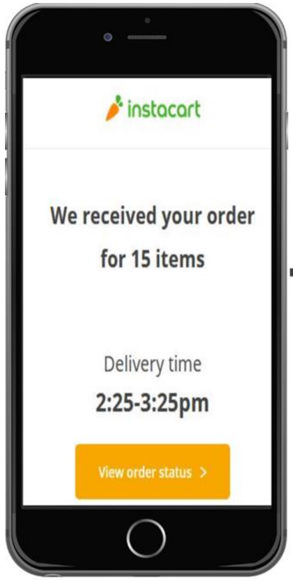
SHOPPING LIST
UNIVERSITY OF CAMBRIDGE


Home
All Categories
Fresh Produce
Snacks


instacart
We received your order for 15 items
Delivery time
2:25-3:25pm
View order status


instacart

What is instacart?

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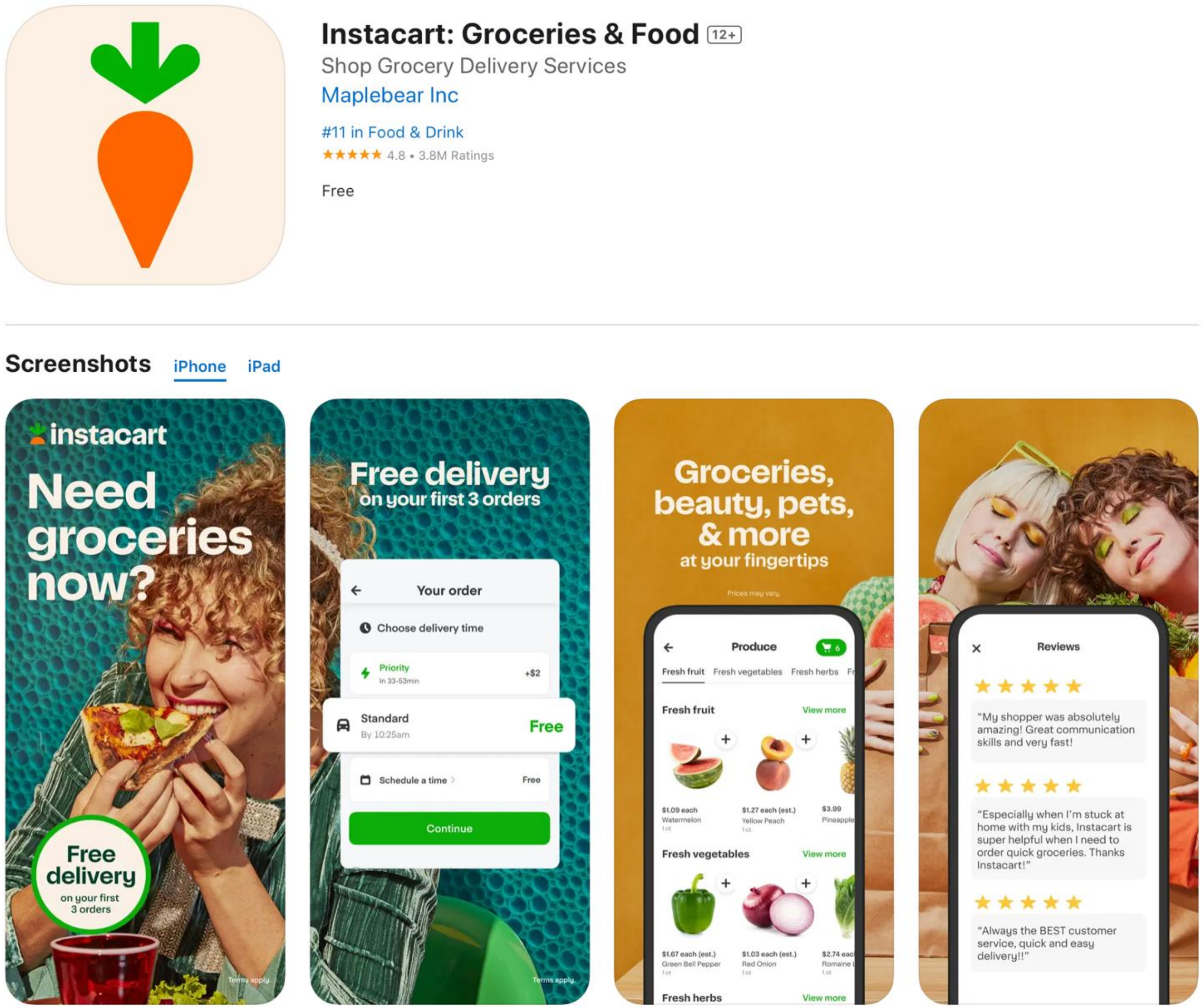
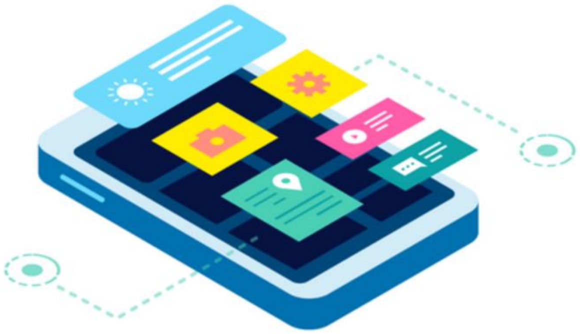


- A **four-sided marketplace** connecting customers, shoppers, drivers, and retail stores
- Fast-growing grocery delivery platform enabling **30-minute delivery**
- Complex coordination between **multiple stakeholders**

How Does instacart Work?



How Does instacart Work?

Shop Owner

Partnership Contract

Listing Groceries On Site / App

Sell

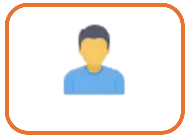
Advertise

How Does

instacart

Work?

Customer



- Sign Up
- Browse Goods
- Add to Cart
- Fill out Details & Make Payment
- Receive Goods

Shop Owner

- Partnership Contract
- Listing Groceries On Site / App
- Sell
- Advertise

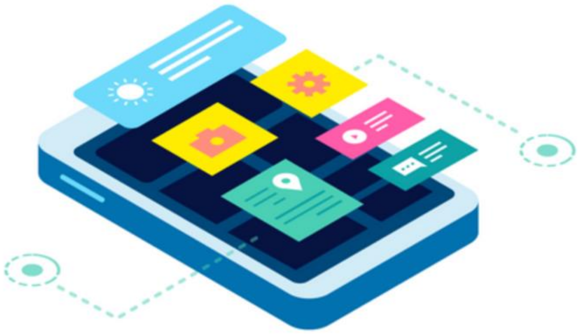
How Does

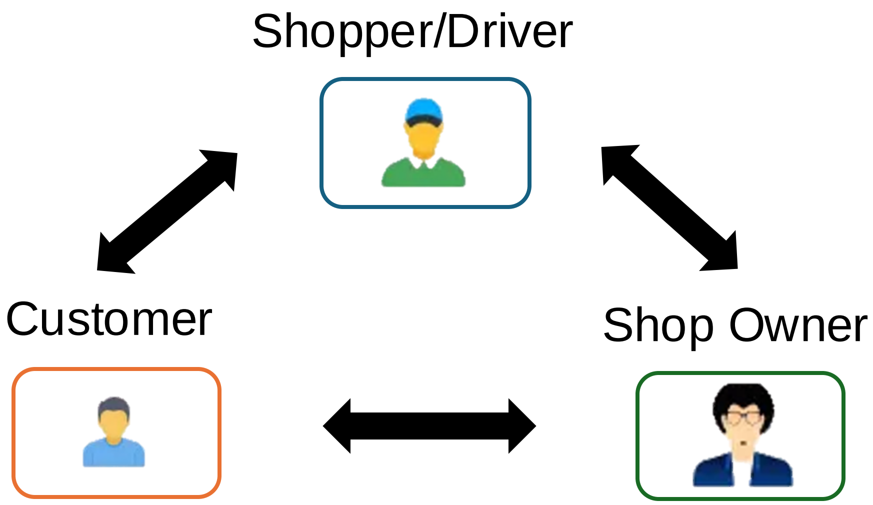
instacart

Work?

Shopper/Driver	Customer	Shop Owner
Register & Create a Profile	Sign Up	Partnership Contract
Receive Orders Via Notifications	Browse Goods	Listing Groceries On Site / App
Shop Goods	Add to Cart	Sell
Deliver Goods	Fill out Details & Make Payment	Advertise
Get Paid	Receive Goods	

How Does instacart Work?





Why use instacart?

- **Higher order values** than food delivery
- **Complex inventory management** (fresh/frozen items)
- Greater **batching potential**

Why instacart in Montreal?

- Complex road network (**one-way streets**)
- Frequent **construction zones**
- **Diverse** neighborhood densities
- **Extreme** seasonal weather
- **High reliance** on public transportation

Apoorva Mehta

Optimization???

Apoorva Mehta

Optimization???

Apoorva Mehta

- Shopper assignment to orders
- Minimizing in-store shopping time
- Batching multiple orders
- Determining the most efficient delivery routes

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Optimization???

Apoorva Mehta

- Shopper assignment to orders
- Determining the most efficient delivery routes

Optimization???

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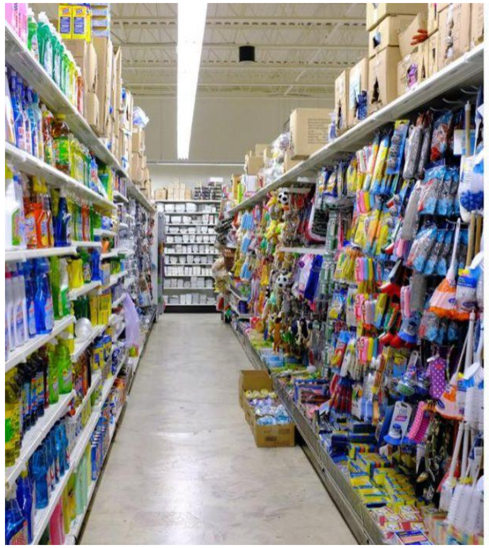


Why Dollarama?

- Well-distributed network across Montreal
- Represents residential, commercial, mix-used, and suburban areas.

Focus on Peak Hours

- Focused on peak hours: 10:00 AM - 3:00 PM
- Simulates busiest period for realistic demand.
- Enables effective modeling of shopper assignments and delivery routes.

Formulating the Optimization Problem

1

Pickup Stage

Assign shoppers to pick up orders from Dollarama stores.

2

Delivery Stage

Determine efficient routes for delivering orders to customers.

Project Objective

Minimize Average Delivery Time

Develop an optimization model for peak hour deliveries.

Key Assumptions

Proximity

Dollarama branches serve nearby customer locations.

Pre-packaged

Orders are ready for pickup at Dollarama stores.

Delivery Capacity

Shoppers can deliver up to two orders per trip.

Pickup Stage Optimization

Pickup Assignment Rules

Two Order Rule

2 orders within 10 minutes → assign the nearest shopper

One Order Rule

1 order and no additional order arrives within 10 minutes → assign the nearest shopper

Delivery Stage Optimization

Four Areas in Montreal

- City is divided into residential, commercial, mixed-use, and suburban zones.
- Each zone accounts for different traffic and geographic challenges

Real-World Traffic Constraints

- High-traffic zones
- Construction sites
- One-way streets

Optimized Delivery Sequence

The sequence of deliveries is optimized by serving the nearest customer first while considering proximity and travel conditions.

Mathematical Model: Variables and Parameters

Notation	Category	Description
S	Set	Set of supplier locations (possible pickup points).
D	Set	Set of demand locations (possible customer locations).
I	Set	Set of driver initial locations.
Δ_S	Set	Set of pickup locations (only 1 element contained)
Δ_D	Set	Set of delivery locations (up to 2 element contained)
T_{ij}	Parameter	Travel time between locations i and j, where $i, j \in S \cup D$.
x_{ij}	Decision Variable	Binary variable: 1 if driver k travels from i to j, 0 otherwise.

Pickup Stage: Objective Function and Constraints

$$\min_{\{x_{ij}\}} \sum_{j} \sum_{i} T_{ij} \cdot x_{ij}$$

s.t.

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} \geqslant 0 \quad \forall i \in I$$

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} \leq 1 \quad \forall i \in I$$

$$\sum_{i \in I} \left(\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} \right) = 1$$

To make sure there's exactly one source node (one driver take the order)

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = 0 \quad \forall i \in N \backslash \{I \cup \Delta_s\}$$

Intermediate Node

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = -1 \quad i = m \in \Delta_S$$

Terminal Node

Delivery Stage: Objective Function and Constraints

$$\min_{\{x_{ij}\}} \sum_{j} \sum_{i} T_{ij} \cdot x_{ij}$$

s.t.

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = \begin{cases} 1 & i = m \\ -1 & i = n \\ 0 & \text{o. w.} \end{cases}$$

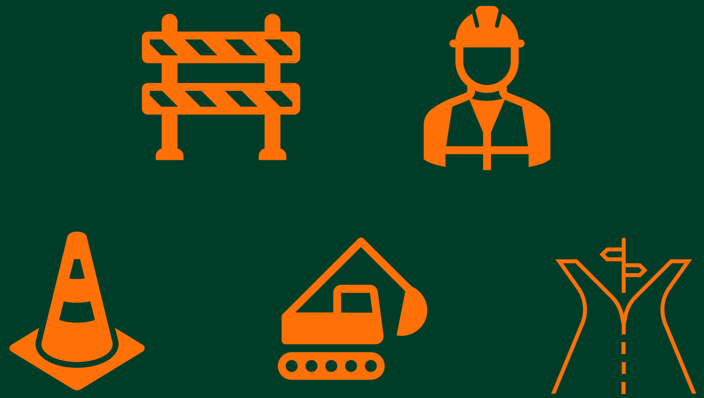
Source Node
Terminal Node
Intermediate Node

For any route starting from m and ending at n, the shortest path is found by solving the above optimization problem. We represent this shortest path as $u(m, n)$. Assuming $\Delta_S = \{a\}$, $\Delta_D = \{b, c\}$, in order to find the shortest path starting from $a \in \Delta_S$, dropping by one of the element of Δ_D, and ending at the other, we need to compare $u(a, b) + u(b, c)$ and $u(a, c) + u(c, b)$.

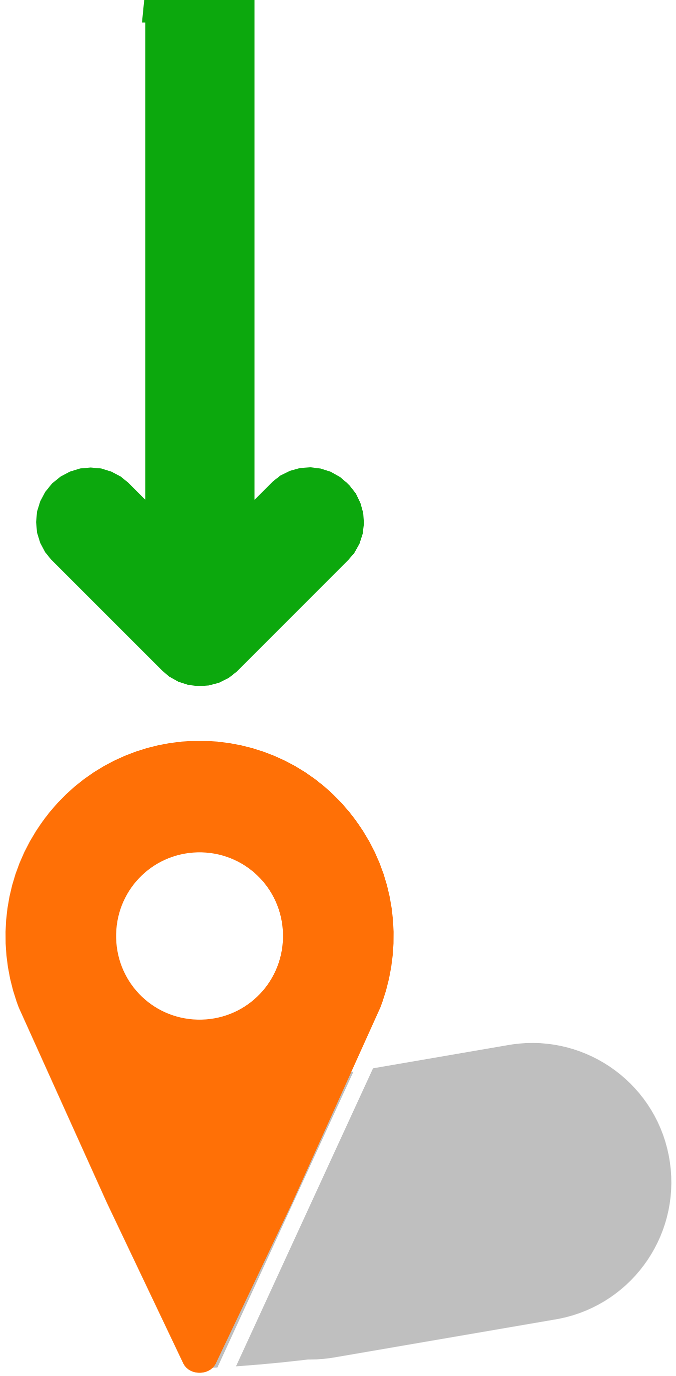
Data Simulation

Location

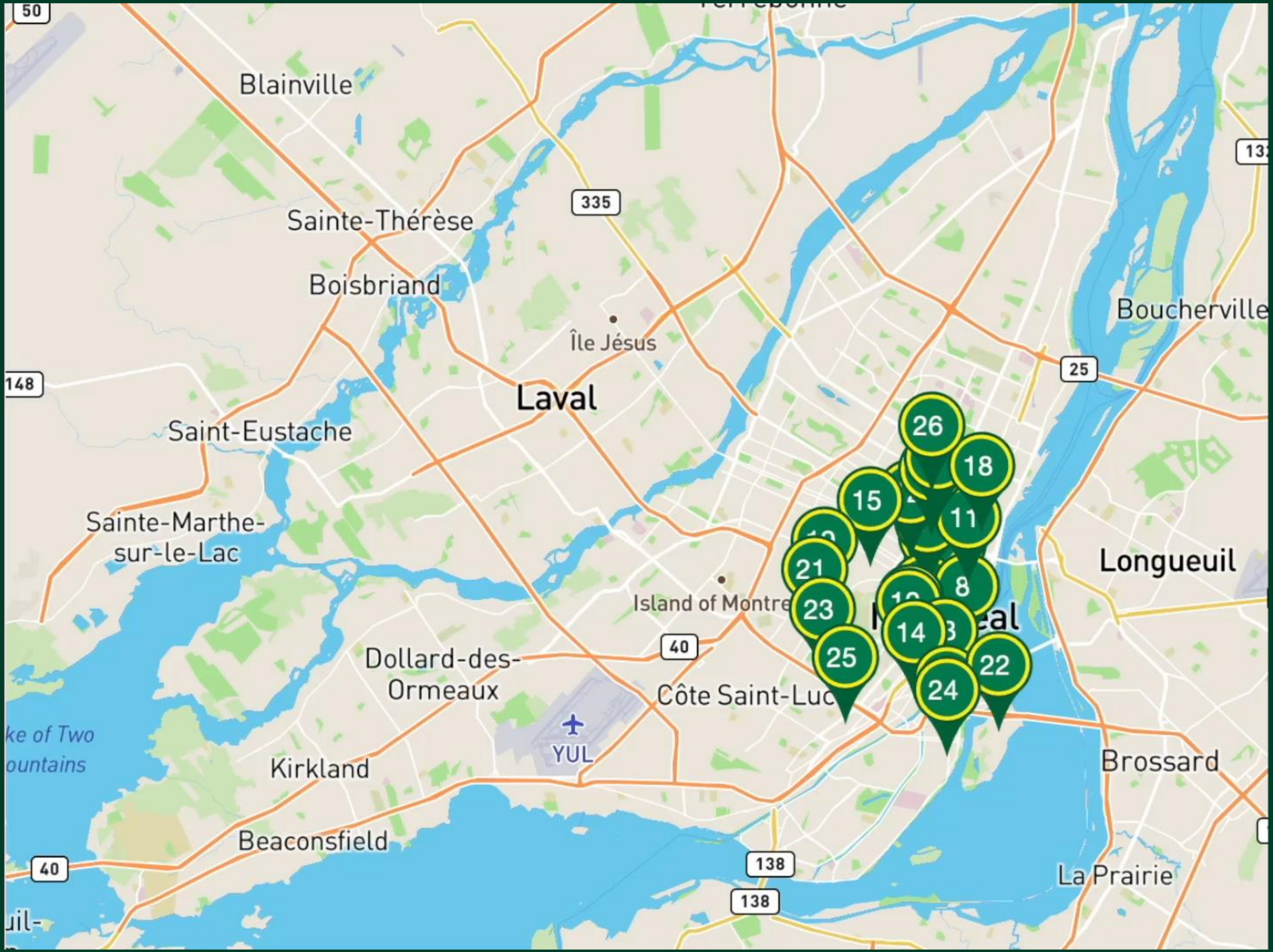
Time

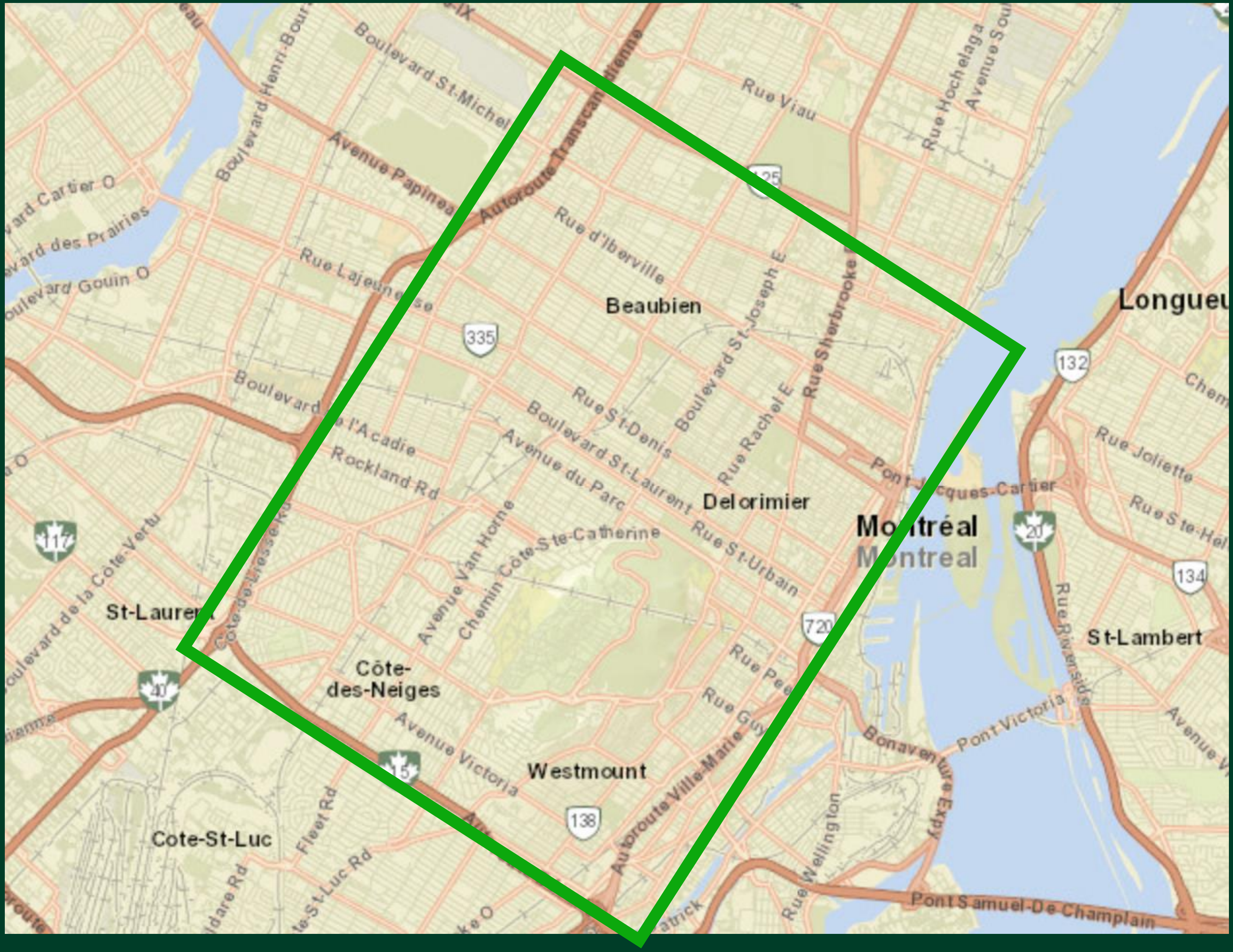


Location Setup



Dollarama Stores





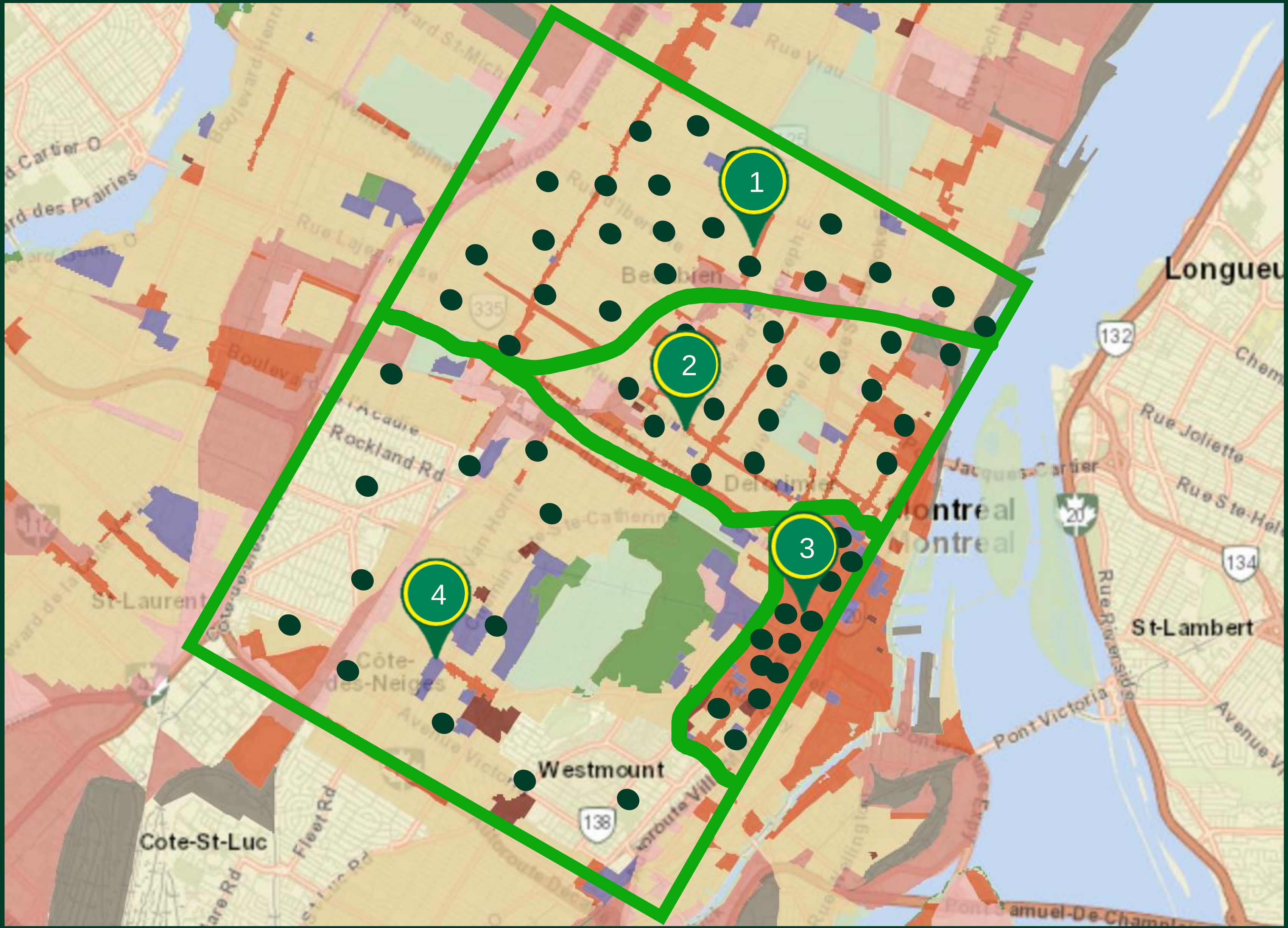
Beaubien
Delorimier
Montréal
Montreal
Longueuil
St-Laurent
Côte-des-Neiges
Westmount
Cote-St-Luc
St-Lambert
Rue Viau
Rue d'Iberville
Rue Lajeunesse
Rue St-Denis
Boulevard St-Laurent
Avenue du Parc
Rue St-Urbain
Boulevard St-Joseph E
Rue Rachel E
Rue Sherbrooke
Avenue Papineau
Boulevard St-Michel
Boulevard Henri-Bourassa
Autoroute Transcanadienne
Boulevard de l'Acadie
Rockland Rd
Avenue Van Horne
Chemin Côte-Ste-Catherine
Avenue Victoria
Rue Peel
Rue Guy
Autoroute Ville-Marie
Pont Jacques-Cartier
Pont Victoria
Pont Samuel-De Champlain
Bonaventure Expy
Rue Wellington
Rue Riverside
Rue Joliette
Boulevard de la Côte-Vertu
Boulevard Gouin O
Boulevard des Prairies
Fleet Rd
Rue Hochelaga

Residential

Mixed-use

Commercial

Suburbs



Schema Summary

Land uses	Percentage of Demand Points	Demand Sites (*D*)	Dollarama Stores (*S*)
Residential	45%	D1~D16	S1
Commercial	25%	D17~D26	S2
Mixed-used	20%	D27~D33	S3
Suburbs	10%	D34~D36	S4

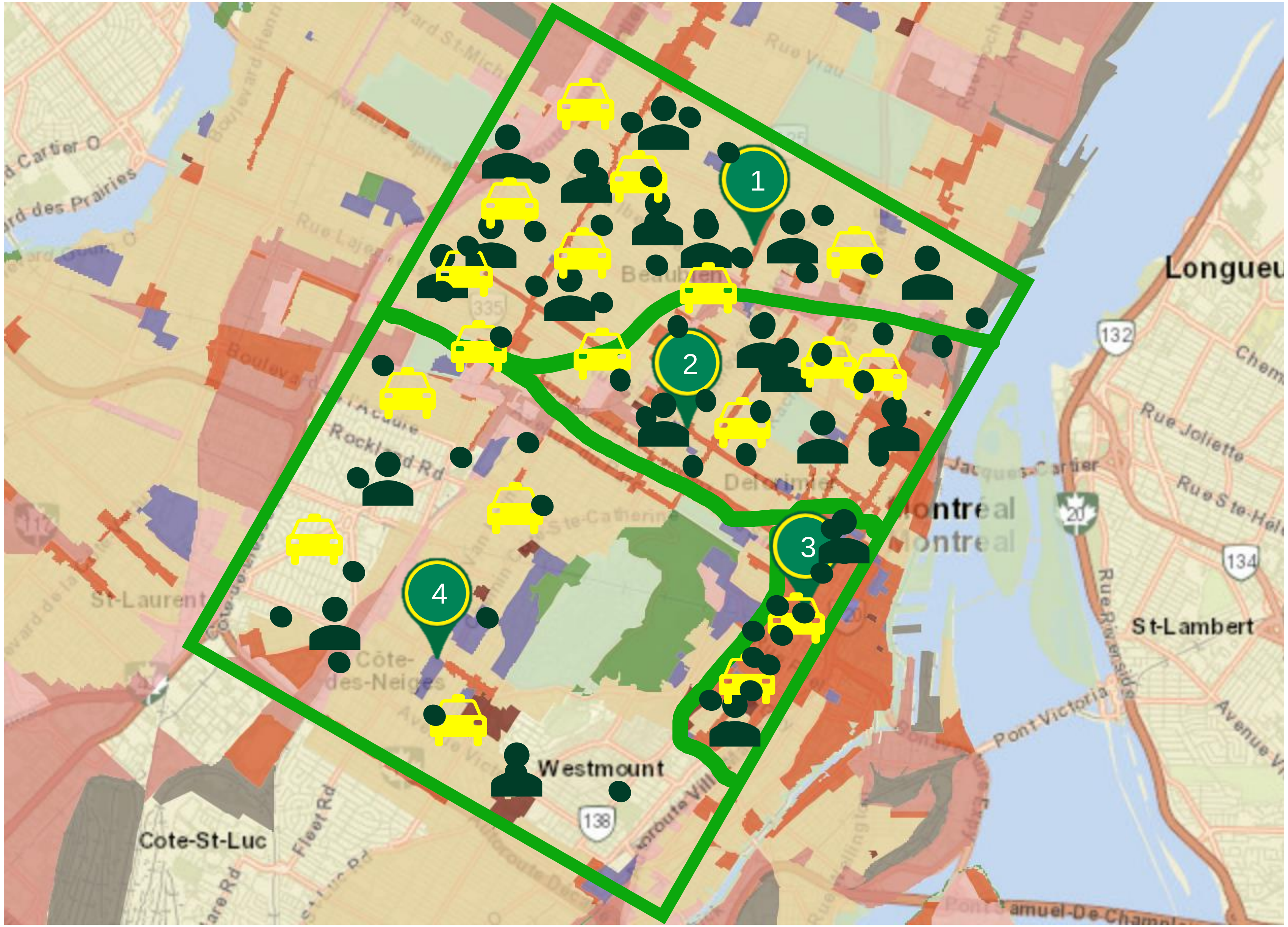
Initial Points for Drivers and Source Locations for Orders

Residential

Mixed-use

Commercial

Suburbs



Order Log

Order Number	Timestamp	S	D
1	10:03:11	S2	D20
2	10:05:42	S2	D25
3	10:07:59	S3	D30
4	10:11:33	S2	D17
5	10:18:42	S2	D19
6	10:24:19	S1	D6
7	10:27:03	S1	D6
8	10:34:28	S2	D19
9	10:37:39	S1	D4
10	10:41:09	S1	D8
11	10:45:00	S3	D31
12	10:46:59	S1	D14
13	10:49:42	S4	D36
14	10:51:37	S2	D18

Time Cost

Setup

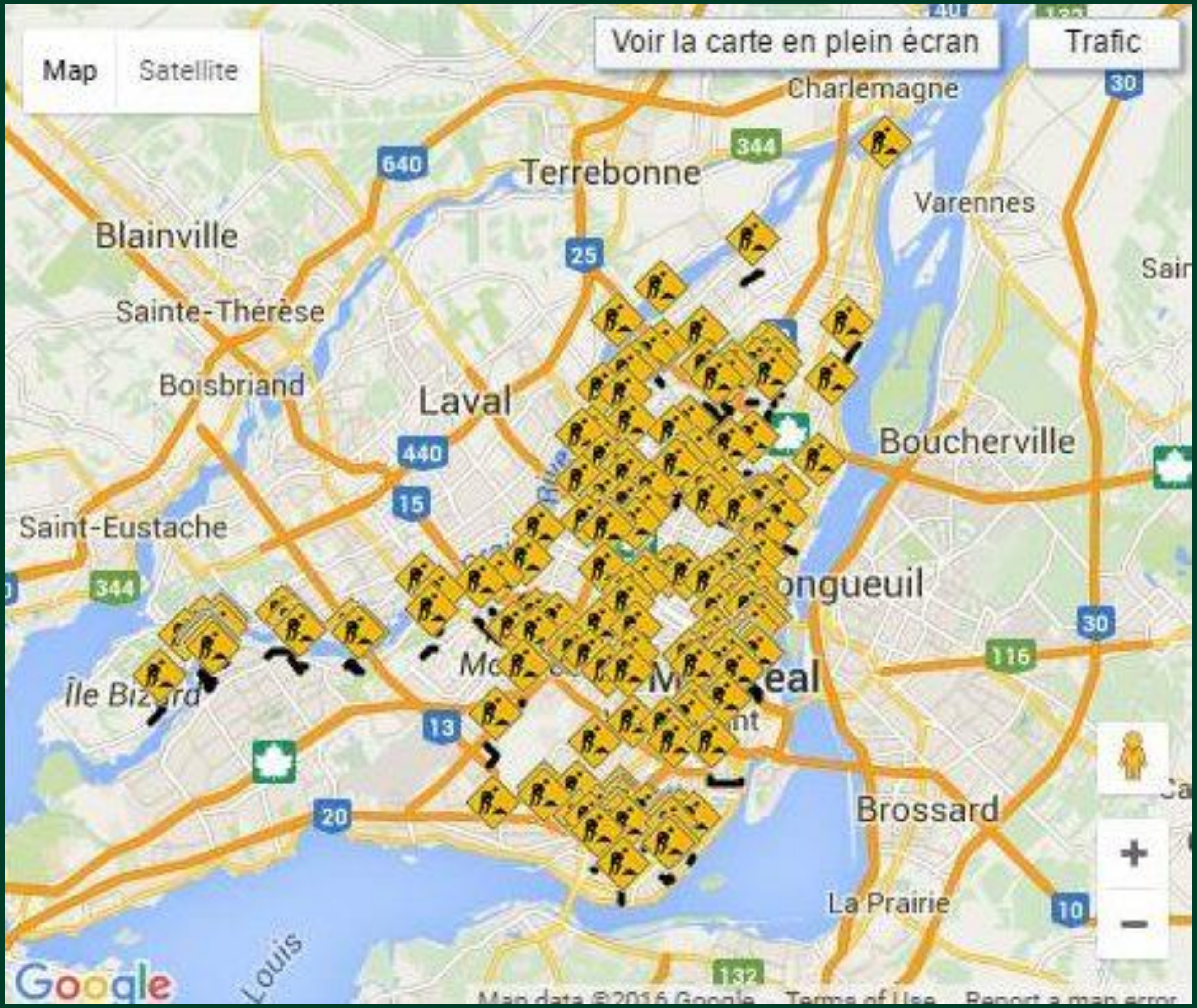
Driving Time Matrix (T_ij base)

(Minutes)	S1	S2	S3	D8	D14	D15	D19
S1	0	18.9	13.25	26.0333333	3.61666667	7.03333333	21.0666667
S2	18.9	0	7.71666667	19.0833333	18.5	22.9333333	9.55
S3	13.25	7.71666667	0	23.7166667	15.3666667	19.6166667	9.51666667
S4	23.0166667	15.8666667	20.7166667	4.6	22.8666667	25.3333333	12.1666667
D1	6.16666667	16.5333333	12.25	27.8	7.9	12.0666667	18.4166667
D2	8.43333333	11	8.61666667	1000023.6	8.21666667	12.3833333	16.65
D3	11.0666667	10.2666667	5.95	24.1666667	11.1333333	15.1833333	15.3166667
D4	22.1333333	16.1	19.8166667	5.75	22.7666667	1000025.23	12.0833333
D5	26.0333333	19.0833333	23.7166667	0	26	28.4666667	15.7333333
D6	3.61666667	18.5	15.3666667	26	0	6.15	20.3
D7	7.03333333	22.9333333	19.6166667	[illegible]	6.15	0	23.1666667
D8	21.0666667	9.55	9.51666667	[illegible]	20.3	23.1666667	0
D9	21.9166667	11.0166667	9.86666667	[illegible]	21.5833333	24.4333333	4.76666667
D10	20.95	16.8833333	15.3	20.05	20.1	22.0166667	10.6333333
D11	19.4833333	24.2333333	23.5833333	[illegible]	19.0166667	16.6333333	21.5
D12	21.8833333	26.6666667	26.0166667	[illegible]	1000021.43	10019.05	23.9166667
D13	18.7833333	23.55	22.9	22.95	18.3333333	15.95	20.8166667
D14	19.2333333	23.8833333	23.35	[illegible]	18.7833333	16.4	21.1333333
D15	18.4166667	20.4166667	21.6666667	26.7166667	17.6666667	20.5	20.5
D16	14.75	17.65	18.9	23.95	14.9166667	17.2833333	17.7333333
D17	15.1	7.78333333	8.51666667	16.2666667	14.35	17.1833333	10.0333333
D18	15.25	7.4	12.65	14.7333333	14.5	17.3166667	12.8833333
D19	16	2.48333333	6.4	18.8666667	15.25	18.0833333	7.88333333

Construction

Road Closure

Passable but slow



One-Way street

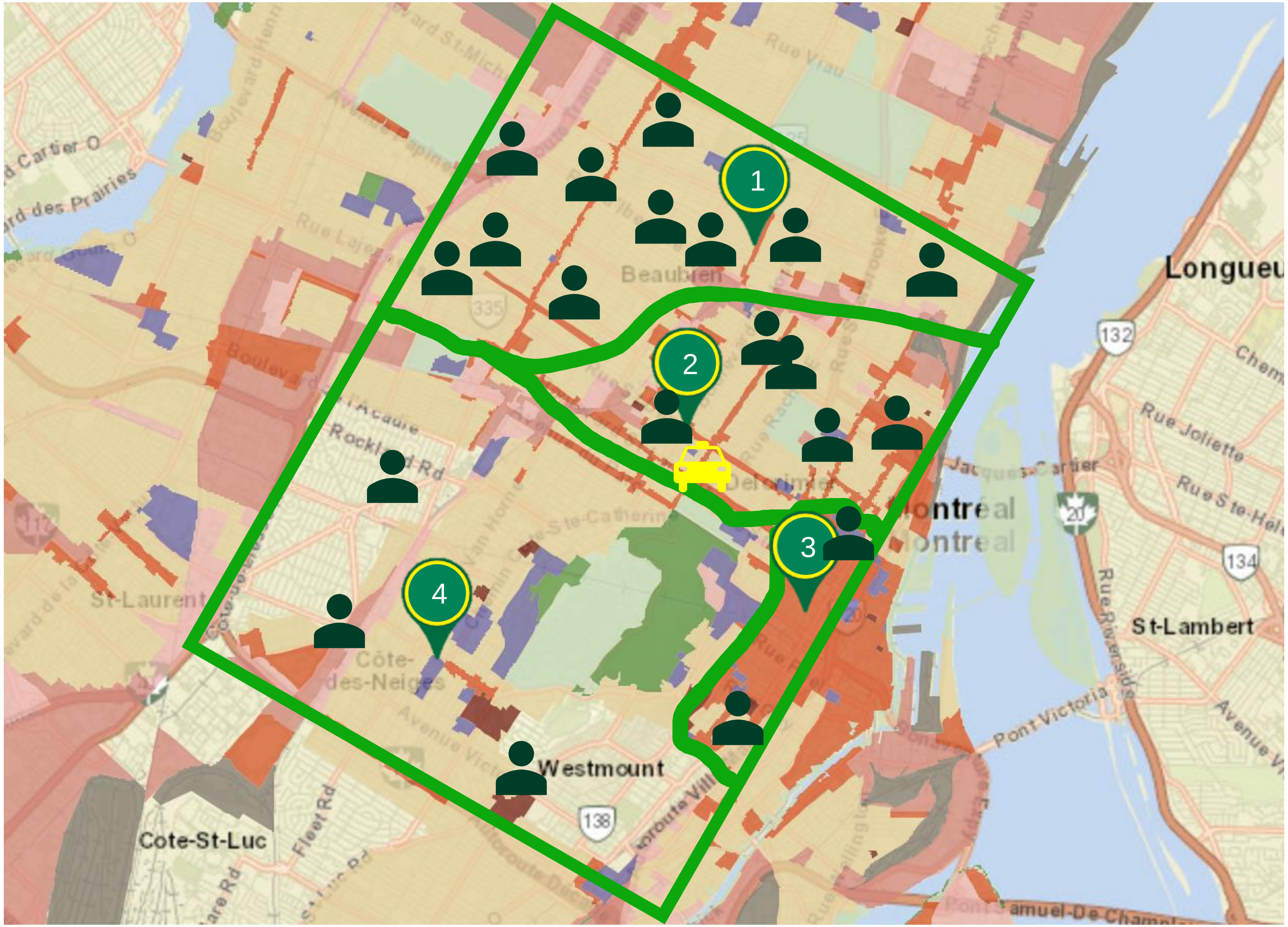
Distance Weighting

Residential

Mixed-use

Commercial

Suburbs



Weighting Schema

	Residential	Commercial	Mixed-used	Suburbs
Residential	Rand (0.05, 0.15)	0.15	0.1	0.05
Commercial	0.15	Rand (0.15, 0.25)	0.15	0.1
Mixed-used	0.1	0.15	Rand (0.1, 0.2)	0.05
Suburbs	0.05	0.1	0.05	0

Time Penalties

A=1 if Road closure

$$T_{ij} = T_{ij}^{\text{base}} + MA_{ij}$$

B=1 if Passable but slow

$$T_{ij} = T_{ij}^{\text{base}} + MB_{ij}$$

C=1 if One-Way street

$$T_{ij} = T_{ij}^{\text{base}} + MC_{ij}$$

Density Penalty

$$T_{ij} = T_{ij}^{\text{base}} \times \left(1 + \beta_{ij}\,\text{density penalty}\right)$$

Total Time Cost

$$T_{ij} = T_{ij}^{\text{base}} \times \left(1 + \beta_{ij}\,\text{density penalty}\right) + MA_{ij} + MB_{ij} + MC_{ij}$$

Preliminary Result

```python
initial_points = ['D1', 'D2', 'D3']
delta_s = ['S19']
delta_d = ['D26', 'D88']
source_node = delta_s[0]

result_p = optimize_shortest_path(tij_matrix, initial_points, delta_s)
result_d = compare_routes(tij_matrix, source_node, delta_d)

# Results
print(f"Shortest Pickup Path: {result_p['path']}")
print(f"Pickup Travel Time: {result_p['total_time']} minutes")
print(f"\nShortest Delivery Path: {result_d['shortest_path']}")
print(f"Delivery Travel Time: {result_d['total_time']} minutes")
print(f"Delivery Route Description: {result_d['route_description']}")
total_travel_time = result_p['total_time']+result_d['total_time']
print(f"\nTotal Travel Time: {total_travel_time} minutes")
```

✓ 0.5s

```
Shortest Pickup Path: [('D3', 'S19')]
Pickup Travel Time: 5.95 minutes

Shortest Delivery Path: ['S19', 'D75', 'D25', 'D26', 'D25', 'D88']
Delivery Travel Time: 49.116666699999996 minutes
Delivery Route Description: u(S19, D26) + u(D26, D88)

Total Travel Time: 55.0666667 minutes
```

Initial Point Chosen: D3
Pickup Path:
D3->S19
Pickup Time = 5.95 min

Delivery Path:
S19->D75->D25->D26->D25->D88
Delivery Time = 49.12 min

Total Time = 55.07 min

Time Window Integration

Why Time Windows?

- Instacart customers often specify preferred delivery intervals.
- Balances operational efficiency with customer-specific delivery preferences.

The New Constraint

- In the Delivery Stage, a time window metric measures the gap between order placement and the desired delivery time.
- Delivery routes prioritize orders nearing time window limits.

Key Benefits

- Improved customer satisfaction encourages loyalty and increases the likelihood of future orders.
- Better alignment with real-world operational settings.

Multi-Supplier & Multi-Store Integration

Why Multi-Supplier & Multi-Store?

- Real-world orders often span multiple suppliers (e.g., Provigo, IGA).
- Shared demand points between suppliers fulfill diverse customer needs.
- Enhances service flexibility and adapts to customer expectations.

New Model Features

- Many to many relationships: Demand points linked with multiple suppliers and shoppers.
- Flexible Shopper Routes: Shoppers can visit multiple suppliers in a single trip, allowing high-priority deliveries to be completed without waiting for all pickups to be finished.

Key Benefits

- Adapts to Montreal’s diverse supplier landscape and customer requirements.

Conclusion: Driving Sustainability Through Optimization

For Customers

Faster and more reliable access to groceries, improving convenience and satisfaction.

For Drivers

Increased earnings with more deliveries and reduced frustration from optimized routes.

For Instacart

Enhanced efficiency, stronger customer loyalty, and sustainable business growth.

For Environment

Reduced energy use and emissions, promoting eco-friendly delivery practices.