CMSE 820 HW9

This HW is due on Nov 17th at 11:59pm.

Question 1: Download HW11.csv from D2L and do the following (you can use existing package):

1. Run hard margin SVM and plot the data together with the estimated hyperplane. Inidicate the support vectors.
2. Change the label "Y" in the second row ($X_1 = 2.376567682$ and $X_2 = 0.11673195$) to "-1" and re-run the hard-margin SVM. Report your results.
3. Using the setting in (2), run a soft margin SVM with $\gamma = 1$ and plot the data together with the estimated hyperplane. Inidicate the support vectors.

Question 2:Let's return to the MNIST data set. Consider again a binary classification problem between 3 and 4. Take the labels to be $Y = \{+1, -1\}$,where $+1$ for"3" digit, and -1 for "4". Let's try hard margin SVM, soft margin SVM (you need to tune the γ using cross validation) and kernel SVM with Gaussian Kernel (you need to tune the σ using cross validation). Report your results in terms of accuracy in the testing set and plot your result using the first two PC directions. For the Kernel SVM, you can make a grids in the subspace of the first two PCs and evaluate them using the classifier learned from kernel PCA (blue for "1" and red for "-1") (You can do a 50% training and 50% testing split. You will do the CV in the training set.)

Question 3: For soft margin SVM, show that the regularized risk form

$$\min_{w,b} \frac{1}{n} \sum_{i=1}^{m} [1 - y_i(\langle w, x_i \rangle + b)]_+ + \frac{\lambda}{2} \|w\|^2.$$

is equivalent to the primal form

$$\begin{aligned} &\min_{w,b} \gamma \sum_{i=1}^{m} \xi_i + \frac{1}{2} \|w\|^2 \\ &\text{subject to } \xi_i \geq 0, \\ &y_i(\langle w, x_i \rangle + b) \geq 1 - \xi_i. \end{aligned}$$

Namely, the solution to the regularized risk form with $\lambda = 1/\gamma$ is the same as that for the primal form.

Question 4: Prove that the equivalence between constrained and Lagrange forms where

$$\text{constrained form: } \min_x f(x) \quad \text{subject to } h(x) \leq t;$$

$$\text{Lagrange form: } \min_x f(x) + \lambda h(x).$$

Namely, we want to prove that the solution of constrained form is also the solution for the Lagrange form, and vice versa