Comp 3320/6370 Computer Graphics

DIAGNOSTIC MATH TEST SOLUTION

Semester 2, 2018

Question 1:

$\frac{d}{dx}(\sin x \cdot \cos x)^2$

$= \frac{d}{dx}(\sin^2 x \cdot \cos^2 x)$

$= \left(\frac{d}{dx} \sin^2 x\right) \cos^2 x + \sin^2 x \left(\frac{d}{dx} \cos^2 x\right)$ (product rule)

$= 2 \sin x \left(\frac{d}{dx} \sin x\right) \cos^2 x + \sin^2 x \left(\frac{d}{dx} \cos^2 x\right)$ (chain rule)

$= 2 \sin x \cos^3 x + \sin^2 x \left(\frac{d}{dx} \cos^2 x\right)$

$= 2 \sin x \cos^3 x + \sin^2 x \cdot 2 \cos x \left(\frac{d}{dx} \cos x\right)$ (chain rule)

$= 2 \sin x \cos^3 x - 2 \sin^3 x \cos x$ (this is fine as solution)

$= 2 \sin x \cos x \cos 2x$ (but this would be faster)

Question 2:

$(3 + 2i)(1 + 4i)$

$= 3 + 12i + 2i - 8$

$= -5 + 14i$

Question 3:

Solution available in practice exercises (exercise number 3).

Question 4:

$(v_1, v_2, v_3) \cdot (w_1, w_2, w_3) = (v_1 w_1 + v_2 w_2 + v_3 w_3)$

$(v_1, v_2, v_3) \times (w_1, w_2, w_3) = (v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1)$

$||\overrightarrow{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$v = (2, 2, 1)$, $w = (1, -2, 0)$

(a) $\overrightarrow{v} \cdot \overrightarrow{w} = (2, 2, 1) \cdot (1, -2, 0)$

$= 2 \cdot 1 + 2(-2) + 1 \cdot 0$

= - 2

(b) $\overrightarrow{v} \times \overrightarrow{w} = (2, 2, 1) \times (1, -2, 0)$

$= (2 \times 0 - 1 \times (-2), \quad 1 \times 1 - 2 \times 0, \quad 2 \times (-2) - 2 \times 1)$

$= (2, 1, -6)$

(c) $(\overrightarrow{v} \times \overrightarrow{w}) \cdot \overrightarrow{v} = (2, 1, -6) \cdot (2, 2, 1)$

$= 2 \cdot 2 + 1 \cdot 2 + (-6) \cdot 1$

$= 0$

(d) $||\overrightarrow{v}|| = \sqrt{2^2 + 2^2 + 1^2}$

$= \sqrt{9}$

$= 3$

Question 5:

$$\det \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$
$$= 1 \cdot \det \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} - 0 \cdot \det \begin{pmatrix} 0 & 0 \\ 2 & 4 \end{pmatrix} + 2 \cdot \det \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$= 1 \cdot 8 - 0 \cdot 0 + 2(-4)$

$= 0$

$$A \cdot B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \cdot 1 + 0 \cdot 0 + 2 \cdot 0 & 1 \cdot 0 + 0 \cdot 1 + 2 \cdot 0 & 1 \cdot 1 + 0 \cdot 0 + 2 \cdot 1 \\ 0 \cdot 1 + 2 \cdot 0 \cdot 0 \cdot 0 & 0 \cdot 0 + 2 \cdot 1 + 0 \cdot 0 & 0 \cdot 1 + 2 \cdot 0 + 0 \cdot 1 \\ 2 \cdot 1 + 0 \cdot 0 + 4 \cdot 0 & 2 \cdot 0 + 0 \cdot 1 + 4 \cdot 0 & 2 \cdot 1 + 0 \cdot 0 + 4 \cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 2 & 0 & 6 \end{pmatrix}$$

Question 6:

The following is a drawing of the curve given by the parameterisation.