MATH1510 Discrete Mathematics
Semester 2, 2017

Andrew Goh/Thomas Murray's workshop, Monday 12-2pm **Quiz 3a**

Name: .... Student ID: ....
If this is not your normal workshop, then please state which one is:

Demonstrator: .... Weekday: .... Time: ....

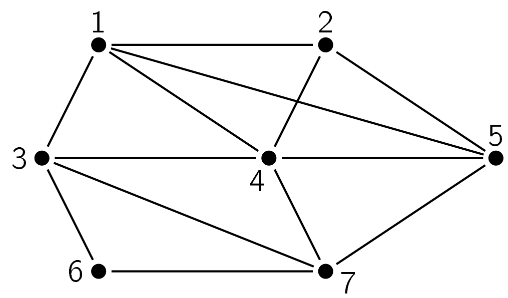
(Demonstrator's use only) Points achieved: out of 4

Instructions: If you run out of space in a question, do use the empty page(s) at the end of the quiz and indicate at the respective question that the working continues at the end of the paper. Always show your working in written answer questions, unless stated otherwise. You have 20 minutes for this quiz.

Solutions

1.

(a) Prove that the following graph G is not Eulerian.



(b) Find an edge e in G such that the graph H which is obtained from G by deleting e is Eulerian. (Hint: There is a unique edge e with this property.)

(c) Write down the vertex sequence for an Euler cycle for H.

1 solution:

(a) The graph does not have an Euler cycle because the vertices 2 and 4 have odd degree.

(b) If we remove the edge $\{2, 4\}$ then the graph becomes Eulerian.

(c) $(1, 2, 5, 7, 6, 3, 1, 5, 4, 7, 3, 4, 1)$.

2. Prove by induction that for every positive integer n,

$$\sum_{i=1}^{n} 3^i = \frac{3^{n+1}-3}{2}.$$

2 solution:

Setup. For every positive integer n, $P(n)$ is the statement

$$\sum_{i=1}^{n} 3^i = \frac{3^{n+1}-3}{2}.$$

Base case. $P(1)$ is

$$\sum_{i=1}^{1} 3^i = 3 = \frac{3^2-3}{2} = \frac{3^{1+1}-3}{2}.$$

Therefore, $P(1)$ is true.

Induction step. Suppose $P(k)$ is true. Then

$$\begin{aligned}\sum_{i=1}^{k+1} 3^i = \sum_{i=1}^{k} 3^i + 3^{k+1} &\overset{\text{using } P(k)}{=} \frac{3^{k+1}-3}{2} + 3^{k+1} \\ &= \frac{3^{k+1}-3+2*3^{k+1}}{2} = \frac{3*3^{k+1}-3}{2} = \frac{3^{(k+1)+1}-3}{2}\end{aligned}$$

So $P(k+1)$ is also true.

Conclusion. By mathematical induction, $P(n)$ is true for every positive integer n.

END OF PAPER