COMP2270/6270 – Theory of Computation
Fifth week

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Exercise 1) (Chapter 7 of Ref. [1]) Show a regular grammar for each of the following languages:

a) $\{w \in \{a, b\}^* : w$ contains an odd number of a's and an odd number of b's$\}$.
b) $\{w \in \{a, b\}^* : w$ does not end in aa$\}$.
c) $\{w \in \{a, b\}^* : w$ contains the substring abb$\}$.

Note: Start by formally defining what a regular grammar is.

Exercise 2) (Exercise 2, of Chapter 7 of Ref. [1]) Consider the following regular grammar G:

$S \to \mathrm{a}T$
$T \to \mathrm{b}T$
$T \to \mathrm{a}$
$T \to \mathrm{a}W$
$W \to \varepsilon$
$W \to \mathrm{a}T$

a) Write a regular expression that generates $L(G)$.
b) Use the procedure *grammartofsm* (see Theorem 7.1 in Chapter 7, of Ref. [1]), to generate a FSM M that accepts $L(G)$.

Exercise 3) Is the following statement True or False: *"For every FSM M there exists a regular grammar G that generates L(M)"*. Justify your answer.

Exercise 4) (Exercise 5, of Chapter 7 of Ref. [1]) Let $L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* :$ every $\mathtt{a}$ in w is immediately followed by at least one $\mathtt{b}\}$.

a) Write a regular expression that describes L.
b) Write a regular grammar that generates L.
c) Construct an FSM that accepts L.

Exercise 5) (Exercise 1, of Chapter 8 of Ref. [1]) For each of the following languages L, state whether or not L is regular. Prove your answer.

a) $\{\mathrm{a}^i\mathrm{b}^j : i, j \geq 0 \text{ and } i + j = 5\}$.
b) $\{\mathrm{a}^i\mathrm{b}^j : i, j \geq 0 \text{ and } i - j = 5\}$.
c) $\{\mathrm{a}^i\mathrm{b}^j : i, j \geq 0 \text{ and } |i - j| \equiv_5 0\}$.
d) $\{w \in \{0, 1, \#\}^* : w = x\#y$, where $x, y \in \{0, 1\}^*$ and $|x|\cdot|y| \equiv_5 0\}$. (Let $\cdot$ mean integer multiplication).

Exercise 6) Could the intersection of two infinite languages be a regular language? Justify your answer.

Exercise 7) When do we say that a binary relation R is *closed under a property* ?

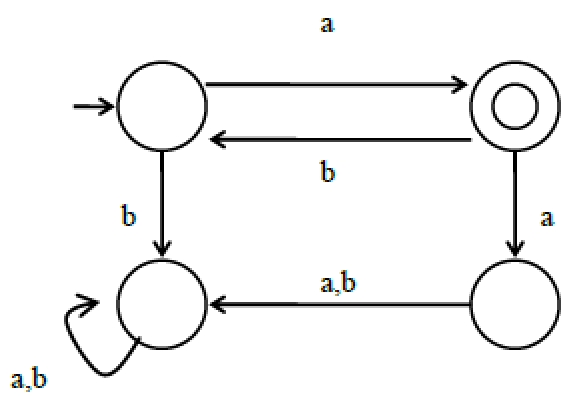
Exercise 8) Give five examples of the previous definition you have given in Exercise 7 (just above) as applied to languages. For instance: *"The set of even length strings of a's and b's is closed under concatenation."* Justify your answers.

Exercise 9) Are regular languages closed under intersection? Justify your answer.

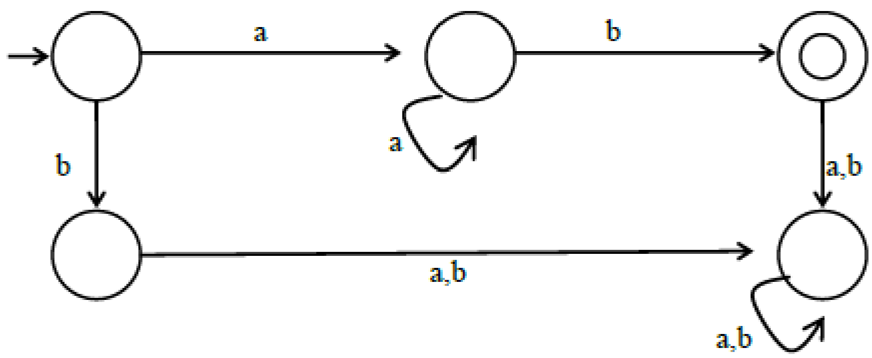
Exercise 10) (Exercise 20, of Chapter 8 of Ref. [1]) Consider the language $L = \{x\mathtt{0}^n y\mathtt{1}^n z : n \geq 0, x \in P, y \in Q, z \in R\}$, where P, Q, and R are nonempty sets over the alphabet $\{\mathtt{0}, \mathtt{1}\}$. Can you find regular languages P, Q, and R such that L is not regular? Can you find regular languages P, Q, and R such that L is regular?

Exercise 11) For the following examples describe informally the languages represented by the FSM and write down their regular expressions. You MUST use the algorithm *fsmtoregex* shown in class (page 142 of Ref[1]) and show your work.

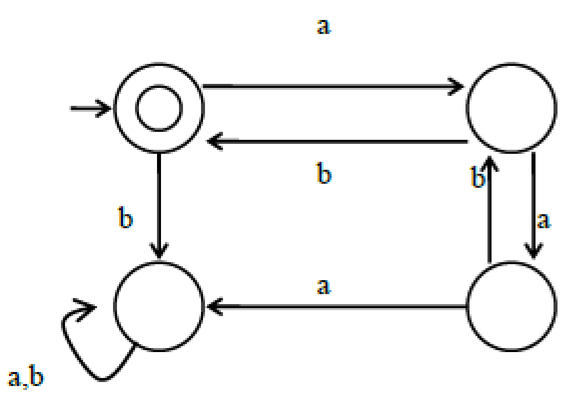
a)



b)



c)



REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008. `