MATH1510 Discrete Mathematics
Semester 2, 2017

Assignment 2

Due Date: This assignment is due in your workshop in week 3. You are also required submit it electronically through Blackboard.

1. Suppose $P(x)$ and $Q(x)$ are propositional functions and D is their domain. Let $A = \{x \in D : P(x) \text{ is true}\}$ and $B = \{x \in D : Q(x) \text{ is true}\}$.

(a) Give an example for a domain D and functions $P(x)$ and $Q(x)$ such that $A \cap B = \emptyset$.

(b) Give an example for a domain D and functions $P(x)$ and $Q(x)$ such that $A \subseteq B$ but $A \neq B$.

(c) Given that $x \in A - B$, what is the truth value of $Q(x)$?

(d) Given that $x \in A - B$, what is the truth value of $P(x) \vee \neg Q(x)$?

2. Show the following are logically equivalent:

(a) $\neg(p \vee q)$ and $\neg p \wedge \neg q$

(b) $\neg(p \rightarrow q)$ and $p \wedge \neg q$

3. Let A, B and C be conditions which can be either *true* or *false*. Suppose we want to write a computer program in which a certain piece of code should be executed if exactly one of the two conditions A and B is true, and in addition C is false. Using the operations $\wedge$, $\vee$ and $\neg$, write down a compound condition which is true only under the described circumstances. Use a truth table to prove that your expression has the required property.

4. For each of the following sequences, find out if there is any simple graph on 6 vertices such that the degrees of its vertices are given by that sequence. If you claim that there is no such graph, provide an argument supporting this claim, otherwise draw a graph with the corresponding degree sequence.

(a) 5, 3, 2, 2, 2	(b) 3, 3, 3, 3, 2	(c) 3, 3, 3, 2, 2
(d) 5, 5, 3, 2, 2, 1	(e) 5, 1, 1, 1, 1, 1	(f) 3, 3, 3, 3, 0, 0.

5. (Challenge question) Does there exist a bipartite graph on 14 vertices with degree sequence

$$3,\ 3,\ 3,\ 3,\ 3,\ 5,\ 6,\ 6,\ 6,\ 6,\ 6,\ 6,\ 6,\ 6\,?$$

6. (Challenge question) Let the two operations $p|q$ (Sheffer function) and $p \downarrow q$ (Peirce function) be defined by the following truth table:

p	q	$p\|q$	$p \downarrow q$
F	F	T	T
F	T	T	F
T	F	T	F
T	T	F	F

Show that the classical operations $\neg p$, $p \wedge q$ and $p \vee q$ (and therefore all truth functions) can be expressed in terms of

(a) the Sheffer function,

(b) the Peirce function.

END OF PAPER