The University of Newcastle
School of Electrical Engineering and Computer Science

COMP2230 Algorithms

Tutorial Week 5

16th – 20th August 2021

Tutorial

The coin-weighing problem:
There are n coins that look identical, but one of them is slightly heavier or slightly lighter than the other coins. The task is to identify the bad coin and to determine whether the coin is heavier or lighter using only a pan balance.

1. Sequential search can be used with the same efficiency in the case the list of elements is implemented as an array or as a linked list. Is this also true for binary search?

2. Write an algorithm to solve the coin-weighing problem for n = 5 using three weighings in the worst case.

3. Prove that the coin weighing problem for $n = 5$ cannot be solved using less than 3 weighings in the worst case.

4. Show the trees that result after the following statements are executed

```
For i=1 to 8
     Makeset1(i)

union1(1,2)
union1(3,4)
union1(5,6)
union1(5,7)
union1(5,8)
union1(4,8)
union1(3,2)
```

```
Makeset1(i){
     parent[i]=i
}

findset1(i){
     while(i!=parent[i])
          i=parent[i]
     return i
}

mergetrees1(i,j){
     parent[i]=j
}

union1(i,j){
     mergetrees1(findset1(i),findset(j))}
```

Workshop

5. What happens if union1 is erroneously called with i and j in the same tree?

6. Write an algorithm to solve the coin-weighing problem for n = 4 using three weighings in the worst case.

7. Prove that the coin weighing problem for n = 4 cannot be solved using less than 3 weighings in the worst case.

8. What is wrong with the following argument?
To solve the coin-weighing problem for n=12, we need at least 4 weighings if we start by weighing 4 coins against 4 coins. If we weigh 4 coins against 4 coins and they balance, we must then determine the bad coin from the remaining 4 coins. But we know from the previous exercise that determining the bad coin from among 4 coins requires at least 3 weighings in the worst case. Thus we require at least 4 weighings.

9. You are given 42 pictures arranged in 7 rows and 6 columns. Your task is to construct a serious of yes/no questions so that by getting correct answers to your questions you can identify the picture with as few questions as possible.

More exercises

10. Write an algorithm to solve the coin-weighing problem for n = 12 using three weighings in the worst case.

11. Prove that the coin weighing problem for n = 12 cannot be solved using less than 3 weighings in the worst case.

12. Write an optimal algorithm to solve the coin-weighing problem for n = 13. Prove that your algorithm is optimal.

13. Show that if m<n, after executing Algorithms makeset1, findeset1 and union1 the maximum height of a tree is m.