MATH1510 - Discrete Mathematics
Sets

University of Newcastle

UoN

Why study sets?

- The language of sets is used to write mathematics
- The theory of sets underpins most modern mathematics
- Sets have interesting structure: they make a "Boolean algebra", relevant to Computer Science

What is a Set?

Definition

- A set is a well-defined collection of distinct objects.
- Objects in a set are called elements.

Example: The set of days of the week has elements: $\boxed{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}}$.

Example: The set of counting numbers (starting at 1) less than six has elements: $\boxed{1, 2, 3, 4, 5}$.

Sets must be *well-defined*

There must be no ambiguity about which objects belong to the set.

Examples of sets:

1. The collection of all students studying Math1510.
2. The collection of stars in our galaxy.
3. The collection of all even numbers.

Examples of collections which are not sets:

4. The collection of all good movies.
5. The collection of all pretty pictures.
6. The collection of all bad weather.

Which of the following is *not* a set?

A The collection of books in the Auchmuty Library

B The collection of boring books in the Auchmuty Library

C The collection of birds in the Southern Hemisphere

D The collection of flightless birds in the Southern Hemisphere

Why?

Notational Conventions

- Capital letters, like A, B, C, represent sets
- Lower-case letters, like a, b, c, represent elements of sets
- We *say* any of
  - a is an element of the set A,
  - a belongs to A,
  - a is in A
- and we *write*
  - $a \in A$

Example:

- if an object 2 is an element of set B, we write $2 \in B$.
- if an object 2 is not an element of set B, we write $2 \notin B$.

Notational Conventions from *logic*

We may also use the following notation:

- $\forall$ means 'for all', called the universal quantifier
- $\exists$ means 'there exists', called the existential quantifier
- $\exists!$ means 'there exists a unique', also $\exists_1$
- $:=$ means 'defined to be equal to'
- $\therefore$ means 'therefore'
- $\square$ indicates the end of a proof
- QED also indicates the end of a proof: *"quod erat demonstrandum"* which translates loosely to "that which was to be demonstrated"

Notational Conventions: order doesn't matter for sets

- Curly brackets enclosing brackets { and } denote a set.

Example: Let T stand for the set of colours used in traffic lights. Then

$$T = \{\text{red}, \text{orange}, \text{green}\}.$$

We could equally have written

$$\boxed{\begin{aligned} T &= \{\text{orange}, \text{green}, \text{red}\} \\ T &= \{\text{red}, \text{red}, \text{orange}, \text{green}\} \end{aligned}}$$

because

- elements may be listed any number of times, and
- in any order.

Note that

$$\boxed{T \neq \{\text{red}, \text{orange}\}.}$$

Three sets are the same. One is different. Which one?

- A $\{1, 2, 2\}$
- B $\{2, 2, 1\}$
- C $\{2, 2, 2\}$
- D $\{2, 1\}$

Set Descriptions

We describe sets into two ways:

- **Roster notation** - listing all elements
- **Set-builder notation** - alternative to listing all elements

Roster notation

In roster notation, also known as tabular form or enumeration, the elements of the set are presented in a complete or implied listing, separated by commas, eg.

1. $\{1, 2, 3, 4\}$
2. $\{2, 3, 5, 7, 11, 13, \ldots\}$
3. $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$

We use ellipsis $(\ldots)$ to show that a pattern continues.

Set-builder Form

In set-builder form the set is described in terms of some properties of the set. We use colon to mean "such that" (though some authors use "|").

1. $\{x : x \text{ is a natural number less than } 5\}$
2. $\{y : y \text{ is a prime number }\}$
3. $\{x : x \text{ is an even number}\}$

Note: often it will be clear from the context what our "universe of discourse" is, in which case we can just write $\{x : x < 5\}$ instead of $\{x : x \text{ is a natural number less than } 5\}$.

Two special sets are

- The empty set, and
- The Universal set

Important sets

Definition

The empty set is the unique set containing no elements. It is denoted by the Norwegian and Danish letter $\varnothing$ (or $\emptyset$) or by a pair of empty braces $\{\}$.

The Universal Set

The Universal Set depends on the context.

Any set which contains all the elements we may wish to discuss may be designated as a universal set. We denote the universal set by U.

1. If we are dealing with whole numbers then an appropriate universal set would be the set of all integers.
2. If the topic being discussed were stars then an appropriate universal set would be the set of all stars in the universe.

Common Sets

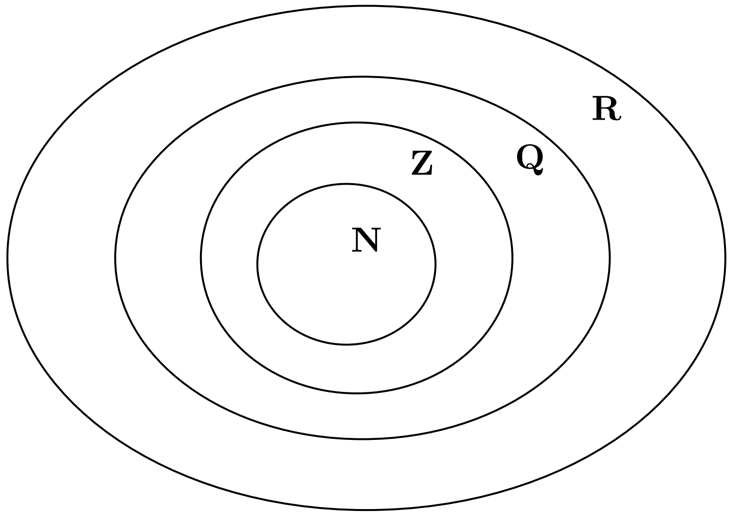
Some sets are so common and important that they have special symbols:

- $\mathbb{Z}$ is the set of integers, $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
- $\mathbb{N}$ is the set of natural numbers, $\{1, 2, 3, \ldots\}$. (Some books say $\{0, 1, 2, 3, \ldots\}$ instead).
- $\mathbb{Q}$ is the set of rational numbers. That is, the set $\{x : x = \frac{p}{q}$, where p and q are integers , $q \neq 0\}$.
- $\mathbb{R}$ is the set of real numbers (the rationals and the irrationals).

- Subscripts and superscripts can restrict the set, for instance to only the positive members. For example, $\mathbb{R}^+$ and $\mathbb{R}_{>0}$ both denote the set of positive real numbers, and $R_{\geq 0}$ is non-negative real numbers.

- Of these common sets, $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$ are part of *Discrete Mathematics*, whereas $\mathbb{R}$ is not. *Why?*

These common sets nest inside each other



Cardinality

Definition

The cardinality of a finite set is the number of elements in the set. The cardinality of set A is denoted $|A|$.

1. If $A = \{x, y, z\}$ then $|A| = 3$.
2. $|\emptyset| = 0$.

Which of the following sets has cardinality <u>not</u> equal to 3?

- A $\{\{1, 2\}, 3\}$
- B $\{1, 2, 3\}$
- C $\{4, 5, 6\}$
- D $\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$

Finite and Infinite Sets

Definition

If a set contains a finite number of elements then we say that the set is finite. Otherwise we say that the set is infinite.

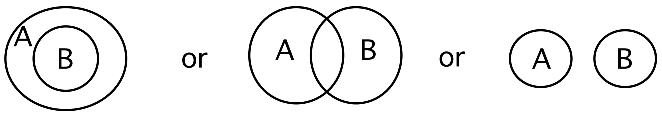
1. The set of stars in our galaxy is finite.
2. The set of all even numbers is infinite.

Venn Diagrams

Definition

A diagram in which sets are represented as circles or more general shapes that may overlap is called a Venn Diagram.

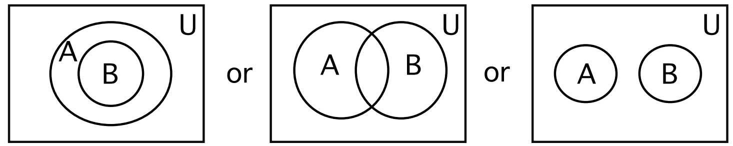
eg.



Venn Diagrams were introduced for visualizing concepts in set theory in the late 1800s by English logician, John Venn, although it is believed that the method originated earlier.

Venn Diagrams with Universal Sets

A Universal set is typically represented as a rectangle containing all the other sets under discussion, eg.



Relationships between sets

We discuss the following relationships:

- Subset $\subseteq$
- Superset $\supseteq$
- Proper subset $\subset$
- Proper superset $\supset$
- Set equality $=$

Subsets and supersets

Definition

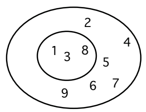
A set A is a subset of a set B if every element of A is also an element of B. We write this as

$$A \subseteq B.$$

If A is a subset of B, then B is called a superset of A.

Examples:

1. $\{1, 3, 8\} \subseteq \{1, 2, \ldots, 9\}$
2. $\{a\} \subseteq \{a, b, c\}$



Some subset relationships that are always true

Theorem

- *The empty set is a subset of every set.*
- *Every set is a subset of itself.*

Proper subsets and proper supersets

Definition

A set A is a proper subset of a set B if $A \subseteq B$ and $A \neq B$. We write this as

$$A \subset B.$$

If A is a proper subset of B then we say B is a proper superset of A.

Examples:

1. $\{a, c\} \subset \{a, b, c\}$
2. $\{b\} \subset \{a, b, c\}$
3. $\emptyset \subset \{a, b, c\}$

Warning: some authors use $\subset$ for subset and $\subsetneq$ for proper subset.

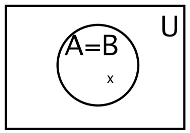
Equality of Sets

Definition

Two sets, A and B, are said to be equal if and only if they contain exactly the same elements. We write this as

$$A = B.$$

More formally, $A = B$ provided: for all $x \in U$, $x \in A$ if and only if $x \in B$.



Which claim is false?

Let $S = \{a, b, c\}$. Exactly one claim is FALSE. Which one?

- A $\{c\} \subseteq S$
- B $a \subset S$
- C $\varnothing \subseteq S$
- D $\{b, a\} \subset S$

How do we find out if two sets are equal?

Eg.

- Let $S = \{x : 1 < x < 9 \text{ and } x \in \mathbb{Z}\}$
- Let $T = \{w : w \leq 7 \text{ and } w \text{ is a prime number}\}$

Which is true?

A $S = T$

B $S \neq T$

How do we find out if two sets are equal?

We can express equality of sets in terms of subsets as follows:

Theorem

Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

This is because the only way we can have both $A \subseteq B$ and $B \subseteq A$ is if they contain the same elements.

1. $\{a, a, b\} = \{a, b\}$
2. $\{a, b, c\} = \{b, c, a\}$

Proving the set equality theorem

Theorem

Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

PROOF

$\Rightarrow$ Suppose that $A = B$. Then since every set is a subset of itself we must have both $A \subseteq B$ and $B \subseteq A$.

$\Leftarrow$ Now suppose that $A \subseteq B$ and $B \subseteq A$. We show that the elements of B are precisely the elements of A. For each element x in our universe either $x \in A$ or $x \notin A$. Suppose $x \in A$, then since $A \subseteq B$, $x \in B$. Suppose $x \notin A$, then since $B \subseteq A$, $x \notin B$. So $x \in A$ precisely when $x \in B$. Hence the two sets are the same. $\square$

Exercise

- Dream up an example in which you would need to use the set equality theorem to show that two sets are equal.

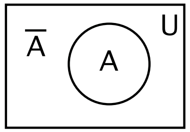
The Complement of a Set

Definition

The complement of a set A in relation to some universal set U is the set

$$\overline{A} = \{x \in U : x \notin A\}.$$

$\overline{A}$ is sometimes written A^c.



- Eg, if $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 3\}$, then $\overline{A} = \{2, 4, 5\}$.

What is $\overline{A}$?

Let $U = \{1, 3, 5, 7, ...\}$ and $A = \{1, 3\}$. Then which is correct?

A $\overline{A} = \{5, 7\}$

B $\overline{A} = \{5, 7, 9, 11, \dots\}$

C $\overline{A} = \{5, 7, 9, 11\}$

D $\overline{A} = \{-1, -3\}$

Power Sets

Definition

The power set of a set A, denoted $\mathcal{P}(A)$, is the set of all subsets of A.

Note that

- the empty set is a subset of any set; and
- any set is a subset of itself.

Example: Let $A = \{1, 2, 3\}$. Then

$$\mathcal{P}(\mathrm{A}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

Notice that in our example A contains 3 elements and $\mathcal{P}(A)$ contains 2^3 elements. This is no accident. For a set containing $n \in \mathcal{N}$ elements, its power set contains 2^n elements. *Why?*

Binary Operations on Sets

In arithmetic we have the binary operations of addition, subtraction, multiplication and division. Similarly there are binary operations for sets.

The four binary set operations that we will consider are

- $\cup$ is *union*
- $\cap$ is *intersection*
- $-$ or $\setminus$ is *difference*
- $\triangle$ is *symmetric difference*

The Union of Sets

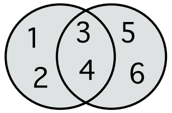
Definition

The union of two sets A and B, written as $A \cup B$, is the set of all elements which belong to either A or B (or both, since we use the "inclusive or" by default.). Symbolically

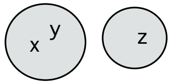
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Examples:

1. $\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, \ldots, 6\}$



2. $\{x, y\} \cup \{z\} = \{x, y, z\}$



Some Properties of the Union of Sets

The following are always true for any two sets A and B.

- $A \cup B = B \cup A$
- $A \subseteq A \cup B$
- $B \subseteq A \cup B$
- $A \cup A = A$
- $A \cup \varnothing = A$
- $A \cup U = U$

Unions of More than Two Sets

We can also take the union of several sets. Since $A \cup (B \cup C) = (A \cup B) \cup C$, we can just write $A \cup B \cup C$ without ambiguity.

Similarly for a union of n sets we can write

$$S_1 \cup S_2 \cup S_3 \cup \cdots \cup S_n.$$

Or we can use the following notation:

$$\bigcup \{S_1, S_2, S_3, \ldots, S_n\}.$$

or sometimes just

$$\bigcup_{i=1}^{n} S_i$$

The Intersection of Sets

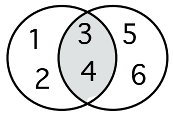
Definition

The intersection of two sets A and B, written as $A \cap B$, is the set of all elements which belong to both A and B. Symbolically

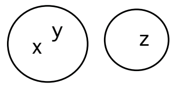
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Examples:

1. $\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$



2. $\{x, y\} \cap \{z\} = \varnothing$



The shaded parts of the Venn Diagrams indicate the intersections. In the second example, the intersection is empty.

Some Properties of the Intersection of Sets

The following are always true for any two sets A and B.

- $A \cap B = B \cap A$
- $A \cap B \subseteq A$
- $A \cap B \subseteq B$
- $A \cap A = A$
- $A \cap \varnothing = \varnothing$
- $A \cap U = A$

Draw Venn Diagrams!

A direct proof of one of these properties

To show that $A \cap B \subseteq A$ we must we must show that for all x, if $x \in A \cap B$, then $x \in A$.

Proof: *Let x be a particular but arbitrary element of $A \cap B$. Then*

$$\begin{aligned} & x \in A \cap B && (1)\\ \textit{which implies that} \quad & x \in A \text{ and } x \in B && (2)\\ \textit{which implies that in particular} \quad & x \in A. && (3) \end{aligned}$$

Since x was an arbitrary element of $A \cap B$, we have shown that all of the elements of $A \cap B$ are contained in A, i.e., $A \cap B \subseteq A$, as required. $\square$

Intersections of More than Two Sets

We can also take the intersection of several sets. Just as in the case of unions, there is no ambiguity if we write $A \cap B \cap C$.

Similarly for an intersection of n sets we can write

$$S_1 \cap S_2 \cap S_3 \cap \cdots \cap S_n.$$

Or we can use the following notation:

$$\bigcap \{S_1, S_2, S_3, \ldots, S_n\}.$$

or sometimes just

$$\bigcap_{i=1}^{n} S_i$$

Disjoint Sets

Definition

If two sets A and B have no elements in common then we say that they are disjoint.

Two sets A and B are disjoint if and only if $A \cap B = \varnothing$.

1. The sets $\{1, 2, 3\}$ and $\{4,5,6\}$ are disjoint.
2. The set of even numbers and the set of odd numbers are disjoint; as are the sets of prime and composite numbers.
3. For any set A, $A \cap \varnothing = \varnothing$, hence every set is disjoint from the empty set.

The Difference of Sets

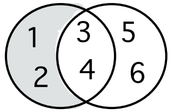
Definition

The difference of two sets A and B, written $A - B$, is the set of all elements which belong to A and which do not belong to B. Symbolically

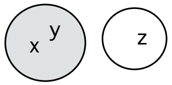
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

$A - B$ is also written as $A \backslash B$.

1. $\{1, 2, 3, 4\} - \{3, 4, 5, 6\} = \{1, 2\}$



2. $\{x, y\} - \{z\} = \{x, y\}$



Some Properties of the Difference of Sets

The following are always true for any two sets A and B.

- $A - B = A \cap \overline{B}$
- $A - B \subseteq A$
- $A - A = \varnothing$
- $A - \varnothing = A$
- $A - U = \varnothing$
- $U - A = \overline{A}$

Note: generally, $A - B \neq B - A$.

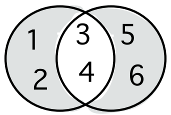
The Symmetric Difference of Sets

Definition

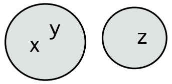
The symmetric difference of two sets A and B, written $A \triangle B$, is the set of all elements which are in exactly one of A or B. Symbolically

$$A \triangle B = (A - B) \cup (B - A).$$

1. $\{1, 2, 3, 4\} \triangle \{3, 4, 5, 6\} = \{1, 2, 5, 6\}$



2. $\{x, y\} \triangle \{z\} = \{x, y, z\}$

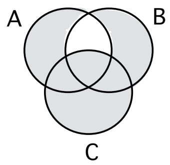


Some Properties of the Symmetric Difference of Sets

The following are always true for any two sets A and B.

- $A \triangle B = B \triangle A$
- $A \triangle B = (A \cup B) - (A \cap B)$
- $A \triangle A = \varnothing$
- $A \triangle \varnothing = A$
- $A \triangle U = \overline{A}$

Which situation does the shaded part of the Venn Diagram below represent?



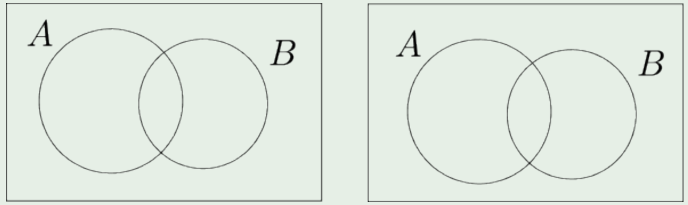
- A $A \cup (B \cap C)$
- B $(A \cup B) \cap C$
- C $(B - C) \cap A$
- D $(A \triangle B) \cup C$

Deciding if sets are equal using Venn Diagrams

One use for Venn diagrams is checking to see if two sets are equal.

Example

Decide if the sets $(A \cup B) - (A \cap B)$ and $(A - B) \cup (B - A)$ are equal.



Yes, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

Laws of the Algebra of Sets

Just as numbers obey laws, so do sets:

a. Associative Laws
   - $(A \cup B) \cup C = A \cup (B \cup C)$
   - $(A \cap B) \cap C = A \cap (B \cap C)$
b. Commutative Laws
   - $A \cup B = B \cup A$
   - $A \cap B = B \cap A$
c. Distributive Laws
   - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Laws of the Algebra of Sets (continued)

d. Identity Laws
   - $A \cup \varnothing = A$
   - $A \cap U = A$
e. Complement Laws
   - $A \cup \overline{A} = U$
   - $A \cap \overline{A} = \varnothing$
f. Idempotent Laws
   - $A \cup A = A$
   - $A \cap A = A$
g. Bound Laws
   - $A \cup U = U$
   - $A \cap \varnothing = \varnothing$

Laws of the Algebra of Sets (continued)

(h) Absorption Laws
- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

(i) Involution Law
- $\overline{(\overline{A})} = A$

(j) 0/1 Laws
- $\overline{\varnothing} = U$
- $\overline{U} = \varnothing$

(k) De Morgan's Laws
- $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
- $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

Check these, either 'in your head' or with a Venn Diagram

An example of proving a property using the Algebra of Sets

Lemma: Let A and B be sets. Then $A \cup B - A \cap B = A \triangle B$.
Proof:

$$
\begin{aligned}
A \cup B - A \cap B &= (A \cup B) \cap (A \cap B)^c, \text{ by Set Difference law} \\
&= (A \cup B) \cap (A^c \cup B^c), \quad \text{by De Morgan's law} \\
&= ((A \cup B) \cap A^c) \cup ((A \cup B) \cap B^c), \text{ by Distributive law} \\
&= (A^c \cap (A \cup B)) \cup (B^c \cap (A \cup B)), \text{ by Commutative law} \\
&= ((A \cap A^c) \cup (B \cap A^c)) \cup ((A \cap B^c) \cup (B \cap B^c)), \text{ Distrib.} \\
&= (\emptyset \cup (B \cap A^c)) \cup ((A \cap B^c) \cup \emptyset), \text{ by Complement laws} \\
&= (B \cap A^c) \cup (A \cap B^c), \text{ by Identity laws} \\
&= (B - A) \cup (A - B), \text{ by Set Difference law} \\
&= (A - B) \cup (B - A), \text{ by Commutative law} \\
&= A \triangle B, \text{ by definition of } \triangle.
\end{aligned}
$$

□

The Principle of Duality

Theorem

For any general theorem on sets involving only the operations of intersection and union, there is another theorem obtained by interchanging both

(i) $\cup$ *with* $\cap$,

and

(ii) $\varnothing$ *with* U.

You can check that each of the previously listed laws has a dual which is also in the list.

A Cover of a Set

Definition

A cover for a set S is a set X of distinct, non-empty subsets of S such that

$$\bigcup X = S.$$

All except one of the following are covers for $\{1, 2, 3, 4, 5\}$. Which is not?

- $\{\{1, 2\}, \{2\}, \{1, 4, 5\}, \{1, 3\}\}$
- A $\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}\}$
- B $\{\{1, 3\}, \{1, 2, 3, 4, 5\}\}$
- $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
- C $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \emptyset\}$
- D $\{\{1, 2\}, \{3, 4, 5\}\}$

A Partition of a Set

Definition

A partition for a set S is a set X of pairwise disjoint, non-empty subsets of S such that

$$\bigcup X = S.$$

In other words, a partition of S is a pairwise disjoint cover of S.

All except one of the following are partitions for $\{1, 2, 3, 4, 5\}$. Which is not?

- $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
- A $\{\{1, 2\}, \{3, 4, 5\}\}$
- B $\{\{1, 3\}, \{1, 2, 3, 4, 5\}\}$
- C $\{\{1, 2, 3\}, \{4, 5\}\}$
- D $\{\{1, 2\}, \{3, 4\}, \{5\}\}$

Which of the following is neither a cover nor a partition for $\{a, b, c, d\}$?

- A $\{\{a, b, c\}, \{a, d\}\}$
- B $\{\{a, c\}, \{b, d\}\}$
- C $\{a, b, c, d\}$
- D $\{\{a, b, c, d\}\}$

The Cartesian Product of 2 Sets

Definition

Given two sets A and B, the *Cartesian product* of A and B, denoted $A \times B$, is the set of **all ordered pairs** (a, b), where a is in A and b is in B.

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$

a is called the first coordinate and b is called the second coordinate.

Example: Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}.$$

Which is false?

- A $\{s_1, s_2\} \times \{t_1, t_2\} = \{(s_1, t_1), (s_1, t_2), (s_2, t_1), (s_2, t_2)\}$
- B $\{s_1, s_2\} \times \{t_1, t_2\} = \{(s_1, t_1), (s_2, t_2)\}$
- C $\{s\} \times \{t_1, t_2\} = \{(s, t_1), (s, t_2)\}$
- D $\{\} \times \{t_1, t_2\} = \{\}$

Equality of ordered pairs

Definition

Two ordered pairs (a_1, a_2) and (b_1, b_2) are equal if and only if

$$a_1 = b_1 \text{ and } a_2 = b_2.$$

Is this true?

$$(1, 2) = (2, 1)?$$

A True

B False

Equality of ordered pairs

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Two ordered pairs (a_1, a_2) and (b_1, b_2) are equal if and only if

$$a_1 = b_1 \text{ and } a_2 = b_2.$$

Is this true?

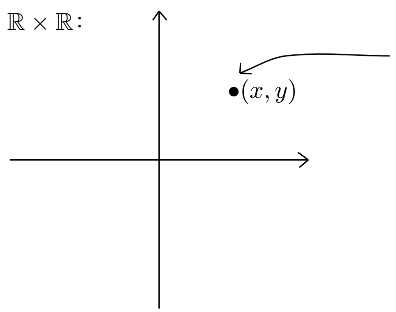
$$(1, 2) = (2, 1)?$$

A True

B **False**, because order matters.

The Cartesian Plane

The Cartesian product $\mathbf{R} \times \mathbf{R}$ is often represented by the *Cartesian Plane:*

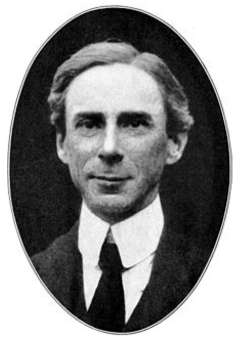


By convention, the first coordinate of an element of the Cartesian Plane is usually labelled x, while the second coordinate is usually called y.

Two points in the plane are equal (i.e. the *same* point) if and only if both coordinates agree.

A bit of history, if we have time

Bertrand Russell, (1872 – 1970)



caused a big stir in 1901 when he discovered **Russell's Paradox:**

> *Is the set of all sets that are not members of themselves a member of itself or not?*

Russell's Paradox in set notation

Let S be the set of all sets which are not elements of themselves, i.e.,

$$S = \{T : T \notin T\}$$

Question: Is $S \in S$?

Think about this. Do you see a problem?

How Russell's Paradox is avoided

- Several different ways out of the dilemma posed by Russell's Paradox have been invented.
- A 'standard solution' is the Zermelo-Fraenkel (ZF) Axioms.
- In the ZF system, sets are built 'from the ground up' according to strict rules which disallow Russell's original construction.

Textbook exercises

Review Exercises Chapter 1.1:

- 1-8, 11, 14, 16-27, 39 (Notation and interpretation)
- 9-10, 12-13, 15 (Understanding and proving)
- 28-38 (Recognizing and naming structures sets obey)

Exercises Chapter 1.1:

- 1-16 (Computations with set operations)
- 17-24 (Computing cardinality)
- 25-40 (Finding/proving equality and subset relationships)
- 41-48 (Notation and interpretation: Venn Diagrams)
- 49-56 (Modelling)
- 73-76 (Computation: partitions)
- 77-82 (Notation and interpretation)
- 83-86 (Computing cardinality of Power Sets)
- 87-90 (Interpreting and understanding structure)
- 91-95 (Notation and Interpretation)