MATH1510 Discrete Mathematics
Semester 2, 2017

Assignment 4

Due Date: This assignment is due in your workshop in week 5. You are also required submit it electronically through Blackboard.

1. Write down the truth table for the compound proposition

$$(p \to q) \to ((p \vee r) \to (q \vee r)).$$

Is this a tautology, a contradiction or neither?

2. Give a truth table which shows that the Elimination argument

$$\begin{array}{rl} & p \vee q \\ & \neg p \\ \therefore & q \end{array}$$

is a valid logical argument.

3. Prove the following by contradiction.

(a) If a and b are integers then $a^2 - 4b \neq 2$.

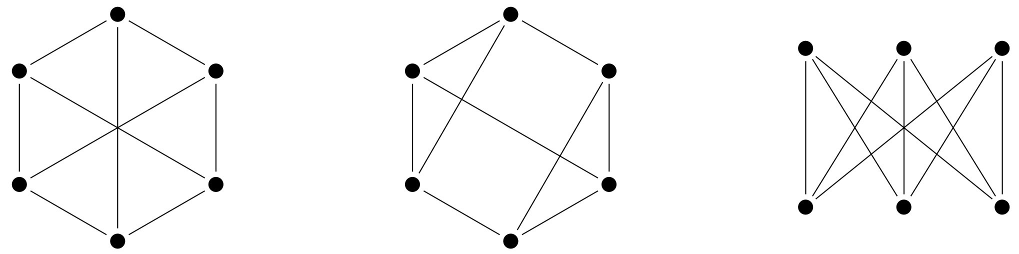
(b) For all sets X and Y, $Y \cap (X - Y) = \emptyset$.

(c) There does not exist a largest negative rational number.

(d) **(Challenge question)** There is no rational number x with $x^3 + x + 1 = 0$.

4. Prove that $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\} : x \mapsto 1/x$ is a bijection.

5. Two of the following graphs are isomorphic, and one is not. Identify the non-isomorphic graph and provide a clear argument why the third one is not isomorphic to the other two.



G_1 $\quad$ G_2 $\quad$ G_3

6. (Challenge Question)

(a) Let $G = (V, E)$ be a simple graph. Prove that there is a partition $V = V_1 \cup V_2$ of the vertex set such that at least $|E|/2$ edges have one endpoint in V_1 and one endpoint in V_2.

(b) Let $G = (V, E)$ be a simple graph with n vertices in which every vertex has degree 3 (such a graph is called *3-regular*). Explain why n has to be even and express the number of edges in terms of n. Prove that there is a partition $V = V_1 \cup V_2$ of the vertex set such that at least n edges have one endpoint in V_1 and one endpoint in V_2.

END OF PAPER