

COMP3260/COMP6360 Data Security

Week 2 Workshop – 7th March 2019

1. Apply Chinese Remainder Theorem to find x in the range $[0, 59]$ such that

$$x \bmod 4 = 3$$

$$x \bmod 3 = 2$$

$$x \bmod 5 = 4$$

2. Using Chinese Remainder Theorem solve for x in the range $[0, n-1]$.

a) $5x \bmod 17 = 1$

b) $19x \bmod 26 = 1$

c) $17x \bmod 100 = 1$

d) $2x \bmod 57 = 1$

3. Using extended Euclid's algorithm, find the solution to the equation $17x \bmod 100 = 1$ in the range $[0, 99]$.

4. Using Euler's theorem and fast exponentiation, solve the following equation for x in the range $[0, n-1]$.

a) $5x \bmod 17 = 1$

b) $19x \bmod 26 = 1$

c) $17x \bmod 100 = 1$

d) $2x \bmod 57 = 1$

5. Find the inverse of $5 \bmod 31$.

6. Find all solutions to the equation $17x \bmod 100 = 10$ in the range $[0, 99]$.

7. Let X be an integer variable represented with 32 bits. Suppose that the probability is $\frac{1}{2}$ that X is in the range $[0, 2^8-1]$, with all such values being equally likely, and $\frac{1}{2}$ that X is in the range $[2^8, 2^{32}-1]$, with all such values being equally likely. Compute $H(X)$.

8. Let X be one of the 6 messages: A, B, C, D, E and F, where:

$$p(A)=p(B)=p(C)=1/4$$

$$p(D)=1/8$$

$$p(E)=p(F)=1/16$$

Compute $H(X)$ and find an optimal binary encoding of the message.

9. Suppose there are 5 possible messages, A, B, C, D and E, with the probabilities $p(A)=0.5$, $p(B)=0.3$, $p(C)=0.1$, $p(D)=0.05$ and $p(E)=0.05$. What is the expected number of bits needed to encode these messages in optimal encoding? (That is, find $H(M)$.) Provide optimal encoding.