COMP2270 – Theory of Computation
Assignment 2
Due on 17/05/2020 23:59

Problem 1) [20 marks, 4+5+5+6] Let A and B be two languages such that $B \subseteq A$. For each of the following statements, indicate whether it must be true for any arbitrary A and B. Prove your answer.

a) If A is finite then B is finite.
b) If A is regular then B is regular.
c) If B is regular then A is regular.
d) If *firstchars*(A) is regular then *firstchars*(B) is regular. The function *firstchars* on language L is defined as
   $\mathit{firstchars}(L) = \{w : \exists y \in L\ (y = cx \wedge c \in \Sigma_L \wedge x \in \Sigma_L^* \wedge w \in c^*)\}$.

Problem 2) [20 marks, 10 marks each] Show a context-free grammar for each of the following languages L:

a) $\{\mathtt{a}^i\mathtt{b}^k : k = 4i + 2 \text{ and } i,k \geq 0\}$.
b) $\{\mathtt{a}^n\mathtt{b}^p : p \geq n,\ p\text{-}n \text{ is odd}\}$.

Problem 3) [20 marks, 10 marks each] The following grammar describes a tiny fragment of English. The symbol *NP* will derive noun phrases; the symbol *VP* will derive verb phrases:

S → *NP VP*
NP → the *Nominal* | a *Nominal* | *Nominal* | *ProperNoun* | *NP PP*
Nominal → *N* | *Adjs N*
N → cat | dogs | bear |girl | chocolate | rifle
ProperNoun → Chris | Fluffy
Adjs → *Adj Adjs* | *Adj*
Adj → young | older | smart
VP → *V* | *V NP* | *VP PP*
V → like | likes | thinks | shot | smells
PP → *Prep NP*
Prep → with

Using this simple English grammar, show two parse trees for each of the following sentences. In each case, indicate which parse tree almost certainly corresponds to the intended meaning of the sentence:

a) `The bear shot Fluffy with the rifle.`
b) `Fluffy likes the girl with the chocolate.`

Problem 4) [15 marks, 7+8 marks] Consider the following context-free grammar G:

$S \rightarrow T\#T$
$T \rightarrow ABA$
$T \rightarrow C$
$A \rightarrow \mathtt{a}A$
$A \rightarrow \varepsilon$
$B \rightarrow \mathtt{b}B$
$B \rightarrow \varepsilon$
$C \rightarrow \mathtt{c}C$
$C \rightarrow \mathtt{c}$

a) Show the leftmost derivation of the string `ab#cc` produced by G.
b) G is ambiguous. Prove this claim by showing a string that has at least two parse trees. Show at least two of the trees.

Problem 5) [20 marks, 10 marks each] Build a PDA to accept each of the following languages L:

a) $\{\mathtt{a}^i\mathtt{b}^k : k = 3i + 3\}$.
b) $\{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k, i > k, 0 \leq j < 3, \; k \geq 0\}$.

Problem 6) [20 marks, 4+5+5+6 marks] Consider the language $L = L_1 \cap L_2$, where $L_1 = \{ww^R : w \in \{\mathtt{a}, \mathtt{b}\}^*$ and $L_2 = \{\mathtt{a}^n\mathtt{b}^*\mathtt{a}^n : n \geq 0\}$.

a) List the first four strings in the lexicographic enumeration of L?
b) Write a context-free grammar to generate L.
c) Show a natural PDA for L. (In other words, don't just build it from the grammar using one of the two-state constructions presented in the book.)
d) Is L a regular language? Prove it.

[**Hint:** It could be easier if you start with a description / definition of L (not just L=L1∩L2)]

Problem 7) [15 marks, 5 marks each] For each of the following claims, state whether it is *True* or *False*. Prove your answer.

a) If $L = L_1^+$ and L is context-free, then L_1 must be context-free.
b) If $L = L_1L_2$ and L is context-free, then L_1 must be context-free.
c) If $L = L_1 \cap L_2$ and L_1 and L_2 are context-free, then L must be context-free.

Problem 8) [20 marks, 10 marks each] For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

a) $L = \{ww : w \in (\mathtt{ab})^n : n \geq 0\}$.
b) $L = \{a^i b^k c^i d^k \mid i,k \geq 0\}$

REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008.