SCHOOL of ELECTRICAL ENGINEERING & COMPUTING
FACULTY of ENGINEERING & BUILT ENVIRONMENT
The UNIVERSITY of NEWCASTLE

Comp3320/6370 Computer Graphics

Semester 2, 2018

Lab Week 4

For this lab check that you have worked though all exercises on the Exercise I and II sheets and then proceed with the lab questions below.

The solutions to this lab sheet are to be worked out in the lab in team work. Please compare your solutions with your fellow students and the tutor and ask us if you have any questions. We do not have worked solutions for everything, but so that you can check your approach we have worked solutions to very similar questions in the Exercise sheets I and II.

These practice questions and the Exercise I and II sheets should help you with your preparation for the mid-semester exam where we will then again use some similar questions. More exercises will be released in the coming weeks.

Lab question 4.1 (intersection of line and plane): Consider 3-dimensional Euclidean space and the parameterised line through point $A = (0, 0, 0)$ in direction of the vector $\mathbf{v} = (1, 2, 5)^T$. Let p be a plane anchored at point $B = (1, 1, 1)$ and normal to vector $\mathbf{n} = (2, 1, 8)$. Calculate the hitpoint P_{hit} where the line hits the plane.

Solution hints: To start draw a schematic graph of the situation. Compare your solution with Exercise 3 on the Exercises I sheet.

Lab question 4.2: Explain the 3D rotation matrices for rotation about the x and the y axes.

Solution hint: There are many points that we have already discussed in the lectures and the Exercise sheets. One interesting detail was the position of the minus sign in the matrix for the rotation about the y-axis. Find your own explanation.

Lab question 4.3: Consider 3D transforms in homogeneous coordinates. We would like to translate a point 3 units in positive direction along the x-axis, rotate $180°$ around the z-axis and then $180°$ around the y-axis and then $180°$ around the x-axis, and then translate it back -3 units along the x-axis. Find the homogeneous transformation matrix which accomplishes this by using the matrix product to combine the transformation matrices of each step.

Solution hint: Write down each individual transform. Then take the product of all of them where the order is right to left with the matrix that gets executed first is on the right side.

Lab question 4.4: How can we see that the dot product of two vectors is a special case of the product of two matrices?

Solution hints: First accept that any vector in $\mathbf{R}^n$ can be regarded as a $(n \times 1)$-matrix (assume $n = 3$ for now). Then the answer becomes clear when we look at the two definitions (the "dot product of two vectors" in Exercise 2 and the product of two matrices in Exercise 8):

From Exercise 2: The "dot product" of vectors $\mathbf{v} = (v_1, v_2, v_3)^T$ and $\mathbf{w} = (w_1, w_2, w_3)^T$ is

$$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

From Exercise 8: Let $A = (a_{ij})_{\substack{i=1,2,3 \\ j=1,..,k}} \in M(3 \times k, \mathbf{R})$ and $B = (b_{ij})_{\substack{i=1,...,k \\ j=1,2,3}} \in M(k \times 3, \mathbf{R})$ be two matrices.

Then we can consider matrix products $C = A \cdot B$ but also $D = B \cdot A$. Both are well-defined if we assume that $m = n = 3$ in the definition of Exercise 8.

According to the definition C becomes a (3×3)-matrix and D a $(k \times k)$-matrix:

$$C = (c_{ij})_{\substack{i=1,2,3 \\ j=1,2,3}} = \left(\sum_{\nu=1,...,k} a_{i\nu} b_{\nu j} \right)_{\substack{i=1,2,3 \\ j=1,2,3}} \quad \text{and } D = (d_{ij})_{\substack{i=1,...,k \\ j=1,...,k}} = \left(\sum_{\nu=1,2,3} b_{j\nu} a_{\nu i} \right)_{\substack{i=1,...,k \\ j=1,...,k}}$$

If we assume that $k = 1$ this simplifies to:

$$C = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} \\ a_{31}b_{11} & a_{31}b_{12} & a_{31}b_{13} \end{pmatrix} \text{ and } D = b_{11}a_{11} + b_{12}a_{21} + b_{13}a_{31}$$

If we set $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \end{pmatrix}$ this means we have the matrix products

$$C = \mathbf{v} \cdot \mathbf{w}^T \text{ and } D = \mathbf{v}^T \cdot \mathbf{w}$$

where $D = \mathbf{v} \cdot \mathbf{w}$ is the dot product of vectors $\mathbf{v}$ and $\mathbf{w}$.

Lab question 4.5: We wish to have floating text on a rectangular board appear above an object in the world at the point $P = (3, 3, 1)$. Let the up-direction of the text on the billboard be $\mathbf{u} = (-1, 0, 1)$ and its normal vector be $\mathbf{n} = (-1, -1, -1)$ (Note: These are perpendicular). Discuss how we could calculate the transformation matrix we need to have so that our text appears flat and upright on the screen.

Solution hint: We can assume our screen's view vector is $\mathbf{v}_{dir} = (0, 0, 1)$ and our screen's up vector is $\mathbf{v}_{up} = (0, -1, 0)$. For more details see topic "Screen Aligned Billboards" on the lecture slides.