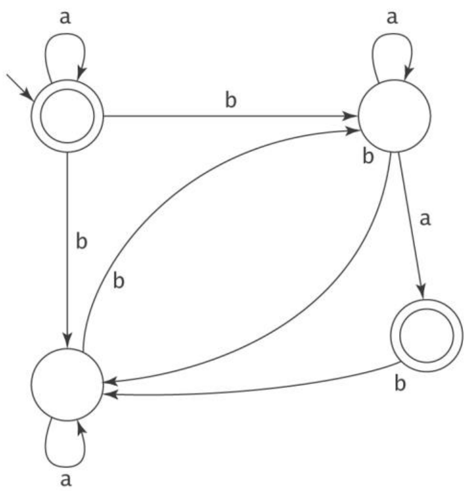
COMP2270/6270 – Theory of Computation
Fourth week

School of Electrical Engineering & Computing
The University of Newcastle

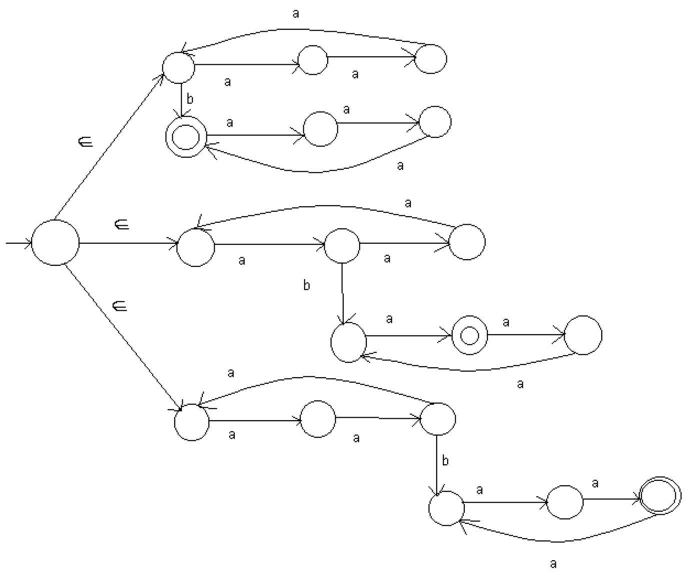
1) Not true. Because we know, "Given an NDFSM M=(K, Σ, Δ, s, A) that accepts some language L there exists an equivalent DFSM that accepts L." Therefore, there could not exists such a language L that is accepted by some NDFSM but no DFSM.
2) Consider the following NDFSM *M*:



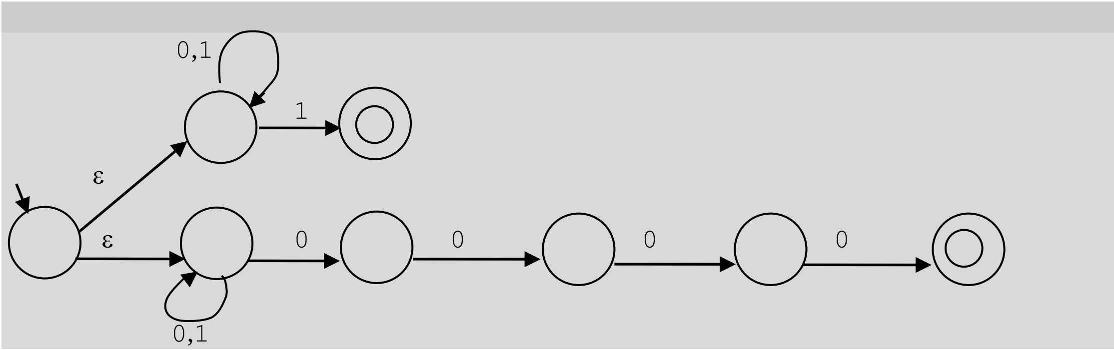
For each of the following strings *w*, determine whether $w \in L(M)$:

a) aabbba. Yes.
b) bab. No.
c) baba. Yes.

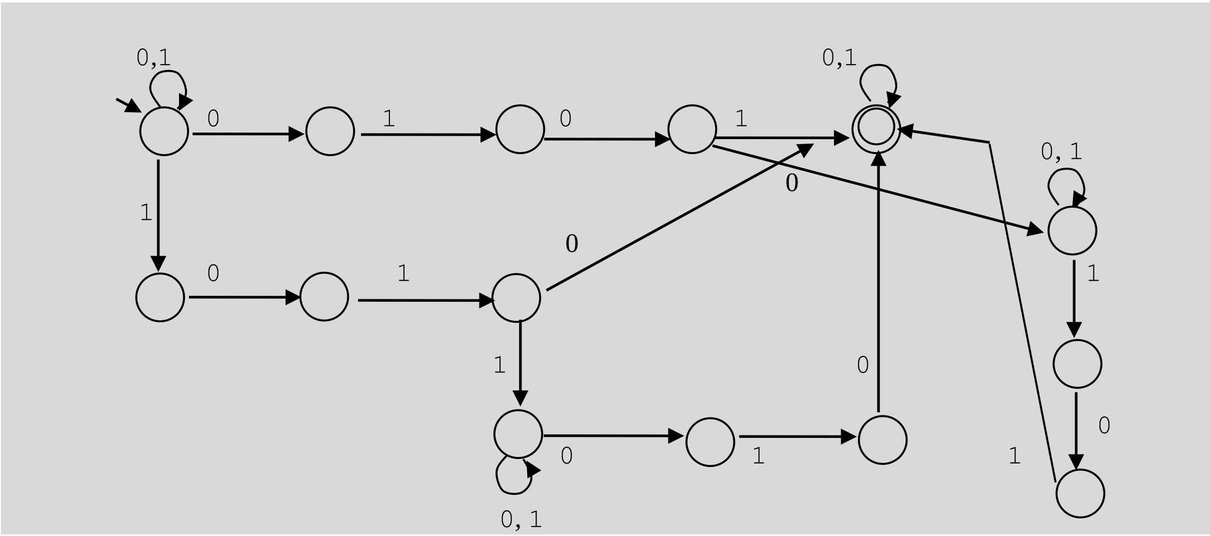
3) Show a possibly nondeterministic FSM to accept each of the following languages:
   a) $\{a^n b a^m : n, m \geq 0, n \equiv_3 m\}$.



b) {$w \in$ {0, 1}* : w corresponds to the binary encoding of a positive integer that is divisible by 16 or is odd}.

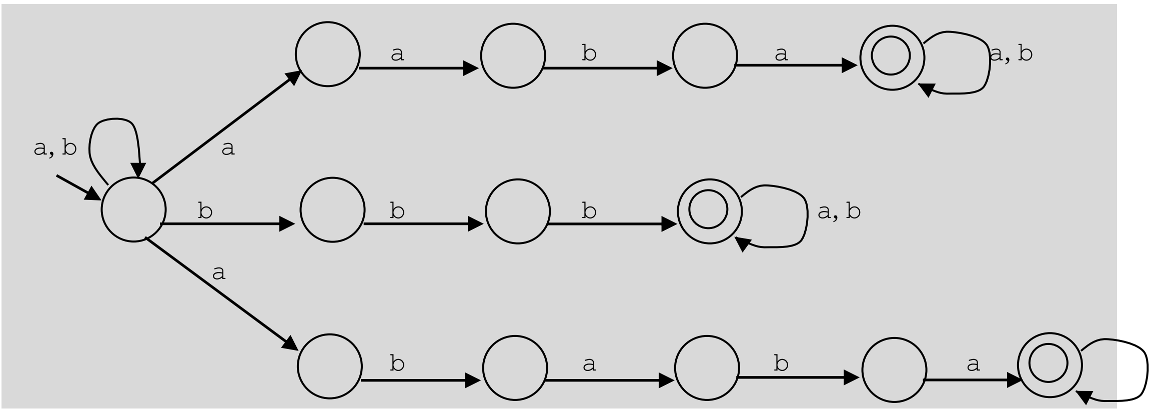


c) {$w \in$ {0, 1}* : w contains both 101 and 010 as substrings}.



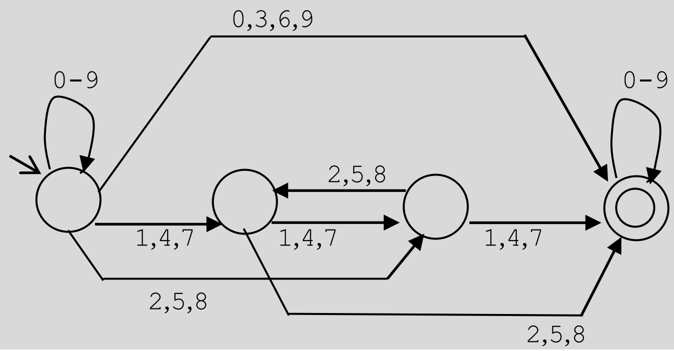
EXTRA from THE BOOK

a) {$w \in$ {a, b}* : w contains at least one instance of aaba, bbb or ababa}.



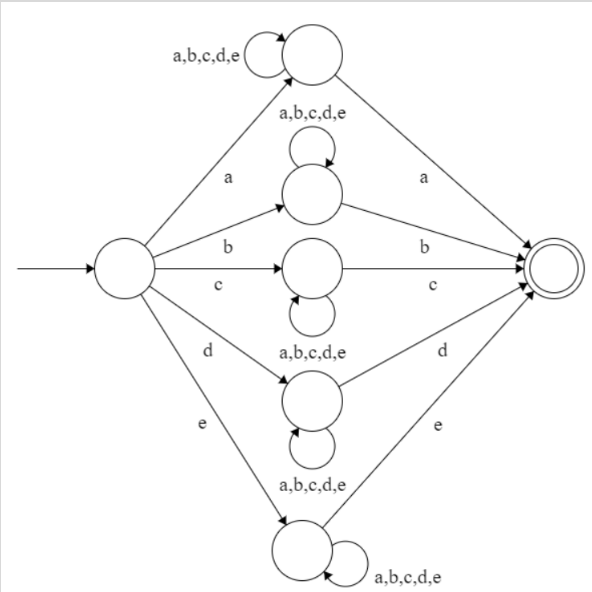
b) $L = \{w \in \{0\text{-}9\}^* : w$ represents the decimal encoding of a natural number whose encoding contains, as a substring, the encoding of a natural number that is divisible by 3$\}$.

Note that 0 is a natural number that is divisible by 3. So any string that contains even one 0, 3, 6, or 9 is in *L*, no matter what else it contains. Otherwise, to be in *L*, there must be a sequence of digits whose sum equals 0 mod 3.



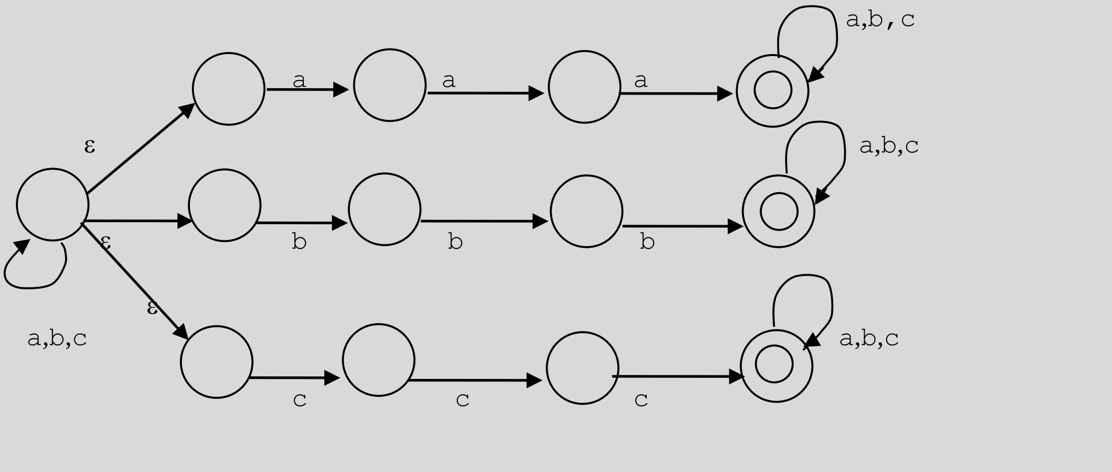
c) $\{w \in \{a, b, c, d, e\}^* : |w| \geq 2$ and w begins and ends with the same symbol$\}$.

Guess which of the five symbols it is. Go to a state for each. Then, from each such state, guess that the next symbol is not the last and guess that it is.



d) Show an FSM (deterministic or nondeterministic) that accepts $L = \{w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* : w$ contains at least one substring that consists of three identical symbols in a row$\}$. For example:

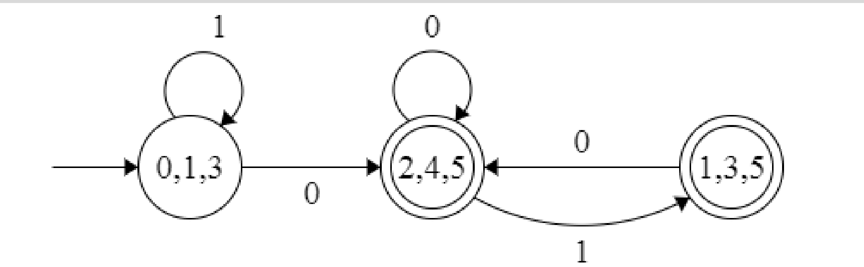
- The following strings are in L: `aabbb`, `baacccbbb`.
- The following strings are not in L: ε, `aba`, `abababab`, `abcbcab`.



4) a)

s	$eps(s)$
q_0	$\{q_0, q_1, q_3\}$
q_1	$\{q_1, q_3\}$
q_2	$\{q_2\}$
q_3	$\{q_3\}$
q_4	$\{q_2, q_4, q_5\}$
q_5	$\{q_5\}$

$\{q_0, q_1, q_3\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{q_0, q_1, q_3\}$
$\{q_2, q_4, q_5\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{q_1, q_3, q_5\}$
$\{q_1, q_3, q_5\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{ \}$



b)

Calculate the epsilon closure of each state:

s	**eps(s)**
q0	{q0}
q1	{q1}
q2	{q2}
q3	{q3}
q4	{q4}

Starting with the only active state {q0}, calculate the δ′

{q0}	a	{q0,q1}
	b	{q0}
{q0,q1}	a	{q0,q1,q2}
	b	{q0,q2}
{q0,q1,q2}	a	{q0,q1,q2,q3}
	b	{q0,q2,q3}
{q0,q2}	a	{q0,q1,q3}
	b	{q0,q3}
{q0,q1,q2,q3}	a	{q0,q1,q2,q3,q4}
	b	{q0,q2,q3,q4}
{ q0,q2,q3}	a	{q0,q1,q3,q4}
	b	{q0,q3,q4}
{q0,q1,q3}	a	{q0,q1,q2,q4}
	b	{q0,q2,q4}
{q0,q3}	a	{q0,q1,q4}
	b	{q0,q4}
{q0,q1,q2,q3,q4}	a	{q0,q1,q2,q3,q4}
	b	{q0,q2,q3,q4}
{q0,q2,q3,q4}	a	{q0,q1,q3,q4}
	b	{q0,q3,q4}
{q0,q1,q3,q4}	a	{q0,q1,q2,q4}
	b	{q0,q2,q4}
{q0,q3,q4}	a	{q0,q1,q4}
	b	{q0,q4}
{q0,q1,q2,q4}	a	{q0,q1,q2,q3}
	b	{q0,q2,q3}
{q0,q2,q4}	a	{q0,q1,q3}
	b	{q0,q3}
{q0,q1,q4}	a	{q0,q1,q2}
	b	{q0,q2}
{q0,q4}	a	{q0,q1}
	b	{q0}

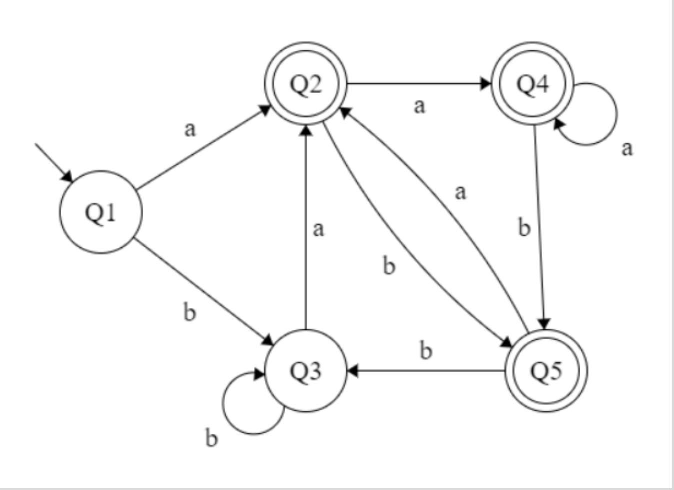
c)

s	$eps(s)$
q_0	$\{q_0, q_1\}$
q_1	$\{q_1\}$
q_2	$\{q_2\}$
q_3	$\{q_3, q_0, q_1\}$
q_4	$\{q_4\}$
q_5	$\{q_5\}$

5)

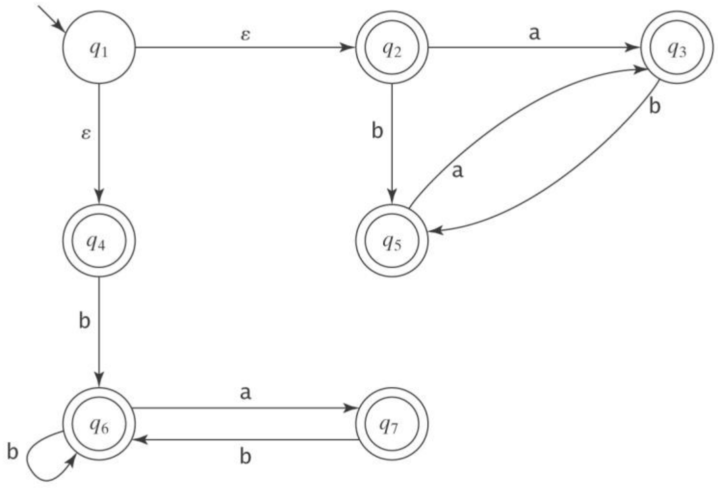
$\{q_0, q_1\} \equiv$ Q1	a	$\{q_2, q_4\} \equiv$ Q2
	b	$\{q_0, q_1, q_3\} \equiv$ Q3
$\{q_2, q_4\} \equiv$ Q2	a	$\{q_1, q_2, q_4\} \equiv$ Q4
	b	$\{q_0, q_1, q_3, q_5\} \equiv$ Q5
$\{q_0, q_1, q_3\} \equiv$ Q3	a	$\{q_2, q_4,\} \equiv$ Q2
	b	$\{q_0, q_1, q_3\} \equiv$ Q3
$\{q_1, q_2, q_4\} \equiv$ Q4	a	$\{q_1, q_2, q_4\} \equiv$ Q4
	b	$\{q_0, q_1, q_3, q_5\} \equiv$ Q5
$\{q_0, q_1, q_3, q_5\} \equiv$ Q5	a	$\{q_2, q_4\} \equiv$ Q2
	b	$\{q_0, q_1, q_3\} \equiv$ Q3

Accepting state is $\{q_0, q_1, q_3, q_5\} \equiv$ Q5, $\{q_2, q_4\} \equiv$ Q2, $\{q_1, q_2, q_4\} \equiv$ Q4.



EXTRA from THE BOOK

Let M be the following NDFSM. Construct (using *ndfsmtodfsm*), a DFSM that accepts $\neg L(M)$.



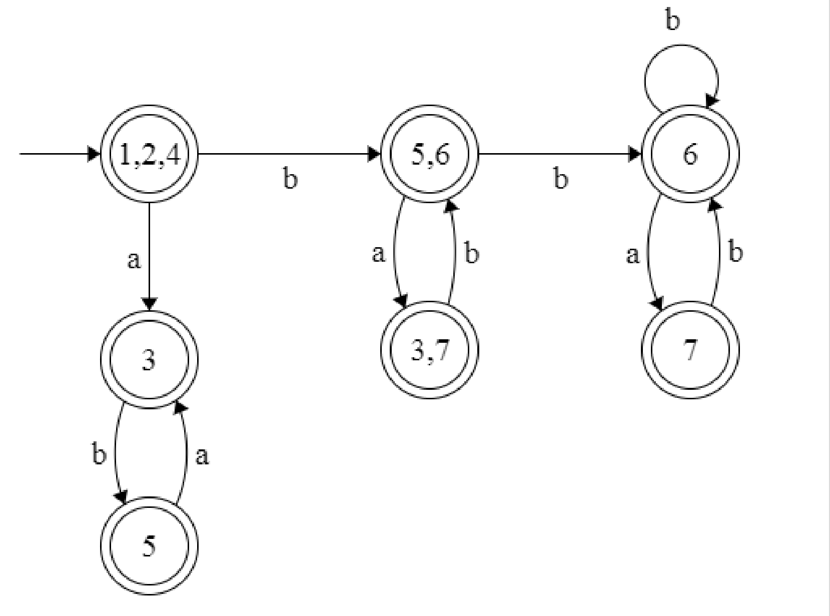
Calculate the epsilon closure of each state:

s	**eps(s)**
q1	{q1, q2, q4}
q2	{q2}
q3	{q3}
q4	{q4}
q5	{q5}
q6	{q6}
q7	{q7}

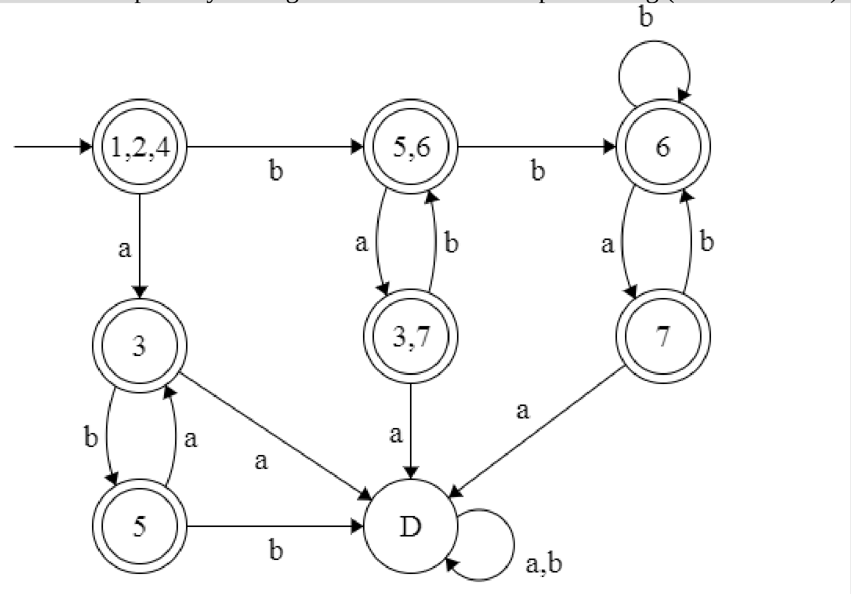
Starting with the only active state {q1, q2, q4}, calculate the δ′

{q1, q2, q4}	a	{q3}
	b	{q5, q6}
{q3}	a	φ
	b	{q5}
{q5,q6}	a	{q3,q7}
	b	{q6}
{q5}	a	{q3}
	b	φ
{q3,q7}	a	φ
	b	{q5, q6}
{q6}	a	{q7}
	b	{q6}
{q7}	a	φ
	b	{q6}

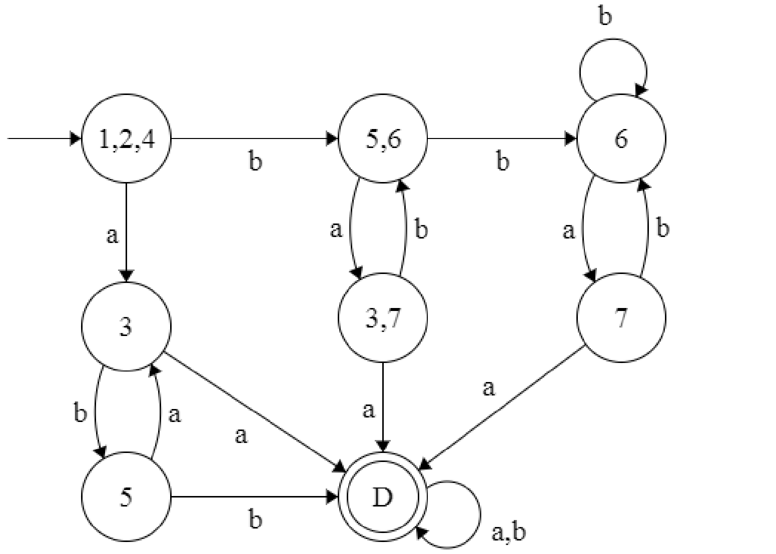
So the DFSM:



Make it complete by adding dead state before complementing (IMPORTANT)



Now complement the DFSM by swapping the accepting and non-accepting states. The resulting DFSM will accept $\neg L(M)$.



5) Describe in English, as briefly as possible, the language defined by each of these regular expressions:

a) (b ∪ ba) (b ∪ a)* (ab ∪ b).

The set of strings of length at least two over the alphabet {a, b} that start and end with b.

b) (((a*b*)*ab) ∪ ((a*b*)*ba))(b ∪ a)*.

The obvious answer is the set of strings over the alphabet {a, b} that contain at least one occurrence of ab or ba. A simpler answer is the set of strings over the alphabet {a, b} that contain at least one a and at least one b.

6) Write a regular expression to describe each of the following languages:

a) {w ∈ {a, b}* : every a in w is immediately preceded and followed by b}.

b (b (ab)*)* ∪ ε

b) {w ∈ {a, b}* : w does not end in ba}.

ε ∪ a ∪ (a ∪ b)* (b ∪ aa)

c) {w ∈ {0, 1}* : ∃y ∈ {0, 1}* (|xy| is even)}.

(0 ∪ 1)*

d) {w ∈ {0, 1}* : w corresponds to the binary encoding, without leading 0's, of natural numbers that are evenly divisible by 4}.

(1(0 ∪ 1)* 00) ∪ 0

7) Simplify each of the following regular expressions:

a) (a ∪ b)* (a ∪ ε) b*.

(a ∪ b)*.

b) (∅* ∪ b) b*.

b*.

c) $(a \cup b)^*a^* \cup b$.

$(a \cup b)^*$.

d) $((a \cup b)^*)^*$.

$(a \cup b)^*$.

e) $a\,(\,(a \cup b)(b \cup a)\,)^* \cup a\,(\,(a \cup b)\,a\,)^* \cup a\,(\,(b \cup a)\,b\,)^*$.

$a\,(\,(a \cup b)(b \cup a)\,)^*$.

8) For each of the following expressions E, answer the following three questions and prove your answer:

(i) Is E a regular expression?

(ii) If E is a regular expression, give a simpler regular expression.

(iii) Does E describe a regular language?

a) $((a \cup b) \cup (ab))^*$.

E is a regular expression. A simpler one is $(a \cup b)^*$. The language is regular.

b) $(a^+ a^n b^n)$.

E is not a regular expression. The language is not regular. It is $\{a^m b^n : m > n\}$.

c) $((ab)^* \varnothing)$.

E is a regular expression. A simpler one is $\varnothing$. The language is regular.

d) $(((ab) \cup c)^* \cap (b \cup c^*))$.

E is not a regular expression because it contains $\cap$. But it does describe a regular language (c^*) because the regular languages are closed under intersection.

e) $(\varnothing^* \cup (bb^*))$.

E is a regular expression. A simpler one is b^*. The language is regular.

9) Let $L = \{a^n b^n : 0 \le n \le 4\}$.

a) Show a regular expression for L.

$(\varepsilon \cup ab \cup aabb \cup aaabbb \cup aaaabbbb)$

b) Show an FSM that accepts *L*.