MATH1510 Discrete Mathematics
Semester 2, 2017

Jordan Shaw-Carmody/Nathan Anstess's workshop, Friday 12-2pm **Quiz 2d**

Name: .... Student ID: ....
If this is not your normal workshop, then please state which one is:

Demonstrator: .... Weekday: .... Time: ....

(Demonstrator's use only) Points achieved: out of 4

Instructions: If you run out of space in a question, do use the empty page(s) at the end of the quiz and indicate at the respective question that the working continues at the end of the paper. Always show your working in written answer questions, unless stated otherwise. You have 20 minutes for this quiz.

Solutions

1. Show that $(p \wedge \neg q) \equiv \neg(p \to q)$, i.e., show that the LHS is logically equivalent to the RHS.

1 solution:

p	q	$p \to q$	$p \wedge \neg q$	$\neg(p \to q)$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Column "$p \wedge \neg q$" is equal to column "$\neg(p \to q)$", and this proves the claim.

2. The Fibonacci sequence $(0, 1, 1, 2, 3, 5, 8, 13, \ldots)$ is defined by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geqslant 2$. Prove that for all positive integers n

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}.$$

2 solution:

Setup. For every positive integer n, $P(n)$ is the statement

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}.$$

Base case. $P(1)$ is

$$1^2 = 1 \times 1,$$

which is true.

Induction step. Assume $P(k)$ is true. Using the definition of the Fibonacci sequence we obtain

$$F_1^2 + F_2^2 + \cdots + F_k^2 + F_{k+1}^2 \overset{\text{using } P(k)}{=} F_k F_{k+1} + F_{k+1}^2 = F_{k+1}(F_k + F_{k+1}) = F_{k+1} F_{k+2},$$

which is precisely $P(k+1)$.

Conclusion. By mathematical induction, $P(n)$ is true for every positive integer n.

END OF PAPER