COMP2270/6270 – Theory of Computation
Ninth Week

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Note: Some exercises belong to Chapters 13 of Ref [1]

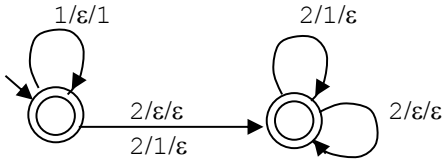
Exercise 1) Consider the following (highly ambiguous) grammar *G* for Boolean expressions (Don't get confused by the fact that the symbol $\rightarrow$ is both a metasymbol in each rule and a terminal symbol in the grammar *G*.):

$E \rightarrow \neg E$
$E \rightarrow E \vee E$
$E \rightarrow E \wedge E$
$E \rightarrow E \rightarrow E$
$E \rightarrow (E)$
$E \rightarrow \texttt{id}$

a) Show the PDA that *cfgtoPDAbottomup* will build on input *G*.
b) Trace the execution of one accepting path of your PDA on the input:
$(\neg \texttt{id} \rightarrow (\texttt{id} \vee \texttt{id}))$

c) Show the PDA that *cfgtoPDAtopdown* will build on input *G*.
d) Prove that *G* is ambiguous.

Exercise 2) Consider the following PDA *M*:



a) Give a concise description of $L(M)$.
b) Show a context-free grammar that generates $L(M)$.
c) Is M deterministic? Justify your answer.
d) Is $L(M)$ deterministic context-free? Justify your answer.

Exercise 3) For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

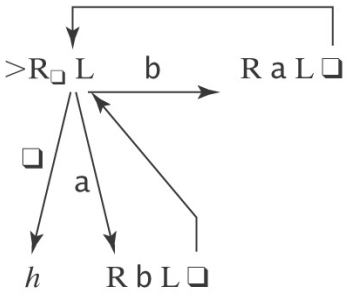
a) $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } j > i + k\}$.
b) $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } j > max(i, k)\}$.
c) $\{ww^R w : w \in \{a, b\}^*\}$.
d) $\{a^n b^m c^k : n, m, k \geq 0 \text{ and } m \leq min(n, k)\}$.

Exercise 4) Are the context-free languages closed under each of the following functions? Prove your answer.

a) $chop(L) = \{w : \exists x \in L\ (x = x_1 c x_2 \wedge x_1 \in \Sigma_L^* \wedge x_2 \in \Sigma_L^* \wedge c \in \Sigma_L \wedge |x_1| = |x_2| \wedge w = x_1 x_2)\}$.

b) $mix(L) = \{w: x, y, z: (x \in L, x = yz, |y| = |z|, w = yz^R)\}$.

Exercise 5) Give a short English description of what each of this Turing machines does. Note the alphabet $\Sigma_M = \{a, b\}$.



REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008.