Nathan Van Maastricht/Mohd Noor Islam's workshop, Tuesday 10-12pm   **Quiz 4c**

Name: .... Student ID: ....
If this is not your normal workshop, then please state which one is:

Demonstrator: .... Weekday: .... Time: ....

(Demonstrator's use only) Points achieved: out of 4

Instructions: If you run out of space in a question, do use the empty page(s) at the end of the quiz and indicate at the respective question that the working continues at the end of the paper. Always show your working in written answer questions, unless stated otherwise. You have 20 minutes for this quiz.

Solutions

1. Prove by contradiction that $\sqrt{3}$ is irrational.

1 solution: Suppose $\sqrt{3} = a/b$ with integers a and b without a common factor. Then

$$3b^2 = a^2 \tag{3}$$

The LHS is divisible by 3, so a^2 is divisible by 3 as well. This implies that a is divisible by 3, and therefore a^2 is divisible by 9. But then the LHS of (3) is divisible by 9 which implies that b^2 is divisible by 3. But then b is divisible by 3 as well, and this is the required contradiction as we have assumed that a and b do not have a common factor.

2. Let f be the function
$$f : \mathbb{Z} \to \mathbb{Z}, \qquad x \mapsto 3x + 2$$

Is f injective? Is f surjective? Is f bijective? Support your answers by arguments.

2 solution: The function is injective because $f(x) = f(y)$ implies $3x + 2 = 3y + 2$, and therefore $x = y$. The function is not surjective, because there is no $x \in \mathbb{Z}$ with $f(x) = 1$. The function is not bijective because it is not surjective.

END OF PAPER

ANSWERS