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**Assignment 1**

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**Due Date:** This assignment is due in your workshop in week 2. You are also required submit it electronically through Blackboard.

**1.** Let  $A = \{\emptyset, \{\emptyset\}, 1, a, \text{cat}, \{1, a, \text{cat}\}\}$ . Determine the following sets.

(a)  $A \setminus \{a, b, c\}$

(b)  $A \cup \{x\}$

(c)  $A \cap \{\text{cat}, \text{dog}, \text{mouse}\}$

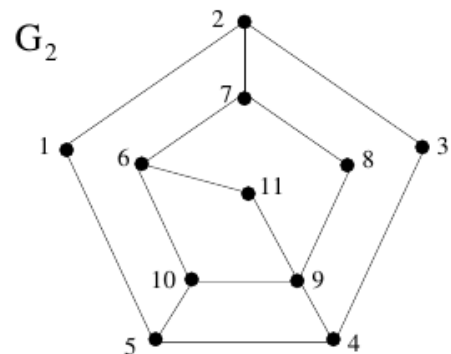
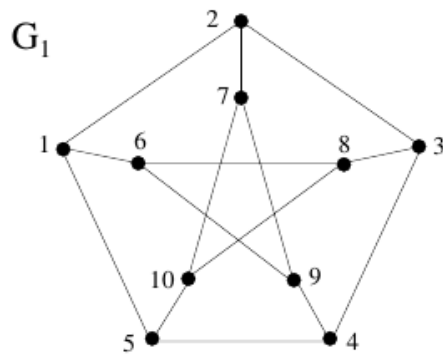
(d)  $A \setminus \emptyset$

(e)  $A \setminus \{\emptyset\}$

(f)  $A \triangle \{a, b, \{1, a, \text{cat}\}\}$

**2.** Prove that  $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$  using the set equality theorem.

3. What are the diameters of the following graphs? Briefly explain your answer.



4. Let  $A_1, \dots, A_5$  be the sets  $A_1 = \{1, 2\}$ ,  $A_2 = \{2, 3\}$ ,  $A_3 = \{3, 4\}$ ,  $A_4 = \{4, 5\}$  and  $A_5 = \{1, 5\}$ , and let  $G = (V, E)$  be the graph with vertex set  $V = \{1, 2, 3, 4, 5\}$  and edge set  $E = \{\{i, j\} : i, j \in V, A_i \cap A_j = \emptyset\}$ .

- Give an explicit description of the edge set  $E$ .
- Draw the graph  $G$ .
- Write down an adjacency matrix for  $G$ .
- Write down an incidence matrix for  $G$ .

5. **(Challenge question)** A *triangle* in a graph  $G$  is a set of three vertices  $u$ ,  $v$  and  $w$  that are adjacent to each other, i.e., such that all three pairs  $\{u, v\}$ ,  $\{u, w\}$  and  $\{v, w\}$  are edges in  $G$ .

- How many edges can a graph  $G$  on 5 vertices have if it does not contain any triangles?
- Try to answer the same question for a graph with 6 vertices, and for a graph with 7 vertices.
- Can you guess what happens in general?

END OF PAPER