
Assignment 3

Due Date: This assignment is due in your workshop in week 4. You are also required submit it electronically through Blackboard.

1. Prove by induction that for every positive integer n ,

$$\sum_{i=1}^n 2^i = 2^{n+1} - 2.$$

2. Prove by induction that for every positive integer n , $3^n - 1$ is divisible by 2.

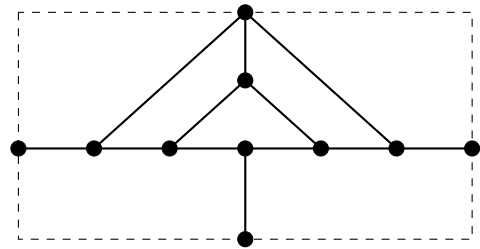
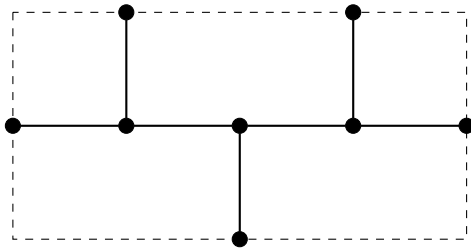
3. Consider the sum $\sum_{i=1}^n 2i$.

(a) Show that if $\sum_{i=1}^k 2i = k(k+1) + 2$, then $\sum_{i=1}^{k+1} 2i = (k+1)(k+2) + 2$.

(b) Is the formula in part (a) true for all positive integers, n ? Why or why not?

4. Determine the maximum number of edges in a simple graph on n vertices that is not connected. Support your maximality claim by an argument.

5. In each of the pictures below there are line segments connecting black dots (7 line segments in the left picture and 12 line segments in the right picture). For both pictures, find out if it is possible to draw a closed (like a cycle instead of a path) continuous line that crosses each of these line segments exactly once, whilst staying inside the dashed rectangle. If it is possible, draw a picture to show how, and if it is not possible provide an argument.



END OF PAPER