MATH1510 Discrete Mathematics
Semester 2, 2017

Assignment 6

Due Date: This assignment is due in your workshop in week 7. You are also required submit it electronically through Blackboard.

1. Define a binary relation R on $\{n \in \mathbb{Z} : n \geqslant 2\}$ such that aRb if and only if a and b have a common factor not equal to 1. Show R is reflexive and symmetric but not an equivalence relation.

2. Let X be a set whose elements are sets, and consider the subset relation $\subseteq$ on X, i.e. for two elements $A, B \in X$, the pair (A, B) is an element of the relation if and only if A is a subset of B.

(a) Prove that $\subseteq$ is a partial order on X.

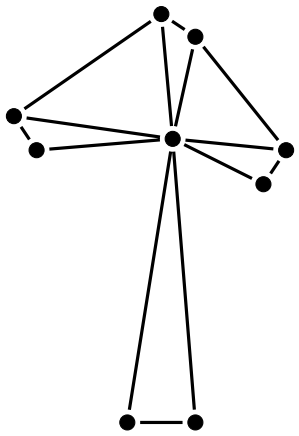
(b) Give an example with $|X| = 4$ where $\subseteq$ is a total order.

(c) Give an example with $|X| = 4$ where the subset relation does not contain any pair (A, B) with $A \neq B$.

(d) Give an example with $|X| = 6$ where the subset relation does not contain any pair (A, B) with $A \neq B$, and all the elements of X are subsets of the set $\{1, 2, 3, 4\}$.

3. Find the smallest equivalence relation R on $M = \{1, 2, 3, 4, 5\}$ which contains the subset $R_0 = \{(1, 1), (1, 2), (2, 4), (3, 5)\}$ and give its equivalence classes.

4. Why is the following graph not a tree? How many edges do you have to delete to make it a tree?



5. Let T be a tree on 12 vertices which has exactly 3 vertices of degree 3 and exactly one vertex of degree 2.

(a) What is the sum of the degrees of the 8 remaining vertices?

(b) What is the maximum degree of this tree? Justify your answer.

(c) What is the degree sequence of T?

(d) Find two non-isomorphic trees with this degree sequence.

END OF PAPER