COMP2270/6270 – Theory of Computation
Eighth Week

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Exercise 1) For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

a) $L = \{xy : x, y \in \{\mathtt{a}, \mathtt{b}\}^* \text{ and } |x| = |y|\}$.
b) $L = \{(\mathtt{ab})^n \mathtt{a}^n \mathtt{b}^n : n > 0\}$.
c) $L = \{\mathtt{a}^i \mathtt{b}^n : i, n > 0 \text{ and } i = n \text{ or } i = 2n\}$.
d) $L = \{\mathtt{a}^i : i \geq 0\}\ \{\mathtt{b}^i : i \geq 0\}\ \{\mathtt{a}^i : i \geq 0\}$.
e) $L(G)$, where $G = S \rightarrow \mathtt{a}S\mathtt{a}$
$S \rightarrow SS$
$S \rightarrow \varepsilon$

Exercise 2) Let $L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* :$ the first, middle, and last characters of w are identical$\}$.
a) Show a context-free grammar for L.
b) Show a natural PDA that accepts L.
c) Prove that L is not regular.

Exercise 3) Without using the Pumping Theorem, prove that $L = \{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* : \#_{\mathtt{a}}(w) = \#_{\mathtt{b}}(w) = \#_{\mathtt{c}}(w)$ and $\#_{\mathtt{a}}(w) > 50\}$ is not context-free.

Exercise 4) Give an example of a context-free language L ($\neq \Sigma^*$) that contains a subset L_1 that is not context-free. Prove that L is context free. Describe L_1 and prove that it is not context-free.

Exercise 5) Let $L_1 = L_2 \cap L_3$.
a) Show values for L_1, L_2, and L_3, such that L_1 is context-free but neither L_2 nor L_3 is.
b) Show values for L_1, L_2, and L_3, such that L_2 is context-free but neither L_1 nor L_3 is.

Exercise 6) Give an example of a context-free language L, other than one of the ones in the book, where $\neg L$ is not context-free.

Exercise 7) Theorem 13.7 tells us that the context-free languages are closed under intersection with the regular languages. Prove that the context-free languages are also closed under union with the regular languages.

REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008.