$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

$$v(t) \neq 0$$

$$= \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$$

$$s = L_\gamma(t) = \int_0^t v(s)\,ds$$

$$\tilde{\gamma} : [0,1] \longrightarrow R^3$$

$$\tilde{\gamma}(s) = \begin{pmatrix} x(L_\gamma^{-1}(s)) \\ y(L_\gamma^{-1}(s)) \\ z(L_\gamma^{-1}(s)) \end{pmatrix}$$

$$\frac{d}{dt} L_\gamma(t) = v(t)$$

Inverse mapping theorem

$v(t) \neq 0 \Rightarrow$ $L_\gamma(t)$ is locally invertible and

$$\frac{d}{ds}\left(L_\gamma^{-1}(s)\right)' = \frac{1}{\left(L_\gamma'(t)\right)} = \frac{1}{v(t)}$$

$$\frac{d}{ds}\tilde{\gamma}(s) = \begin{pmatrix} x'(L_\gamma^{-1}(s)) \\ y'(L_\gamma^{-1}(s)) \\ z'(L_\gamma^{-1}(s)) \end{pmatrix} \cdot \frac{d}{ds} L_\gamma^{-1}(s)$$

$$v_{\tilde{\gamma}}(s) = \left| \frac{d}{ds}\tilde{\gamma}(s) \right| = \left| \gamma'(t) \right| \cdot \frac{1}{v(t)} = \frac{v(t)}{v(t)} = 1.$$