An *ordered tree* $(V, E, v_0, \succ)$ is specified by the following data:

- a tree $T = (V, E)$ with vertex set V and edge set E,
- a root vertex $v_0 \in V$,
- for every vertex $v \in V$ and ordering $w_1 \succ w_2 \cdots \succ w_k$ of the children of v.

In a drawing of an ordered tree, the ordering is represented by drawing v to the left of w whenever $v \succ w$. For example, for the tree T_1 in Figure 1 we have

- $V = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $E = \{\{0, 1\}, \{0, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 6\}, \{2, 7\}\}$,
- $v_0 = 0$
- $1 \succ 2, \qquad 3 \succ 4 \succ 5, \qquad 6 \succ 7,$

while for the tree T_2 in Figure 2 we have

- $V = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $E = \{\{0, 1\}, \{0, 2\}, \{1, 3\}, \{1, 7\}, \{2, 4\}, \{2, 5\}, \{2, 6\}\}$,
- $v_0 = 0$
- $2 \succ 1, \qquad 4 \succ 6 \succ 5, \qquad 3 \succ 7.$

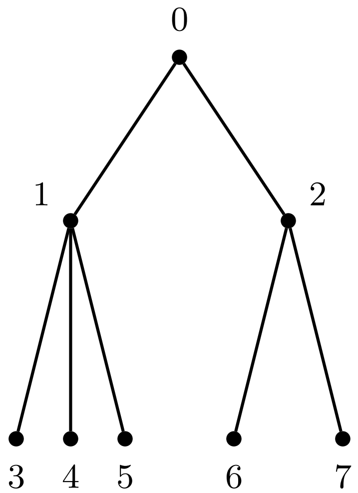

Figure 1: T_1.

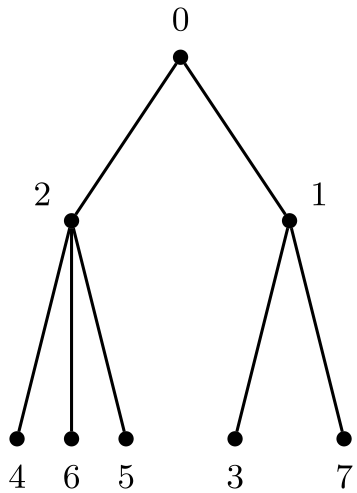

Figure 2: T_2.

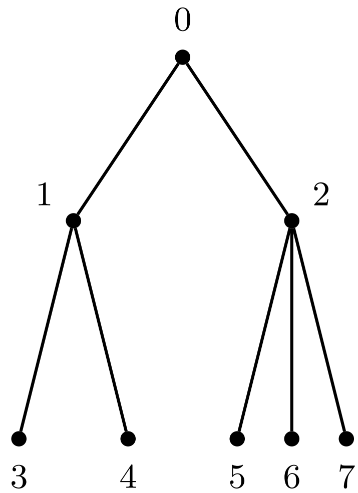

Figure 3: T_3.

Definition 1. Two ordered trees $(V, E, v_0, \succ)$ and $(V', E', v_0', \succ')$ are isomorphic if there exists a bijection $f : V \to V'$ which "preserves the structure", i.e.,

- $\{v, w\} \in E \iff \{f(v), f(w)\} \in E'$ (This means $G = (V, E)$ and $G' = (V', E')$ are isomorpic as graphs), and
- $f(v_0) = v_0'$ (This means (V, E, v_0) and (V', E', v_0') are isomorphic as rooted trees), and
- $v \succ w \iff f(v) \succ' f(w)$.

With this definition the ordered trees T_1 and T_2 are isomorphic which is witnessed by the bijection

$$f(0) = 0, \quad f(1) = 2, \quad f(2) = 1, \quad f(3) = 4, \quad f(4) = 6, \quad f(5) = 5, \quad f(6) = 3, \quad f(7) = 7.$$

On the other hand, T_1 and T_3 are not isomorphic due to the following argument. Suppose there is an isomorphism, i.e., a bijection $f : \{0, 1, \ldots, 7\} \to \{0, 1, \ldots, 7\}$ with the required properties. Then $f(0) = 0$, because f has to preserve the root (the second condition in Definition 1). The first condition in Definition 1 then implies that f maps $\{1, 2\}$ to $\{1, 2\}$. But we also know that graph isomorphisms preserve degrees, hence we must have $f(1) = 2$ and $f(2) = 1$. Now f violates the third condition in Definition 1, because $1 \succ 2$ (in T_1), but $f(1) \not\succ' f(2)$ (in T_2). This proves that there is no isomorphism of ordered trees between T_1 and T_3.