COMP2270/6270 – Theory of Computation
Second week

School of Electrical Engineering & Computing
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Exercise 1) Let $\Sigma = \{a, b, c\}$. How many elements are there in Σ^*?

Exercise 2) (Chapter 2, Exercise 1 of Ref. [1])
Consider the language $L = \{1^n2^n : n > 0\}$. Is the string 122 in L? Why?

Exercise 3) (Chapter 2, Exercise 2 of Ref. [1])
Let $L_1 = \{a^nb^n : n > 0\}$. Let $L_2 = \{c^n : n > 0\}$. For each of the following strings, state whether or not it is an element of L_1L_2:

a) ε.
b) aabbcc.
c) abbcc.
d) aabbcccc.

Justify your decision in each case.

Exercise 4) Give a formal definition of the ***Kleene star*** **?**

Exercise 5) (Chapter 2, Exercise 3 of Ref. [1])
Let L_1 = {peach, apple, cherry} and L_2 = {pie, cobbler, ε}.
List the elements of L_1L_2 in lexicographic order.

Exercise 6) With L_1 and L_2 as defined above in Exercise 5, give the Cartesian product of these two languages ?

Exercise 7) Again, referring to L_1 and L_2 in Exercise 5, give the *power set* of both languages ? Give the formal definition of a *power set* of a language and answer, is the intersection of both power sets empty or not ? (Justify your answer)

Exercise 8) (Chapter 2, Exercise 5 of Ref. [1])
Consider the language L of all strings drawn from the alphabet {a, b} with at least two different substrings of length 2.
a. Describe L by writing a sentence of the form $L = \{w \in \Sigma^* : P(w)\}$, where Σ is a set of symbols and P is a first-order logic formula. You may use the function $|s|$ to return the length of s. You may use all the standard relational symbols (e.g., =, ≠, <, etc.), plus the predicate *Substr*(s, t), which is *True* iff s is a substring of t.

b. List the first six elements of a lexicographic enumeration of L.

Exercise 9) (Chapter 2, Exercise 6 of Ref. [1])
For each of the following languages L, give a simple English description. Show two strings that are in L and two that are not (unless there are fewer than two strings in L or two not in L, in which case show as many as possible).

a) $L = \{w \in \{a, b\}^* : \text{exactly one prefix of } w \text{ ends in a}\}$.
b) $L = \{w \in \{a, b\}^* : \text{all prefixes of } w \text{ end in a}\}$.
c) $L = \{w \in \{a, b\}^* : \exists x \in \{a, b\}^+ (w = axa)\}$.

Exercise 10) (Chapter 2, Exercise 7 of Ref. [1])
Are the following sets closed under the following operations? If not, what are their respective closures?

a) The language {a, b} under concatenation.
b) The odd length strings over the alphabet {a, b} under Kleene star.
c) $L = \{w \in \{a, b\}^*\}$ under reverse.
d) $L = \{w \in \{a, b\}^* : w \text{ starts with a}\}$ under reverse.
e) $L = \{w \in \{a, b\}^* : w \text{ ends in a}\}$ under concatenation.

Exercise 11) (Chapter 2, Exercise 8 of Ref. [1])
For each of the following statements, state whether it is *True* or *False*. Prove your answer.

a) $\forall L_1, L_2\ (L_1 = L_2 \text{ iff } L_1^* = L_2^*)$.
b) $(\varnothing \cup \varnothing^*) \cap (\neg\varnothing - (\varnothing\varnothing^*)) = \varnothing$ (where $\neg\varnothing$ is the complement of $\varnothing$).
c) Every infinite language is the complement of a finite language.
d) $\forall L\ ((L^R)^R = L)$.
e) $\forall L_1, L_2\ ((L_1 L_2)^* = L_1^* L_2^*)$.
f) $\forall L_1, L_2\ ((L_1^* L_2^* L_1^*)^* = (L_2 \cup L_1)^*)$.
g) $\forall L_1, L_2\ ((L_1 \cup L_2)^* = L_1^* \cup L_2^*)$.
h) $\forall L_1, L_2, L_3\ ((L_1 \cup L_2) L_3 = (L_1 L_3) \cup (L_2 L_3))$.
i) $\forall L_1, L_2, L_3\ ((L_1 L_2) \cup L_3 = (L_1 \cup L_3) (L_2 \cup L_3))$.
j) $\forall L\ ((L^+)^* = L^*)$.
k) $\forall L\ (\varnothing L^* = \{\varepsilon\})$.
l) $\forall L\ (\varnothing \cup L^+ = L^*)$.
m) $\forall L_1, L_2\ ((L_1 \cup L_2)^* = (L_2 \cup L_1)^*)$.

Exercise 12) Using mathematical induction:

(a) Prove that $n! \leq n^n$ for any integer $n \geq 1$.

(b) Prove that $3^{2n} - 1$ is divisible by 8 for any integer $n \geq 0$.

REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008.