COMP2270/6270 – Theory of Computation
Sixth week

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Exercise 1) Define the function *twice*(L) = $\{w : \exists x \in L$ (x can be written as $c_1c_2 \ldots c_n$, for some $n \geq 1$, where each $c_i \in \Sigma_L$, and $w = c_1c_1c_2c_2 \ldots c_nc_n)\}$.

a) Let $L = (1 \cup 0)^*1$. Write a regular expression for *twice*(L).
b) Are the regular languages closed under *twice*? Prove your answer.

Exercise 2) For each of the following claims, state whether it is *True* or *False*. Prove your answer.:

a) The union of an infinite number of regular languages must be regular.
b) The union of an infinite number of regular languages is never regular.
c) If L_1 and L_2 are regular languages and $L_1 \subseteq L \subseteq L_2$, then L must be regular.
d) The intersection of two nonregular languages must not be regular.
e) The intersection of an infinite number of regular languages must be regular.
f) If L is a language that is not regular, then L^* is not regular.
g) If L^* is regular, then L is regular.
h) Every subset of a regular language is regular.

Exercise 3) For each of the following languages L, state whether L is regular or not and prove your answer:

a) $\{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* :$ in each prefix x of w, $\#_{\mathtt{a}}(x) = \#_{\mathtt{b}}(x) = \#_{\mathtt{c}}(x))\}$.
b) $\{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* : \exists$ some prefix x of w $(\#_{\mathtt{a}}(x) = \#_{\mathtt{b}}(x) = \#_{\mathtt{c}}(x))\}$.
c) $\{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* : \exists$ some prefix x of w $(x \neq \varepsilon$ and $\#_{\mathtt{a}}(x) = \#_{\mathtt{b}}(x) = \#_{\mathtt{c}}(x))\}$.

Exercise 4) Define the following two languages:

$L_{\mathtt{a}} = \{w \in \{\mathtt{a}, \mathtt{b}\}^* :$ in each prefix x of w, $\#_{\mathtt{a}}(x) \geq \#_{\mathtt{b}}(x)\}$.
$L_{\mathtt{b}} = \{w \in \{\mathtt{a}, \mathtt{b}\}^* :$ in each prefix x of w, $\#_{\mathtt{b}}(x) \geq \#_{\mathtt{a}}(x)\}$.

a) Let $L_1 = L_{\mathtt{a}} \cap L_{\mathtt{b}}$. Is L_1 regular? Prove your answer.
b) Let $L_2 = L_{\mathtt{a}} \cup L_{\mathtt{b}}$. Is L_2 regular? Prove your answer.

Exercise 5) For each of the following languages L, state whether L is regular or not and prove your answer:

a) $\{uww^Rv : u, v, w \in \{\mathtt{a}, \mathtt{b}\}^+\}$.
b) $\{xyzy^Rx : x, y, z \in \{\mathtt{a}, \mathtt{b}\}^+\}$.

Exercise 6) Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. For the languages that are defined by each of the following grammars, do each of the following:

i. List five strings that are in L.
ii. List five strings that are not in L.
iii. Describe L concisely. You can use regular expressions, expressions using variables (e.g., $\mathtt{a}^n\mathtt{b}^n$, or set
theoretic expressions (e.g., $\{x: \ldots\}$)
iv. Indicate whether or not L is regular. Prove your answer.

a) $S \rightarrow \mathtt{a}S \mid S\mathtt{b} \mid \varepsilon$
b) $S \rightarrow \mathtt{a}S\mathtt{a} \mid \mathtt{b}S\mathtt{b} \mid \mathtt{a} \mid \mathtt{b}$

Exercise 7) Consider the following context free grammar G:

$S \rightarrow \mathtt{a}S\mathtt{a}$
$S \rightarrow T$
$S \rightarrow \varepsilon$
$T \rightarrow \mathtt{b}T$
$T \rightarrow \mathtt{c}T$
$T \rightarrow \varepsilon$

One of these rules is redundant and could be removed without altering $L(G)$. Which one?

Exercise 8) Show a context-free grammar for each of the following languages L:

a) BalDelim = {w : where w is a string of delimeters: (,), [,], {, }, that are properly balanced}.
b) $\{\mathtt{a}^i\mathtt{b}^j : 2i \neq 3j + 1\}$.
c) $\{w \in \{\mathtt{a}, \mathtt{b}\}^* : \#_{\mathtt{a}}(w) = 2\, \#_{\mathtt{b}}(w)\}$.

REFERENCES

[1] Elaine Rich, Automata Computatibility and Complexity: Theory and Applications, Pearson, Prentice Hall, 2008.