COMP2230/6230 Algorithms
Workshop 1

1. Simplify the following expressions.
   (a) $\log_{10}(1000)$
   (b) $\log_2\left(\sqrt{128}\right)$
   (c) $\log_e\left(e^{100}\right)$
   (d) $\log_{10}\left(\frac{\sqrt{x}\sin(x)}{x+4}\right)$

2. Solve each of the following equations for x.
   (a) $100 = 50e^{-x}$
   (b) $\frac{1}{5} = 5^{3x-2}$
   (c) $\log(2x+5) = 0$
   (d) $\log_x(6) = \frac{1}{3}$

3. Find the particular solution to the Fibonacci recurrence relation, that is
$$F_n = F_{n-1} + F_{n-2}$$
with $F_1 = 1$ and $F_2 = 1$. Be careful with the square roots and negative signs, it will get messy!

4. Prove that
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
using induction.

5. Prove that
$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$
using induction.

6. Prove that, for all $n \geq 1$,
$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}.$$

7. For each of the following sequences, determine if they are *increasing*, *decreasing*, *non increasing*, *non decreasing*, or none of them.
   (a) $2, 3, 88, 89, 100$
   (b) $2, 3, 3, 88, 89, 100$
   (c) 2
   (d) $2, 1$
   (e) $2, 1, 3, 4, 7, 11, 18$
   (f) $a_n = a_{n-1} + a_{n-2}$ with $a_1 = 2$ and $a_2 = 1$

8. For each of the following sequences, determine if they are *eventually increasing*, *eventually decreasing*, or neither.
   (a) $a_n = 3a_{n-1}$ with $a_1 = 1$
   (b) $a_n = 3a_{n-1} + 2a_{n-2}$ with $a_1 = 3$ and $a_2 = 2$
   (c) $a_n = (-1)^n 3a_{n-1}$
   (d) $a_n = \log(n) - n^{\frac{5}{4}} \sin\left(\frac{1}{n}\right)$

9. What type of sequences are both *non increasing* and *non decreasing*?

10. Verify the two De Morgan's Laws for logic by using a truth table.
   (a) $\overline{(p \wedge q)} = \overline{p} \vee \overline{q}$
   (b) $\overline{(p \vee q)} = \overline{p} \wedge \overline{q}$

11. Using only OR, AND, and $\neg$, construct a logic expression for two Boolean variables, p and q that is the same as the following truth table. It is called XOR and generally denoted as $\oplus$.

p	q	$p \oplus q$
0	0	0
1	0	1
0	1	1
1	1	0

12. Write the double sum from theorem 9 in the lecture notes,
$$a_n = \sum_{i=1}^{k} \sum_{j=0}^{m_i - 1} c_{ij} n^j r_i^n,$$
as a nested set of loops, similar to the single loop for the single sum. Assume that there are variables `int[] m`, `int[][] c`, `int[] r`, and they are all filled with the correct data and are of the correct length.

13. Solve the following recurrence relations.

    (a) $a_n = 3a_{n-1} - 2a_{n-2}$

    (b) $b_n = 4b_{n-1} - 4b_{n-2}$

    (c) $c_n = 8c_{n-1} - 21c_{n-2} + 18c_{n-3}$. It might be useful to know that $x^3 - 8x^2 + 21x - 18 = (x-2)(x-3)^2$.

14. Prove the following

    (a)
    $$\sum_{i=1}^{n}(2i-1) = n^2$$

    (b)
    $$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

    (c)
    $$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

15. **Challenge 1:** Consider an array of length An, with integer $A \geq 2$ where the first n entries have data. We wish to move this data to the last n entries, but do not care if the order stays the same. If a bit of data starts at index j and ends at index $\sigma(j)$, what is the value of
    $$\sum_{j=0}^{n-1} \sigma(j) - j$$
    in terms of A and n? Does the order the data ends up in change this sum?

16. **Challenge 2:** For what values of α does the following sequence go to $-\infty$?
    $$a_n = \log(n) - n^{\frac{\alpha+1}{\alpha}} \sin\left(\frac{1}{n}\right).$$
    Provide a proof.