The University of Newcastle
School of Information and Physical Sciences

COMP2230/6230 Algorithms

Tutorial Week 9

13^{th} – 17^{th} September 2021

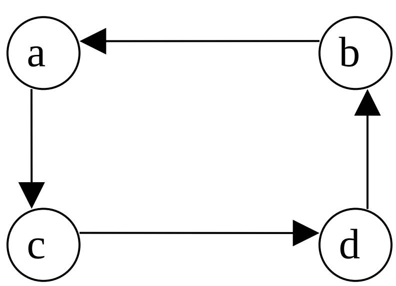
Tutorial

1. Write a dynamic programming algorithm for computing the n^{th} Fibonacci number *f(n)*. Trace your algorithm for $n = 7$. Compare the time complexity of your algorithm to the time complexity of a recursive algorithm for computing the n^{th} Fibonacci number *f(n)*. Refine your algorithm so that it does not use extra space. Trace the refined algorithm for $n = 7$.

2. Write a dynamic programming algorithm for computing binomial coefficient $C(n,k) = \binom{n}{k}$. What is the complexity of your algorithm? Trace the algorithm for *C(5,3)*.

3. The following is a pseudocode of Warshall's algorithm for computing the transitive closure of a digraph, where the transitive closure of a directed graph with n vertices is an n × n Boolean matrix TC such that TC(i,j) is 1 if there is a directed path from vertex i to vertex j, and 0 otherwise. Trace the algorithm for the digraph below.

```
//Input: The adjacency matrix A of a digraph D with n vertices.
//Output: The transitive closure of D.
TC(0) = A
for k=1 to n
    for i=1 to n
            for j=1 to n
                    TC(k) [i,j] = TC(k-1) [i,j] or (TC(k-1)[i,k] and TC(k-1)[k,j])
return TC(n)
```

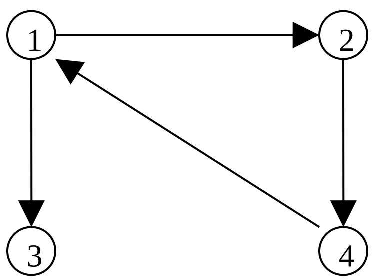


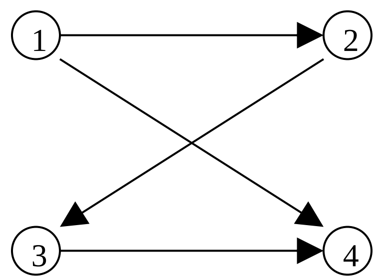
4. Trace Floyd's algorithm for the following digraph.

$$A = \begin{matrix} 0 & 5 & 1 & \infty \\ 1 & 0 & \infty & 10 \\ \infty & 3 & 0 & 1 \\ 7 & 1 & \infty & 0 \end{matrix}$$

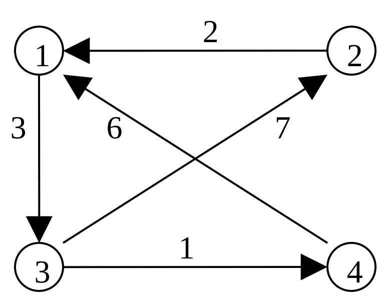
Homework

5. Trace Warshall's algorithm on the diagraphs below.





6. Trace Floyd's algorithm for the following digraph.

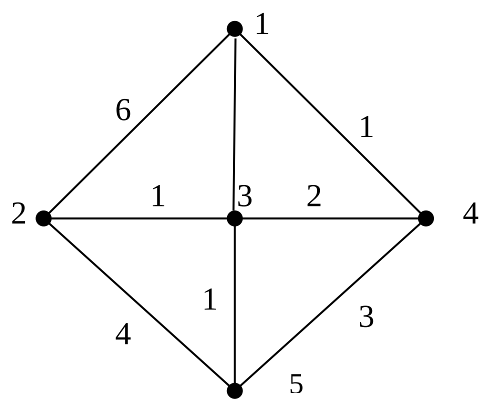


7. Write a pseudocode of a dynamic programming algorithm for Knapsack problem and trace it on the following instance: a_1 ($w_1 = 2$, $v_1 = 5$), a_2($w_2 = 3$, $v_2 = 8$), a_3($w_3 = 1$, $v_3 = 7$), a_4 ($w_4 = 2$, $v_4 = 15$) and W=5.

8. Prove that when *Fibonacci_recurs* computes F_n, $n>=3$, F_n computations are required for the base cases.

More exercise

9. Trace Floyd's algorithm for the following graph.



10. There are six permutations of Floyds algorithm of the lines

for k = 1 to n

for i = 1 to n

for j = 1 to n

Which ones give a correct algorithm?

11. Suppose that G is directed, weighted graph in which some weights are negative. Write an algorithm that determines whether G contains a cycle of negative weight.

12. Explain why Warshall's algorithm can compute the matrices $A^{(k)}$ in place? What is the running time of Warshall's algorithm?