COMP 6660 Fall 2022 Assignment 1d

Joshua Boyd
jcb0216@auburn.edu

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Contents

1 Island Model EA Results

1.1 Green Uni-Circle

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/green1d_uni_circle_config.txt`. Best fitness values per run are given in table 1 on page 4. The best solution found was given a fitness score of **56500000**. This fitness was hit in the **13th run**. The bridge created for this individual is shown in figure 1 on page 5. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 2 on page 6.

Run Number	Best Solution Fitness Score
1	30000000
2	18000000
3	38500000
4	38000000
5	30500000
6	37000000
7	39000000
8	40000000
9	32000000
10	43500000
11	29500000
12	26500000
13	56500000
14	36000000
15	29500000
16	35000000
17	45500000
18	33500000
19	25500000
20	30000000
21	33500000
22	24000000
23	47000000
24	40000000
25	43000000
26	41500000
27	37500000
28	29000000
29	28000000
30	25000000

Table 1: Green Uni-Circle Best Fitness Score Per Run

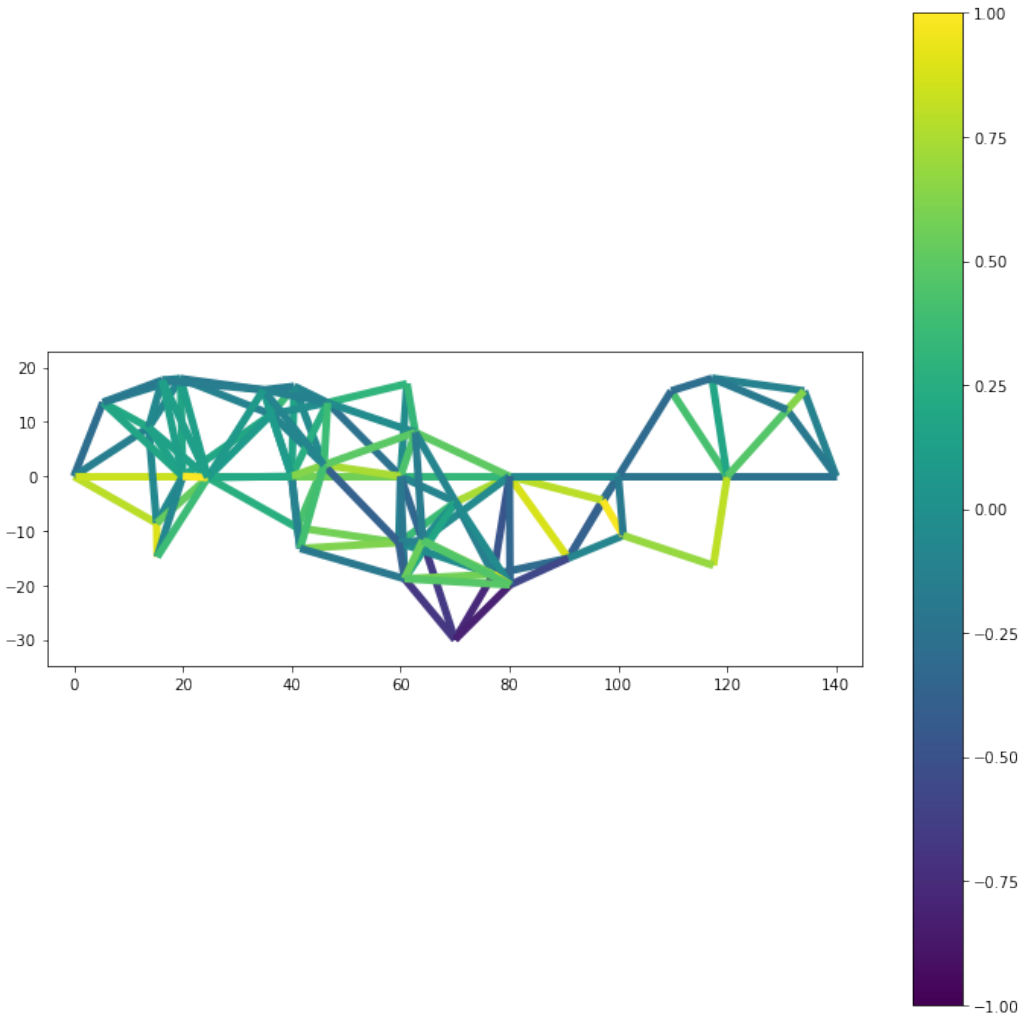

Figure 1: Green Uni-Circle Best Fitness Bridge

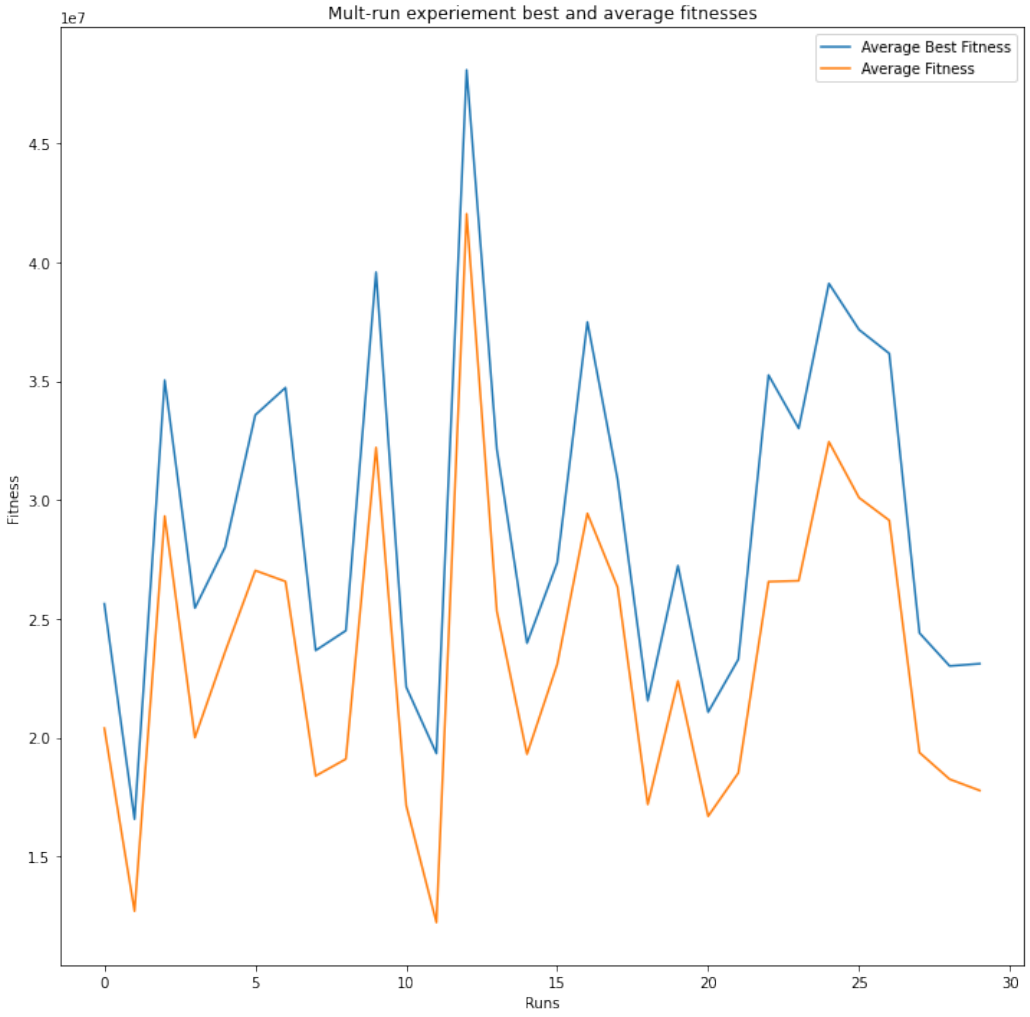

Figure 2: Green Uni-Circle Best and Average Fitness

1.2 Green Bi-Circle

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/green1d_bi_circle_config.txt`. Best fitness values per run are given in table 2 on page 8. The best solution found was given a fitness score of **59000000**. This fitness was hit in the **16th run**. The bridge created for this individual is shown in figure 3 on page 9. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 4 on page 10.

Run Number	Best Solution Fitness Score
1	27500000
2	14500000
3	35000000
4	42000000
5	29000000
6	40000000
7	43500000
8	21000000
9	41000000
10	53000000
11	35000000
12	24500000
13	27500000
14	28000000
15	58000000
16	59000000
17	37500000
18	53000000
19	51000000
20	48000000
21	40000000
22	29500000
23	34500000
24	50000000
25	42500000
26	39000000
27	45000000
28	29000000
29	24500000
30	23500000

Table 2: Green Bi-Circle Best Fitness Score Per Run

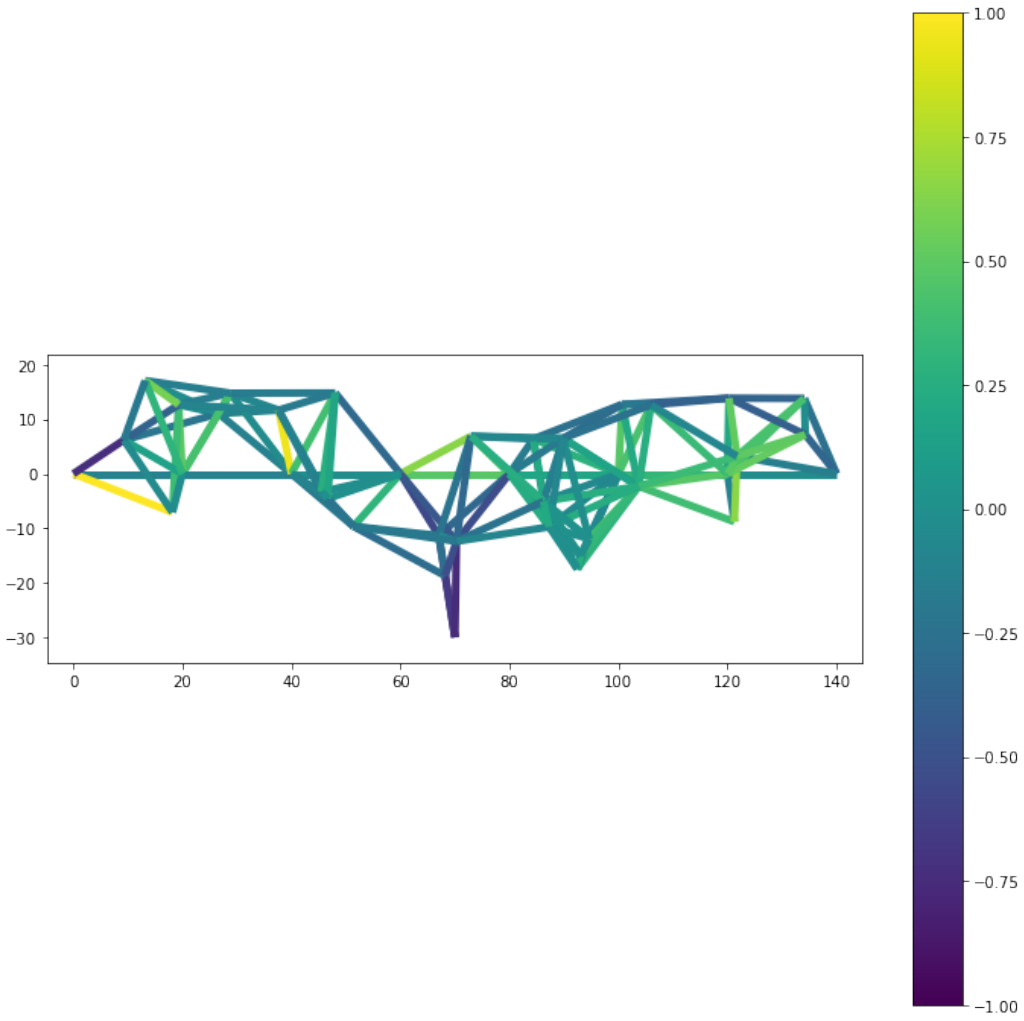

Figure 3: Green Bi-Circle Best Fitness Bridge

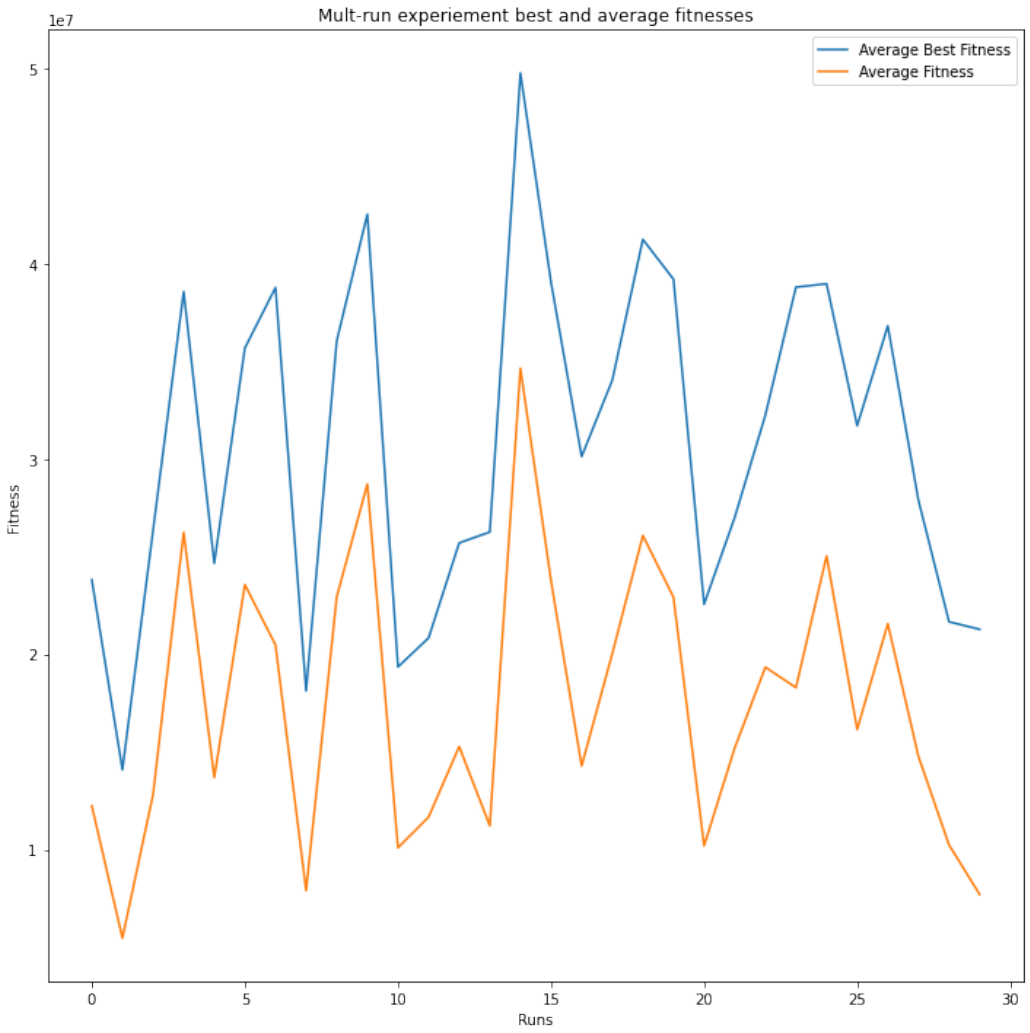

Figure 4: Green Bi-Circle Best and Average Fitness

1.3 Green All-to-All

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/green1d_all_to_all_config.txt`. Best fitness values per run are given in table 3 on page 12. The best solution found was given a fitness score of **83000000**. This fitness was hit in the **12th run**. The bridge created for this individual is shown in figure 5 on page 13. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 6 on page 14.

Run Number	Best Solution Fitness Score
1	56000000
2	43500000
3	40500000
4	67000000
5	62500000
6	60000000
7	65500000
8	43000000
9	42500000
10	50500000
11	55500000
12	83000000
13	64000000
14	39500000
15	47500000
16	67500000
17	47500000
18	50500000
19	59000000
20	64500000
21	58000000
22	50500000
23	61000000
24	36500000
25	42000000
26	54000000
27	52000000
28	49000000
29	67500000
30	50500000

Table 3: Green All-to-All Best Fitness Score Per Run

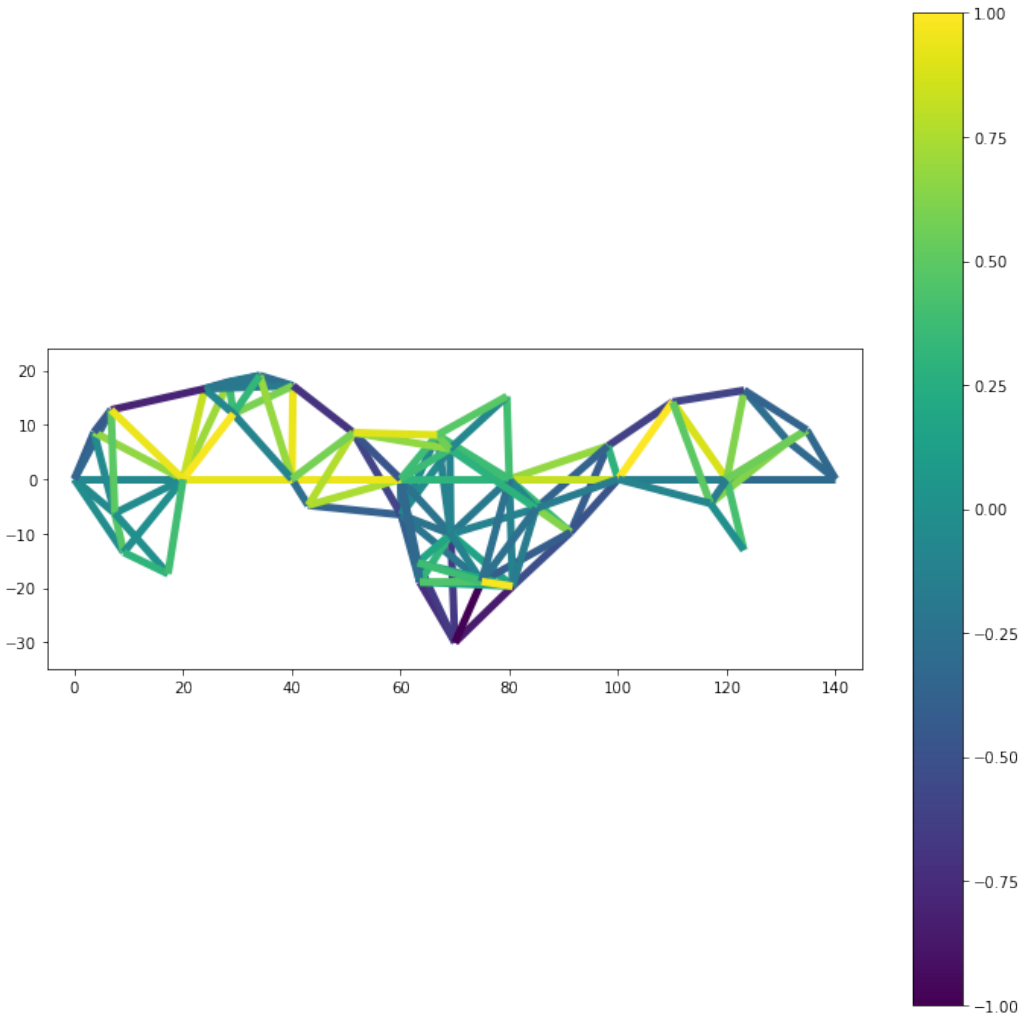

Figure 5: Green All-to-All Best Fitness Bridge

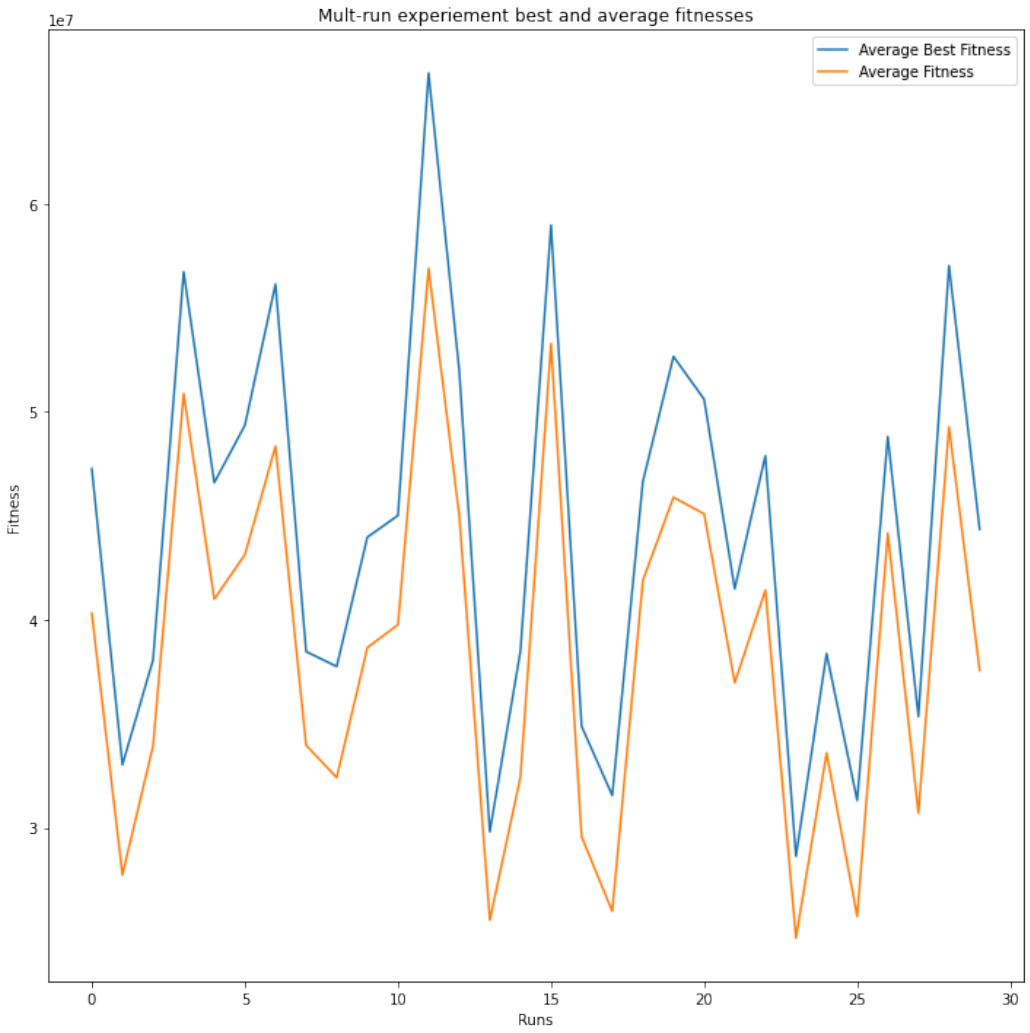

Figure 6: Green All-to-All Best and Average Fitness

1.4 Green Toroid

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/green1d_torid_config.txt`. Best fitness values per run are given in table 4 on page 16. The best solution found was given a fitness score of **83000000**. This fitness was hit in the **12th run**. The bridge created for this individual is shown in figure 7 on page 17. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 8 on page 18.

Run Number	Best Solution Fitness Score
1	31000000
2	33500000
3	42500000
4	31500000
5	55000000
6	27500000
7	42000000
8	63500000
9	36500000
10	33000000
11	26500000
12	39000000
13	36000000
14	49000000
15	46000000
16	34500000
17	53000000
18	49500000
19	29000000
20	33000000
21	39000000
22	30500000
23	30500000
24	36500000
25	42000000
26	54000000
27	52000000
28	49000000
29	67500000
30	50500000

Table 4: Green Toroid Best Fitness Score Per Run

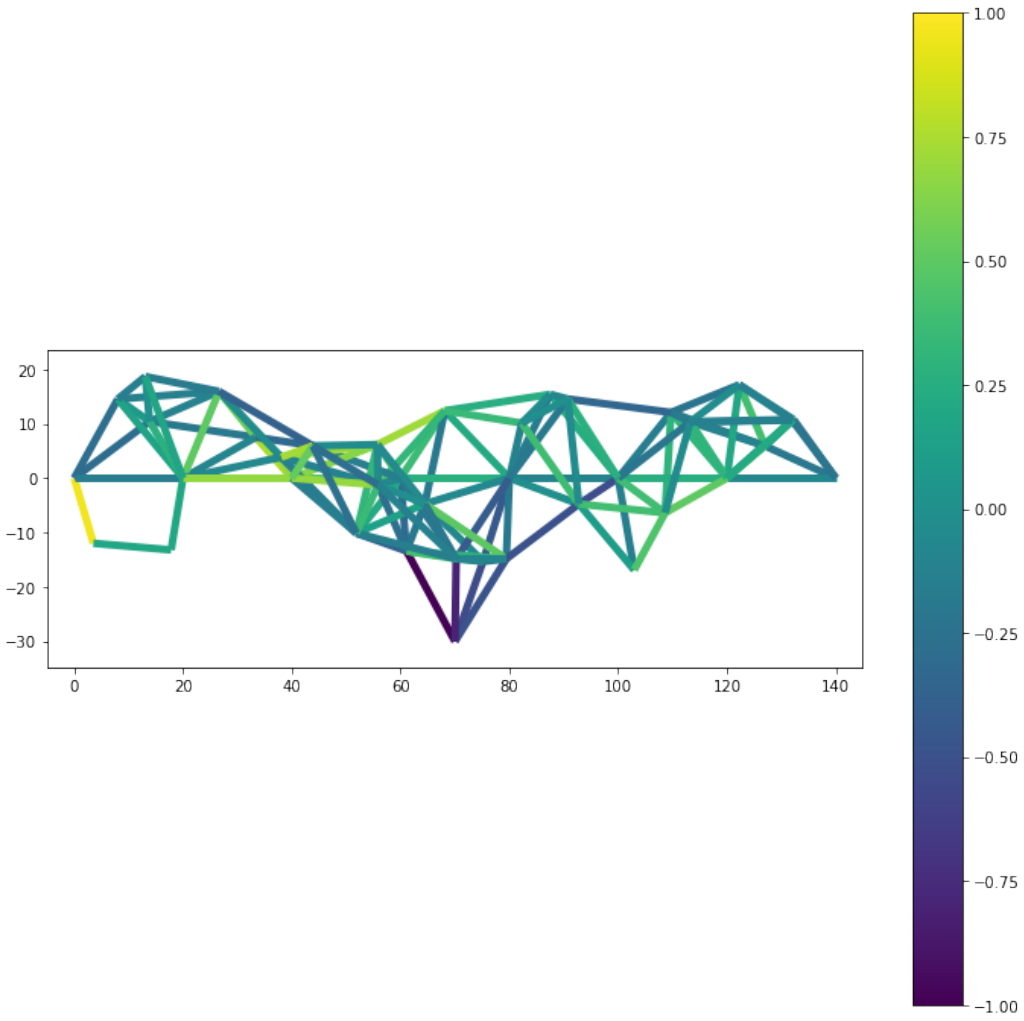

Figure 7: Green Toroid Best Fitness Bridge

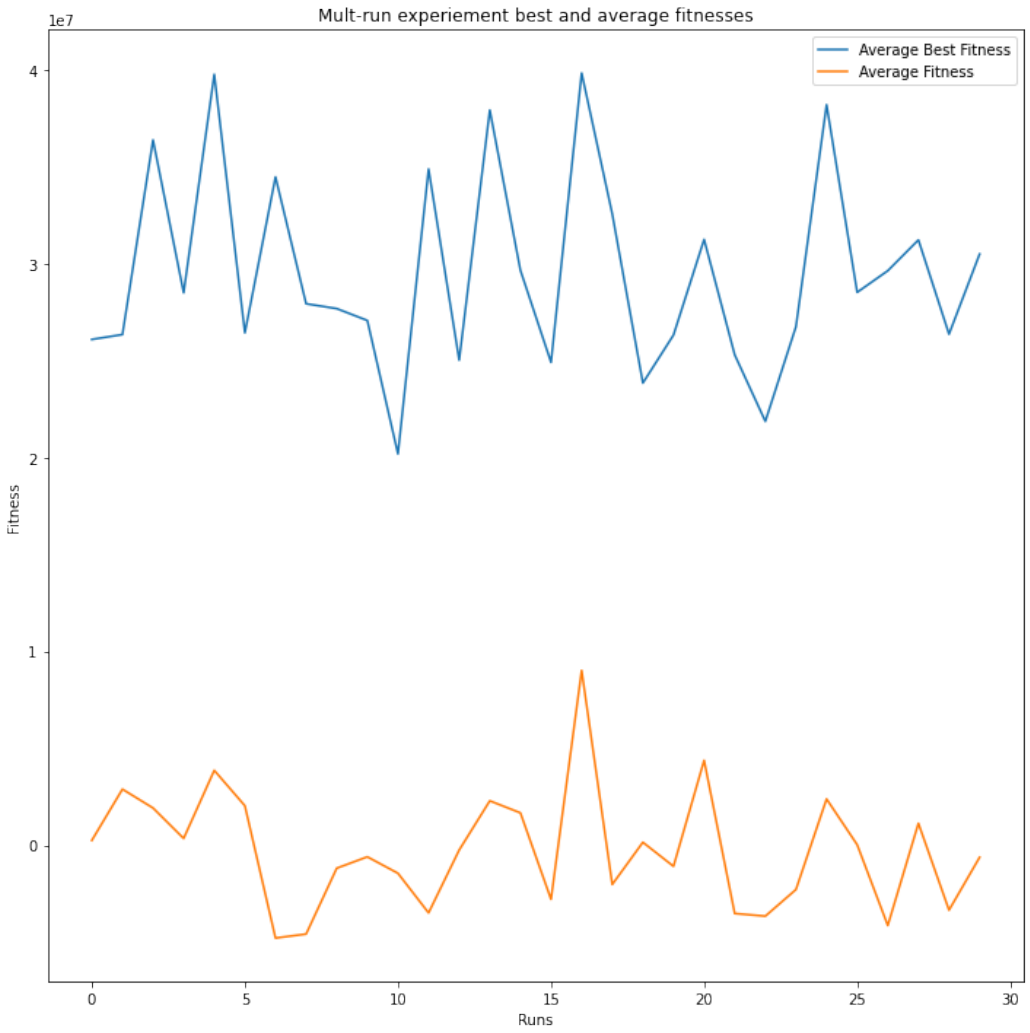

Figure 8: Green Toroid Best and Average Fitness

1.5 Yellow MOEA

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/yellow1d_uni_circle_config.txt`. Best fitness values per run are given in table 5 on page 20. The best solution found was given a fitness score of **20**. This fitness was hit in the **6th run**. The objective scores for this individual was **18000000, 18.35421782594822**. The bridge created for this individual is shown in figure 9 on page 21. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 10 on page 22.

Run Number	Best Solution Fitness Score
1	14
2	6
3	4
4	7
5	7
6	20
7	7
8	7
9	6
10	2
11	7
12	2
13	7
14	11
15	4
16	3
17	6
18	6
19	6
20	12
21	13
22	2
23	8
24	4
25	11
26	6
27	12
28	6
29	5
30	9

Table 5: Yellow Uni-Circle MOEA Best Fitness Score Per Run

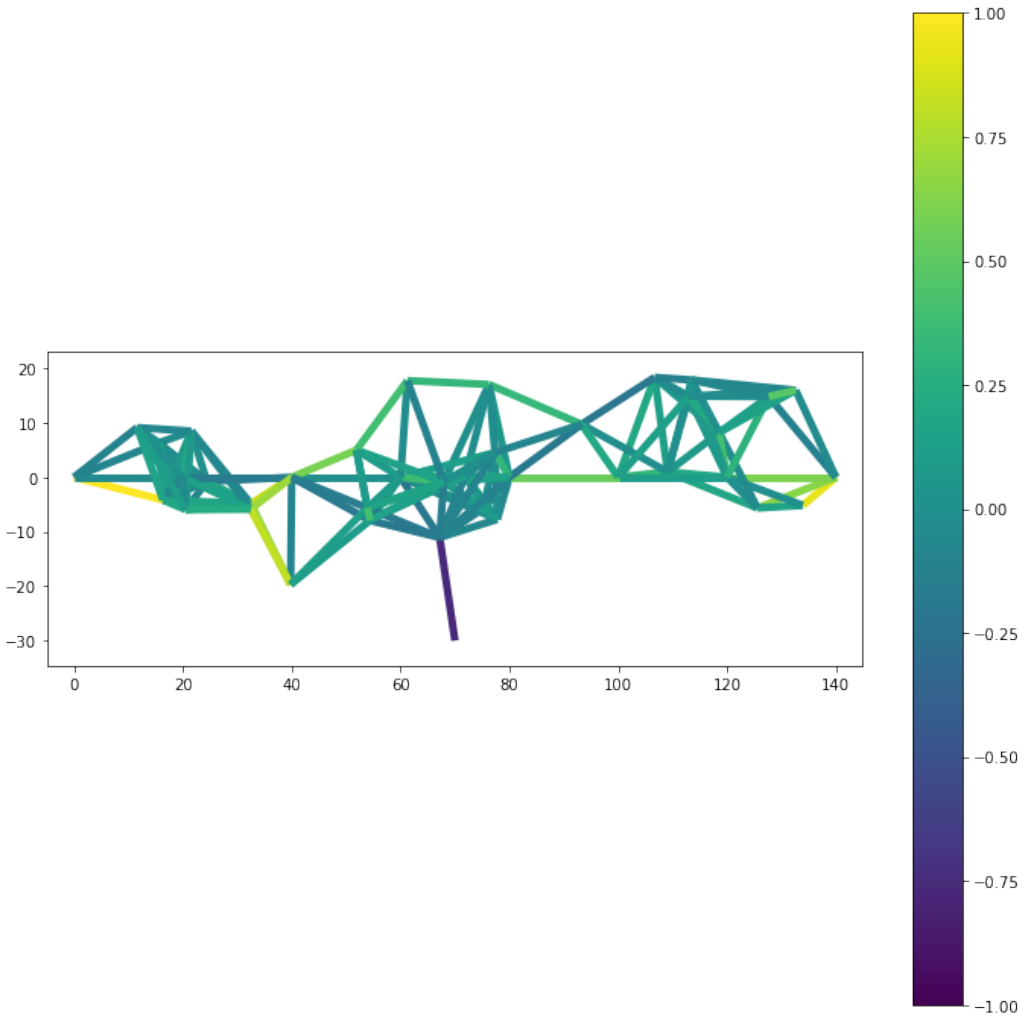

Figure 9: Yellow MOEA Best Fitness Bridge

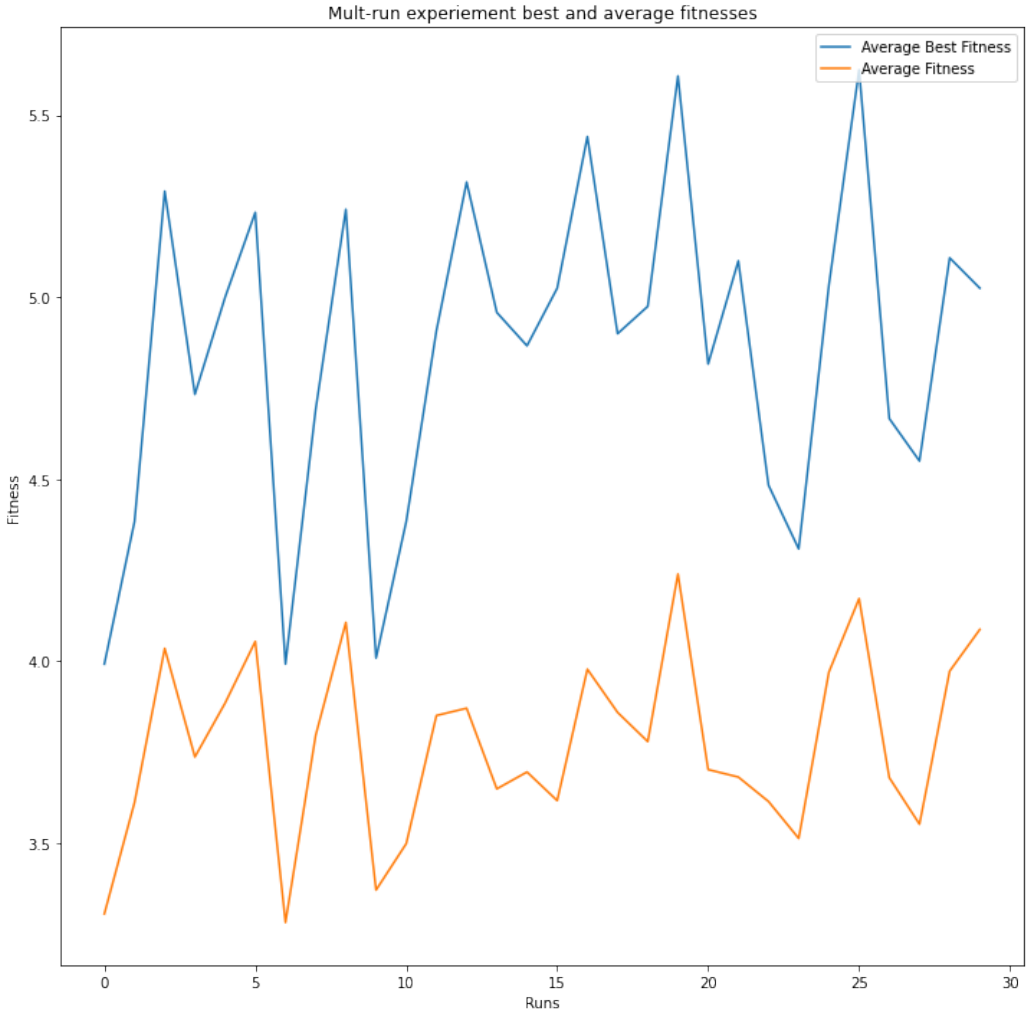

Figure 10: Yellow MOEA Best and Average Fitness

1.6 Red 1 (Different Island Topology)

The new topology that was used can be found in figure 11 on page 23.

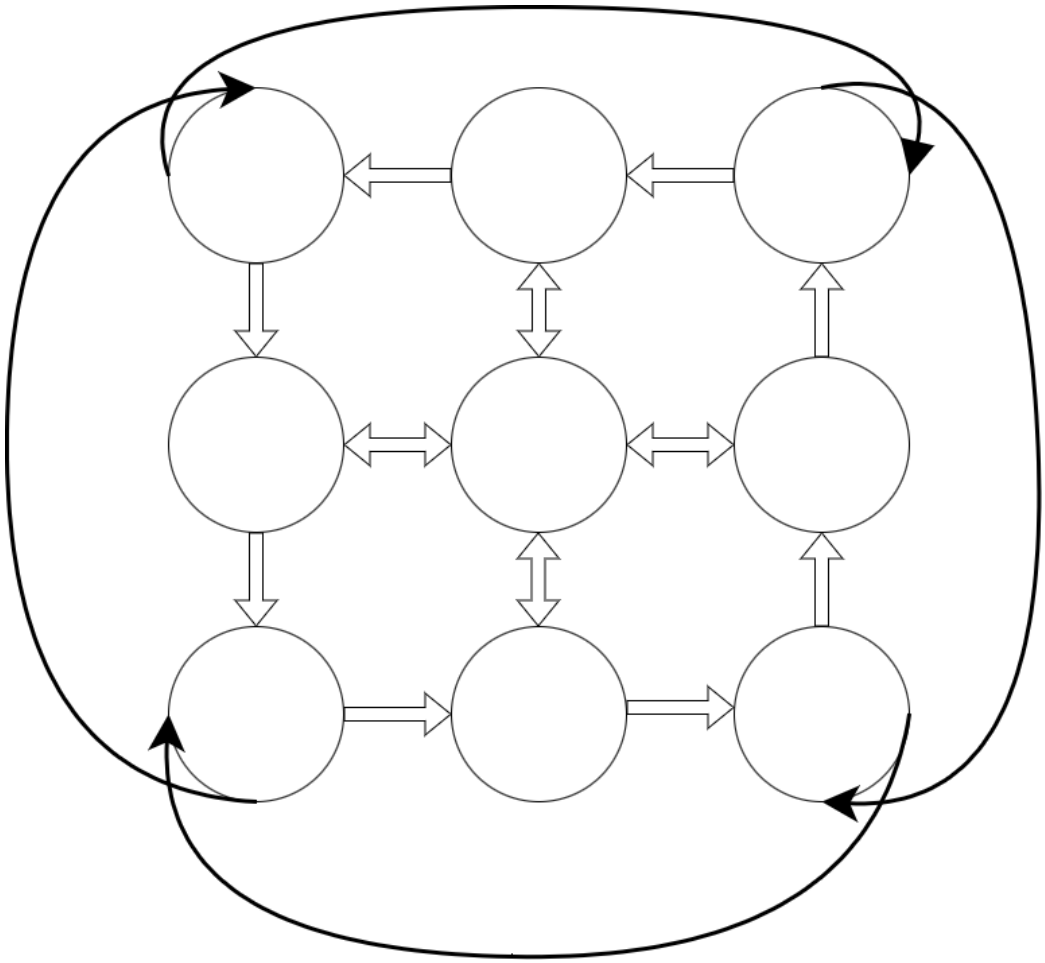

Figure 11: Red 1 New Topology

In this topology, migration is able to occur in either direction to and from the center island. However, on the outer islands, migration is only able to occur in a counter-clockwise direction unless returning back to the center island. In addition, outer islands are also able to loop back with respect to their directional axis.

The experiment was run 30 times for 5000 evaluations per run. The config file used for the experiment can be found at `config/red1d_config.txt`. Best fitness values per run are given in table 6 on page 25. The best solution found was given a fitness score of **51000000**. This fitness was hit in the **11th run**. The bridge created for this individual is shown in figure 12 on page 24. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 13 on page 26.

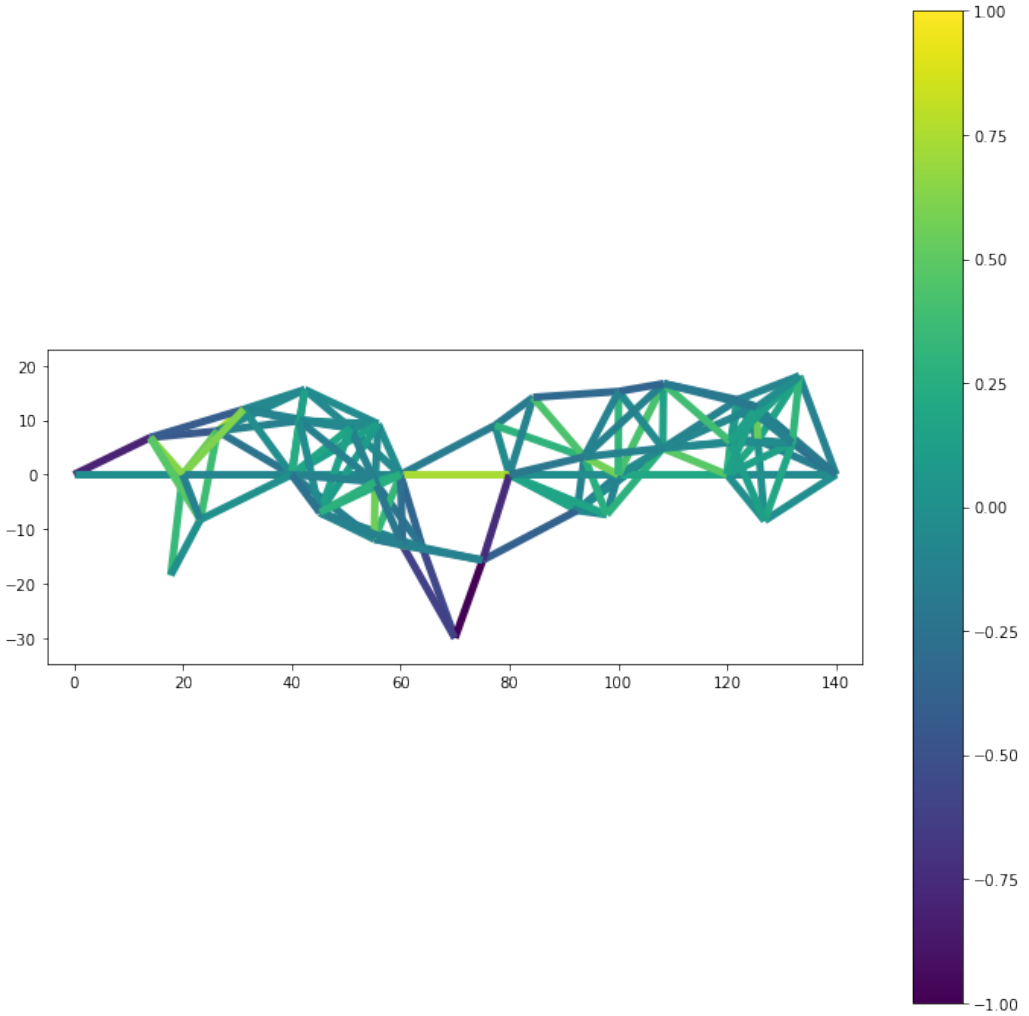

Figure 12: Red 1 Different Topology Best Fitness Bridge

Run Number	Best Solution Fitness Score
1	22500000
2	35500000
3	31500000
4	35500000
5	25500000
6	28500000
7	38000000
8	35000000
9	42500000
10	32500000
11	51000000
12	26000000
13	39000000
14	34500000
15	26500000
16	34500000
17	31000000
18	38500000
19	32000000
20	40500000
21	38000000
22	34000000
23	39000000
24	35000000
25	26000000
26	27000000
27	32500000
28	44000000
29	25500000
30	41500000

Table 6: Red 1 Different Topology Best Fitness Score Per Run

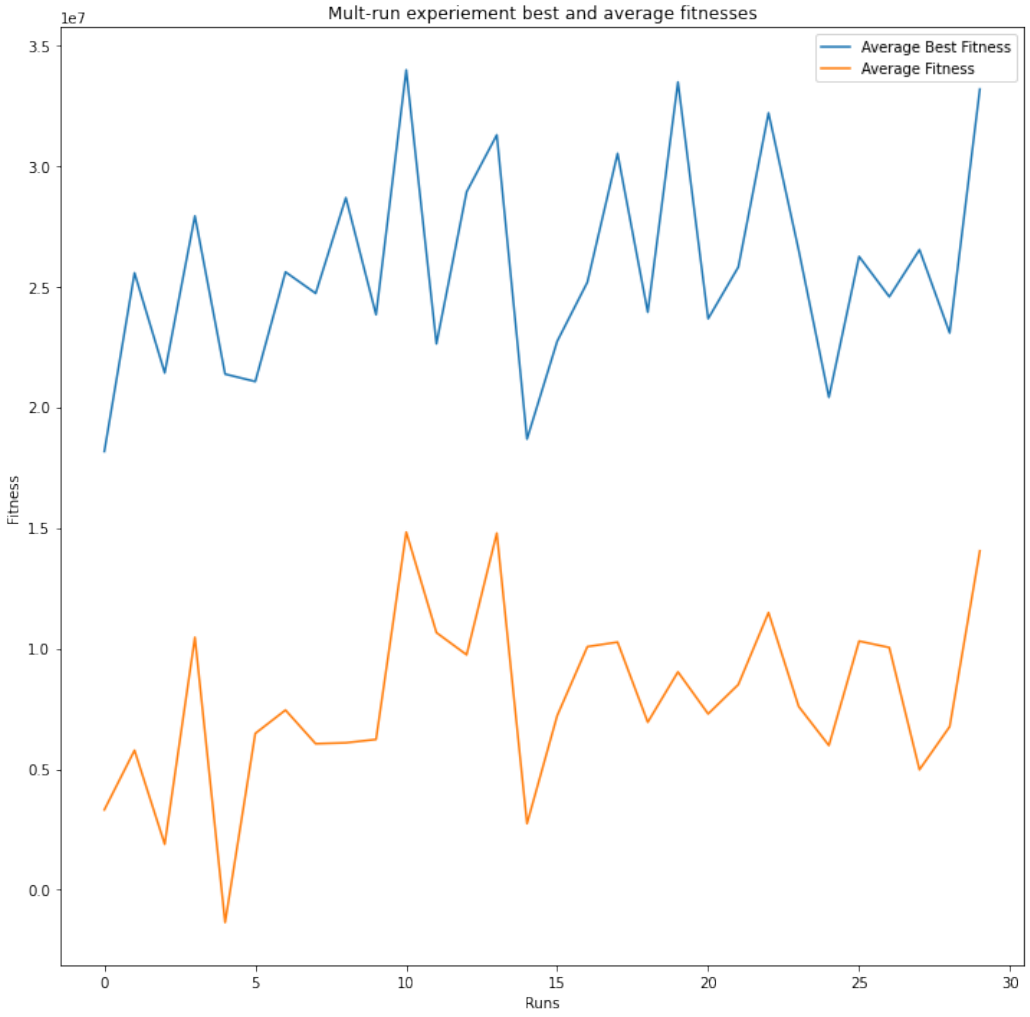

Figure 13: Red 1 Different Topology Best and Average Fitness

1.7 Red 2 (Heterogeneous Islands)

The experiment was run 30 times for 5000 evaluations per run. The different configurations was handled by creating an `EA_configs_1`, `EA_configs_2`, and `EA_configs_3` section in the config file. These three configurations were cycled through for each island. The config file used for the experiment can be found at `config/red2d_config.txt`. Best fitness values per run are given in table 7 on page 28. The best solution found was given a fitness score of **61500000**. This fitness was hit in the **24th run**. The bridge created for this individual is shown in figure 14 on page 29. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 15 on page 30.

Run Number	Best Solution Fitness Score
1	34000000
2	35000000
3	26500000
4	34500000
5	34500000
6	37000000
7	28000000
8	34500000
9	26000000
10	29000000
11	33000000
12	37000000
13	29000000
14	34000000
15	31500000
16	32500000
17	22000000
18	28000000
19	22000000
20	25000000
21	26000000
22	20500000
23	38000000
24	61500000
25	40500000
26	26500000
27	31000000
28	50500000
29	37500000
30	31000000

Table 7: Red 2 Heterogeneous Islands Best Fitness Score Per Run

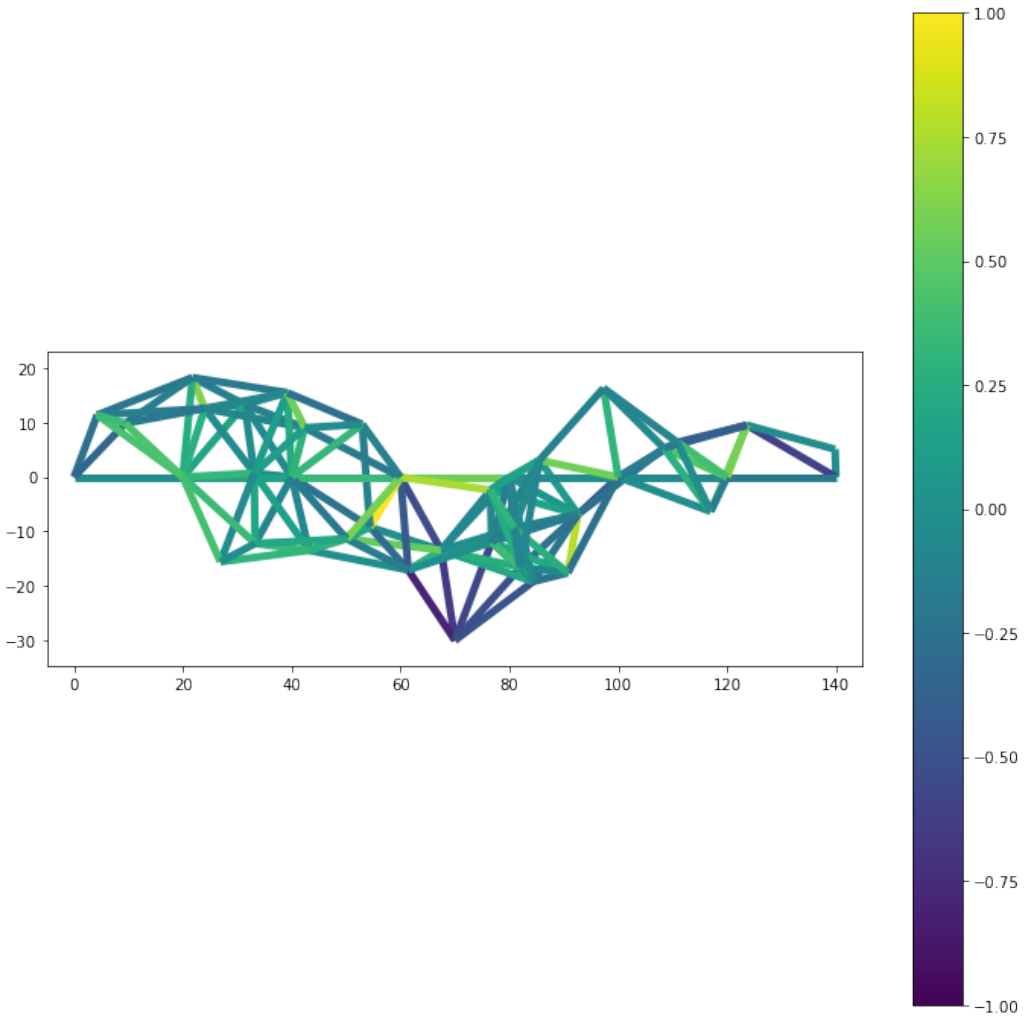

Figure 14: Red 2 Heterogeneous Islands Best Fitness Bridge

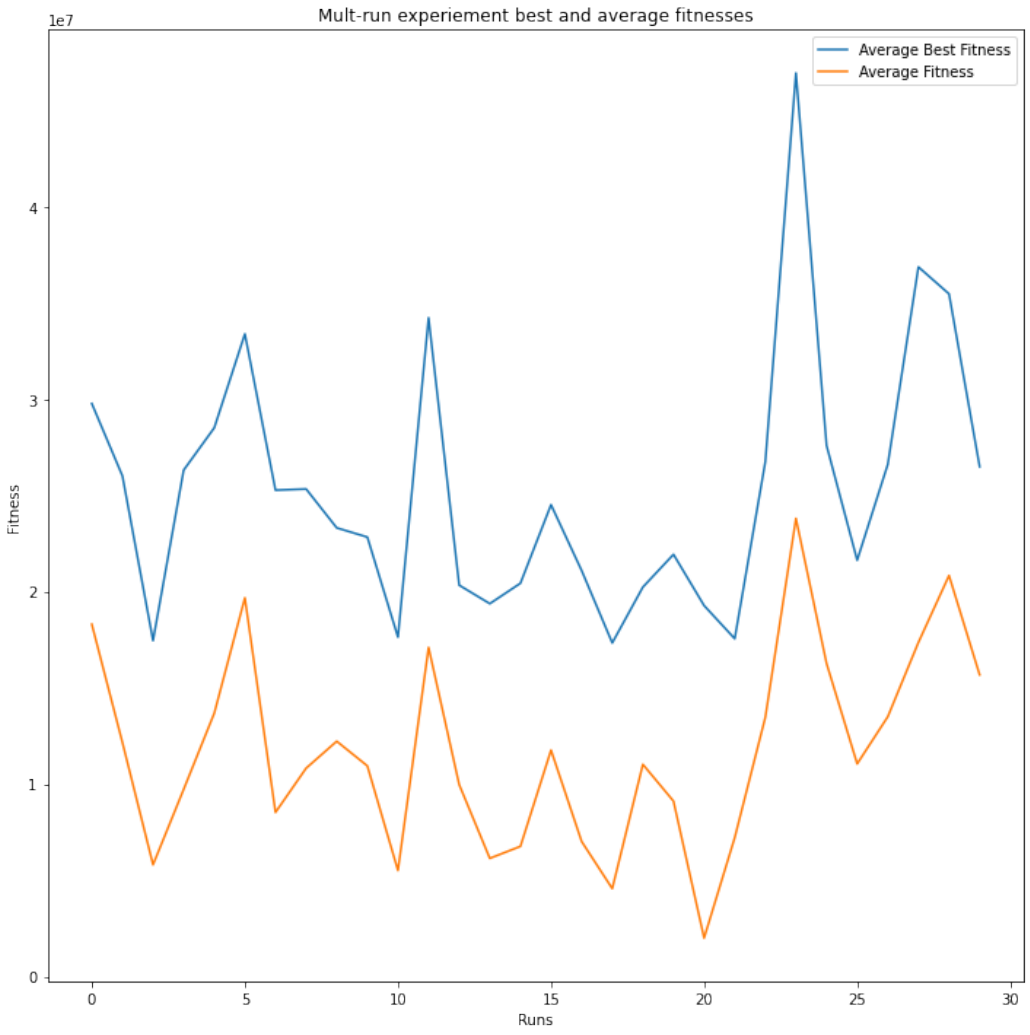

Figure 15: Red 2 Heterogeneous Islands Best and Average Fitness

1.8 Red 3 (All Individual Non-domination Fitness Assignment)

The experiment was run 30 times for 5000 evaluations per run. These three configurations were cycled through for each island. The config file used for the experiment can be found at `config/red3d_config.txt`. Best fitness values per run are given in table 8 on page 32. The best solution found was given a fitness score of **127**. This fitness was hit in the **5th run**. The objective scores for this individual was **8500000, 18.89958101987468**. The bridge created for this individual is shown in figure 16 on page 33. A plot of the best fitness and average fitness progress for the 30 runs is shown in figure 17 on page 34.

Run Number	Best Solution Fitness Score
1	15
2	14
3	16
4	19
5	127
6	9
7	16
8	12
9	19
10	119
11	21
12	13
13	13
14	14
15	19
16	5
17	14
18	7
19	23
20	13
21	10
22	104
23	13
24	11
25	11
26	27
27	21
28	7
29	9
30	22

Table 8: Red 3 All Individual Non-domination Fitness Assignment Best Fitness Score Per Run

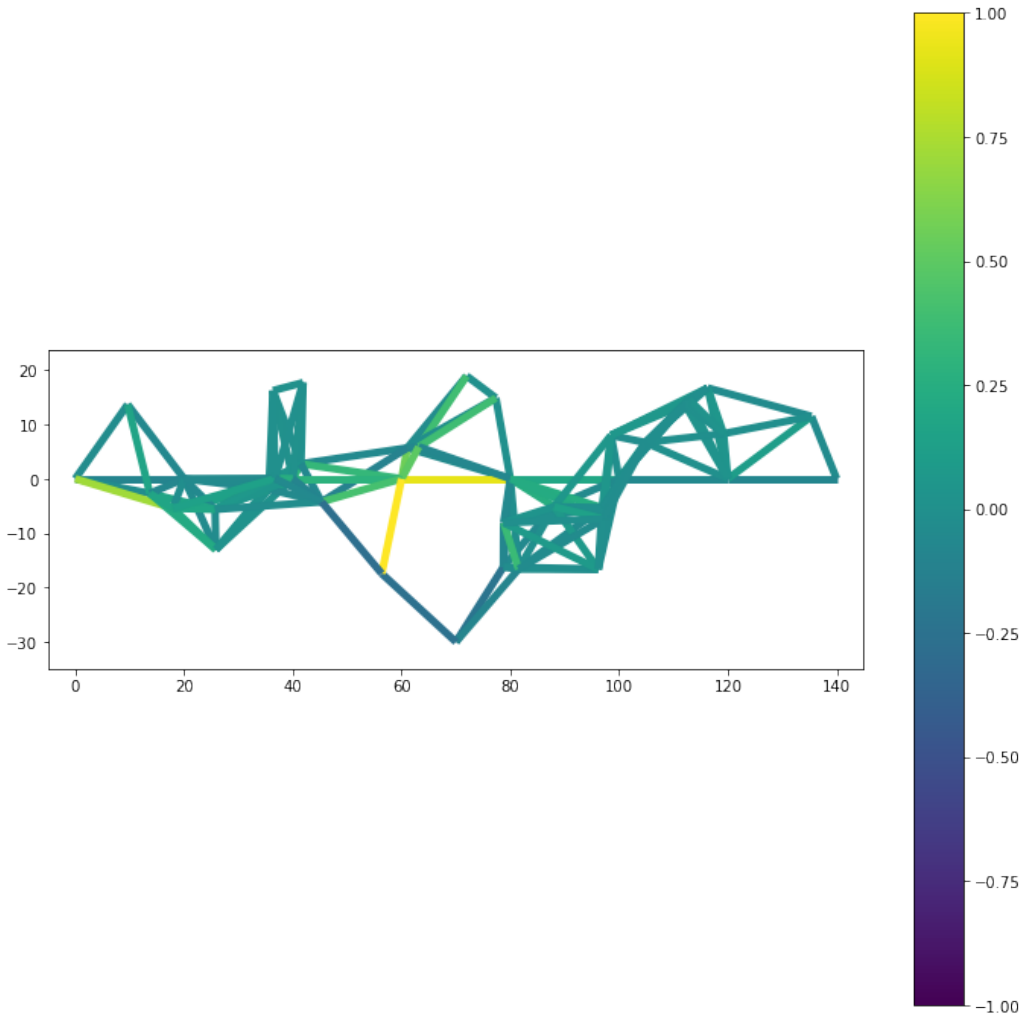

Figure 16: Red 3 All Individual Non-domination Fitness Assignment Best Fitness Bridge

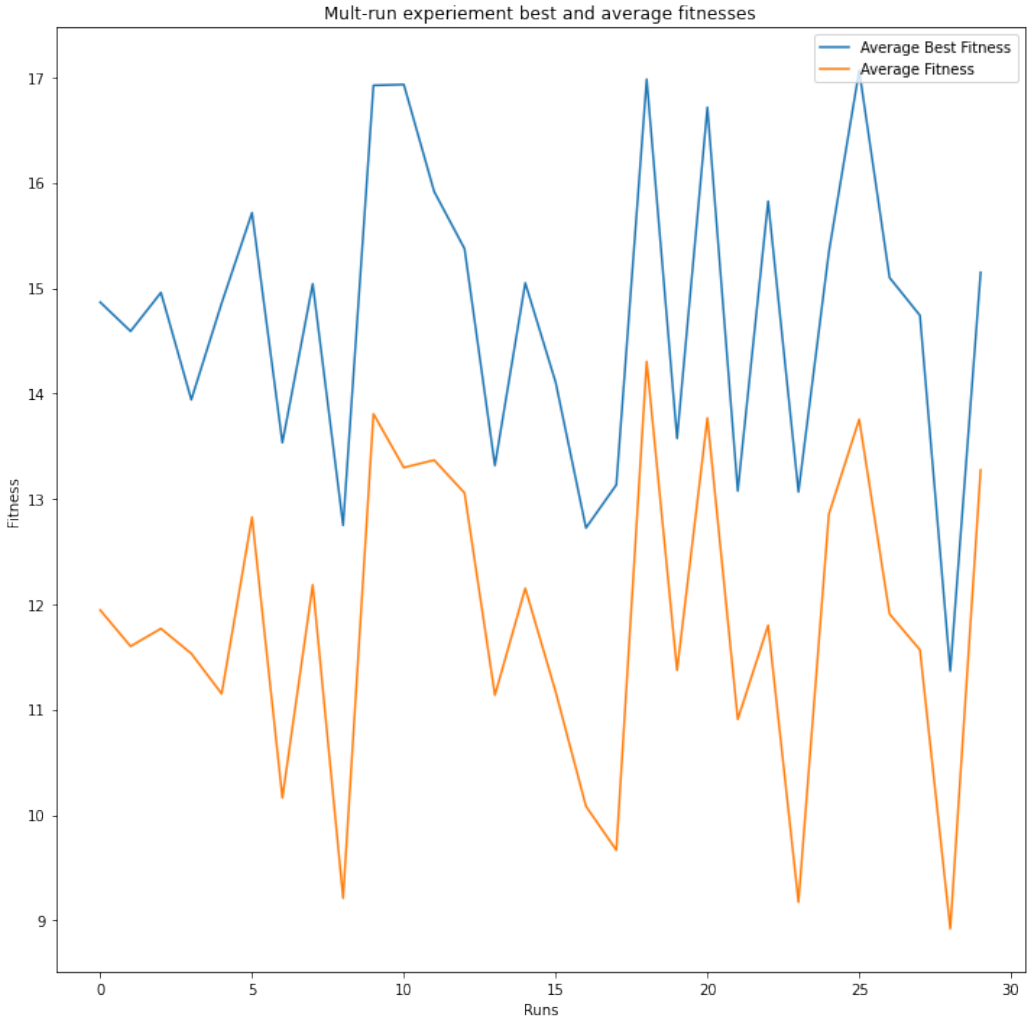

Figure 17: Red 3 All Individual Non-domination Fitness Assignment Best and Average Fitness

2 Statistical Analysis

2.1 Green Deliverables

2.1.1 Green Uni-Circle vs Bi-Circle

In this analysis, the best fitness per run data from the Uni-Circle experiment was compared against the best fitness per run data from the Bi-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 18 on page 35. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **65,288,793,103,448.3** and the variance for sample 2 was **132,129,022,988,506**. The calculated test statistic F was valued at **0.494129084032718**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is smaller than one and but is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	34750000	37516666.67
Variance	6.52888E+13	1.32129E+14
Observations	30	30
df	29	29
F	0.494129084	
P(F<=f) one-tail	0.031225695	
F Critical one-tail	0.475964774	

Figure 18: Green Uni-Circle vs Bi-Circle F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure

19 on page 36. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **34,750,000** and the sample mean for sample 2 was **37,516,666.6666667**. The sample variance for sample 1 was **65,288,793,103,448.3** and the sample variance for sample 2 was **132,129,022,988,506**. The calculated test statistic t was found to be **-1.07851029085424**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	34750000	37516666.67
Variance	6.52888E+13	1.32129E+14
Observations	30	30
Pooled Variance	9.87089E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-1.078510291	
P(T<=t) one-tail	0.142635552	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.285271105	
t Critical two-tail	2.001717484	

Figure 19: Green Uni-Circle vs Bi-Circle t-Test

2.1.2 Green Bi-Circle vs All-To-All

In this analysis, the best fitness per run data from the Bi-Circle experiment was compared against the best fitness per run data from the All-To-All experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 20 on page 37. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **132,129,022,988,506** and the variance for sample 2 was **113,330,172,413,793**. The calculated test statistic F was valued at **1.16587683733573**. The nearest critical value to F(F Critical one-tail) was **2.10099581728421**. According to the test results, the value of F is greater than one but is less than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	37516666.67	54350000
Variance	1.32129E+14	1.1333E+14
Observations	30	30
df	29	29
F	1.165876837	
P(F<=f) one-tail	0.3410884	
F Critical one-tail	2.100995817	

Figure 20: Green Bi-Circle vs All-To-All F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 21 on page 38. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **37,516,666.6666667** and the sample mean for sample 2 was **54,350,000**. The sample variance for sample 1 was **132,129,022,988,506** and the sample variance for sample

2 was **113,330,172,413,793**. The calculated test statistic t was found to be **-5.88492723895094**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	37516666.67	54350000
Variance	1.32129E+14	1.1333E+14
Observations	30	30
Pooled Variance	1.2273E+14	
Hypothesized Mean Difference	0	
df	58	
t Stat	-5.884927239	
P(T<=t) one-tail	1.05404E-07	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	2.10809E-07	
t Critical two-tail	2.001717484	

Figure 21: Green Bi-Circle vs All-To-All t-Test

2.1.3 Green All-To-All vs Toroid

In this analysis, the best fitness per run data from the All-To-All experiment was compared against the best fitness per run data from the Toroid experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 22 on page 39. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **113,330,172,413,793** and the variance for sample 2 was **117,236,781,609,195**. The calculated test statistic F was valued at **0.966677614808425**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one but is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	54350000	41433333.33
Variance	1.1333E+14	1.17237E+14
Observations	30	30
df	29	29
F	0.966677615	
P(F<=f) one-tail	0.463958075	
F Critical one-tail	0.475964774	

Figure 22: Green All-To-All vs Toroid F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 23 on page 40. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **54,350,000** and the sample mean for sample 2 was **41,433,333.3333333**. The sample variance for sample 1 was **113,330,172,413,793** and the sample variance for sample

2 was **117,236,781,609,195**. The calculated test statistic t was found to be **4.65921282102235**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	54350000	41433333.33
Variance	1.1333E+14	1.17237E+14
Observations	30	30
Pooled Variance	1.15283E+14	
Hypothesized Mean Difference	0	
df	58	
t Stat	4.659212821	
P(T<=t) one-tail	9.50552E-06	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	1.9011E-05	
t Critical two-tail	2.001717484	

Figure 23: Green All-To-All vs Toroid t-Test

2.1.4 Green All-To-All vs Uni-Circle

In this analysis, the best fitness per run data from the All-To-All experiment was compared against the best fitness per run data from the Uni-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 24 on page 41. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **113,330,172,413,793** and the variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic F was valued at **1.73582887700535**. The nearest critical value to F(F Critical one-tail) was **2.10099581728421**. According to the test results, the value of F is greater than one but is less than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	54350000	34750000
Variance	1.1333E+14	6.52888E+13
Observations	30	30
df	29	29
F	1.735828877	
P(F<=f) one-tail	0.071722197	
F Critical one-tail	2.100995817	

Figure 24: Green All-To-All vs Uni-Circle F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 25 on page 42. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **54,350,000** and the sample mean for sample 2 was **34,750,000**. The sample variance for sample 1 was **113,330,172,413,793** and the sample variance for sample 2

was **65,288,793,103,448.3**. The calculated test statistic t was found to be **8.03254031284316**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	54350000	34750000
Variance	1.1333E+14	6.52888E+13
Observations	30	30
Pooled Variance	8.93095E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	8.032540313	
P(T<=t) one-tail	2.701E-11	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	5.402E-11	
t Critical two-tail	2.001717484	

Figure 25: Green All-To-All vs Uni-Circle t-Test

2.1.5 Green Toroid vs Uni-Circle

In this analysis, the best fitness per run data from the Toroid experiment was compared against the best fitness per run data from the Uni-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 26 on page 43. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **117,236,781,609,195** and the variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic F was valued at **1.79566470808301**. The nearest critical value to F(F Critical one-tail) was **2.10099581728421**. According to the test results, the value of F is greater than one but is less than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	41433333.33	34750000
Variance	1.17237E+14	6.52888E+13
Observations	30	30
df	29	29
F	1.795664708	
P(F<=f) one-tail	0.060355661	
F Critical one-tail	2.100995817	

Figure 26: Green Toroid vs Uni-Circle F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 27 on page 44. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **41,433,333.3333333** and the sample mean for sample 2 was **34,750,000**. The sample variance for sample 1 was **117,236,781,609,195** and the sample variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic t was found to be **2.70951704438931**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	41433333.33	34750000
Variance	1.17237E+14	6.52888E+13
Observations	30	30
Pooled Variance	9.12628E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	2.709517044	
P(T<=t) one-tail	0.004422311	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.008844623	
t Critical two-tail	2.001717484	

Figure 27: Green Toroid vs Uni-Circle t-Test

2.1.6 Green Toroid vs Bi-Circle

In this analysis, the best fitness per run data from the Toroid experiment was compared against the best fitness per run data from the Bi-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 28 on page 45. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **117,236,781,609,195** and the variance for sample 2 was **132,129,022,988,506**. The calculated test statistic F was valued at **0.887290157434935**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one but is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	41433333.33	37516666.67
Variance	1.17237E+14	1.32129E+14
Observations	30	30
df	29	29
F	0.887290157	
P(F<=f) one-tail	0.374813884	
F Critical one-tail	0.475964774	

Figure 28: Green Toroid vs Bi-Circle F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 29 on page 46. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **41,433,333.3333333** and the sample mean for sample 2 was **37,516,666.6666667**. The sample variance for sample 1 was **117,236,781,609,195** and the sample variance

for sample 2 was **132,129,022,988,506**. The calculated test statistic t was found to be **1.35849733229386**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	41433333.33	37516666.67
Variance	1.17237E+14	1.32129E+14
Observations	30	30
Pooled Variance	1.24683E+14	
Hypothesized Mean Difference	0	
df	58	
t Stat	1.358497332	
P(T<=t) one-tail	0.089783193	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.179566385	
t Critical two-tail	2.001717484	

Figure 29: Green Toroid vs Bi-Circle t-Test

2.2 Red 1 (Different Island Topology)

2.2.1 vs Green Uni-Circle

In this analysis, the best fitness per run data from the Red 1 (Different Island Topology) experiment was compared against the best fitness per run data from the Green Uni-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 30 on page 47. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **42,506,896,551,724.1** and the variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic F was valued at **0.651059615765498**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one but is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	34100000	34750000
Variance	4.25069E+13	6.52888E+13
Observations	30	30
df	29	29
F	0.651059616	
P(F<=f) one-tail	0.126871852	
F Critical one-tail	0.475964774	

Figure 30: Red 1 (Different Island Topology) vs Uni-Circle F-Test

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 31 on page 48. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **34,100,000** and

the sample mean for sample 2 was **34,750,000**. The sample variance for sample 1 was **42,506,896,551,724.1** and the sample variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic t was found to be **-0.34290458039864**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	34100000	34750000
Variance	4.25069E+13	6.52888E+13
Observations	30	30
Pooled Variance	5.38978E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-0.34290458	
P(T<=t) one-tail	0.366454986	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.732909972	
t Critical two-tail	2.001717484	

Figure 31: Red 1 (Different Island Topology) vs Uni-Circle t-Test

2.2.2 vs Green Bi-Circle

In this analysis, the best fitness per run data from the Red 1 (Different Island Topology) experiment was compared against the best fitness per run data from the Green Bi-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 32 on page 49. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **42,506,896,551,724.1** and the variance for sample 2 was **132,129,022,988,506**. The calculated test statistic F was valued at **0.321707491588899**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is

less than one and is less than F Critical one-tail. This means the null hypothesis of equal variances is rejected, and the two sets are not shown to be equal.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	34100000	37516666.67
Variance	4.25069E+13	1.32129E+14
Observations	30	30
df	29	29
F	0.321707492	
P(F<=f) one-tail	0.001576509	
F Critical one-tail	0.475964774	

Figure 32: Red 1 (Different Island Topology) vs Bi-Circle F-Test

With the variances of the data sets not shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming unequal variances was used next for comparison. The table showing the results of this test can be found in figure 33 on page 50. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **34,100,000** and the sample mean for sample 2 was **37,516,666.6666667**. The sample variance for sample 1 was **42,506,896,551,724.1** and the sample variance for sample 2 was **132,129,022,988,506**. The calculated test statistic t was found to be **-1.41610824173332**. The upper critical value (t Critical two-tail) was **2.01289559891943**.

t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	34100000	37516666.67
Variance	4.25069E+13	1.32129E+14
Observations	30	30
Hypothesized Mean Difference	0	
df	46	
t Stat	-1.416108242	
P(T<=t) one-tail	0.081739949	
t Critical one-tail	1.678660414	
P(T<=t) two-tail	0.163479898	
t Critical two-tail	2.012895599	

Figure 33: Red 1 (Different Island Topology) vs Bi-Circle t-Test

2.2.3 vs Green All-To-All

In this analysis, the best fitness per run data from the Red 1 (Different Island Topology) experiment was compared against the best fitness per run data from the Green All-To-All experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 34 on page 51. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **42,506,896,551,724.1** and the variance for sample 2 was **113,330,172,413,793**. The calculated test statistic F was valued at **0.375071312840875**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one and is less than F Critical one-tail. This means the null hypothesis of equal variances is rejected, and the two sets are not shown to be equal.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	34100000	54350000
Variance	4.25069E+13	1.1333E+14
Observations	30	30
df	29	29
F	0.375071313	
P(F<=f) one-tail	0.005107867	
F Critical one-tail	0.475964774	

Figure 34: Red 1 (Different Island Topology) vs All-To-All F-Test

With the variances of the data sets not shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming unequal variances was used next for comparison. The table showing the results of this test can be found in figure 35 on page 52. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **34,100,000** and the sample mean for sample 2 was **54,350,000**. The sample variance

for sample 1 was **42,506,896,551,724.1** and the sample variance for sample 2 was **113,330,172,413,793**. The calculated test statistic t was found to be **-8.88485346005707**. The upper critical value (t Critical two-tail) was **2.01063475762423**.

t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	34100000	54350000
Variance	4.25069E+13	1.1333E+14
Observations	30	30
Hypothesized Mean Difference	0	
df	48	
t Stat	-8.88485346	
P(T<=t) one-tail	5.24103E-12	
t Critical one-tail	1.677224196	
P(T<=t) two-tail	1.04821E-11	
t Critical two-tail	2.010634758	

Figure 35: Red 1 (Different Island Topology) vs All-To-All t-Test

2.2.4 vs Green Toroid

In this analysis, the best fitness per run data from the Red 1 (Different Island Topology) experiment was compared against the best fitness per run data from the Green Toroid experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 36 on page 53. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **42,506,896,551,724.1** and the variance for sample 2 was **117,236,781,609,195**. The calculated test statistic F was valued at **0.362573042080082**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one and is less than F Critical one-tail. This means the null hypothesis of equal variances is rejected, and the two sets are not shown to be equal.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	34100000	41433333.33
Variance	4.25069E+13	1.17237E+14
Observations	30	30
df	29	29
F	0.362573042	
P(F<=f) one-tail	0.003979484	
F Critical one-tail	0.475964774	

Figure 36: Red 1 (Different Island Topology) vs Toroid F-Test

With the variances of the data sets not shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming unequal variances was used next for comparison. The table showing the results of this test can be found in figure 37 on page 54. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **34,100,000** and the sample mean for sample 2 was **41,433,333.3333333**. The sample variance

for sample 1 was **42,506,896,551,724.1** and the sample variance for sample 2 was **117,236,781,609,195**. The calculated test statistic t was found to be **-3.17797307556812**. The upper critical value (t Critical two-tail) was **2.01063475762423**.

t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	34100000	41433333.33
Variance	4.25069E+13	1.17237E+14
Observations	30	30
Hypothesized Mean Difference	0	
df	48	
t Stat	-3.177973076	
P(T<=t) one-tail	0.001297451	
t Critical one-tail	1.677224196	
P(T<=t) two-tail	0.002594901	
t Critical two-tail	2.010634758	

Figure 37: Red 1 (Different Island Topology) vs Toroid t-Test

2.3 Red 2 (Heterogeneous Islands)

2.3.1 vs Green Uni-Circle

In this analysis, the best fitness per run data from the Red 2 (Heterogeneous Islands) experiment was compared against the best fitness per run data from the Green Uni-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 38 on page 55. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **68,809,195,402,298.9** and the variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic F was valued at **1.05392046829955**. The nearest critical value to F(F Critical one-tail) was **2.10099581728421**. According to the test results, the value of F is greater than one and is less than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	34750000
Variance	6.88092E+13	6.52888E+13
Observations	30	30
df	29	29
F	1.053920468	
P(F<=f) one-tail	0.444256991	
F Critical one-tail	2.100995817	

Figure 38: Red 2 (Heterogeneous Islands) vs Uni-Circle F-Test

With the variances of the data sets shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 39 on page 56. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **32,533,333.3333333**

and the sample mean for sample 2 was **34,750,000**. The sample variance for sample 1 was **68,809,195,402,298.9** and the sample variance for sample 2 was **65,288,793,103,448.3**. The calculated test statistic t was found to be **-1.04845521990073**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	34750000
Variance	6.88092E+13	6.52888E+13
Observations	30	30
Pooled Variance	6.7049E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-1.04845522	
P(T<=t) one-tail	0.149389168	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.298778336	
t Critical two-tail	2.001717484	

Figure 39: Red 2 (Heterogeneous Islands) vs Uni-Circle t-Test

2.3.2 vs Green Bi-Circle

In this analysis, the best fitness per run data from the Red 2 (Heterogeneous Islands) experiment was compared against the best fitness per run data from the Green Bi-Circle experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 40 on page 57. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **68,809,195,402,298.9** and the variance for sample 2 was **132,129,022,988,506**. The calculated test statistic F was valued at **0.520772755644192**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one and is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	37516666.67
Variance	6.88092E+13	1.32129E+14
Observations	30	30
df	29	29
F	0.520772756	
P(F<=f) one-tail	0.042108841	
F Critical one-tail	0.475964774	

Figure 40: Red 2 (Heterogeneous Islands) vs Bi-Circle F-Test

With the variances of the data sets shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 41 on page 58. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **32,533,333.3333333** and the sample mean for sample 2 was **37,516,666.6666667**. The sample variance for sample 1 was **68,809,195,402,298.9** and the sample variance

for sample 2 was **132,129,022,988,506**. The calculated test statistic t was found to be **-1.92552557642542**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	37516666.67
Variance	6.88092E+13	1.32129E+14
Observations	30	30
Pooled Variance	1.00469E+14	
Hypothesized Mean Difference	0	
df	58	
t Stat	-1.925525576	
P(T<=t) one-tail	0.029535539	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.059071079	
t Critical two-tail	2.001717484	

Figure 41: Red 2 (Heterogeneous Islands) vs Bi-Circle t-Test

2.3.3 vs Green All-To-All

In this analysis, the best fitness per run data from the Red 2 (Heterogeneous Islands) experiment was compared against the best fitness per run data from the Green All-To-All experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 42 on page 59. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **68,809,195,402,298.9** and the variance for sample 2 was **113,330,172,413,793**. The calculated test statistic F was valued at **0.607156893321062**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one and is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	54350000
Variance	6.88092E+13	1.1333E+14
Observations	30	30
df	29	29
F	0.607156893	
P(F<=f) one-tail	0.092547675	
F Critical one-tail	0.475964774	

Figure 42: Red 2 (Heterogeneous Islands) vs All-to-All F-Test

With the variances of the data sets shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 43 on page 60. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **32,533,333.3333333** and the sample mean for sample 2 was **54,350,000**. The sample variance for sample 1 was **68,809,195,402,298.9** and the sample variance for sample

2 was **113,330,172,413,793**. The calculated test statistic t was found to be **-8.85415480979046**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	54350000
Variance	6.88092E+13	1.1333E+14
Observations	30	30
Pooled Variance	9.10697E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-8.85415481	
P(T<=t) one-tail	1.15487E-12	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	2.30974E-12	
t Critical two-tail	2.001717484	

Figure 43: Red 2 (Heterogeneous Islands) vs All-to-All t-Test

2.3.4 vs Green Toroid

In this analysis, the best fitness per run data from the Red 2 (Heterogeneous Islands) experiment was compared against the best fitness per run data from the Green Toroid experiment. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 44 on page 61. The chosen value for α was **0.025**. The sample size for each sample was 30. The variance for sample 1 was **68,809,195,402,298.9** and the variance for sample 2 was **117,236,781,609,195**. The calculated test statistic F was valued at **0.586924977450097**. The nearest critical value to F(F Critical one-tail) was **0.475964774310031**. According to the test results, the value of F is less than one and is greater than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	41433333.33
Variance	6.88092E+13	1.17237E+14
Observations	30	30
df	29	29
F	0.586924977	
P(F<=f) one-tail	0.078645478	
F Critical one-tail	0.475964774	

Figure 44: Red 2 (Heterogeneous Islands) vs Toroid F-Test

With the variances of the data sets shown to be equal according to the F-Test, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 45 on page 62. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was **32,533,333.3333333** and the sample mean for sample 2 was **41,433,333.3333333**. The sample variance for sample 1 was **68,809,195,402,298.9** and the sample variance for sample 2 was **117,236,781,609,195**. The calculated test statistic t was found to be **-3.57388432983147**. The upper critical value (t Critical two-tail) was **2.00171748414524**.

3 Conclusions

3.1 Green Deliverables

3.1.1 Green Uni-Circle vs Bi-Circle

According to the t Test results, t Stat was less than 0 but greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments.

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	32533333.33	41433333.33
Variance	6.88092E+13	1.17237E+14
Observations	30	30
Pooled Variance	9.3023E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-3.57388433	
P(T<=t) one-tail	0.000358186	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.000716371	
t Critical two-tail	2.001717484	

Figure 45: Red 2 (Heterogeneous Islands) vs Toroid t-Test

3.1.2 Green Bi-Circle vs All-To-All

According to the t Test results, t Stat was less than 0 and less than -t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the All-To-All experiment outperformed the Bi-Circle experiment.

3.1.3 Green All-To-All vs Toroid

According to the t Test results, t Stat was greater than 0 and greater than t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the All-To-All experiment outperformed the Toroid experiment.

3.1.4 Green All-To-All vs Uni-Circle

According to the t Test results, t Stat was greater than 0 and greater than t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the All-To-All experiment outperformed the Uni-Circle experiment.

3.1.5 Green Toroid vs Uni-Circle

According to the t Test results, t Stat was greater than 0 and greater than t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the Toroid experiment outperformed the Uni-Circle experiment.

3.1.6 Green Toroid vs Bi-Circle

According to the t Test results, t Stat was greater than 0 but less than t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments.

3.2 Red Deliverables

3.2.1 Red 1 (Different Island Topology) vs Green Uni-Circle

According to the t Test results, t Stat was less than 0 but greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments.

3.2.2 Red 1 (Different Island Topology) vs Green Bi-Circle

According to the t Test results, t Stat was less than 0 but greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments.

3.2.3 Red 1 (Different Island Topology) vs Green All-To-All

According to the t Test results, t Stat was less than 0 and less than -t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the All-To-All experiment outperformed the Red 1 experiment.

3.2.4 Red 1 (Different Island Topology) vs Green Toroid

According to the t Test results, t Stat was less than 0 and less than -t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the Toroid experiment outperformed the Red 1 experiment.

3.2.5 Red 2 (Heterogeneous Islands) vs Green Uni-Circle

According to the t Test results, t Stat was less than 0 but greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments

3.2.6 Red 2 (Heterogeneous Islands) vs Green Bi-Circle

According to the t Test results, t Stat was less than 0 but greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded to find that there was no significant difference detected between the two experiments

3.2.7 Red 2 (Heterogeneous Islands) vs Green All-To-All

According to the t Test results, t Stat was less than 0 and less than -t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the All-to-All experiment outperformed the Red 2 experiment.

3.2.8 Red 2 (Heterogeneous Islands) vs Green Toroid

According to the t Test results, t Stat was less than 0 and less than -t Critical two-tail. Thus the t-test found that the two experiments produced a significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Based on this, the Toroid experiment outperformed the Red 2 experiment.

3.2.9 Red 3 (All Individual Non-domination Fitness Assignment)

In comparing objectives between Red 3 experiment and Yellow MOEA experiment, the objectives for Red 3 were **8500000, 18.89958101987468**, while the objectives for Yellow MOEA was **18000000, 18.35421782594822**. With the height objective being very similar between both algorithms, the attention is turned to the fitness objective. This objective is higher in the Yellow MOEA, leading to an informal conclusion that the Yellow MOEA outperformed the Red 3 (All Individual Non-domination Fitness Assignment) algorithm.