COMP 6660 Fall 2022 Assignment 1c

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Constraint Satisfaction EA Results

Constraint Ignore Bridge Generation

The experiment was ran for a total of 30 runs, with each run processing 5000 generations each. In this experiment, Fitness Proportionate Selection was used for parent selection, while k Tournament Without Replacement was used for survival selection. The parent selection amount was set to 5. The recombine method used was 1-point crossover. The experiment started with 50 parents/mu and generated 5 children at a mutation rate of 50%. The file used for this configuration can be found at `config/green1c0_ignore_config.txt`. Best fitness values per run are given in table 1 on page 2. The best solution found was given a fitness score of **1000000**. This fitness was first hit in the **15th run**. The bridge created for this solution is shown in figure 1 on page 11. A plot of the best fitness progress for the 30 runs is shown in figure 3 on page 13. A plot of the average fitness progress for the 30 runs is shown in figure 2 on page 12.

Constraint Penalty Bridge Generation

The experiment was ran for a total of 30 runs, with each run processing 5000 generations each. The configuration used was the same as the run without constraint penalty, except with the addition of the constraint penalty itself. The constraint penalty coefficient was set to 100000.0. The file used for this configuration can be found at `config/green1c0_penalty_config.txt`. Best fitness values per run are given in table 2 on page 4. The best solution

Run Number	Best Solution Fitness Score
1	-54000000.0
2	0
3	0
4	-46000000.0
5	0
6	0
7	-4000000.0000000037
8	0
9	0
10	-57000000.00000001
11	-62000000.0
12	0
13	-75000000.0
14	-54000000.0
15	1000000
16	-18999999.999999996
17	-17999999.999999993
18	500000
19	0
20	-18999999.999999996
21	-17999999.999999993
22	-79000000.0
23	-24000000.0
24	-61000000.0
25	-44999999.99999999
26	-14000000.000000002
27	0
28	-64000000.0
29	-65999999.99999999
30	-12000000.0

Table 1: Constraint Ignore Best Fitness Score Per Run

found was given a fitness score of **14000000**. This fitness was first hit in the **13th run**. The bridge created for this solution is shown in figure 4 on page

14. A plot of the best fitness progress for the 30 runs is shown in figure 6 on page 16. A plot of the average fitness progress for the 30 runs is shown in figure 5 on page 15.

Statistical Analysis

The best fitness per run data for no constraint penalty was compared against the best fitness per run data with the constraint penalty. The distribution of the data is not known to be normal and the sample size is greater than 29. Therefore, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 7 on page 17. The chosen value for α was 0.025. The sample size for each sample was 30. The variance for sample 1 was 780,387,643,678,161 and the variance for sample 2 was 20,183,000,000,000. The calculated test statistic F was valued at 38.6655920169529. The nearest critical value to F(F Critical one-tail) was 2.10099581728421. According to the test results, the value of F is greater than one and is greater than F Critical one-tail. This means that the test has rejected the null hypothesis of equal variances, and determined that the two populations have significantly unequal variances.

With the variances of the data sets shown as being significantly unequal, a Two-tailed two-sample t-test assuming unequal variances was used next for comparison. The table showing the results of this test can be found in figure 8 on page 17. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was -26,316,666.6666666 and the sample mean for sample 2 was 4,090,000. The sample variance for sample 1 was 780,387,643,678,161 and the sample variance for sample 2 was 20,183,000,000,000. The calculated test statistic t was found to be -5.88612675427251. The upper critical value (t Critical two-tail) was 2.04227245630124. The t Stat was less than 0 and less than -t Critical two-tail. Therefore, the two experiments produced significantly different mean fitness. The experiment with the highest sample mean can then be assumed to produce a significantly higher mean fitness than the other. Since experiment 2 produced a higher sample mean, we can conclude that experiment 2 is statistically significantly better than experiment 1.

Run Number	Best Solution Fitness Score
1	1900000
2	3900000
3	6000000
4	400000
5	3000000
6	4100000
7	800000
8	1000000
9	600000
10	11300000
11	-900000
12	13100000
13	14000000
14	8700000
15	1700000
16	1300000
17	13700000
18	700000
19	600000
20	2700000
21	-100000
22	2800000
23	6100000
24	1400000
25	1400000
26	400000
27	5400000
28	3000000
29	11900000
30	1800000

Table 2: Constraint Penalty Best Fitness Score Per Run

Multi-Objective EA Results

Green Results

The experiment was ran for a total of 30 runs, with each run processing 5000 generations each. In this experiment, k Tournament with Replacement

was used for parent selection, while k Tournament Without Replacement was used for survival selection. The recombine method used was 1-point crossover. The experiment started with 150 parents/mu and generated 15 children at a mutation rate of 50%. The file used for this configuration can be found at `config/green1c1_config.txt`. Best weight and material values per run are given in table 3 on page 6. The best solution found was given a weight score of **16500000** and a material score of **-139.9991000053771**. These scores were first hit in the **21st run**. For the average material per run, a plot is in figure 9 given on page 18. Additionally, the plot for average weight per run is given in figure 10 on page 18. A table with the final Pareto Front values is given in table 4 on page 7. The plot for this final Pareto Front is given in figure 11 on page 18. The bridges for the final Pareto Front are also included, starting on page 18.

Yellow Results

The experiment was ran for a total of 30 runs, with each run processing 5000 generations each. The experiment was ran with the same configurations as the green experiment. The file used for this configuration can be found at `config/green1c1_config.txt` Best weight and material values per run are given in table 5 on page 8. The best solution found was given a weight score of **15000000** and a material score of **-139.99910017880262**. These scores were first hit in the **17th run**. For the average material per run, a plot is in figure 19 given on page 20. Additionally, the plot for average weight per run is given in figure 20 on page 20. A table with the final Pareto Front values is given in table 6 on page 9. For the yellow deliverable, a diversity metric was used to better choose the optimal Pareto Front. The diversity metric used was the *Crowded-Comparison Operator* from the NSGA-II paper. The plot for this final Pareto Front is given in figure 21 on page 21. The bridges for the final Pareto Front are also included, starting on page 21.

Yellow Statistical Analysis

Green Objective One vs Yellow Objective One

The first objective of the best results from the green experiment is compared to the first objective of the best results for the yellow experiment that used crowding selection for the Pareto Front. A statistical F-test and T-test

MOEA Green Run Number	Best Solution Weight Score	Best Solution Material Score
1	0	-139.99910008670443
2	4000000	-139.99910005127066
3	500000	-139.9991000958547
4	2000000	-139.99910000604757
5	1000000	-139.9991000770155
6	0	-139.9991000070293
7	500000	-139.9991000269347
8	2500000	-139.99910008083097
9	2000000	-139.9991001934784
10	500000	-139.999100023952
11	0	-139.99910016597883
12	2500000	-139.99910000896847
13	5000000	-139.9991002957964
14	1000000	-139.99910015215198
15	3500000	-139.99910010008733
16	0	-139.99910004764988
17	2000000	-139.99910001727392
18	1500000	-139.99910003706856
19	13000000	-139.99910006546781
20	500000	-139.9991000092847
21	16500000	-139.9991000053771
22	1000000	-139.99910001240474
23	1000000	-139.99910000428838
24	0	-139.99910000299906
25	5500000	-139.99910298187777
26	0	-139.9991002303587
27	4500000	-139.99910008671765
28	1500000	-139.99910021494594
29	0	-139.99910000390344
30	500000	-139.99910000369732

Table 3: Green MOEA Best Weight and Material Score Per Run

were used for the objective to compare which algorithm performed the best.

Individual #	Weight Score	Material Score
1	0	-547.6200878113531
2	-90000000.0	-160.00512542173263
3	-14000000.000000002	-236.2907193807521
4	-99000000.0	-139.99911607347244
5	-8999999.999999996	-434.63893145857
6	-53000000.0	-190.44279849297448
7	-7999999.999999996	-465.33708657920454

Table 4: Green MOEA Final Pareto Front Individuals

Since the distribution of the data is not known to be normal and the data size is greater than 29, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 25 on page 22. The chosen value for α was 0.025. The sample size for each sample was 30. The variance for sample 1 was 13,932,471,264,367.8 and the variance for sample 2 was 11,391,091,954,023. The calculated test statistic F was valued at 1.22310234353321. The nearest critical value to F(F Critical one-tail) was 2.10099581728421. According to the test results, the value of F is greater than one but is less than F Critical one-tail. This means that there is no significant differences in the variances of the two data sets.

With the variances of the data sets showing no significant differences, a Two-tailed two-sample t-test assuming equal variances was used next for comparison. The table showing the results of this test can be found in figure 26 on page 22. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was 2,416,666.66666667 and the sample mean for sample 2 was 2,616,666.66666667. The sample variance for sample 1 was 13,932,471,264,367.8 and the sample variance for sample 2 was 11,391,091,954,023. The calculated test statistic t was found to be -0.217684855516355. The upper critical value (t Critical two-tail) was 2.00171748414524. According to the t-test, the t Stat was less than 0 but is greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded and there is no significant difference detected between the two experiments. However, an advantage of using the crowding distance sorting could produce results with less repetitive results and a more unique solution

MOEA Yellow Run Number	Best Solution Weight Score	Best Solution Material Score
1	4000000	-139.99910012955547
2	5000000	-139.99910004029834
3	3000000	-139.99910000372955
4	2500000	-139.99910009829435
5	500000	-139.99910039794594
6	0	-139.99910004604044
7	4000000	-139.99910000674782
8	2500000	-139.99910009620447
9	0	-139.99910004271425
10	0	-139.99910001202792
11	1500000	-139.99910001695446
12	7500000	-139.99910003415042
13	2000000	-139.9991002629337
14	2000000	-139.99910003061328
15	0	-139.99910000394829
16	1000000	-139.9991001013791
17	15000000	-139.99910017880262
18	500000	-139.99910007612806
19	10500000	-139.99910013738707
20	1000000	-139.99910000957985
21	1500000	-139.99910001790943
22	4000000	-139.99910012320106
23	5000000	-139.99910007010385
24	1000000	-139.99910001867474
25	2500000	-139.99910003297376
26	500000	-139.9991000369465
27	0	-139.9991001142511
28	0	-139.9991000454171
29	0	-139.99910004243387
30	1500000	-139.99910004663465

Table 5: Yellow MOEA Best Weight and Material Score Per Run

space.

Individual #	Weight Score	Material Score
1	-99000000.0	-139.99911889939344
2	-95000000.0	-160.20596095449596
3	4000000	-536.8154663083089

Table 6: Yellow MOEA Final Pareto Front Individuals

Green Objective Two vs Yellow Objective Two

The second objective of the best results from the green experiment is compared to the second objective of the best results for the yellow experiment that used crowding selection for the Pareto Front. A statistical F-test and T-test were used for the objective to compare which algorithm performed the best. Since the distribution of the data is not known to be normal and the data size is greater than 29, a two sample F-Test was performed for variances. The table showing the results of this test can be found in figure 27 on page 23. The chosen value for α was 0.025. The sample size for each sample was 30. The variance for sample 1 was 0.000000000000288289780707131 and the variance for sample 2 was 0.000000000000007183652990664280. The calculated test statistic F was valued at 40.1313622862611. The nearest critical value to F(F Critical one-tail) was 2.10099581728421. According to the test results, the value of F is greater than one and is greater than F Critical one-tail. This means that the test has rejected the null hypothesis of equal variances, and determined that the two populations have significantly unequal variances.

With the variances of the data sets shown as being significantly unequal, a Two-tailed two-sample t-test assuming unequal variances was used next for comparison. The table showing the results of this test can be found in figure 28 on page 23. The chosen value for α was 0.05. The sample size for each sample was 30. The sample mean for sample 1 was -139.9991002 and the sample mean for sample 2 was -139.9991001. The sample variance for sample 1 was 2.88289780707131E-13 and the sample variance for sample 2 was 7.18365299066428E-15. The calculated test statistic t was found to be -0.94765403. The upper critical value (t Critical two-tail) was 2.042272456. According to the t-test, the t Stat was less than 0 and is greater than -t Critical two-tail. Therefore, the null hypothesis can be concluded and there

is no significant difference detected between the two experiments. However, an advantage of using the crowding distance sorting could produce results with less repetitive results and a more unique solution space.

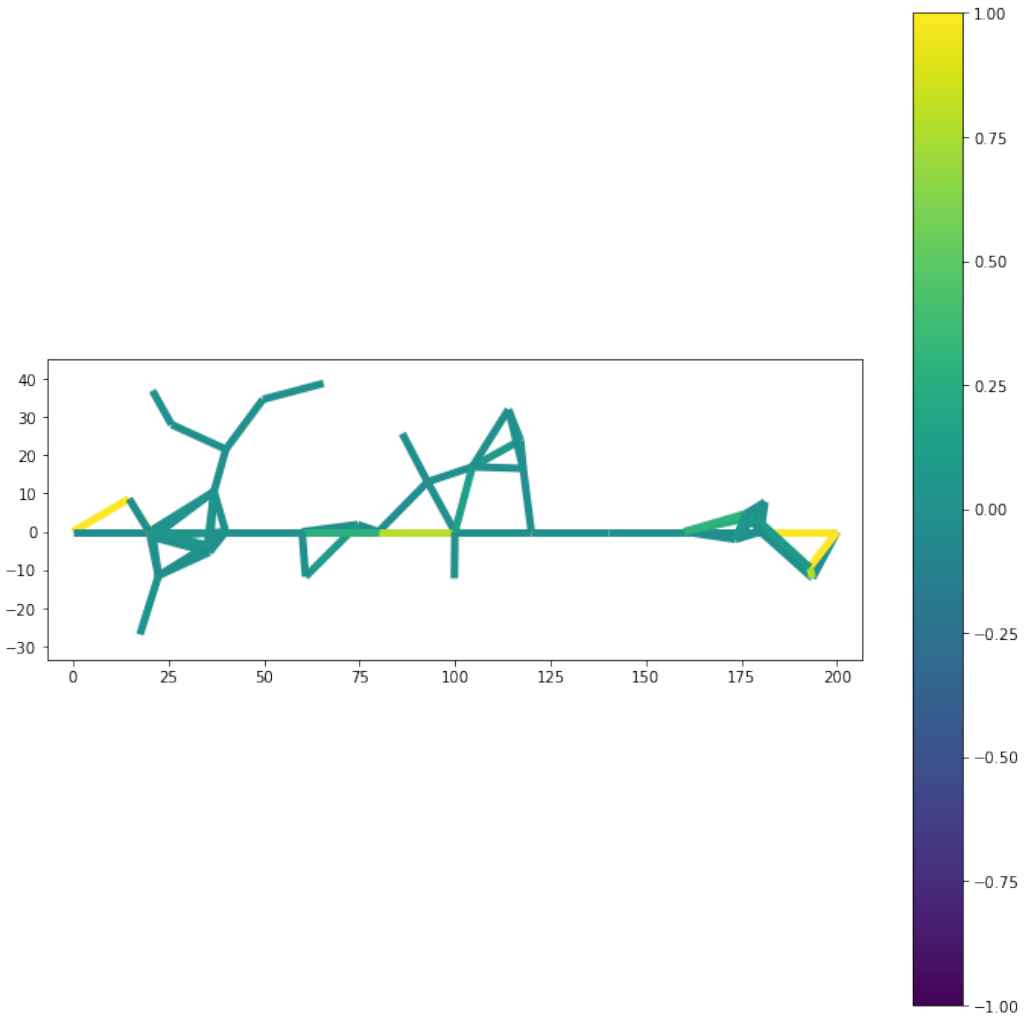

Figure 1: Constraint Ignore Best Fitness Bridge

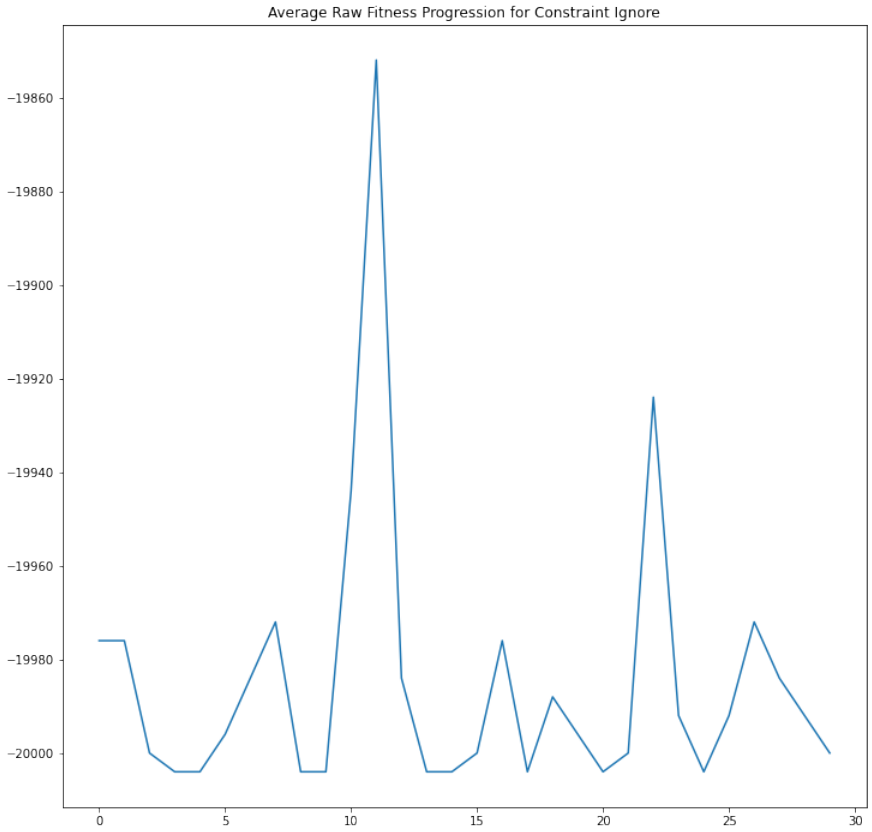

Figure 2: Constraint Ignore Average Raw Fitness Per Run

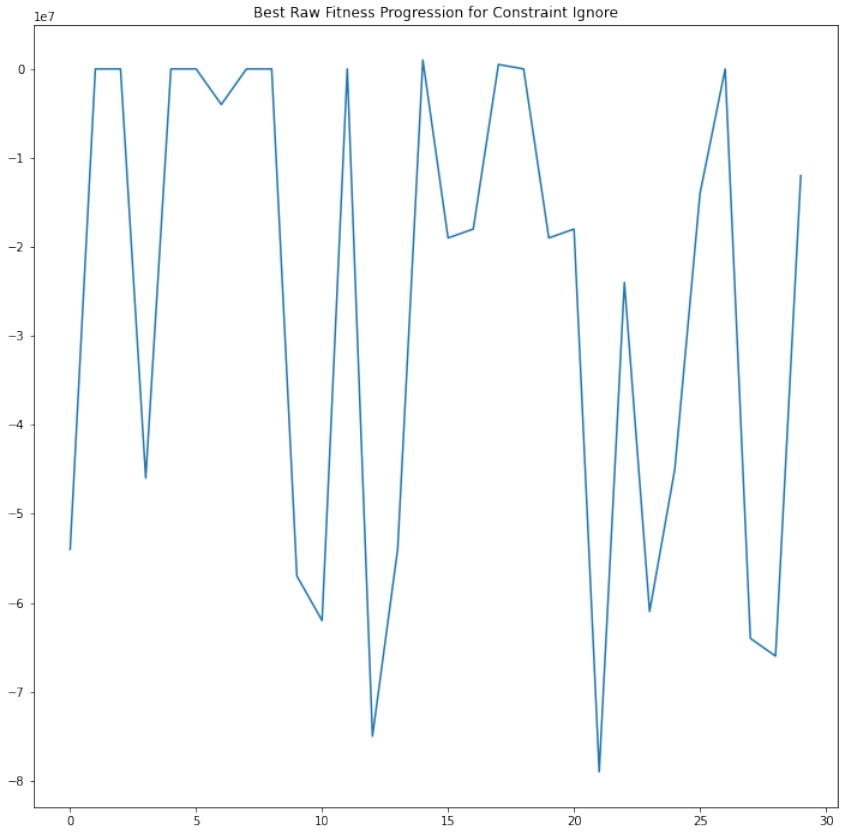

Figure 3: Constraint Ignore Best Raw Fitness Per Run

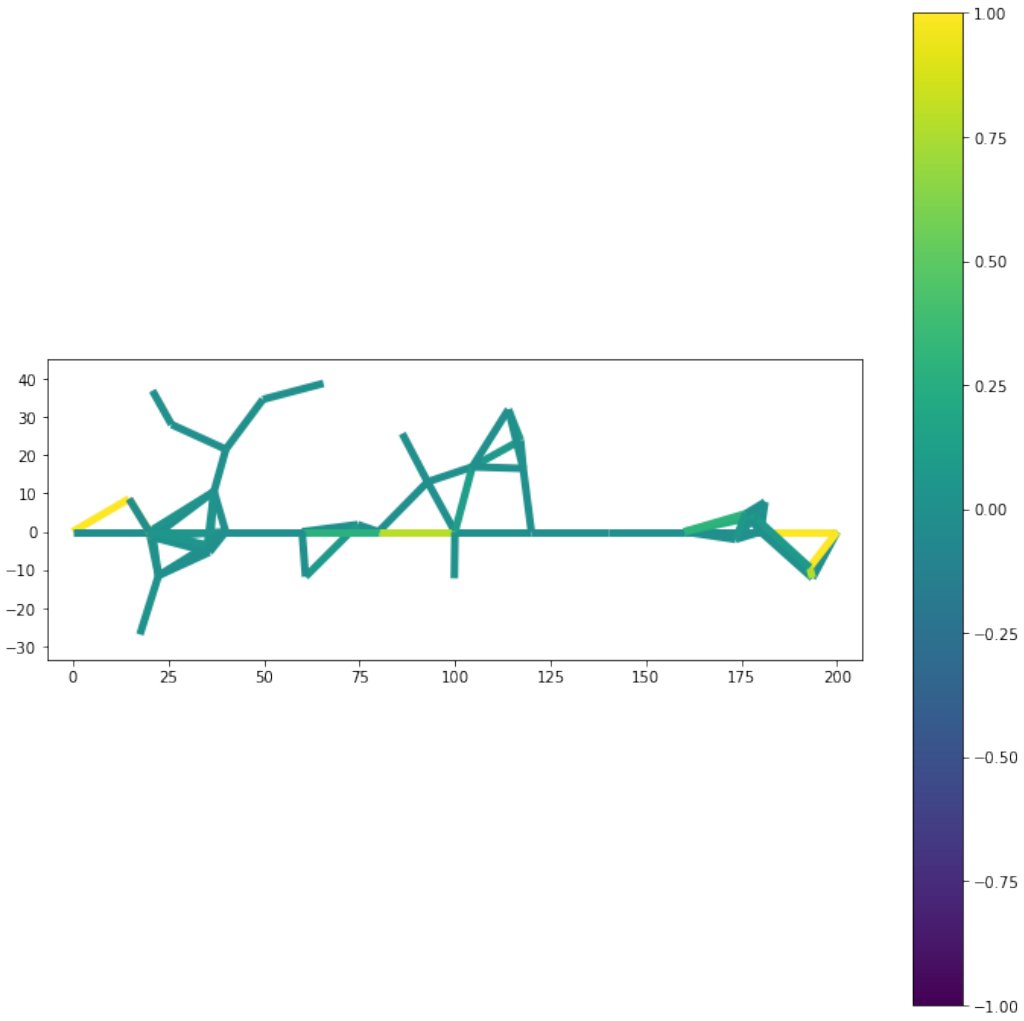

Figure 4: Constraint Penalty Best Fitness Bridge

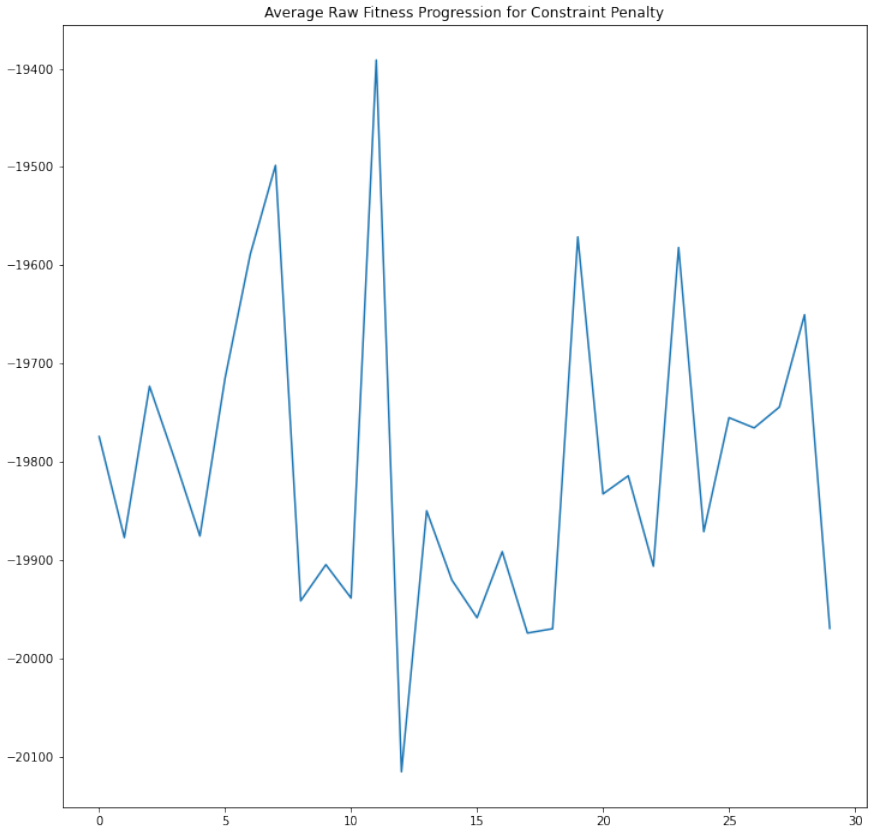

Figure 5: Constraint Penalty Average Raw Fitness Per Run

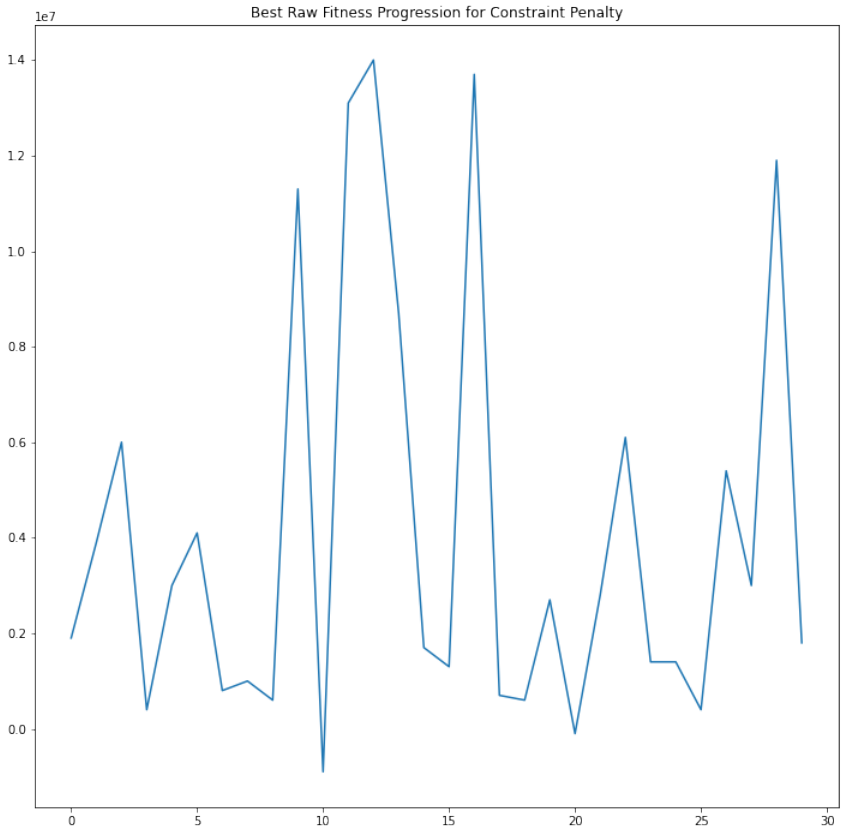

Figure 6: Constraint Penalty Best Raw Fitness Per Run

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	-26316666.7	4090000
Variance	7.80388E+14	2.0183E+13
Observations	30	30
df	29	29
F	38.66559202	
P(F<=f) one-tail	1.90106E-16	
F Critical one-tail	2.100995817	

Figure 7: Constraint Satisfaction F Test

t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	-26316666.7	4090000
Variance	7.80388E+14	2.0183E+13
Observations	30	30
Hypothesized Mean Difference	0	
df	30	
t Stat	-5.88612675	
P(T<=t) one-tail	9.58807E-07	
t Critical one-tail	1.697260887	
P(T<=t) two-tail	1.91761E-06	
t Critical two-tail	2.042272456	

Figure 8: Constraint Satisfaction t Test

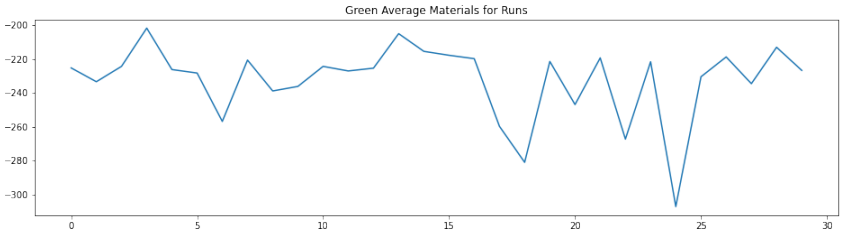

Figure 9: Green MOEA Average Material Score Per Run

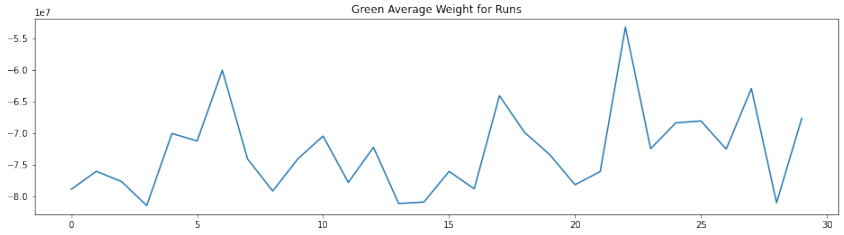

Figure 10: Green MOEA Average Weight Score Per Run

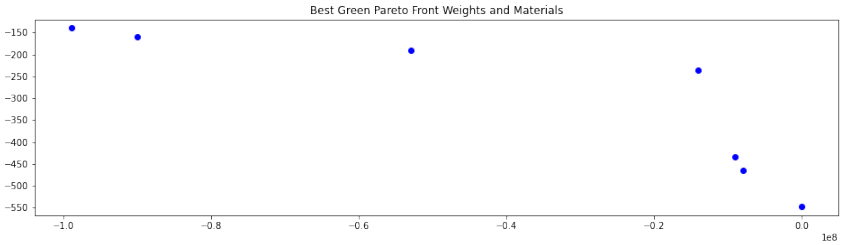

Figure 11: Green MOEA Final Pareto Front

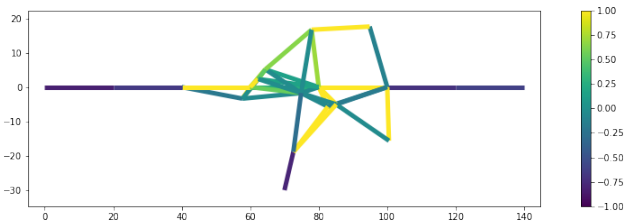

Figure 12: Green MOEA Pareto Bridge 0

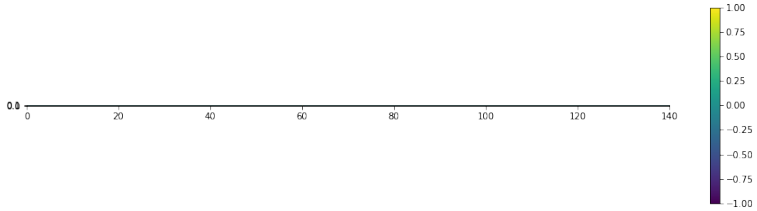

Figure 13: Green MOEA Pareto Bridge 1

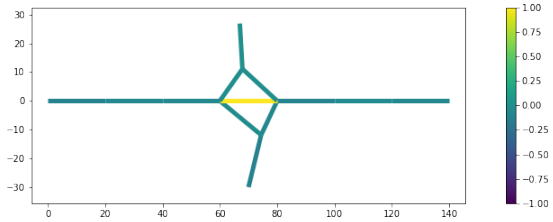

Figure 14: Green MOEA Pareto Bridge 2

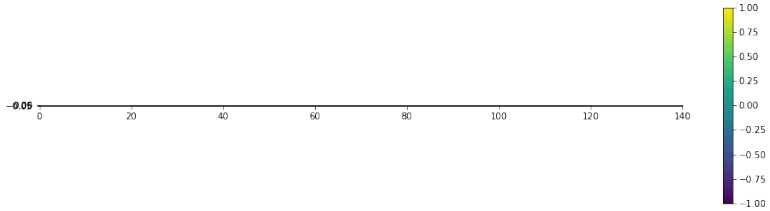

Figure 15: Green MOEA Pareto Bridge 3

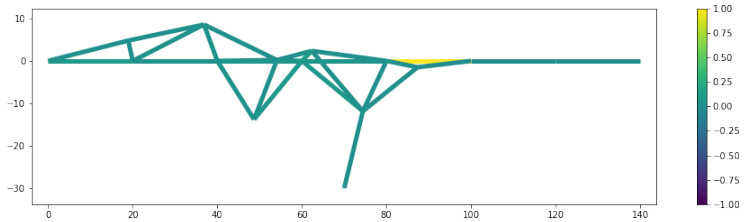

Figure 16: Green MOEA Pareto Bridge 4

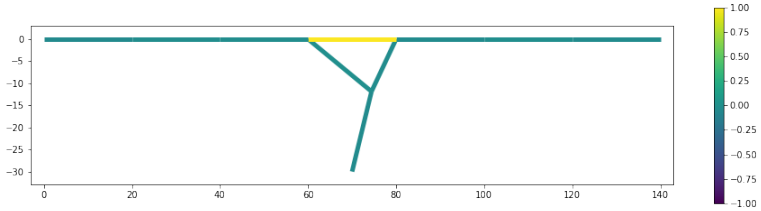

Figure 17: Green MOEA Pareto Bridge 5

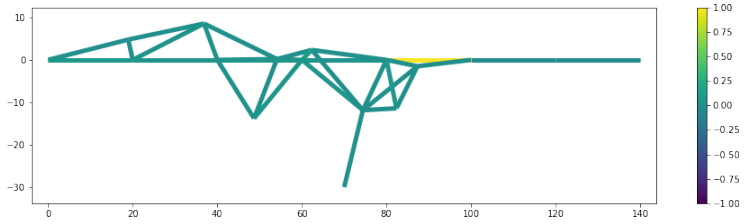

Figure 18: Green MOEA Pareto Bridge 6

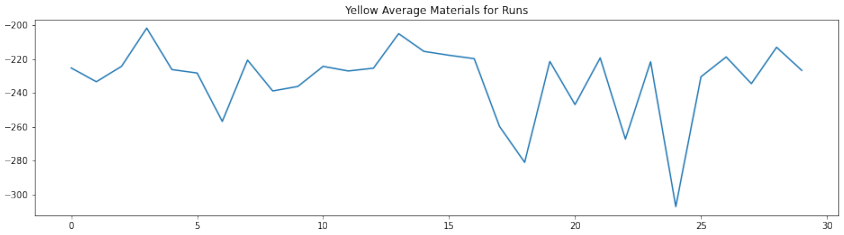

Figure 19: Yellow MOEA Average Material Score Per Run

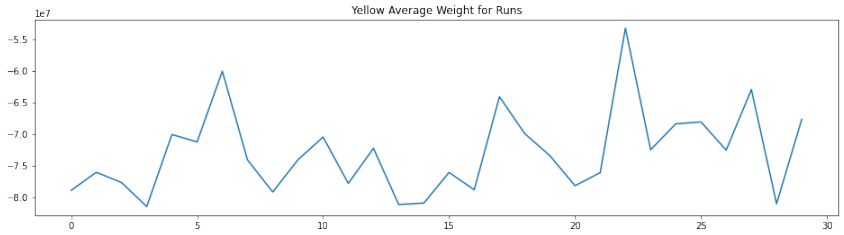

Figure 20: Yellow MOEA Average Weight Score Per Run

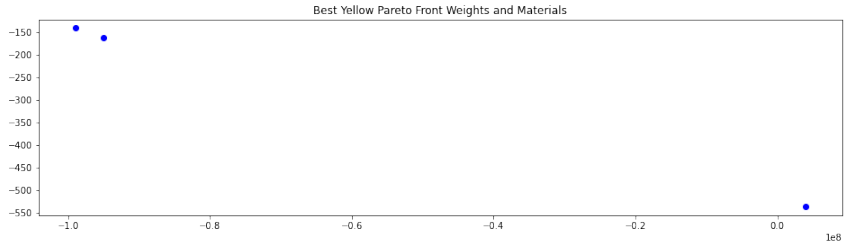

Figure 21: Yellow MOEA Final Pareto Front

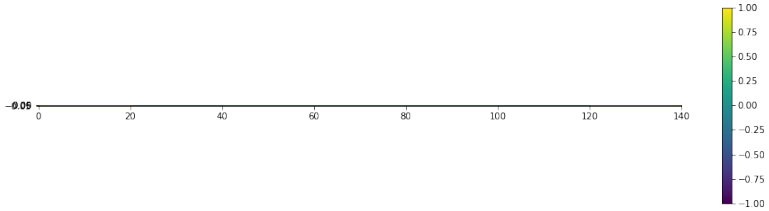

Figure 22: Yellow MOEA Pareto Bridge 0

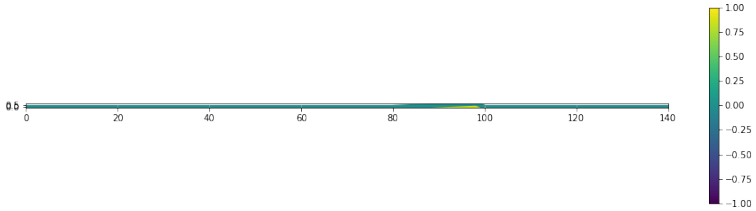

Figure 23: Yellow MOEA Pareto Bridge 1

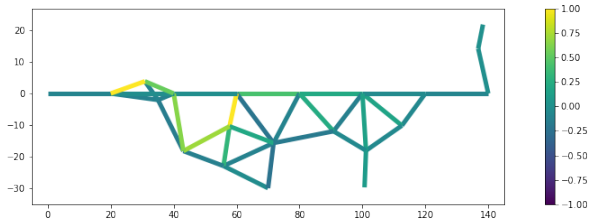

Figure 24: Yellow MOEA Pareto Bridge 2

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	2416666.667	2616666.667
Variance	1.39325E+13	1.13911E+13
Observations	30	30
df	29	29
F	1.223102344	
P(F<=f) one-tail	0.295579341	
F Critical one-tail	2.100995817	

Figure 25: MOEA Objective One F Test

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	2416666.667	2616666.667
Variance	1.39325E+13	1.13911E+13
Observations	30	30
Pooled Variance	1.26618E+13	
Hypothesized Mean Difference	0	
df	58	
t Stat	-0.217684856	
P(T<=t) one-tail	0.414219345	
t Critical one-tail	1.671552762	
P(T<=t) two-tail	0.82843869	
t Critical two-tail	2.001717484	

Figure 26: MOEA Objective One t Test

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	-139.9991002	-139.9991001
Variance	0.000000000000288289780707131	0.000000000000007183652990664280
Observations	30	30
df	29	29
F	40.13136229	
P(F<=f) one-tail	1.13638E-16	
F Critical one-tail	2.100995817	

Figure 27: MOEA Objective Two F Test

t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	-139.9991002	-139.9991001
Variance	2.88E-13	7.18E-15
Observations	30	30
Hypothesized Mean Difference	0	
df	30	
t Stat	-0.94765403	
P(T<=t) one-tail	0.17543858	
t Critical one-tail	1.697260887	
P(T<=t) two-tail	0.350877159	
t Critical two-tail	2.042272456	

Figure 28: MOEA Objective Two t Test