HW6. Stokes second problem

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```
In [ ]: import numpy as np
        import matplotlib.pyplot as plt
        from mpl_toolkits.mplot3d import Axes3D
        from scipy import linalg
        import pandas as pd
        from scipy.linalg import solve_banded
```

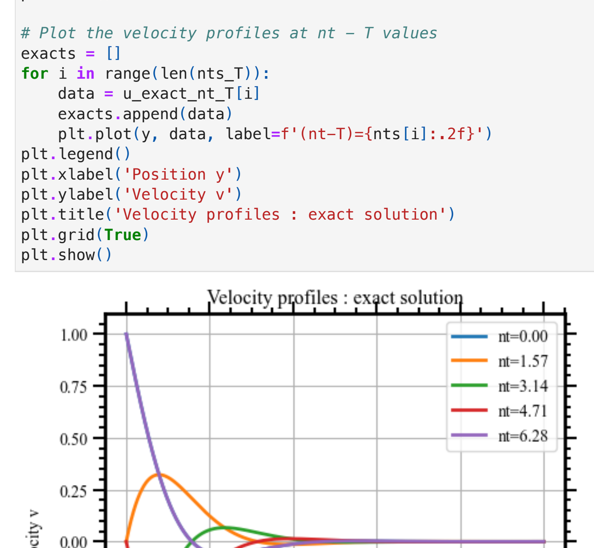
```
In [ ]: # Set plot params
        plt.rcParams['figure.figsize'] = [5,5]
        plt.rcParams['font.size'] = 10
        plt.rcParams['font.family'] = 'Times New Roman'
        plt.rcParams['axes.linewidth'] = 2
        plt.rcParams['lines.linewidth'] = 2
        plt.rcParams['xtick.direction'] = 'out'
        plt.rcParams['ytick.direction'] = 'out'
        plt.rcParams['xtick.minor.visible'] = True
        plt.rcParams['ytick.minor.visible'] = True
        plt.rcParams['xtick.major.size'] = 7
        plt.rcParams['ytick.major.size'] = 7
        plt.rcParams['xtick.minor.size'] = 3.5
        plt.rcParams['ytick.minor.size'] = 3.5
        plt.rcParams['xtick.major.width'] = 1.5
        plt.rcParams['ytick.major.width'] = 1.5
        plt.rcParams['xtick.minor.width'] = 1.5
        plt.rcParams['ytick.minor.width'] = 1.5
        plt.rcParams['xtick.top'] = True
        plt.rcParams['ytick.right'] = True
```

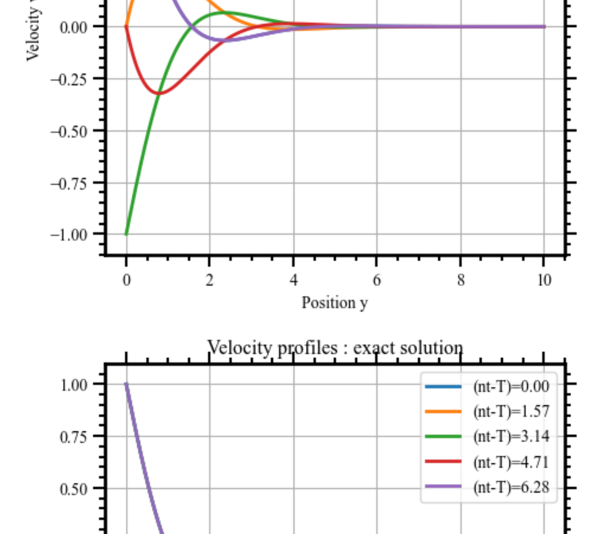
Consider an infinitely extended flat wall carrying out harmonic oscillation in its own plane (see Figure 1). Due to the no-slip condition, the flow velocity at the wall (where $y = 0$) is $u(0,t) = U_0 cos(nt)$.

Check the Exact solution

```
In [ ]: def exact_solution(n, nu, y, t):
            etas = np.sqrt(n/(2*nu)) * y
            return np.cos(n*t - etas) * np.exp(-etas)
```

```
In [ ]: import numpy as np
        import matplotlib.pyplot as plt

        # Given parameters
        nu = 1
        n = 2
        U0 = 1
        L = 10
        T = 10 * np.pi
        dt = 0.001
        dy = 0.01

        # Discretize the spatial domain
        y = np.arange(0, L+dy, dy)

        # Calculate eta
        eta = np.sqrt(n / (2 * nu)) * y

        # Time values for nt and nt - T
        nts = [0, np.pi/2, np.pi, 3*np.pi/2, 2*np.pi]
        nts_T = [value + T for value in nts]

        # Calculate the exact solution at nt values
        u_exact_nt = [U0 * np.exp(-eta) * np.cos(t - eta) for t in nts]
        u_exact_nt_T = [U0 * np.exp(-eta) * np.cos(n*t - eta) for t in nts_T]

        # Plot the velocity profiles at nt values
        for i in range(len(nts)):
            plt.plot(y, u_exact_nt[i], label=f'nt={nts[i]:.2f}')
        plt.legend()
        plt.xlabel('Position y')
        plt.ylabel('Velocity v')
        plt.title('Velocity profiles : exact solution')
        plt.grid(True)
        plt.show()

        # Plot the velocity profiles at nt - T values
        exacts = []
        for i in range(len(nts_T)):
            data = u_exact_nt_T[i]
            exacts.append(data)
            plt.plot(y, data, label=f'(nt-T)={nts[i]:.2f}')
        plt.legend()
        plt.xlabel('Position y')
        plt.ylabel('Velocity v')
        plt.title('Velocity profiles : exact solution')
        plt.grid(True)
        plt.show()
```





2. (Numerical analysis)

Consider two infinitely long plates placed at y = 0 and y = L. The bottom plate (y = 0) is oscillating with u(0,t) = cos(nt), while the top plate (y = L) is stationary. We aim to obtain velocity profiles u(y, t) between two plates by solving Eq. 1 under the assumptions ν = 1, n = 2, U0 = 1 and L = 10.

(1) Solve Eq.(1) numerically using the first-order forward difference in time and second-order central difference in space (FTCS scheme). Plot the velocity profiles at nt = 0, π/2, π, 3π/2, 2π. Also plot the quasi-steady state velocity profiles at (nt − T) = 0, π/2, π, 3π/2, 2π. Note that T represents the transient period required to reach the quasi-steady state solutions. You may use T = 10π.

FTCS

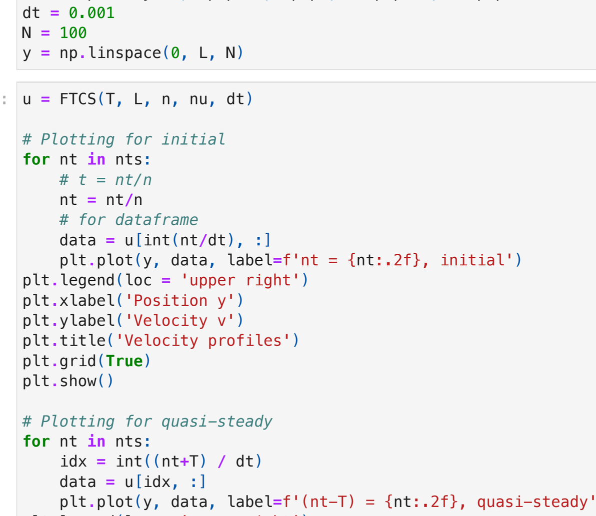
$$\frac{u(i+1,j)-u(i,j)}{\Delta t} = \frac{\nu*(u(i,j+1)-2u(i,j)+u(i,j-1))}{(\Delta y)^2}$$

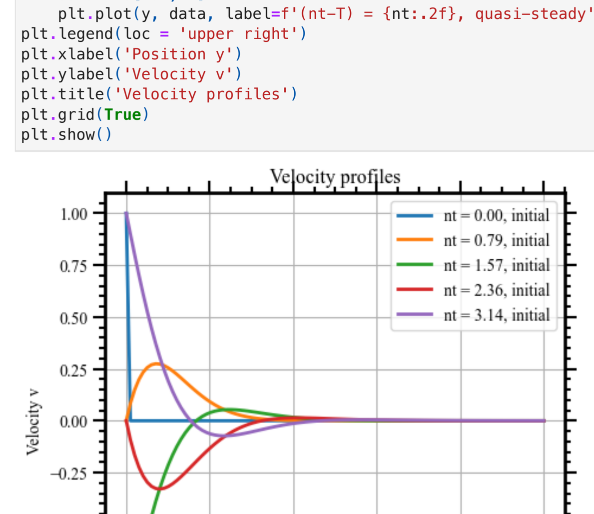
$$u(i+1,j) = u(i,j) + \frac{\nu*\Delta t*(u(i,j+1)-2u(i,j)+u(i,j-1))}{\Delta y^2}$$

```
In [ ]: def FTCS(T, L, n, nu, dt):
            """
            Forward-Time Central-Space (FTCS) scheme for solving a 1D heat equation.

            Parameters:
            - T: Total simulation time
            - L: Length of the domain
            - n: Oscillation frequency
            - nu: Diffusion coefficient
            - dt: Time step size

            Returns:
            - u: Solution array
            """

            N = 100
            M = int(2 * T / dt)
            y = np.linspace(0, L, N)
            t = np.linspace(0, 2 * T, M)
            dy = y[1] - y[0]

            # Initialize u array with boundary conditions
            u = np.zeros((M, N))
            u[:, 0] = np.cos(n * t)
            u[:, -1] = 0

            # FTCS scheme
            for i in range(M - 1):
                for j in range(1, N - 1):
                    u[i + 1, j] = u[i, j] + nu * (dt / dy**2) * (u[i, j + 1] - 2 * u[i, j] + u[i, j - 1])

            return u
```

```
In [ ]: # Given parameters
        nu = 1
        n = 2
        U0 = 1
        L = 10
        T = 10*np.pi
        nts = np.array([0, np.pi/2, np.pi, 3*np.pi/2, 2*np.pi])
        dt = 0.001
        N = 100
        y = np.linspace(0, L, N)
```

```
In [ ]: u = FTCS(T, L, n, nu, dt)

        # Plotting for initial
        for nt in nts:
            # t = nt/n
            nt = nt/n
            # for dataframe
            data = u[int(nt/dt), :]
            plt.plot(y, data, label=f'nt = {nt:.2f}, initial')
        plt.legend(loc = 'upper right')
        plt.xlabel('Position y')
        plt.ylabel('Velocity v')
        plt.title('Velocity profiles')
        plt.grid(True)
        plt.show()

        # Plotting for quasi-steady
        for nt in nts:
            idx = int((nt+T) / dt)
            data = u[idx, :]
            plt.plot(y, data, label=f'(nt-T) = {nt:.2f}, quasi-steady')
        plt.legend(loc = 'upper right')
        plt.xlabel('Position y')
        plt.ylabel('Velocity v')
        plt.title('Velocity profiles')
        plt.grid(True)
        plt.show()
```

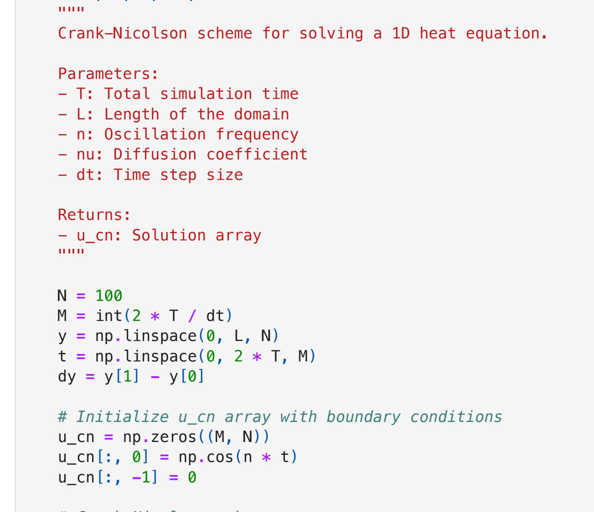


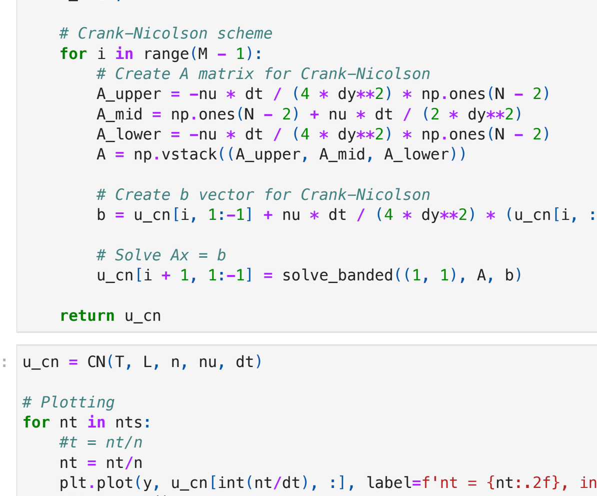


Crank-Nicolson scheme

(3) Repeat (2) using the Crank-Nicolson (C-N) scheme in time.

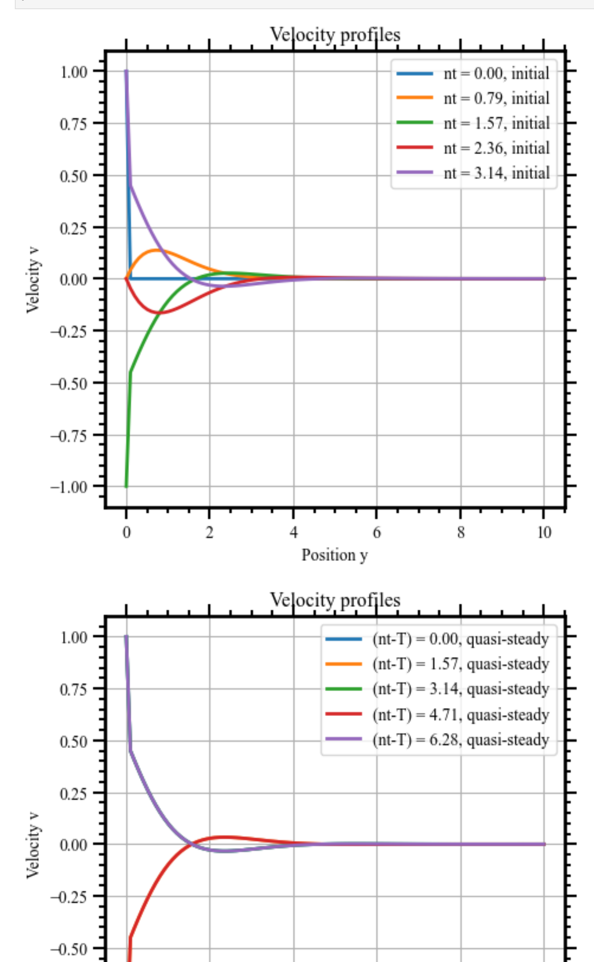
[illegible]

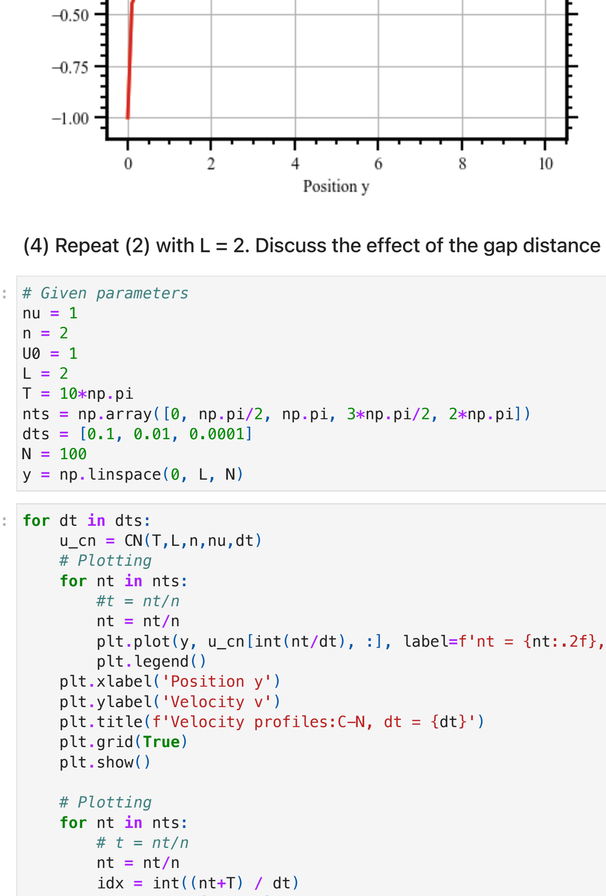


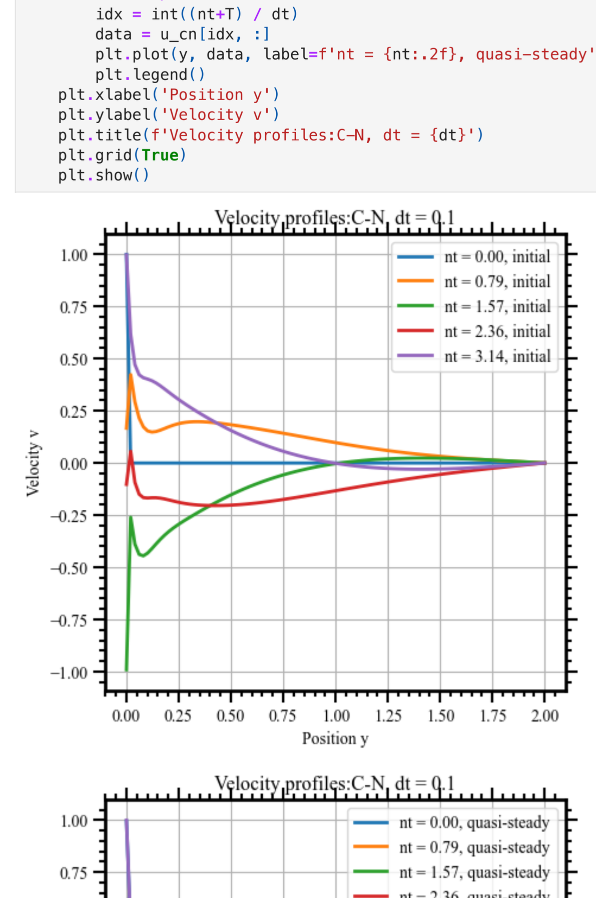


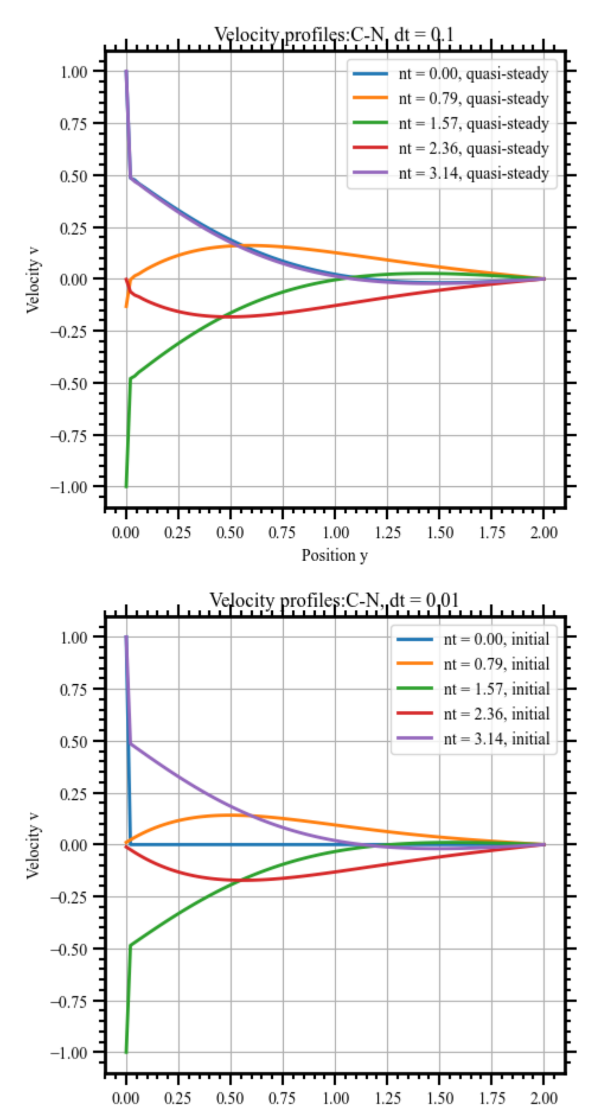
(4) Repeat (2) with L = 2. Discuss the effect of the gap distance between the two plates on the velocity profiles.

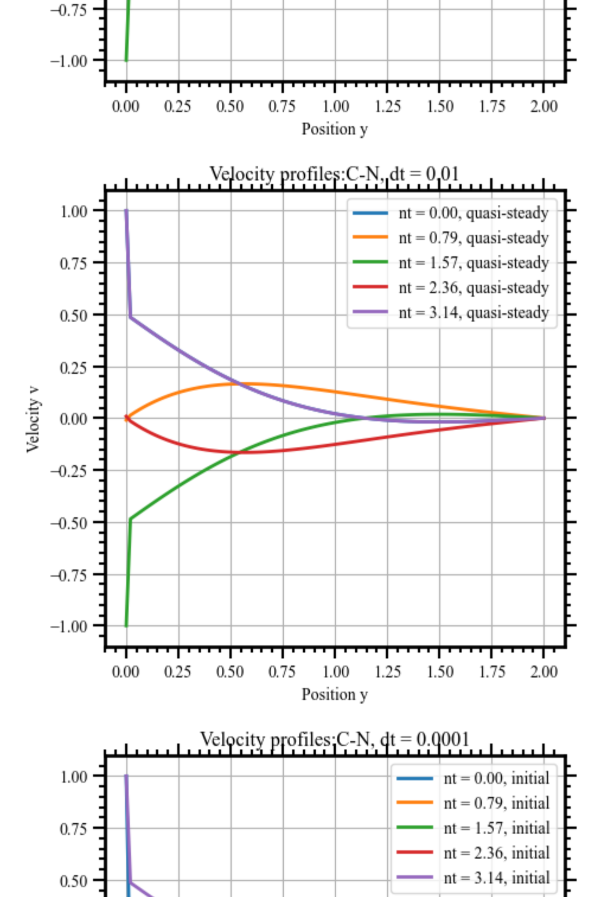
[illegible]

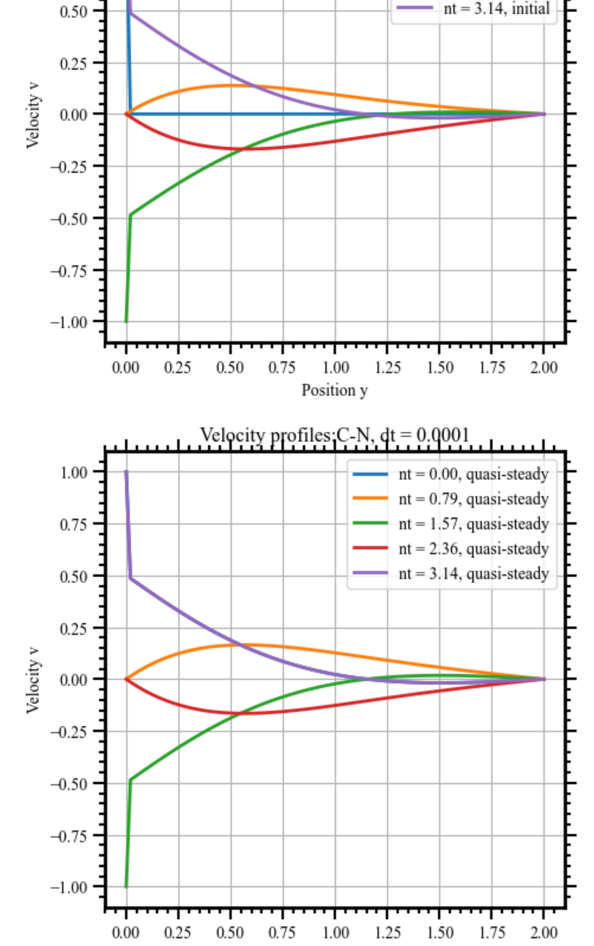












gap distance가 줄어들면 진동이 강해진다..?