CSE5004 Scientific Computation with Python

HW3. Numerical Interpolation

Due date: April 26, 2023

1. Consider the function $f(x, y) = 0.26(x^2 + y^2) - 0.48xy$, where $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.
   (1) Find and plot the Lagrange interpolating polynomial $p(x, y)$ using equally spaced nodes with $h = 0.2$.
   (2) Compare the computed result in Question 1-(1) with the exact function value.

2. Consider the function $f(x, y) = \sin(\pi x)\sin(\pi y)$, where $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.
   (1) Find and plot the Lagrange interpolating polynomial $p(x, y)$ using equally spaced nodes with $h = 0.2$.
   (2) Compare the computed result in Question 2-(1) with the exact function value.
   (3) Do the same work with the Chebyshev nodes,

$$x_i = \cos\left(\frac{2i+1}{2n+2}\pi\right), \quad y_j = \cos\left(\frac{2j+1}{2n+2}\pi\right), \quad i = 0, 1, \ldots, n,\ j = 0, 1, \ldots, m.$$

   (4) Discuss why the Chebyshev nodes are generally better than equally spaced nodes in polynomial interpolation. [Hint: Plot the functions $\prod_{i=0}^{n} |x - x_i|$ for uniform and Chebyshev nodes.]