CSE5004 Scientific Computation with Python

HW2. Root finding

Due date: April 12, 2023

Consider a non-linear equation system, $\mathbf{F}(\mathbf{x}) = \mathbf{b}$,

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} x^2 + xyz + y^2z^3 \\ xy^2 - yz^2 - 2x^2 \\ x^2y + y^2z + z^4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}.$$

1. Using Newton's method, find a root of the system $\mathbf{F}(\mathbf{x}) = \mathbf{b}$ when an initial guess is $(x_0, y_0, z_0) = (1, 2, 3)$ or $(-1, 1, 1)$ (with line-by-line code and using the NumPy library).

2. Explain the behavior of Newton's method when an initial guess is $(x_0, y_0, z_0) = (0, 0, 1)$.

3. Using `broyden2` in the SciPy library, find a root of the system $\mathbf{F}(\mathbf{x}) = \mathbf{b}$ when an initial guess is $(x_0, y_0, z_0) = (1, 2, 3)$ or $(-1, 1, 1)$, and compare the result obtained in Question 1.