CSE5004 Scientific Computation with Python

HW7. Heat equations

Due date: June 7, 2023

Consider the 2D heat equation with a source term in the domain $-1 \leq x \leq 1$ $-1 \leq y \leq 1$:

$$\frac{\partial \phi}{\partial t} = \alpha \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + S(x, y),$$

where α is the thermal conductivity and assumed to be 1. The equation is subject to homogeneous initial and boundary conditions, namely, $\phi(x, y, 0) = 0$, $\phi(\pm 1, y, t) = 0$, and $\phi(x, \pm 1, t) = 0$.

Complete the following tasks:

1. Determine the exact steady-state solution of ϕ when the source term is given by $S(x, y) = 2(2 - x^2 - y^2)$.
2. Employ the Crank-Nicolson method for time stepping and a second-order central difference scheme for the spatial derivative to solve the equation up to steady state on a uniform grid. Afterwards, plot both the exact and numerical steady-state solutions, considering parameters like time step Δt and the number of grid points in the x and y directions, N and M respectively.
3. Based on your numerical findings, provide a discussion about the order of accuracy in both time and space.