Consider the following compartmental model waning natural immunity, death, and a waning vaccine with partial effectiveness against infection and death.

$$
\begin{aligned}
\frac{dS_t}{dt} &= -\frac{\beta S_t(I_t^s + I_t^v)}{N} + \omega R_t - \nu S_t + \sigma V_t \\
\frac{dI_t^s}{dt} &= \frac{\beta S_t(I_t^s + I_t^v)}{N} - \gamma_s I_t^s \\
\frac{dI_t^v}{dt} &= (1-\theta)\frac{\beta V_t(I_t^s + I_t^v)}{N} - \gamma_v I_t^v \\
\frac{dR_t}{dt} &= (1-\mu_s)\gamma I_t^s + (1-\mu_v)\gamma I_t^v - \omega R_t \\
\frac{dV_t}{dt} &= \nu S_t - (1-\theta)\frac{\beta V_t(I_t^s + I_t^v)}{N} - \sigma V_t \\
\frac{dD_t^s}{dt} &= \mu_s \gamma_s I_t^s \\
\frac{dD_t^v}{dt} &= \mu_v \gamma_v I_t^v
\end{aligned}
\tag{1}
$$

where N is population size, S_t is the number of susceptibles, I_t^s is the number of naturally infections, I_t^v is the number of vaccine breakthrough infections, R_t are the recovered (due to infection), V_t are the vaccinated, and D_t^s and D_t^v are deaths among the vaccinated and unvaccinated at time t.

Table 1: Chosen parameter values for compartmental model

Parameter	Value	Description
S_0	10000	initial susceptibles
I_0	1	initial infected
β	0.3	infection rate
γ_s	0.08	recovery rate
γ_v	0.08	recovery rate for breakthroughs
μ_s	0.1	CFR
μ_v	0.005	CFR for breakthroughs
ω	0.005	rate of waning natural immunity
σ	0.01	rate of waning vaccine immunity
ν	varied	vaccination rate
θ	0.5	vaccine efficacy against infection

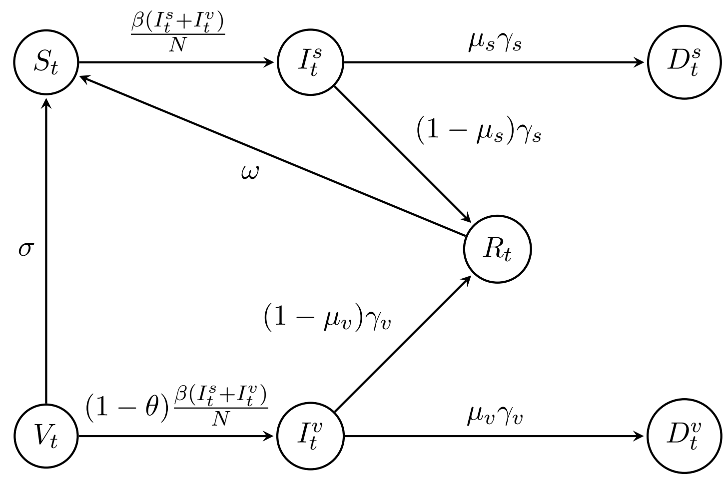

Figure 1: Compartmental model with waning natural immunity, death, and a waning vaccine with partial effectiveness against infection and death.

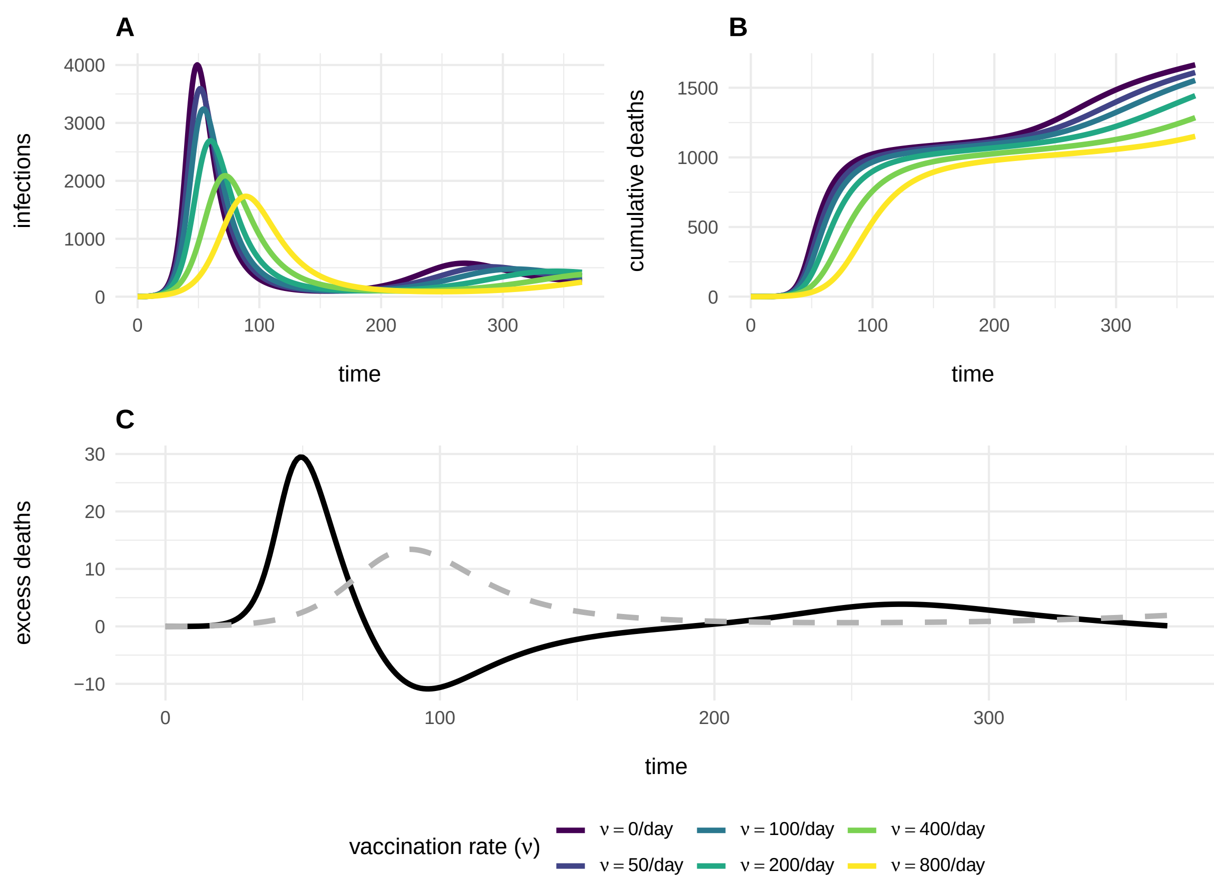

Figure 2: Modeled infections (A), cumulative deaths (B), and excess deaths (C) under different vaccination regimes. In panel C we compare 800/day to 0/day strategies and calculate excess deaths using the naive method (dotted line) and modeled differences (solid line).