

# Dummy Titlepage

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# Chapter 1

## Introduction

### 1.1 Relevant work

#### 1.1.1 Cerezo et al. (2021)

Variation quantum algorithms (VQAs) are envisioned as the most likely candidate for quantum advantage to be achieved. By optimising a set of parameters that describe the quantum circuit, classical optimisation techniques are applicable, and only using the quantum hardware for what can be interpreted as function calls, limits the circuit depths needed. Running the same circuit many times with slightly different parameters and inputs in a classical-quantum-hybrid fashion, rather than a complete quantum implementation, means that the quantum operations can be simple enough for the noise and decoherence to be manageable.

Generally, VQAs start with defining a cost function, depending on some input data (states) and the parametrised circuit, to be minimised with respect to the parameters of the quantum circuit. For example, the cost function for the variational quantum eigensolver (VQE) is the expectation value of some Hamiltonian, which is the energy of a system. The cost function should be meaningful in the sense that the minimum coincides with the optimal solution to the problem, and that lower values generally implies better solutions. Additionally, the cost function should be complicated enough to warrant quantum computation by not being easily calculated on classical hardware, while still having few enough parameters to be efficiently optimised.

The optimisation of the cost function is often done with gradient descent methods. To evaluate the gradient of the quantum circuit w.r.t. the parameters, the very convenient parameter shift rule is often used. Though appearing almost as a finite difference scheme, relying on evaluating the circuit with slightly shifted parameters, it is indeed an exact formula. Furthermore, it may be used recursively to evaluate higher order derivatives, which is useful for optimisation methods that require the Hessian.

VQA's applications are numerous. The archetypical example is finding the ground state of a Hamiltonian for a molecule. Such problems are exponential in the particle count, and thus intractable on classical hardware for larger molecules, while the problem of evaluating the Hamiltonian on quantum hardware is typically polynomial. VQAs are also well suited for general mathematical problems and optimisation, even machine learning, another common example being QAOA for the max-cut problem.

Still, there are many difficulties when applying VQAs. Barren plateaus are a common occurrence, making the optimisation futile. The choosing of the ansatz determines the performance and feasibility of the algorithms, and there are many strategies and options. Some rely on ex-

exploiting the specific quantum hardware’s properties, while some use the specifics of the problem at hand. Finally, the inherent noise and errors on near-term hardware will still be a problem and limit circuit depths.

### 1.1.2 Moll et al. (2018)

The computational performance of quantum computers is decided by five main factors. Naturally, the total qubit count is important, but also their connectivity (if they are not connected, intermediate operations like swapping is needed). How many gates/operations can be used before decoherence, noise and errors ruins the result also determines what programmes are feasible. Furthermore, which physical gates are available also matters, as transpiling to native gates will increase the circuit depth. Lastly, the degree of gate parallelisation can allow for shallower circuits and increased performance.

With all these factors in mind, the metric of quantum volume is defined, giving a single number describing the performance. It is effectively defined as the largest rectangular circuit of two-qubits a quantum computer may execute.

### 1.1.3 Torlai et al. (2020)

Due to the probabilistic nature of quantum computers and their exponentially great number of states, measuring complex observables accurately requires many samples. By post-processing the measurements using an artificial neural network, the variance of the samples are significantly reduced, though at the cost of some increased bias.

### 1.1.4 Schuld et al. (2019)

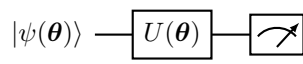
In optimising the parameters of variational circuits, having access to the gradient of the cost function (with respect to the parameters) is beneficial. The individual measurements are probabilistic, but the expectation is a deterministic value whose gradient can be calculated. Often, this is possible exactly using the parameter shift rule, allowing for evaluating the gradient using the same circuit with changed parameters. For circuits containing gates whose derivatives are not as nice, a method of linear combination of unitaries can be used. This method requires an extended circuit including an ancillary qubit.

### 1.1.5 Pesah et al. (2021)

The problem of barren plateaus plagues the optimisation of variational circuits and quantum neural network; for randomly initialised ansätze, the gradient of the cost function may exhibit exponentially small gradients, prohibiting gradient based optimisation. Under certain assumptions, it is shown that for quantum convolutional neural networks, the gradient of the cost function is no worse than polynomially small, such that the networks can be trainable.

## 1.2 Quantikz

Admire fig. 1.1. It is a quantum circuit. It is drawn using the package `quantikz`.



**Figure 1.1:** Test figure.

## Chapter 2

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# References

- [1] MD SAJID ANIS et al. *Qiskit: An Open-source Framework for Quantum Computing*. 2021. DOI: 10.5281/zenodo.2573505.
- [2] M. Cerezo, Andrew Arrasmith, Ryan Babbush, Simon C. Benjamin, Suguru Endo, Keisuke Fujii, Jarrod R. McClean, Kosuke Mitarai, Xiao Yuan, Lukasz Cincio and Patrick J. Coles. ‘Variational quantum algorithms’. In: *Nature Reviews Physics* 3.9 (Aug. 2021), pp. 625–644. DOI: 10.1038/s42254-021-00348-9. URL: <https://doi.org/10.1038/s42254-021-00348-9>.
- [3] Nikolaj Moll, Panagiotis Barkoutsos, Lev S Bishop, Jerry M Chow, Andrew Cross, Daniel J Egger, Stefan Filipp, Andreas Fuhrer, Jay M Gambetta, Marc Ganzhorn, Abhinav Kandala, Antonio Mezzacapo, Peter Müller, Walter Riess, Gian Salis, John Smolin, Ivano Tavernelli and Kristan Temme. ‘Quantum optimization using variational algorithms on near-term quantum devices’. In: *Quantum Science and Technology* 3.3 (June 2018), p. 030503. DOI: 10.1088/2058-9565/aab822. URL: <https://doi.org/10.1088/2058-9565/aab822>.
- [4] Arthur Pesah, M. Cerezo, Samson Wang, Tyler Volkoff, Andrew T. Sornborger and Patrick J. Coles. ‘Absence of Barren Plateaus in Quantum Convolutional Neural Networks’. In: *Physical Review X* 11.4 (Oct. 2021). DOI: 10.1103/physrevx.11.041011. URL: <https://doi.org/10.1103/physrevx.11.041011>.
- [5] Maria Schuld, Ville Bergholm, Christian Gogolin, Josh Izaac and Nathan Killoran. ‘Evaluating analytic gradients on quantum hardware’. In: *Phys. Rev. A* 99 (3 Mar. 2019), p. 032331. DOI: 10.1103/PhysRevA.99.032331. URL: <https://link.aps.org/doi/10.1103/PhysRevA.99.032331>.
- [6] Giacomo Torlai, Guglielmo Mazzola, Giuseppe Carleo and Antonio Mezzacapo. ‘Precise measurement of quantum observables with neural-network estimators’. In: *Phys. Rev. Research* 2 (2 June 2020), p. 022060. DOI: 10.1103/PhysRevResearch.2.022060. URL: <https://link.aps.org/doi/10.1103/PhysRevResearch.2.022060>.