

# COMS W4705: Natural Language Processing (Fall 2018)

## Problem Set #4

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### Problem 1

Using the raw co-occurrence counts:

- Which word is the most similar to 'animal' using euclidean distance?  
'dog'.

$$\text{dis}(\text{dog}, \text{animal}) = \sqrt{(0-2)^2 + (4-3)^2 + (0-0)^2 + (4-3)^2 + (2-0)^2 + (2-3)^2} = \sqrt{11}$$

$$\text{dis}(\text{cat}, \text{animal}) = \sqrt{(4-2)^2 + (0-3)^2 + (0-0)^2 + (3-3)^2 + (3-0)^2 + (10-3)^2} = \sqrt{71}$$

$$\text{dis}(\text{computer}, \text{animal}) = \sqrt{(0-2)^2 + (0-3)^2 + (0-0)^2 + (5-3)^2 + (0-0)^2 + (5-3)^2} = \sqrt{21}$$

$$\text{dis}(\text{run}, \text{animal}) = \sqrt{(4-2)^2 + (3-3)^2 + (5-0)^2 + (0-3)^2 + (3-0)^2 + (4-3)^2} = 4\sqrt{3}$$

$$\text{dis}(\text{mouse}, \text{animal}) = \sqrt{(2-2)^2 + (10-3)^2 + (5-0)^2 + (4-3)^2 + (3-0)^2 + (0-3)^2} = \sqrt{93}$$

- Which word is the most similar to 'animal' using cosine similarity?

$$\begin{aligned}\text{dis}(\text{dog}, \text{animal}) &= \frac{\vec{v}_{\text{dog}} \cdot \vec{v}_{\text{animal}}}{\|\vec{v}_{\text{dog}}\| \|\vec{v}_{\text{animal}}\|} \\ &= \frac{0 * 2 + 4 * 3 + 0 * 0 + 4 * 3 + 2 * 0 + 2 * 3}{\sqrt{0^2 + 4^2 + 0^2 + 4^2 + 2^2 + 2^2} \sqrt{2^2 + 3^2 + 0^2 + 3^2 + 0^2 + 3^2}} \\ &= 0.851942751369\end{aligned}$$

$$\begin{aligned}\text{dis}(\text{cat}, \text{animal}) &= \frac{\vec{v}_{\text{cat}} \cdot \vec{v}_{\text{animal}}}{\|\vec{v}_{\text{cat}}\| \|\vec{v}_{\text{animal}}\|} \\ &= \frac{4 * 2 + 0 * 3 + 0 * 0 + 3 * 3 + 3 * 0 + 10 * 3}{\sqrt{4^2 + 0^2 + 0^2 + 3^2 + 3^2 + 10^2} \sqrt{2^2 + 3^2 + 0^2 + 3^2 + 0^2 + 3^2}} \\ &= 0.729230142593\end{aligned}$$

$$\begin{aligned}
\text{dis}(\text{computer}, \text{animal}) &= \frac{\vec{v}_{\text{computer}} \cdot \vec{v}_{\text{animal}}}{\|\vec{v}_{\text{computer}}\| \|\vec{v}_{\text{animal}}\|} \\
&= \frac{0 * 2 + 0 * 3 + 0 * 0 + 5 * 3 + 0 * 0 + 5 * 3}{\sqrt{0^2 + 0^2 + 0^2 + 5^2 + 0^2 + 5^2} \sqrt{2^2 + 3^2 + 0^2 + 3^2 + 0^2 + 3^2}} \\
&= 0.762000762
\end{aligned}$$

$$\begin{aligned}
\text{dis}(\text{run}, \text{animal}) &= \frac{\vec{v}_{\text{run}} \cdot \vec{v}_{\text{animal}}}{\|\vec{v}_{\text{run}}\| \|\vec{v}_{\text{animal}}\|} \\
&= \frac{4 * 2 + 3 * 3 + 5 * 0 + 0 * 3 + 3 * 0 + 4 * 3}{\sqrt{4^2 + 3^2 + 5^2 + 0^2 + 3^2 + 4^2} \sqrt{2^2 + 3^2 + 0^2 + 3^2 + 0^2 + 3^2}} \\
&= 0.601431982944
\end{aligned}$$

$$\begin{aligned}
\text{dis}(\text{mouse}, \text{animal}) &= \frac{\vec{v}_{\text{mouse}} \cdot \vec{v}_{\text{animal}}}{\|\vec{v}_{\text{mouse}}\| \|\vec{v}_{\text{animal}}\|} \\
&= \frac{2 * 2 + 10 * 3 + 5 * 0 + 4 * 3 + 3 * 0 + 0 * 3}{\sqrt{2^2 + 10^2 + 5^2 + 4^2 + 3^2 + 0^2} \sqrt{2^2 + 3^2 + 0^2 + 3^2 + 0^2 + 3^2}} \\
&= 0.665758353436
\end{aligned}$$

## Problem 2

### Problem 3

a)

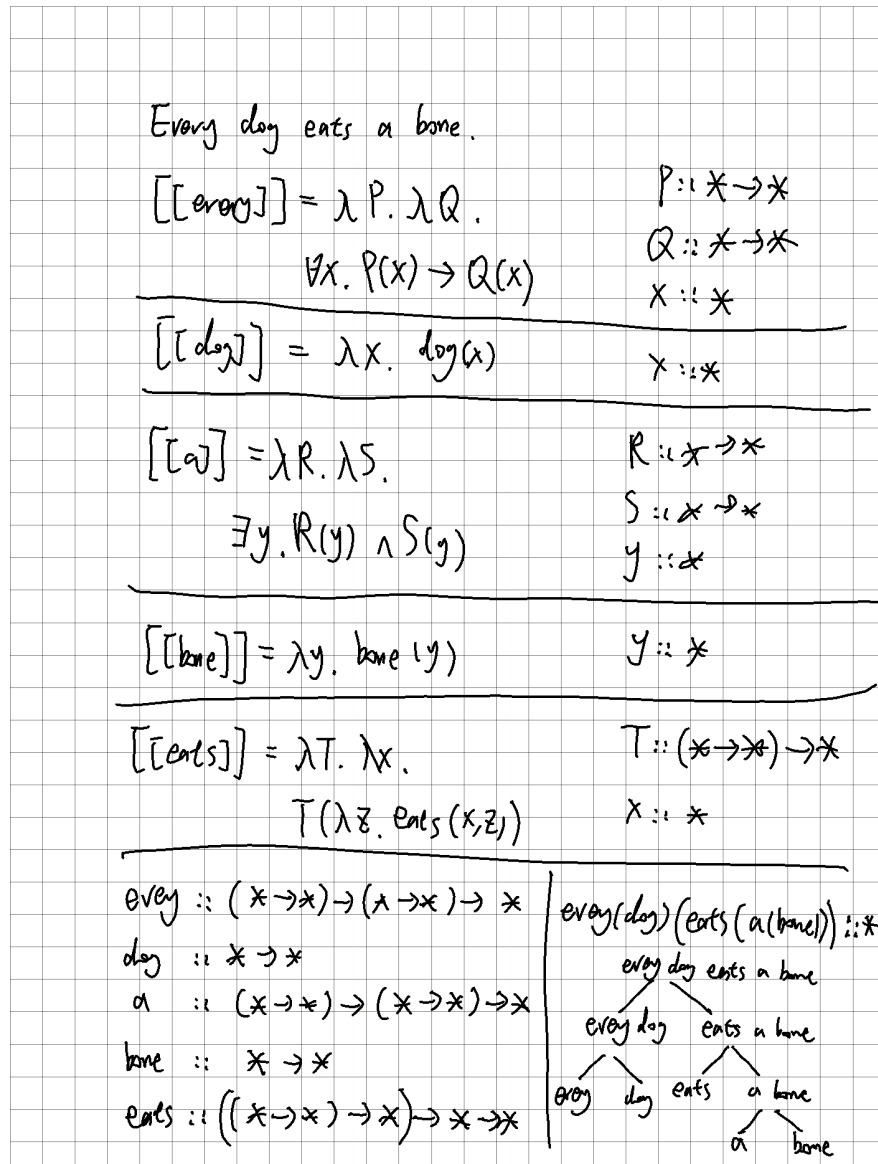


Figure 1: problem 3 (a)

b)

"each dog eats a distinct bone":  
 $\forall x. \text{dog}(x) \rightarrow (\exists y. \text{bone}(y) \wedge \text{eats}(x, y))$

"there exists a bone, that every dog eats"  
 $\exists y. \forall x. \text{dog}(x) \rightarrow (\text{bone}(y) \wedge \text{eats}(x, y))$

Figure 2: problem 3 (b)