

1.37 draw a machine for  $n=3$

1.48 try to think of a short description of the language

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Complexity Theory

Computability Theory     algorithmic

Turing/Church/Gödel

Automata Theory     mathematical model for computation.

Set

	Relation		equivalence	→	① Reflexive
	Function				② Symmetric
					③ Transitive

Function → no repeated first element.

Russell's paradox

alphabet any nonempty finite set

string finite sequence of elements from the alphabet

language any set of strings over an alphabet

Substring, empty string ( $\epsilon$ ), concatenation  
string with length of 0.  $u \circ v = uv$

# Ch1 Regular languages

Def A finite automaton (FA, FSA, DFSA) is a 5-tuple  $(Q, \Sigma, \delta, q_0, F)$

1)  $Q$  is a finite set ( $\Omega$  states)

2)  $\Sigma$  is a finite set (alphabet)

3)  $\delta : Q \times \Sigma \rightarrow Q$  is a function (transition)

4)  $q_0 \in Q$  (initial state)

5)  $F \subseteq Q$  (the set of accepting/final states)

store Deterministic  
↙ ↘  
no missing arrow  
extra

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Def If  $M$  is a FA, then  $L(M)$ , the language of  $M$ , is the set of all strings accepted by  $M$ .

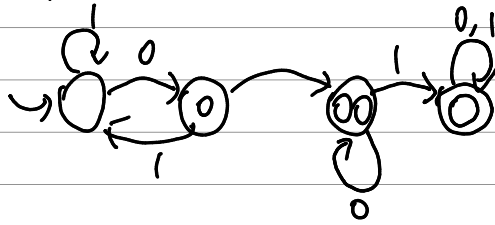
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Def A string  $w = w_1 w_2 \dots w_n$  is accepted by a FA,  $M$ , if there exists a sequence of states  $r_0, r_1, \dots, r_n$  such that 1)  $r_0 = q_0$  2)  $r_n \in F$  3)  $\delta(r_i, w_{i+1}) = r_{i+1}$  for  $i = 0, \dots, n-1$

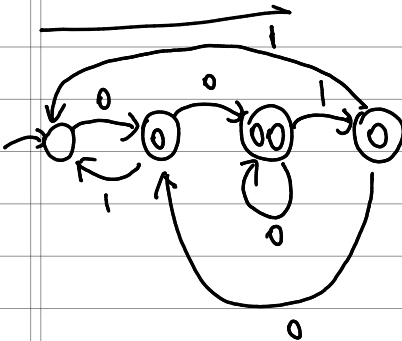
Odd # of 1



Contains subset of 001

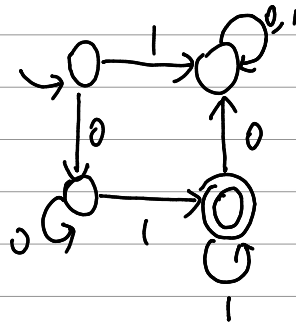


End in 001



1) Contains at least one 0 and one 1

2) all 0's come before all 1's



Contains more 0's than 1's

not regular  $\Rightarrow$  impossible to achieve  
in FA

## Regular Operations (on languages)

Union:  $A \cup B = \{x \mid x \in A \vee x \in B\}$

Concatenation:  $A \circ B = \{w \mid w = xy \text{ where } x \in A \wedge y \in B\}$

Star:  $A^* = \{w \mid w = x_1 x_2 \dots x_n \text{ for some } n \geq 0, \text{ } x_i \in A \text{ for all } i=0,1,\dots,n\}$   
(Kleene star)

Null string  $\epsilon \in A^*$ , always.

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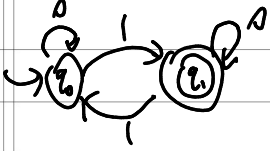
## Closure Properties

A set  $S$  is closed under an operation (op) if  
 $(s \text{ op } t) \in S$  for all  $s, t \in S$ .

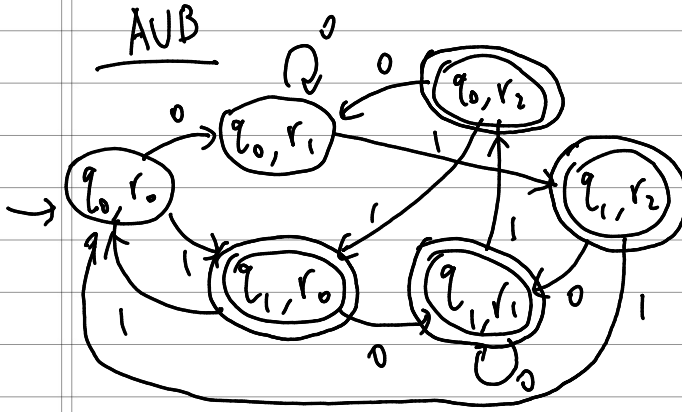
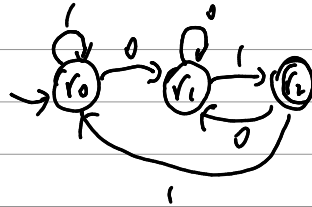
Theorem If  $A$  and  $B$  are regular languages,  
then so is  $A \cup B$ .

(regular languages are closed under union (U).)

A = odd # of 1's



B = ends in 01



$$A = \{Q, \Sigma, q_0, \delta_A, F_A\}$$

$$B = \{R, \Sigma, r_0, \delta_B, F_B\}$$

if  $\delta = F_A \times F_B$ ,  
then under  
the product

$$A \cup B = \{Q \times R, \Sigma, (q_0, r_0), \delta, \overbrace{F_A \times R \cup Q \times F_B}^{\text{if } \delta = F_A \times F_B, \text{ then under the product}}\}$$

$$\delta :: (Q \times R) \times \Sigma \rightarrow (Q \times R)$$

$$\delta((q, r), s) = (\delta_A(q, s), \delta_B(r, s))$$