Universal TM, U

$$U(M; w) = M(w)$$

$$M = (Q, \Sigma, \delta, s)$$

- $\Sigma = \{1, 2, \ldots, |\Sigma|\}$
- $Q = \{|\Sigma|+1, \ldots, |\Sigma| + |Q|\}$
- $s = |\Sigma| + 1$
- $\{\leftarrow, \rightarrow, -, h, yes, no\} = \{|\Sigma|+|Q|+1, \ldots, |\Sigma|+|Q|+6\}$
- $\delta :: Q \times \Gamma \to Q \times \Gamma \times \{\leftarrow | \rightarrow\}$

$$\cong \{((q, \delta), (p, \sigma, D)) \mid \forall q, \delta, p, \sigma, D.\ \delta(q, \delta) = (p, \delta, D)\}$$

$$L = \{\langle\langle M\rangle, w\rangle \mid \forall M \in \mathbb{M}, w \in \mathbb{L}^{\Sigma^*}, M(w) = 1\}$$

Assume $H(w) = \begin{cases} 1 ; & \text{if } w \in L \\ 0 ; & \text{otherwise} \end{cases}$ $\left(\begin{array}{c} H :: \Sigma^* \to \{0,1\} \\ \cong \mathbb{L} \end{array} \right)$

Construct $D(w) = \begin{cases} 1 ; & \text{if } w \notin \{\langle M\rangle \mid \forall M \in \underbrace{\mathbb{M}}_{\text{recursively enumerable}}\} \\ \neg H(\langle w, w\rangle) ; & \text{otherwise} \end{cases}$

① $\underline{D \in \mathbb{M}}$? ② $\langle D, \langle D\rangle\rangle \in L$?

For ①,

Assume $D \in \mathbb{M}$

For ②,

Assume $\langle D, \langle D \rangle \rangle \in L \Rightarrow D(\langle D \rangle) = 1$

but $D(\langle D \rangle) = \neg \underbrace{H(\langle D, \langle D \rangle \rangle)}_{=1}$

$= 0$

Contradiction

Assume $\underbrace{\langle D, \langle D \rangle \rangle \notin L} \Rightarrow D(\langle D \rangle) \neq 1$

$(D :: \mathbb{L} \to \{0, 1\}) \Rightarrow D(\langle D \rangle) = 0$

but $D(\langle D \rangle) = \neg \underbrace{H(\langle D, \langle D \rangle \rangle)}_{=0}$

$= 1$

Contradiction

~~Assume $D \notin \mathbb{M}$~~

by behavior as a binary classifier over $\mathbb{L}$