COMS 4236: Computational Complexity (Fall 2018) Problem Set #2

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Problem 4

Problem. Problem 8.15 on page 303 of TC (5 points if you can show the following problem is in PSPACE; 10 points if you can show if it is in P!): The cat-and-mouse game is played by two players, "CAT" and "Mouse", on an arbitrary undirected graph. At a given point each player occupies a node of the graph. The players take turns moving to a node adjacent to the one that they currently occupy. A special node of the graph is called "Hole". Terminal conditions of the game:

- **Cat wins** if the two players ever occupy the same node.

- **Mouse wins** if it reaches the Hole before the preceding happens.

- The game is a **draw** if the two players ever simultaneously reach positions that they previously occupied.

Let

HAPPY-CAT = $\{(G, c, m, h) \mid$ G,c,m,h are respectively a graph, and
positions of the Cat, Mouse, and Hole, such that
Cat has a wining strategy, if Cat moves first.$\}$

Show that HAPPY-CAT is in PSPACE (and in P for more points).

Solution. *HAPPY-CAT is in P.*
Given undirected graph $G(V, E)$, where $|V| = n$.
The state space of the game has $2n^2$ vertices:

$$\text{State} =$$
$$\{ \quad \text{Cat\_position} :: V$$
$$, \quad \text{Mouse\_position} :: V$$
$$, \quad \text{turn\_player} :: \{\text{Cat}, \text{Mouse}\}$$
$$\}$$

Annotate the states by Cat's win condition:

$$\forall V_0 \in V, (\{V_0, V_0, \text{Cat}\}, \text{Cat\_Win}) \text{ and } (\{V_0, V_0, \text{Mouse}\}, \text{Cat\_Win})$$

HAPPY-CAT then can be formalized as a reachability problem:
Given Cat's initial position $V_{Cat\_init} \in V$, Mouse's initial position $V_{Mouse\_init} \in V$, and initial turn player Cat, whether there's a state annotated by Cat_Win can be reached.   □