The more challenging problems are marked with $*$. Be concise when describing a Turing machine. It is like writing pseudocodes. It suffices to present the most important ideas behind your Turing machines. You do not need to give all the details, e.g., the set of states and the transition function. Check the course website for more info about homeworks. CC: Computational Complexity. TC: Introduction to the Theory of Computation. MA: Computational Complexity: A Modern Approach.

1. Exercises 7.7 on page 271 and Problem 7.14 on page 272 of TC (6 points): Show that NP is closed under union, concatenation, and star operation. (Xi: You may want to use the alternative view of NP as languages decided by polynomial-time verifiers. Can you also show that P is closed under the star operation?) Exercise 7.11 on page 272 of TC (4 points): Call graphs G and H isomorphic if the nodes of G may be reordered so that it is identical to H. Let

$$\text{ISO} = \big\{(G, H) \mid G \text{ and } H \text{ are isomorphic graphs}\big\}.$$

Show that ISO $\in$ NP.

2. Problem 8.20 on page 304 of TC (10 points): An undirected graph is bipartite if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it doesn't contain a cycle that has an odd number of nodes. Let

$$\textsc{Bipartite} = \big\{G \mid G \text{ is a bipartite graph}\big\}.$$

Show that $\textsc{Bipartite} \in$ NL. (Xi: What is the more natural complexity class to which $\textsc{Bipartite}$ belongs? as suggested by the fact mentioned, and what do we know about this class?)

3. (10 points) Show that P $\neq$ SPACE(n). (Hint: Assume P = SPACE(n). Use the space hierarchy theorem to derive a contradiction. You may find the function pad, defined in Problem 9.18, to be helpful.)

4.* Problem 8.15 on page 303 of TC (5 points if you can show the following problem is in PSPACE; 10 points if you can show it is in P!): The cat-and-mouse game is played by two players, "Cat" and "Mouse," on an arbitrary undirected graph. At a given point each player occupies a node of the graph. The players take turns moving to a node adjacent to the one that they currently occupy. A special node of the graph is called "Hole." Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is a draw if the two players ever simultaneously reach positions that they previously occupied. Let

$$\begin{aligned}\textsc{Happy-Cat} = \Big\{(G, c, m, h) \mid\ & G, c, m, h \text{ are respectively a graph, and} \\ & \text{positions of the Cat, Mouse, and Hole, such that} \\ & \text{Cat has a winning strategy, if Cat moves first.}\Big\}\end{aligned}$$

Show that $\textsc{Happy-Cat}$ is in PSPACE (and in P for more points).