prove L is NP-complete

1. $L \in NP$
2. $L \in NP\text{-Hard}$

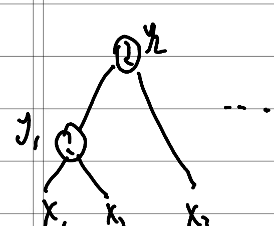
- $\forall A \in NP$, $A \leq_p L$. $\quad$ ($\exists A \in \mathbb{L}$, $A \in NP\text{-complete}$, $A \leq_p L$)

poly-time reduction

Circuit-SAT

1. Poly-time function to generate certificates (assignment of n variables)
2. verifier ~ Boolean Circuit.

Circuit-SAT $\leq_p$ 3-SAT (n variables, m clauses)



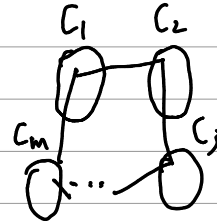
$(x_i, y_j, x_k) \leftarrow x_i \,?\, x_k$

$y_i = \neg x_i \Rightarrow (y_i \vee x_i) \wedge (\overline{y_i} \vee \overline{x_i})$

$y_i = x_i \wedge x_j \Rightarrow (\overline{y_i} \vee x_i) \wedge (\overline{y_i} \vee x_j) \wedge (y_i \vee \overline{x_i} \vee \overline{x_j})$

$(y_i \vee x_i) \xrightarrow[z]{} (y_i \vee x_i \vee z) \wedge (y_i \vee x_i \vee \overline{z})$

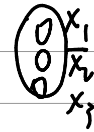
3-SAT $\le_p$ CLIQUE



(n variables
m clauses)

(G, m) named
w/ constraint on vertices

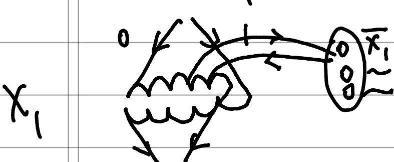
- no edge between opposite literals (opposite assignment of the same variable)
- no intraconnection within a clauses



CLIQUE $\le_p$ Independent Set

flip the connectivity of G.

3-SAT $\le_p$ Hamiltonian Path (directed)



3-SAT $\le_p$ NAE 3-SAT

(global)
false

$$(x_i \lor x_j \lor x_k) \to (x_i \lor x_j \lor c_i) \land (x_k \lor \overline{c_i} \lor Z)$$

Satisfiable

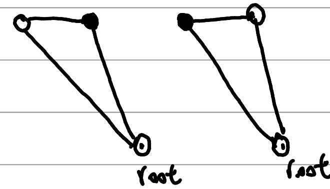
- $\Rightarrow$ Let Z = false
- $\Leftarrow$ if Z = true, flip all assignments
  if Z = false, stay the same.

NAE 3-SAT $\leq_p$ 3-Coloring

assignment

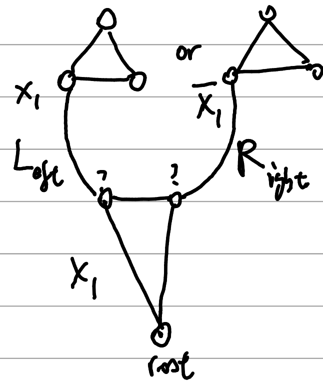
x_1

(=0) (=1)



clause satisfiability

copy color from



why

$x_1 = 0 \Rightarrow x_1 = 0$, $\bar{x}_1 = 1$ | $x_1 = 1 \Rightarrow x_1 = 1$, $\bar{x}_1 = 0$

3-SAT $\leq_p$ Subset-Sum

n variables, m clauses.

base-10 encoding,
if base-2, widen the gaps between meaningful bits to make sure no carry bits.

assignment of variable

	n digits			m digits		
x_1	1	0 ... 0	c_1	c_2	----	
x_2	0	1	0 ... 0	...		
⋮						
c_1	0 ---- 0		1	0 ---- 0		
	0 ---- 0		1	0 ---- 0		

$$c_1 = \begin{cases} 0 ; & \text{if } x_1 = 0 \\ 1 ; & \text{if } x_1 = 1 \end{cases}$$

for c_1 to be satisfied,
at least one literal is T,
at most three are T.
$\sim [1,3]$
$\sim 3-[1,3] = +[0,2]$

$+0$ or $+1$ or $+2 \sim +[0,2]$