

Monoid on set

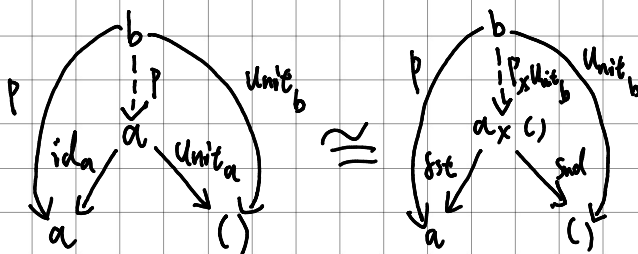
Monoid (ham-set,  $(\circ)$ )

binary operator / morphism:  $(X) :: a \rightarrow b \rightarrow a \times b$

unit object:  $() \leftarrow$  terminal object

prove terminal object is the unit object under product  $(X)$ .

proof



Goal prove  $a$  is isomorphic to the product of  $a$  and  $()$ .

by definition of terminal object

There is a unique morphism from any object to terminal object:

$\text{Unit} :: \text{forall } a. a \rightarrow ()$ .

by definition of Category

There is a unique morphism from any object to itself:

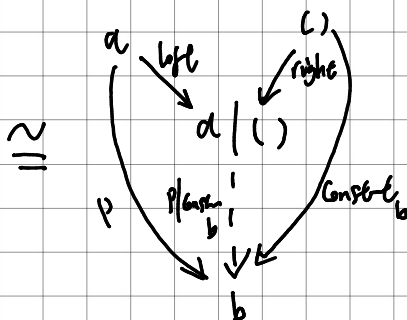
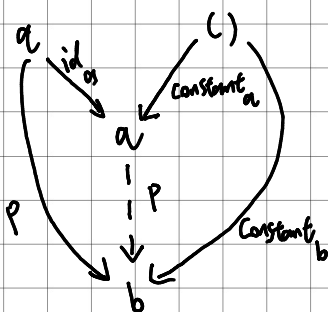
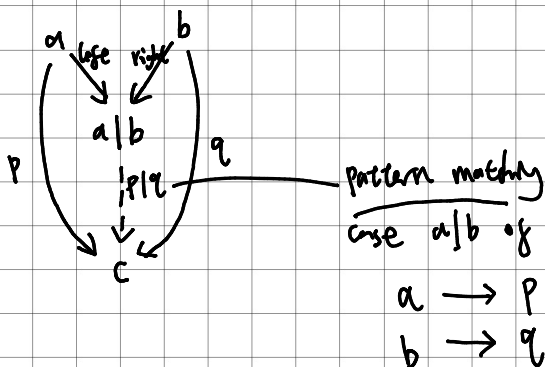
$\text{id} :: \text{forall } a. a \rightarrow a$ .

forall candidate object  $b$  which has an arrow  $p$  to  $a$ ,  
an arrow  $\text{unit}_b$  to  $()$ .

There is a unique morphism from  $b$  to  $a$ :  $p$ .

$p = \text{id}_a \circ p$ ,  $\text{unit}_b = \text{unit}_a \circ p$ .

## Coproduct



Both Product and Coproduct are Bifunctors:  $C \times C \rightarrow C$

(x) (1)

and have a unit object: the terminal object of  $C$ .

$\Rightarrow$  Both Product Category and Coproduct Category are Monoidal Categories.

Monoidal Category is a Category  
of one object, which is  
a set.

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category.

$\otimes, 1$