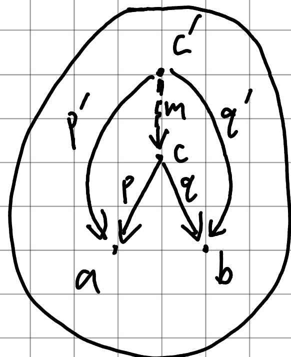
# Produce

generalized from Cartesian Product of two sets.




C

> * fundamentally,
> objects and morphisms have no name.
> (·) (→)
> It's we, as human beings, name different things differently.

pattern ← a "query" to a "database" that narrows down the focus to one distinct object

Given 2 objects in Category C, a and b (can be the same object)

Conditions

1. an object that has an arrow to a and an arrow to b

   (in this example, c and c' are two candidates)

2. among all such objects following condition 1, an object that all objects have a unique arrow (m) pointing it such that

$$\begin{cases} p \circ m = p' \\ q \circ m = q' \end{cases}$$

once locate this distinct object (Product of a and b), which is an universal construction (given the promise, in this case any two objects, such a object either doesn't exist or is unique up to isomorphism), we can rename the object to "$a \times b$" (or "$(a, b)$") and the projections $p$ and $q$ to "fst" and "snd".

$$
\begin{tikzcd}
 & (a,b) \arrow[dl, "fst"'] \arrow[dr, "snd"] & \\
a & & b
\end{tikzcd}
\qquad
\begin{tikzcd}
 & c' \arrow[d, "m"] \arrow[ddl, bend right, "p'"'] \arrow[ddr, bend left, "q'"] & \\
 & (a,b) \arrow[dl, "fst"'] \arrow[dr, "snd"] & \\
a & & b
\end{tikzcd}
$$

__prove__ other objects following condition 1 always have an unique morphism to $(a,b)$.

__idea__ $m$ is isomorphic to a Product of $p'$ and $q'$ in the category of hom-sets.

i.e. $m \cong (p', q')$

$$
\begin{tikzcd}
 & (p',q') \arrow[dl] \arrow[dr] & \\
p' & & q'
\end{tikzcd}
$$

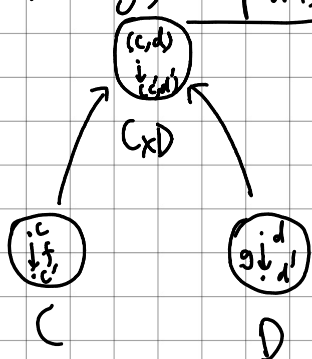
Category of hom-sets

# Bifunctor

| Cat |
|---|
| - objects : Categories |
| - morphisms : Functors |

In the category of Category (Cat),

a Product of 2 categories is also a category.
(easier to define than product of 2 objects in a Category,
because all objects in a "small" category form a set,
so objects in the Product of 2 categories
are elements in the cartesian product of 2 sets of objects.
Similarly, morphisms are elements in the cartesian product
of 2 hom-sets.)




$$(c,d) \xrightarrow{(f,g)} (c',d')$$

$$(f',g') \circ (f,g) = (f' \circ f, g' \circ g)$$

$$(id_a, id_b) = id_{(a,b)}$$

C×D is a Product Category.

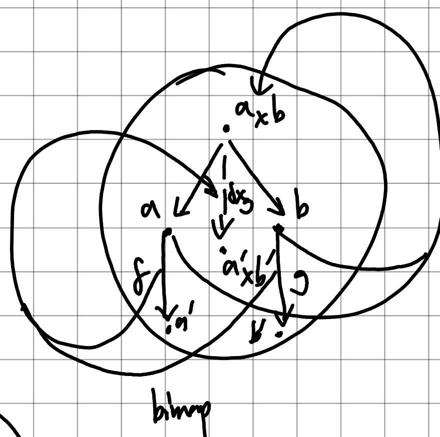
Bifunctor : $C \times D \to E$

from a product Category to any Category (can also be a Product Category)

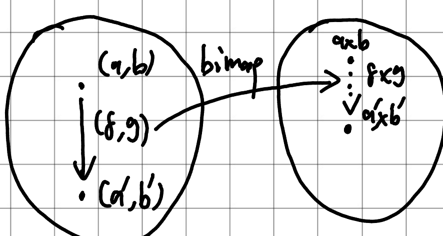
a Category where Product is defined
for every pair of objects
are called <u>Cartesian Category</u>.

$$\begin{tikzcd}
 & a \times b \arrow[dl, "p"'] \arrow[dr, "q"] \arrow[d, dashed] \arrow[ddl, "f \circ p"'] \arrow[ddr, "g \circ q"] & \\
a \arrow[d, "f"'] & a' \times b' \arrow[dl, "p'"] \arrow[dr, "q'"'] & b \arrow[d, "g"] \\
a' & & b'
\end{tikzcd}$$

an unique morphism,
call it "$f \times g$"




$$\underset{C \times C}{(a,b)} \longrightarrow \underset{C}{a \times b}$$




Product Category Cartesian Category

product operator in Cartesian Category
is isomorphic to a Bifunctor.