p2

$0 < \varepsilon_1 < \varepsilon_2 < 0$. Let $\varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2}$

$\forall x \in L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] \geq \varepsilon_2$

$\forall x \notin L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] \leq \varepsilon_1$

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chernoff bound.

$x_i \sim I.D. \{0,1\}$ $\begin{cases} \Pr[x_i=1] = p_i \\ \Pr[x_i=0] = 1-p_i \end{cases}$

① $\Delta > 0$,

i) $\Pr[X - \mu \geq \Delta] \leq e^{-\frac{\Delta^2}{2n}}$

where $X = \sum_{i=1}^{n} x_i$, $\mu = E[X] = \sum_{i=1}^{n} p_i$

ii) $\Pr[|X-\mu| \geq \Delta] \leq 2 \cdot e^{-\frac{\Delta^2}{2n}}$

② $\delta > 0$,

$\Pr[X - \mu \geq \delta\mu] \leq e^{-\frac{\delta^2 \mu}{4}}$

$\Pr[X - \mu \leq \delta\mu] \leq e^{-\frac{\delta^2 \mu}{2}}$

$\{y_1, y_2, \dots, y_m\} \Rightarrow \Pr\left[X - \mu \geq \frac{\varepsilon_2 - \varepsilon_1}{2}\right] \leq e^{-\frac{\left(\frac{\varepsilon_2-\varepsilon_1}{2}\right)^2}{2m}}$ $\boxed{\cancel{\leq \frac{1}{2} - \frac{1}{p(n)}}}$ $-\left(\frac{\varepsilon_2-\varepsilon_1}{2}\right)^2$

$\Rightarrow m \geq \frac{\quad}{2\log(\frac{1}{\delta})}$

$\Rightarrow \Pr\left[\underbrace{X - \mu \leq \frac{\varepsilon_2 - \varepsilon_1}{2}}_{D' =}\right] \geq 1 - e^{-\frac{\left(\frac{\varepsilon_2-\varepsilon_1}{2}\right)^2}{2m}}$

P1 $NP \subseteq BPP \Rightarrow RP = NP$

BPP $\forall x \in L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] \geq \frac{2}{3}$

$\forall x \notin L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] \leq \frac{1}{3}$

RP $\forall x \in L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] \geq \frac{1}{2}$

$\forall x \notin L \Rightarrow \Pr_{y \in \{0,1\}^{q(n)}} [D(x,y)=1] = 0$

guard against the case where $x \notin SAT$ but $\exists y . D(x,y)=1$.

known $P \subseteq RP \subseteq BPP$

goal $SAT \in NP\text{-cplete} \subseteq NP \subseteq BPP$

$\Rightarrow SAT \in RP$.

idea Let A be the machine in BPP that decides SAT.

DFS with access to A to find an assignment,
if $\phi(\ldots) = 0$, reject.

NP $\exists V \in$ all DTMs, $\forall x \in L$,

$\exists c \in \Sigma^* \wedge |c| = q(n)$,

$V(x, c) = 1$

BPP $\exists V \in$ all DTMs, $\forall x \in L$, | **RP**

$\Pr_{c \in \Sigma^* \wedge |c| = q(n)} [V(x,c) = 1] \geq 1 - \frac{1}{q(n)}$ | $\geq \frac{1}{2}$

$\leq \frac{1}{q(n)}$ | $= 0$.

If find a satisfying assignment $\Rightarrow \phi \in SAT$

how probable?

A $(n+1)$ times with $\underline{\varepsilon, \{x_1\}, \{x_1, x_2\}, \ldots, \{x_1, x_2, \ldots, x_n\}}$.

$c \sim \phi(\varepsilon), \phi(x_1), \phi(x_1, x_2), \ldots, \phi(x_1, x_2, \ldots, x_n)$
all $\in SAT$.

$\Pr[V(x, \phi(\varepsilon)) = 1 \wedge \ldots \wedge V(x, \phi(x_1, \ldots, x_n))]$

$= 1 - \Pr[\cdots \vee \cdots \vee \cdots] = 1 - \sum_{i=0}^{n} \Pr[V(\;) = 0] = 1 - \frac{n+1}{q(n)} \leq \frac{1}{2}$.