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Spike-and-Slab Additive Models And Fast Algorithms For High-Dimensional Data Analysis

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Dissertation Committee

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Spline Model Development

Spline Model Development

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Spline Model Development

Spline Model Development

"It is extremely unlikely that the true (effect) function f(X) (on the outcome) is actually linear in X."

*— Hastie, Tibshirani, and Friedman (2009) PP. 139*

Question

How to model nonlinear effects?

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Spline Model Development

Spline Functions

A *spline* function is a piece-wise
polynomial function

$$B(x) = \sum_{k=1}^{K} \beta_k b_k(x) \equiv \boldsymbol{X}^T \boldsymbol{\beta}$$

$b_k(x)$ are the *basis functions*, possibly
truncated power basis and b-spline basis.
(Simon N. Wood 2017)
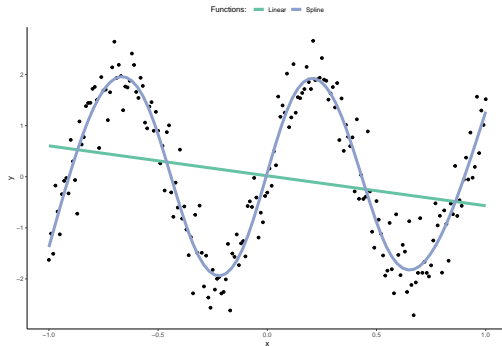
▶ For simplicity, we assume all functions have K basis functions and knots of
functions are equidistance.

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Spline Model Development

Generalized Additive Models with Splines

Generalized additive model (Hastie and Tibshirani 1987) is expressed

$$y_i \overset{\text{iid}}{\sim} EF(\mu_i, \phi), \quad i = 1, \ldots, n$$
$$g(\mu_i) = \beta_0 + B(x_i) = \beta_0 + \boldsymbol{X}_i^T \beta, \quad \mathbb{E}\left[B(X)\right] = 0$$

where $B(x_i)$ is the spline function, $g(\cdot)$ is a link function, ϕ is the dispersion parameter

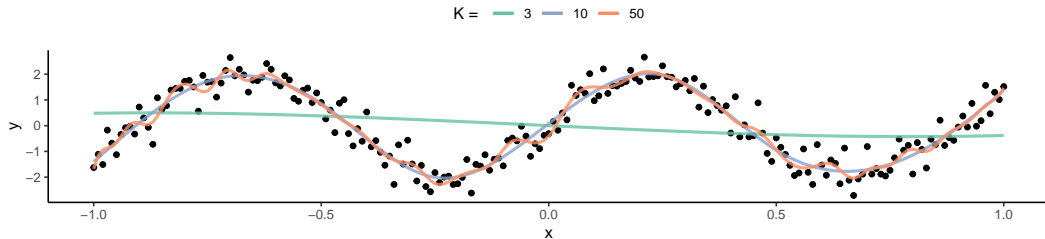▶ Model fitting follows the generalized linear models, e.g. ordinary least square for Gaussian outcome

$$\hat{\beta} = \arg\min \sum_{i=1}^{n} \left[y_i - \beta_0 - \boldsymbol{X}_i^T \beta\right]^2$$

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Spline Model Development

Problem: Function Smoothness

Question

How to mathematically define and estimate the smoothness of spline functions?

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Bayesian Regularization

Bayesian Regularization

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Bayesian Regularization

Smoothing Spline Model

- ▶ Smoothing penalty $\lambda \int B''(X)^2 dx = \lambda \beta^T \mathbf{S} \beta$
  - ▶ The smoothing penalty matrix $\mathbf{S}$ is known given $\mathbf{X}$
  - ▶ $\mathbf{S}$ is symmetric and positive semi-definite
- ▶ Penalized Least Square for Gaussian Outcome

$$\hat{\beta} = \arg\min \sum_{i=1}^{n} \sum_{i=1}^{n} \left[y_i - \beta_0 - \mathbf{X}_i^T \beta \right]^2 + \lambda \beta^T \mathbf{S} \beta$$

- ▶ The smoothing parameter λ is a tuning parameter, selected via cross-validation

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Bayesian Regularization

Problem: Multiple Predictor Model

When a model contains multiple spline functions for variables $X_1, \ldots, X_p$, the penalized least square estimator is

$$\hat{\boldsymbol{\beta}} = \arg\min \sum_{i=1}^{n} \left[y_i - \beta_0 - \sum_{j=1}^{p} \boldsymbol{X}_{ij}^T \beta_j \right]^2 + \sum_{j=1}^{p} \lambda_j \beta_j^T \boldsymbol{S}_j \beta_j$$

Question

How to choose λ_i for $i = 1, \ldots, p$?

- ▶ Global smoothing: $\lambda_1 = \cdots = \lambda_p$
- ▶ Adaptive smoothing: unique λ_i for $i = 1, \ldots, p$

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Bayesian Regularization

Bayesian Regularization

▶ Bayesian regularization is the Bayesian analogy of penalized models by using regularizing priors

$$\text{Bayesian ridge:} \quad \beta \sim N(0, \tau^2) \rightarrow \lambda = \sigma^2/\tau^2$$

▶ Adaptive shrinkage with hierarchical priors

$$\tau_j^2 \stackrel{\text{iid}}{\sim} IG(a, b)$$

▶ Adaptive smoothing
  ▶ Random walk prior on b-spline bases with IG hyperprior (Lang and Brezger 2004)
  ▶ Log-normal spline model for coefficient variance (Baladandayuthapani, Mallick, and Carroll 2005)

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Bayesian Variable Selection

Bayesian Variable Selection

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Bayesian Variable Selection

Problem: Functional Selection

In the context of variable selection and high-dimensional statistics, we always assume some variables are not effective or predictive of the outcome.

Question

How to statistically detect

- if a variable is predictive to the outcome, $B_j(X_j) = 0$
- if a variable has a nonlinear relationship with the outcome, $B_j(X_j) = \beta_j X_j$

Bi-level selection is the procedure that simultaneously addresses the two questions above

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Bayesian Variable Selection

Spike-and-Slab Priors

Spike-and-slab priors are a family of mixture distributions that deploys a characterizing structure

$$\beta|\gamma \sim (1 - \gamma)f_{spike}(\beta) + \gamma f_{slab}(\beta)$$

▶ Latent indicator γ follows a Bernoulli distribution with probability θ
▶ Spike density $f_{spike}(x)$ concentrates around 0 for small effects
▶ Slab density $f_{slab}(x)$ is a flat density for large effects
▶ Natural procedure to select variables via posterior distribution of γ
▶ Markov chain Monte Carlo is not compelling for high-dimensional data analysis

Bayesian Variable Selection

Spike-and-Slab LASSO Priors

▶ Double exponential distributions as the spike and slab distributions

$$\beta|\gamma \sim (1-\gamma)DE(0, s_0) + \gamma DE(0, s_1), 0 < s_0 < s_1$$

  ▶ Computation advantages via Expectation-Maximization (EM) algorithms
  ▶ Seamless variable selection as coefficients shrinkage to 0
▶ Group spike-and-slab LASSO prior
  ▶ Structure among predictors, e.g. gene pathways, bases of a spline function
  ▶ Structured prior $\gamma_k|\theta_j \overset{\text{iid}}{\sim} Binomial(1, \theta_j), k \in j$

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Bayesian Variable Selection

Problem: High-dimensional Spline Model

Question

How to jointly model signal sparsity and function smoothness, while capable of bi-level selection?

▶ Excess shrinkage due to negligence of smooth penalty
  ▶ Group lasso penalty (Ravikumar et al. 2009; Huang, Horowitz, and Wei 2010), group SCAD penalty (Wang, Chen, and Li 2007; Xue 2009)
▶ All-in-all-out selection
  ▶ Failed to select function as whole, e.g. group spike-and-slab LASSO prior
  ▶ Can not detect if a function is linear, e.g. spike-and-slab grouped LASSO prior (Bai et al. 2020; Bai 2021)

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Objectives

▶ To develop statistical models that improve curve interpolation and outcome prediction
  ▶ Local adaption of sparse penalty and smooth penalty
  ▶ Bi-level selection for linear and nonlinear effect
▶ To develop a fast and scalable algorithm
▶ To implement a user-friendly statistical software

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Projects

- ▶ **Guo, B.**, Jaeger, B. C., Rahman, A. F., Long, D. L., Yi, N. (2022). Spike-and-Slab least absolute shrinkage and selection operator generalized additive models and scalable algorithms for high-dimensional data analysis. *Statistics in Medicine*. doi: https://doi.org/10.1002/sim.9483

- ▶ **Guo, B.**, Jaeger, B. C., Rahman, A. F., Long, D. L., Yi, N. (2022). A scalable and flexible Cox proportional hazard model for high-dimensional survival prediction and functional selection *arXiv*. doi: https://doi.org/10.48550/arXiv.2205.11600

- ▶ **Guo, B.**, Yi, N. (2022). BHAM: An R Package to Fit Bayesian Hierarchical Additive Models for High-dimensional Data Analysis *Work in Progress*

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Bayesian Hierarchical Additive Models

Bayesian Hierarchical Additive Models

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Bayesian Hierarchical Additive Models

Generalized Additive Model

Given the data $\{y_i, x_{i1}, \ldots, x_{ip}\}_{i=1}^{n}$ where $p >> n$

$$y_i \overset{\text{i.i.d.}}{\sim} EF(\mu_i, \phi),$$
$$g(\mu_i) = \beta_0 + \sum_{j=1}^{p} B_j(x_{ij}), \quad i = 1, \ldots, n.$$

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Bayesian Hierarchical Additive Models

Spline Function Reparameterization

▶ Smoothing penalty from smoothing spline regression (Simon N. Wood 2017)

$$\lambda_j \int B_j''(x)dx = \lambda_j \beta_j^T \boldsymbol{S}_j \beta_j,$$

where S_j is a known smoothing penalty matrix.

▶ Isolate the linear and nonlinear components via eigendecomposing S_j

$$\boldsymbol{X}\beta = X^0\beta + \boldsymbol{X}^*\beta^*$$

▶ Benefits
   ▶ Motivate bi-level selection
   ▶ Implicit modeling of function smoothness
   ▶ Reduce computation load with conditionally independent prior of basis coefficients

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Bayesian Hierarchical Additive Models

Two-part Spike-and-slab LASSO (SSL) Prior

▶ SSL prior for the linear coefficient and group SSL priors for nonlinear coefficients

$$\beta_j | \gamma_j, s_0, s_1 \sim DE(0, (1 - \gamma_j)s_0 + \gamma_j s_1)$$

$$\beta_{jk}^* | \gamma_j^*, s_0, s_1 \overset{\text{iid}}{\sim} DE(0, (1 - \gamma_j^*)s_0 + \gamma_j^* s_1), k = 1, \dots, K_j$$
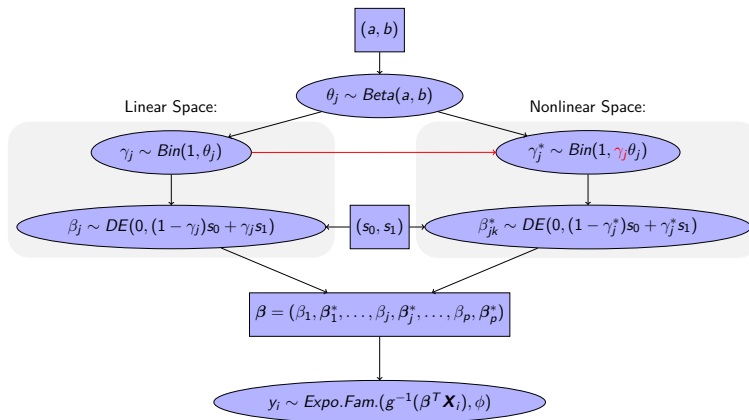
▶ Effect hierarchy enforced latent inclusion indicators γ_j and γ_j^* for bi-level selection

$$\gamma_j | \theta_j \sim Bin(\gamma_j | 1, \theta_j), \quad \gamma_j^* | \gamma_j, \theta_j \sim Bin(1, \gamma_j \theta_j),$$

▶ Local adaptivity of signal sparsity and function smoothness

$$\theta_j \sim \text{Beta}(a, b)$$

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Bayesian Hierarchical Additive Models

Visual Representation

Linear Space:

Nonlinear Space:

(a, b)

$\theta_j \sim Beta(a, b)$

$\gamma_j \sim Bin(1, \theta_j)$

$\gamma_j^* \sim Bin(1, \gamma_j \theta_j)$

$\beta_j \sim DE(0, (1 - \gamma_j)s_0 + \gamma_j s_1)$

(s_0, s_1)

$\beta_{jk}^* \sim DE(0, (1 - \gamma_j^*)s_0 + \gamma_j^* s_1)$

$\beta = (\beta_1, \beta_1^*, \ldots, \beta_j, \beta_j^*, \ldots, \beta_p, \beta_p^*)$

$y_i \sim Expo.Fam.(g^{-1}(\beta^T \mathbf{X}_i), \phi)$

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Bayesian Hierarchical Additive Models

EM-Cooridante Descent Algrithm for Scalable Model Fitting

We are interested in estimating $\Theta = \{\boldsymbol{\beta}, \boldsymbol{\theta}, \phi\}$ using optimization based algorithm for scalability purpose

▶ Basic Ideas
  ▶ Treat γs as the "missing data" in the EM procedure
  ▶ Quantify the expectation of log posterior density function of Θ with respect to γ conditioning on $\Theta^{(t-1)}$
  ▶ Maximize two parts of the objective function independently

▶ Previous applications in high-dimensional data analysis
  ▶ EMVS (Ročková and George 2014), Spike-and-slab lasso (Ročková and George 2018)
  ▶ BhGLM (Yi et al. 2019)

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Bayesian Hierarchical Additive Models

Decomposition of Objective Function

We aim to maximize the log posterior density of Θ by averaging over all possible values of γ

$$\log f(\Theta, \gamma | \mathbf{y}, \mathbf{X}) = Q_1(\boldsymbol{\beta}, \phi) + Q_2(\gamma, \boldsymbol{\theta}),$$

▶ L_1-penalized likelihood function of $\boldsymbol{\beta}, \phi$

$$Q_1 \equiv Q_1(\boldsymbol{\beta}, \phi) = \log f(\mathbf{y}|\boldsymbol{\beta}, \phi) + \sum_{j=1}^{p} \left[\log f(\beta_j | \gamma_j) + \sum_{k=1}^{K_j} \log f(\beta_{jk}^* | \gamma_{jk}^*) \right]$$

▶ Posterior density of θ given data points γs

$$Q_2 \equiv Q_2(\gamma, \boldsymbol{\theta}) = \sum_{j=1}^{p} \left[(\gamma_j + \gamma_j^*) \log \theta_j + (2 - \gamma_j - \gamma_j^*) \log(1 - \theta_j) \right] + \sum_{j=1}^{p} \log f(\theta_j).$$

▶ Q_1 and Q_2 are independent conditioning on γs

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Bayesian Hierarchical Additive Models

Summary of EM-Coordinate Descent Algorithm

▶ E-step
  ▶ Formulate $E_{\gamma|\Theta^{(t)}}\left[Q(\Theta, \gamma)\right] = E(Q_1) + E(Q_2)$
    ▶ $E(Q_1)$ is a penalized likelihood function of β, ϕ
    ▶ $E(Q_2)$ is a posterior density of θ given $E(\gamma)$
    ▶ $E(Q_1)$ and $E(Q_2)$ are conditionally independent
  ▶ Calculate $E(\gamma_j)$, $E(\gamma_j^*)$ and the penalties parameters by Bayes' theorem
▶ M-step:
  ▶ Use Coordinate Descent to fit the penalized model in $E(Q_1)$ to update β, ϕ
  ▶ Closed form calculation via $E(Q_2)$ to update θ

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Bayesian Hierarchical Additive Models

Tuning Parameter Selection

- ▶ s_0 and s_1 are tuning parameters
- ▶ Empirically, s_1 has extremely small effect on changing the estimates
- ▶ Focus on tuning s_0
- ▶ Consider a sequence of L ordered values $\{s_0^l\} : 0 < s_0^1 < s_0^2 < \cdots < s_0^L < s_1$
- ▶ Cross-validation to choose optimal value for s_0

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Bayesian Hierarchical Additive Models

Simulation Study

▶ Follow the data generating process introduced in Bai et al. (2020).

▶ $n_{train} = 500$, $n_{test} = 1000$

▶ $p = 4, 10, 50, 200$

$$\mu = 5\sin(2\pi x_1) - 4\cos(2\pi x_2 - 0.5) + 6(x_3 - 0.5) - 5(x_4^2 - 0.3),$$

▶ $f_j(x_j) = 0$ for $j = 5, \ldots, p$.

▶ 2 types of outcome: Gaussian ($\phi = 1$), Binomial

▶ Splines are constructed using 10 knots

▶ 50 Iterations

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Bayesian Hierarchical Additive Models

Comparison & Metircs

▶ Methods of comparison
  ▶ Proposed model BHAM
  ▶ Linear LASSO model as the benchmark
  ▶ mgcv (S. N. Wood 2004)
  ▶ COSSO (Zhang and Lin 2006) and adaptive COSSO(Storlie et al. 2011)
  ▶ Sparse Bayesian GAM (Bai 2021)
  ▶ spikeSlabGAM (Scheipl, Fahrmeir, and Kneib 2012)

▶ Metrics
  ▶ Prediction: R^2 for continuous outcomes, out-of-sample AUC for binary outcomes
  ▶ Variable Selection: positive predictive value (precision), true positive rate (recall), and Matthews correlation coefficient (MCC)

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Bayesian Hierarchical Additive Models

Prediction Performance

▶ Linear LASSO Model performs bad and mgcv performs well
▶ BHAM performs better than COSSO, adaptive COSSO and spikeSlabGAM
▶ BHAM performs better than SB-GAM in low-dimensional case but slightly worse in the high-dimensional setting
▶ BHAM is much faster than SB-GAM in fitting models

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Bayesian Hierarchical Additive Models

Variable Selection Performance

- ▶ SB-GAM has the best variable selection performance
- ▶ BHAM has conservative selection
- ▶ BHAM and spikeSlabGAM have trade-offs for bi-level selection
  - ▶ spikeSlabGAM tends to select either linear or nonlinear components of the function
  - ▶ BHAM is more likely to select both parts

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Additive Cox Proportional Hazards Model

Additive Cox Proportional Hazards Model

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Additive Cox Proportional Hazards Model

Model & Objective Functions

▶ Cox proportional hazard model with event time t_i

$$h(t_i) = h_0(t_i) \exp(\sum_{j=1}^{p} B_j(x_{ij})), \quad i = 1, \ldots, n.$$

  ▶ No intercept term because of the baseline hazard function
▶ Model fitting
  ▶ Replace likelihood function with partial likelihood function

$$\hat{h}_0(t_i|\beta) = d_i / \sum_{i' \in R(t_i)} exp(X_{i'}\beta).$$

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Additive Cox Proportional Hazards Model

Two-part Spike-and-slab LASSO (SSL) Prior

▶ SSL prior for the linear coefficient and group SSL priors for nonlinear coefficients

$$\beta_j|\gamma_j, s_0, s_1 \sim DE(0, (1-\gamma_j)s_0 + \gamma_j s_1)$$

$$\beta_{jk}^*|\gamma_j^*, s_0, s_1 \overset{\text{iid}}{\sim} DE(0, (1-\gamma_j^*)s_0 + \gamma_j^* s_1), k = 1, \ldots, K_j$$

▶ Effect hierarchy enforced latent inclusion indicators γ_j and γ_j^* for bi-level selection

$$\gamma_j|\theta_j \sim Bin(\gamma_j|1, \theta_j), \quad \gamma_j^*|\gamma_j, \theta_j \sim Bin(1, \gamma_j \theta_j),$$

▶ Local adaptivity of signal sparsity and function smoothness

$$\theta_j \sim \text{Beta}(a, b)$$

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Additive Cox Proportional Hazards Model

Summary of EM-Coordinate Descent Algorithm

- ▶ E-step
  - ▶ Formulate $E_{\gamma|\Theta^{(t)}}\left[Q(\Theta, \gamma)\right] = E(Q_1) + E(Q_2)$
    - ▶ $E(Q_1)$ is a penalized likelihood function of β, ϕ
    - ▶ $E(Q_2)$ is a posterior density of θ given $E(\gamma)$
    - ▶ $E(Q_1)$ and $E(Q_2)$ are conditionally independent
  - ▶ Calculate $E(\gamma_j)$, $E(\gamma_j^*)$ and the penalties parameters by Bayes' theorem
- ▶ M-step:
  - ▶ Use Coordinate Descent to fit the penalized model in $E(Q_1)$ to update β, ϕ
  - ▶ Closed form calculation via $E(Q_2)$ to update θ

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Additive Cox Proportional Hazards Model

Simulation Study

- ▶ Follow the data generating process introduced in Bai et al. (2020).
- ▶ $n_{train} = 500$, $n_{test} = 1000$
- ▶ $p = 4, 10, 50, 100, 200$
- ▶ Survival and censoring time follow Weibull distribution

$$\log \eta = (x_1 + 1)^2/5 + \exp(x_2 + 1)/25 + 3\sin(x_3)/2 + (1.4x_4 + 0.5)/2$$

- ▶ Censoring rate is controlled at $\{0.15, 0.3, 0.45\}$
- ▶ Splines are constructed using 10 knots
- ▶ 50 Iterations

Additive Cox Proportional Hazards Model

Comparison & Metircs

▶ Methods of comparison
    ▶ Proposed model BHAM
    ▶ Linear LASSO model as the benchmark
    ▶ mgcv (S. N. Wood 2004)
    ▶ COSSO (Zhang and Lin 2006) and adaptive COSSO(Storlie et al. 2011)

▶ Metrics
    ▶ Out-of-sample deviance & Concordance

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Additive Cox Proportional Hazards Model

Prediction Performance

▶ Linear LASSO Model performs bad in general
▶ Low dimensional settings:
    ▶ mgcv performs the best
    ▶ BHAM performs as good as mgcv
▶ High dimensional setting:
    ▶ BHAM performs better than COSSO models as p increases and more censoring events

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Additive Cox Proportional Hazards Model

Emipirical Performance: Emory Cardiovascular Biobank

- ▶ predicting all-cause mortality among patents undergoing cardiac catheterization
- ▶ Sample size N=454 and number of features p=200
- ▶ 5-knot cubic spline

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R Package BHAM

R Package BHAM

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R Package BHAM

R Package BHAM

▶ Model formulation for high-dimensional data
▶ Model fitting and tuning
▶ Model summary and variable selection
▶ Spline function visualization
▶ Website via *boyiguo1.github.io/BHAM*

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R Package BHAM

Design Matrix of Spline Fucntions

▶ Flexible spline function formulation for high-dimensional data

```
spline_df <- dplyr::tribble(
    ~Var, ~Func, ~Args,
    "X1",  "s", "bs='cr', k=5",
    "X2",  "s", NA,
    "X3",  "s", "")
spline_df <- data.frame(
    Var = setdiff(names(dat), "y"),
    Func = "s",
    Args ="bs='cr', k=7")
train_sm_dat <- BHAM::construct_smooth_data(spline_df, dat)
```

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R Package BHAM

Model Fitting Functions

▶ Model fitting via `bamlasso`
  ▶ Argument `family` for generalized and survival outcomes
  ▶ Argument `ss` for spike-and-slab LASSO scale parameters
  ▶ Argument `group` for group structures among predictors

▶ Model tuning via `tune`
  ▶ Argument `nfolds`, `ncv` for nested cross-validation
  ▶ Argument `s0` for tuning candidates

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R Package BHAM

Post Fitting Functions

- ▶ Bi-level selection via `bamlasso_var_selection`
- ▶ Make prediction data for splines `make_predict_dat`
- ▶ Plot spline functions via `plot_smooth_term`

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Modeling Interactions

Modeling Interactions

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Modeling Interactions

Varying coefficient models

- ▶ Assume the coefficient of a variable X_j is a function of a covariate Z_j
  - ▶ linear model: $\beta(Z_j) = \beta$
  - ▶ VC model: $\beta(Z_j) = B(Z_j)$
- ▶ Replace each spline function $B(z_{ij})$ with $B(z_{ij})x_{ij} \equiv (x_{ij}\mathbf{Z}_{ij}^T)\beta_j$
- ▶ Model fitting with EM-Coordinate Descent

Question

Can Z_j be continuous? Is it possible to have a more flexible model?

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Modeling Interactions

Smooth Surface Fitting

▶ Tensor product of spline functions

$$B_{js}(x_{ij}, x_{is}) = \sum_{P\rho=1}^{K} \sum_{v=1}^{K} \beta_{jspv} b_{j\rho}(x_{ij}) b_{sv}(x_{is})$$

▶ Smooth Surface

$$B_j(x_{ij}) + B_s(x_{is}) + B_{js}(x_{ij}, x_{is}),$$

Question

Can we have a generalized model that accounts fixed effects, nonlinear curves, smooth surfaces, and random effects?

Modeling Interactions

Structural Additive Model

High-dimensional structural additive model can be formulated as

$$g(\mathbb{E}(y_i)) = \boldsymbol{x}_i^T \boldsymbol{\theta} + \boldsymbol{u}_i^T \boldsymbol{\gamma} + B(z_{i1}) + B(z_{i2}, z_{i3}) + B_{spat}(s_i)$$

- ▶ Un-regularized predictors $\boldsymbol{x}_i$
- ▶ Regularized predictors $\boldsymbol{u}_i$
- ▶ Predictors with nonlinear effects $\boldsymbol{z}_i$
- ▶ Spatial random effects with coordinates s_i

Spike-and-slab LASSO prior motivates a seamless process of variable/functional selection and a scalable optimization-based model fitting algorithm

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Conclusion

- ▶ Identify challenges in high-dimensional GAM with spline functions
  - ▶ Balance between signal sparsity and function smoothness
  - ▶ Bi-level selection to automatically detect linear and nonlinear effects
- ▶ Statistical contribution
  - ▶ Two-part spike-and-slab LASSO prior for spline functions
  - ▶ Scalable EM-Coordinate Descent algorithms for generalized and survival outcomes
  - ▶ R package BHAM
- ▶ Future Research
  - ▶ Extension of spike-and-slab LASSO prior in structured additive model

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