

Spike-and-Slab Additive Models And Fast Algorithms For High-Dimensional Data Analysis

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Dissertation Committee

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Outline

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- ▶ Background
 - ▶ Spline Model Development
 - ▶ Bayesian Regularization
 - ▶ Bayesian Variable Selection
- ▶ Dissertation
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 - ▶ Additive Cox Proportional Hazards Model
 - ▶ R package BHAM
- ▶ Conclusion
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 - ▶ Questions & Answers
 - ▶ Closing Statement & Acknowledgment

Background

Spline Model Development

Spline Model Development

“It is extremely unlikely that the true (effect) function $f(X)$ (on the outcome) is actually linear in X .”

— *Hastie, Tibshirani, and Friedman (2009) PP. 139*

Question

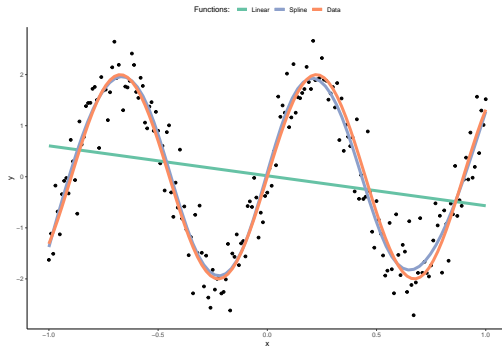
How to model nonlinear effects?

Spline Functions

A *spline* function is a piece-wise polynomial function

$$B(x) = \sum_{k=1}^K \beta_k b_k(x) \equiv \mathbf{x}^T \boldsymbol{\beta}$$

$b_k(x)$ are the *basis functions*, possibly truncated power basis and b-spline basis.
(Simon N. Wood 2017)



- For simplicity, we assume all functions have K basis functions and knots of functions are equidistance.

Generalized Additive Models with Splines

Generalized additive model (Hastie and Tibshirani 1987) is expressed

$$y_i \stackrel{\text{iid}}{\sim} EF(\mu_i, \phi), \quad i = 1, \dots, n$$
$$g(\mu_i) = \beta_0 + B(x_i) = \beta_0 + \mathbf{x}_i^T \boldsymbol{\beta}, \quad \mathbb{E}[B(X)] = 0$$

where $B(x_i)$ is the spline function, $g(\cdot)$ is a link function, ϕ is the dispersion parameter

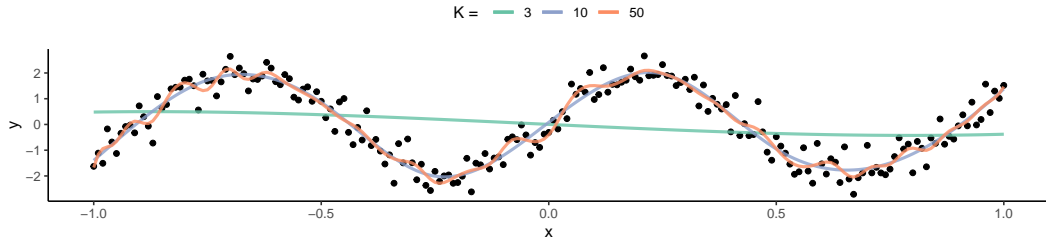
- Model fitting follows the generalized linear models, e.g. ordinary least square for Gaussian outcome

$$\hat{\boldsymbol{\beta}} = \arg \min \sum_{i=1}^n \left[y_i - \beta_0 - \mathbf{x}_i^T \boldsymbol{\beta} \right]^2$$

Problem: Function Smoothness

Question

How to decide K , the number of basis function ?



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Bayesian Regularization

Bayesian Regularization

Smoothing Spline Model

- ▶ Smoothing penalty $\lambda \int B''(X)^2 dx = \lambda \boldsymbol{\beta}^T \mathbf{S} \boldsymbol{\beta}$
 - ▶ The smoothing penalty matrix \mathbf{S} is known given \mathbf{X}
 - ▶ \mathbf{S} is symmetric and positive semi-definite
- ▶ Penalized Least Square for Gaussian Outcome

$$\hat{\boldsymbol{\beta}} = \arg \min \sum_{i=1}^n \sum_{i=1}^n \left[y_i - \beta_0 - \mathbf{x}_i^T \boldsymbol{\beta} \right]^2 + \lambda \boldsymbol{\beta}^T \mathbf{S} \boldsymbol{\beta}$$

- ▶ The smoothing parameter λ is a tuning parameter, selected via cross-validation

Problem: Multiple Predictor Model

When a model contains multiple spline functions for variables X_1, \dots, X_p , the penalized least square estimator is

$$\hat{\beta} = \arg \min \sum_{i=1}^n \left[y_i - \beta_0 - \sum_{j=1}^p \mathbf{x}_{ij}^T \beta_j \right]^2 + \sum_{j=1}^p \lambda_j \beta_j^T \mathbf{S}_j \beta_j$$

Question

How to choose λ_i for $i = 1, \dots, p$?

- ▶ Global smoothing: $\lambda_1 = \dots = \lambda_p$
- ▶ Adaptive smoothing: unique λ_i for $i = 1, \dots, p$

Bayesian Regularization

- ▶ Bayesian regularization is the Bayesian analogy of penalized models by using regularizing priors

$$\text{Bayesian ridge: } \beta \sim N(0, \tau^2) \rightarrow \lambda = \sigma^2 / \tau^2$$

- ▶ Adaptive shrinkage with hierarchical priors

$$\tau_j^2 \stackrel{\text{iid}}{\sim} IG(a, b)$$

- ▶ Adaptive smoothing
 - ▶ Random walk prior on b-spline bases with IG hyperprior
 - ▶ Normal prior on truncated power bases with a log-normal spline model for variance

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Bayesian Variable Selection

Bayesian Variable Selection

Problem: Functional Selection

In the context of variable selection and high-dimensional statistics, we always assume some variables are not effective or predictive of the outcome.

Question

How to statistically detect

- ▶ if a variable is predictive to the outcome, $B_j(X_j) = 0$
- ▶ if a variable has a nonlinear relationship with the outcome, $B_j(X_j) = \beta_j X_j$

Bi-level selection is the procedure that simultaneously addresses the two questions above

Spike-and-Slab Priors

Spike-and-slab priors are a family of mixture distributions that deploys a characterizing structure

$$\beta|\gamma \sim (1 - \gamma)f_{spike}(\beta) + \gamma f_{slab}(\beta)$$

- ▶ Latent indicator γ follows a Bernoulli distribution with probability θ
- ▶ Spike density $f_{spike}(x)$ concentrates around 0 for small effects
- ▶ Slab density $f_{slab}(x)$ is a flat density for large effects
- ▶ Natural procedure to select variables via posterior distribution of γ
- ▶ Markov chain Monte Carlo is not compelling for high-dimensional data analysis

Spike-and-Slab LASSO Priors

- ▶ Double exponential distributions as the spike and slab distributions

$$\beta|\gamma \sim (1 - \gamma)DE(0, s_0) + \gamma DE(0, s_1), 0 < s_0 < s_1$$

- ▶ Computation advantages via Expectation-Maximization (EM) algorithms
 - ▶ Seamless variable selection as coefficients shrinkage to 0
- ▶ Group spike-and-slab LASSO
 - ▶ Structure underlying predictors, e.g. gene pathways, bases of a spline function
 - ▶ Structured prior $\gamma_k|\theta_j \stackrel{\text{iid}}{\sim} \text{Binomial}(1, \theta_j), k \in j$

Problem: High-dimensional Spline Model

Question

How to jointly model signal sparsity and function smoothness, while capable of bi-level selection?

- ▶ Excess shrinkage due to ignoring smooth penalty completely
 - ▶ Group lasso penalty (Ravikumar et al. 2009; Huang, Horowitz, and Wei 2010), group SCAD penalty (Wang, Chen, and Li 2007; Xue 2009)
- ▶ All-in-all-out selection
 - ▶ Can not detect if a function is linear, e.g. spike-and-slab grouped LASSO prior (Bai et al. 2020; Bai 2021)
 - ▶ Failed to select function as whole, e.g. group spike-and-slab LASSO prior

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Dissertation

Objectives

- ▶ To develop statistical models that improve curve interpolation and outcome prediction
 - ▶ Local adaption of sparse penalty and smooth penalty
 - ▶ Bi-level selection for linear and nonlinear effect
- ▶ To develop a fast and scalable algorithm
- ▶ To implement a user-friendly statistical software

Projects

- ▶ **Guo, B.**, Jaeger, B. C., Rahman, A. F., Long, D. L., Yi, N. (2022). Spike-and-Slab least absolute shrinkage and selection operator generalized additive models and scalable algorithms for high-dimensional data analysis. *Statistics in Medicine*. doi: <https://doi.org/10.1002/sim.9483>
- ▶ **Guo, B.**, Jaeger, B. C., Rahman, A. F., Long, D. L., Yi, N. (2022). A scalable and flexible Cox proportional hazard model for high-dimensional survival prediction and functional selection *arXiv*. doi: <https://doi.org/10.48550/arXiv.2205.11600>
- ▶ **Guo, B.**, Yi, N. (2022). BHAM: An R Package to Fit Bayesian Hierarchical Additive Models for High-dimensional Data Analysis *Work in Progress*

Two-part Spike-and-slab LASSO (SSL) Prior for Smooth Functions

Generalized Additive Model

Given the data $\{y_i, x_{i1}, \dots, x_{ip}\}_{i=1}^n$ where $p \gg n$

$$y_i \stackrel{\text{i.i.d.}}{\sim} EF(\mu_i, \phi),$$

$$g(\mu_i) = \beta_0 + \sum_{j=1}^p B_j(x_{ij}), \quad i = 1, \dots, n.$$

Smoothing Function Reparameterization

- ▶ Smoothing penalty from Smoothing spline regression (Simon N. Wood 2017)

$$\lambda_j \int B_j''(x) dx = \lambda_j \beta_j^T \mathbf{S}_j \beta_j,$$

where S_j is a known smoothing penalty matrix.

- ▶ Isolate the linear and nonlinear components via eigendecomposing S_j

$$\mathbf{X}\beta = \mathbf{X}^0\beta + \mathbf{X}^*\beta^*$$

- ▶ Benefits
 - ▶ Motivate bi-level selection
 - ▶ Implicit modeling of function smoothness
 - ▶ Reduce computation load with conditionally independent prior of basis coefficients

Two-part Spike-and-slab LASSO (SSL) Prior

- ▶ SSL prior for the linear coefficient and group SSL priors for nonlinear coefficients

$$\beta_j | \gamma_j, s_0, s_1 \sim DE(0, (1 - \gamma_j)s_0 + \gamma_j s_1)$$

$$\beta_{jk}^* | \gamma_j^*, s_0, s_1 \stackrel{\text{iid}}{\sim} DE(0, (1 - \gamma_j^*)s_0 + \gamma_j^* s_1), k = 1, \dots, K_j$$

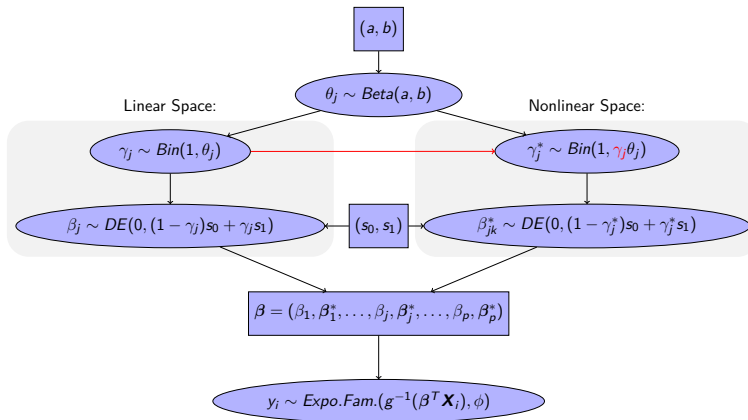
- ▶ Effect hierarchy enforced latent inclusion indicators γ_j and γ_j^* for bi-level selection

$$\gamma_j | \theta_j \sim \text{Bin}(\gamma_j | 1, \theta_j), \quad \gamma_j^* | \gamma_j, \theta_j \sim \text{Bin}(1, \gamma_j \theta_j),$$

- ▶ Local adaptivity of signal sparsity and function smoothness

$$\theta_j \sim \text{Beta}(a, b)$$

Visual Representation



EM-Coordinate Descent Algorithm for Scalable Model Fitting

We are interested in estimating $\Theta = \{\beta, \theta, \phi\}$ using optimization based algorithm for scalability purpose

- ▶ Basic Ideas

- ▶ Treat γ s as the “missing data” in the EM procedure
- ▶ Quantify the expectation of log posterior density function of Θ with respect to γ conditioning on $\Theta^{(t-1)}$
- ▶ Maximize two parts of the objective function independently

- ▶ Previous applications in high-dimensional data analysis

- ▶ EMVS (Ročková and George 2014), Spike-and-slab lasso (Ročková and George 2018)
- ▶ BhGLM (Yi et al. 2019)

Decomposition of Objective Function

We aim to maximize the log posterior density of Θ by averaging over all possible values of γ

$$\log f(\Theta, \gamma | \mathbf{y}, \mathbf{X}) = Q_1(\beta, \phi) + Q_2(\gamma, \theta),$$

- ▶ L_1 -penalized likelihood function of β, ϕ

$$Q_1 \equiv Q_1(\beta, \phi) = \log f(\mathbf{y} | \beta, \phi) + \sum_{j=1}^p \left[\log f(\beta_j | \gamma_j) + \sum_{k=1}^{K_j} \log f(\beta_{jk}^* | \gamma_{jk}^*) \right]$$

- ▶ Posterior density of θ given data points γ s

$$Q_2 \equiv Q_2(\gamma, \theta) = \sum_{j=1}^p \left[(\gamma_j + \gamma_j^*) \log \theta_j + (2 - \gamma_j - \gamma_j^*) \log(1 - \theta_j) \right] + \sum_{j=1}^p \log f(\theta_j).$$

- ▶ Q_1 and Q_2 are independent conditioning on γ s

Summary of EM-Coordinate Descent Algorithm

- ▶ E-step
 - ▶ Formulate $E_{\gamma|\Theta^{(t)}} [Q(\Theta, \gamma)] = E(Q_1) + E(Q_2)$
 - ▶ $E(Q_1)$ is a penalized likelihood function of β, ϕ
 - ▶ $E(Q_2)$ is a posterior density of θ given $E(\gamma)$
 - ▶ $E(Q_1)$ and $E(Q_2)$ are conditionally independent
 - ▶ Calculate $E(\gamma_j)$, $E(\gamma_j^*)$ and the penalties parameters by Bayes' theorem
- ▶ M-step:
 - ▶ Use Coordinate Descent to fit the penalized model in $E(Q_1)$ to update β, ϕ
 - ▶ Closed form calculation via $E(Q_2)$ to update θ

Tuning Parameter Selection

- ▶ s_0 and s_1 are tuning parameters
- ▶ Empirically, s_1 has extremely small effect on changing the estimates
- ▶ Focus on tuning s_0
- ▶ Consider a sequence of L ordered values $\{s_0^l\} : 0 < s_0^1 < s_0^2 < \cdots < s_0^L < s_1$
- ▶ Cross-validation to choose optimal value for s_0

Simulation Study

- ▶ Follow the data generating process introduced in Bai et al. (2020).
- ▶ $n_{train} = 500$, $n_{test} = 1000$
- ▶ $p = 4, 10, 50, 200$

$$\mu = 5 \sin(2\pi x_1) - 4 \cos(2\pi x_2 - 0.5) + 6(x_3 - 0.5) - 5(x_4^2 - 0.3),$$

- ▶ $f_j(x_j) = 0$ for $j = 5, \dots, p$.
- ▶ 2 types of outcome: Gaussian ($\phi = 1$), Binomial
- ▶ Splines are constructed using 10 knots
- ▶ 50 Iterations

Comparison & Metrics

- ▶ Methods of comparison
 - ▶ Proposed model BHAM
 - ▶ Linear LASSO model as the benchmark
 - ▶ mgcv (S. N. Wood 2004)
 - ▶ COSSO (Zhang and Lin 2006) and adaptive COSSO (Storlie et al. 2011)
 - ▶ Sparse Bayesian GAM (Bai 2021)
 - ▶ spikeSlabGAM (Scheipl, Fahrmeir, and Kneib 2012)
- ▶ Metrics
 - ▶ Prediction: R^2 for continuous outcomes, out-of-sample AUC for binary outcomes
 - ▶ Variable Selection: positive predictive value (precision), true positive rate (recall), and Matthews correlation coefficient (MCC)

Prediction Performance

- ▶ Linear LASSO Model performs bad and mgcv performs well
- ▶ BHAM performs better than COSSO, adaptive COSSO and spikeSlabGAM
- ▶ BHAM performs better than SB-GAM in low-dimensional case but slightly worse in the high-dimensional setting
- ▶ BHAM is much faster than SB-GAM in fitting models

Variable Selection Performance

- ▶ SB-GAM has the best variable selection performance
- ▶ BHAM has conservative selection
- ▶ BHAM and spikeSlabGAM have trade-offs for bi-level selection
 - ▶ spikeSlabGAM tends to select either linear or nonlinear components of the function
 - ▶ BHAM is more likely to select both parts

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Additive Cox Proportional Hazards Model

Additive Cox Proportional Hazards Model

Model & Objective Functions

- ▶ Cox proportional hazard model with event time t_i

$$h(t_i) = h_0(t_i) \exp\left(\sum_{j=1}^p B_j(x_{ij})\right), \quad i = 1, \dots, n.$$

- ▶ No intercept term because of the baseline hazard function
- ▶ Model fitting
 - ▶ Replace likelihood function with partial likelihood function

$$\hat{h}_0(t_i|\beta) = d_i / \sum_{i' \in R(t_i)} \exp(X_{i'}\beta).$$

Two-part Spike-and-slab LASSO (SSL) Prior

- ▶ SSL prior for the linear coefficient and group SSL priors for nonlinear coefficients

$$\beta_j | \gamma_j, s_0, s_1 \sim DE(0, (1 - \gamma_j)s_0 + \gamma_j s_1)$$

$$\beta_{jk}^* | \gamma_j^*, s_0, s_1 \stackrel{\text{iid}}{\sim} DE(0, (1 - \gamma_j^*)s_0 + \gamma_j^* s_1), k = 1, \dots, K_j$$

- ▶ Effect hierarchy enforced latent inclusion indicators γ_j and γ_j^* for bi-level selection

$$\gamma_j | \theta_j \sim \text{Bin}(\gamma_j | 1, \theta_j), \quad \gamma_j^* | \gamma_j, \theta_j \sim \text{Bin}(1, \gamma_j \theta_j),$$

- ▶ Local adaptivity of signal sparsity and function smoothness

$$\theta_j \sim \text{Beta}(a, b)$$

Summary of EM-Coordinate Descent Algorithm

- ▶ E-step
 - ▶ Formulate $E_{\gamma|\Theta^{(t)}} [Q(\Theta, \gamma)] = E(Q_1) + E(Q_2)$
 - ▶ $E(Q_1)$ is a penalized likelihood function of β, ϕ
 - ▶ $E(Q_2)$ is a posterior density of θ given $E(\gamma)$
 - ▶ $E(Q_1)$ and $E(Q_2)$ are conditionally independent
 - ▶ Calculate $E(\gamma_j)$, $E(\gamma_j^*)$ and the penalties parameters by Bayes' theorem
- ▶ M-step:
 - ▶ Use Coordinate Descent to fit the penalized model in $E(Q_1)$ to update β, ϕ
 - ▶ Closed form calculation via $E(Q_2)$ to update θ

Simulation Study

- ▶ Follow the data generating process introduced in Bai et al. (2020).
- ▶ $n_{train} = 500$, $n_{test} = 1000$
- ▶ $p = 4, 10, 50, 100, 200$
- ▶ Survival and censoring time follow Weibull distribution

$$\log \eta = (x_1 + 1)^2/5 + \exp(x_2 + 1)/25 + 3\sin(x_3)/2 + (1.4x_4 + 0.5)/2$$

- ▶ Censoring rate is controlled at $\{0.15, 0.3, 0.45\}$
- ▶ Splines are constructed using 10 knots
- ▶ 50 Iterations

Comparison & Metrics

- ▶ Methods of comparison
 - ▶ Proposed model BHAM
 - ▶ Linear LASSO model as the benchmark
 - ▶ mgcv (S. N. Wood 2004)
 - ▶ COSSO (Zhang and Lin 2006) and adaptive COSSO (Storlie et al. 2011)
- ▶ Metrics
 - ▶ Out-of-sample deviance & Concordance

Prediction Performance

- ▶ Linear LASSO Model performs bad in general
- ▶ Low dimensional settings:
 - ▶ mgcv performs the best
 - ▶ BHAM performs as good as mgcv
- ▶ High dimensional setting:
 - ▶ BHAM performs better than COSSO models as p increases and more censoring events

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R Package BHAM

R Package BHAM

R Package BHAM

- ▶ Model formulation for high-dimensional data
- ▶ Model fitting and tuning
- ▶ Model summary and variable selection
- ▶ Spline function visualization
- ▶ Website via *boyiguo1.github.io/BHAM*

Design Matrix of Spline Functions

- Flexible spline function formulation for high-dimensional data

```
spline_df <- dplyr::tribble(  
  ~Var, ~Func, ~Args,  
  "X1",  "s",  "bs='cr', k=5",  
  "X2",  "s",  NA,  
  "X3",  "s",  "")  
spline_df <- data.frame(  
  Var = setdiff(names(dat), "y"),  
  Func = "s",  
  Args = "bs='cr', k=7")  
train_sm_dat <- BHAM::construct_smooth_data(spline_df, dat)
```

Model Fitting Functions

- ▶ Model fitting via `bamlasso`
 - ▶ Argument `family` for generalized and survival outcomes
 - ▶ Argument `ss` for spike-and-slab LASSO scale parameters
 - ▶ Argument `group` for group structures among predictors
- ▶ Model tuning via `tune`
 - ▶ Argument `nfolds`, `ncv` for nested cross-validation
 - ▶ Argument `s0` for tuning candidates

Post Fitting Functions

- ▶ Bi-level selection via `bamlasso_var_selection`
- ▶ Make prediction data for splines `make_predict_dat`
- ▶ Plot spline functions via `plot_smooth_term`

Future Research

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Modeling Interactions

Modeling Interactions

Varying coefficient models

- ▶ Assume the coefficient of a variable X_j is a function of a covariate Z_j
 - ▶ linear model: $\beta(Z_j) = \beta$
 - ▶ VC model: $\beta(Z_j) = B(Z_j)$
- ▶ Replace each smooth function $B(z_{ij})$ with $B(z_{ij})x_{ij} \equiv (x_{ij}\mathbf{Z}_{ij}^T)\beta_j$
- ▶ Model fitting with EM-Coordinate Descent

Question

Can Z_j be continuous? Is it possible to have a more flexible model?

Smooth Surface Fitting

- ▶ Tensor product of spline functions

$$B_{js}(x_{ij}, x_{is}) = \sum_{\rho=1}^K \sum_{v=1}^K \beta_{jspv} b_{j\rho}(x_{ij}) b_{sv}(x_{is})$$

- ▶ Smooth Surface

$$B_j(x_{ij}) + B_s(x_{is}) + B_{js}(x_{ij}, x_{is}),$$

Question

Can we have a generalized model that accounts fixed effects, nonlinear curves, smooth surfaces, and random effects?

Structural Additive Model

High-dimensional structural additive model can be formulated as

$$g(\mathbb{E}(y_i)) = \mathbf{x}_i^T \boldsymbol{\theta} + \mathbf{u}_i^T \boldsymbol{\gamma} + B(z_{i1}) + B(z_{i2}, z_{i3}) + B_{\text{spat}}(s_i)$$

- ▶ Un-regularized predictors \mathbf{x}_i
- ▶ Regularized predictors \mathbf{u}_i
- ▶ Predictors with nonlinear effects \mathbf{z}_i
- ▶ Spatial random effects with coordinates s_i

Spike-and-slab LASSO prior motivates a seamless process of variable/functional selection and a scalable optimization-based model fitting algorithm

Conclusion

Conclusion

- ▶ Identify challenges in high-dimensional GAM with spline functions
 - ▶ Balance between signal sparsity and function smoothness
 - ▶ Bi-level selection to automatically detect linear and nonlinear effects
- ▶ Statistical contribution
 - ▶ Two-part spike-and-slab LASSO prior for smooth functions
 - ▶ Scalable EM-Coordinate Descent algorithms for generalized and survival outcomes
 - ▶ R package BHAM
- ▶ Future Research
 - ▶ Extension of spike-and-slab LASSO prior in structured additive model

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 - ▶ Collaborators
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