Варіант № 5

1 $z=\sqrt{4-x^2+y^2}$

$4-x^2+y^2\geq 0$

$4-x^2+y^2=0$

$y^2-x^2=-4 \quad |:(-4)$

$\frac{x^2}{4}-\frac{y^2}{4}=1$ a=2 дійсна

в=2 уявна

т.A $(-4;-1)$ $4-(-4)^2+(-1)^2=$

$=4-16-1=-11$ т.A $\notin Z$

т.O $(0;0)$ $4-0+0=4>0$ т.O $\in Z$

2) $\{(x;y)\in R^2 : |x\leq\sqrt{4+y^2}\}$

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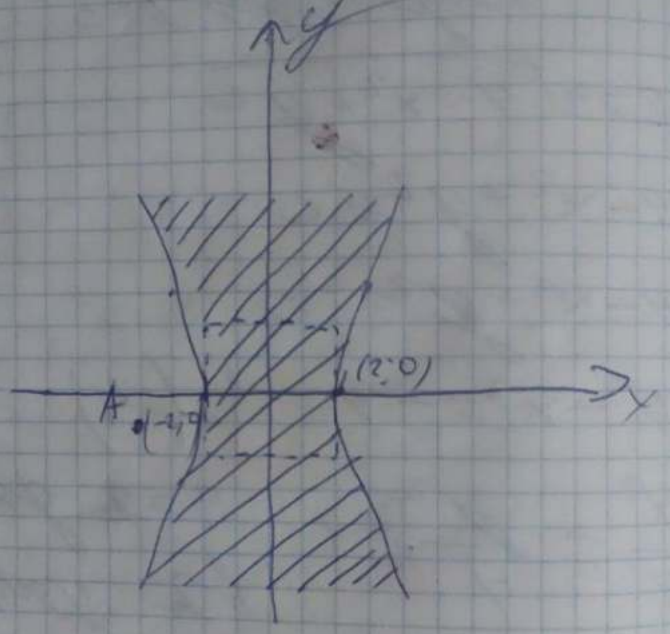
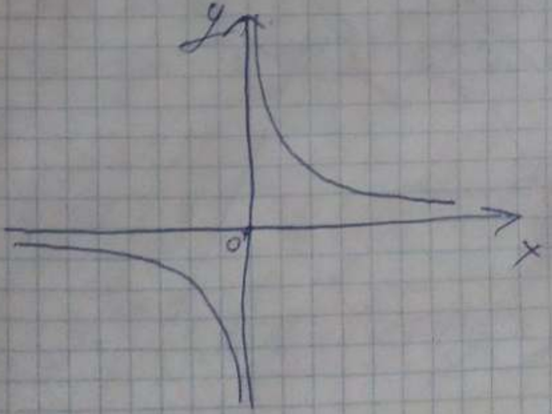

y
(2;0)
x
A
(-2;0)

2. $Z = e^{\frac{xy}{2}}$ $M(2;1)$

$e^{\frac{xy}{2}} = e$ $e^{\frac{2\cdot 1}{2}} = e$

$e = e$ $e^{\frac{xy}{2}} = e$ $\frac{xy}{2} = 1$

$xy = 2$ $y = \frac{2}{x}$



3. ~~Zx~~ $z = y^{x+1}$

$z'_x = y^{x+1}(\ln y \cdot (x+1)') = \ln y \cdot y^{x+1}$

$z'_y = (x+1)\, y^x = x y^x + y^x$

$z''_{xx} = y^{x+1} \cdot \ln y \cdot \ln y\, (x+1)' = \ln^2 y \cdot y^{x+1}$

$z''_{yy} = x \cdot x y^{x-1} + x y^{x-1} = x^2 y^{x-1} + x y^{x-1}$

$z''_{xy} = (\ln y)' y^{x+1} + (y^{x+1})' \cdot \ln y =$

$= \frac{1}{y} \cdot y^{x-1} + \ln y \cdot (x+1) y^x = y^x + \ln y \cdot$

$\cdot (x+1) \cdot y^x$

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4 $u = x^2 z^3 + \sqrt{y^3}$ $\bar{a}(-5; 12; 0)$

$M(2; 2; 4)$

$u'_x = 2xz^3 + 0 = 2xz^3$

$u'_y = \frac{3y}{2\sqrt{y^3}} = \frac{3y}{2\sqrt{y}} = \frac{3\sqrt{y}}{2}$

$u'_z = 3z^2x^2 + 0 = 3x^2z^2$

$f'_x(M) = 2\cdot 2\cdot 4^3 = 4^4 = 256$

$f'_y(M) = \frac{3\cdot 2}{2\cdot\sqrt{2}} = \frac{3}{\sqrt{2}}$

$f'_z(M) = 3\cdot 2^2\cdot 4^2 = 4^3\cdot 3 = 192$

$\cos\alpha = \frac{-5}{\sqrt{12^2+5^2}} = \frac{-5}{13}$

$\cos\beta = \frac{12}{\sqrt{12+5^2}} = \frac{12}{13}$

$\cos\gamma = \frac{0}{\sqrt{12+5^2}} = 0$

$\frac{\partial u(M)}{\partial \bar{a}}$

$= \frac{96}{13\sqrt{2}}$

a) $f'_x(M$

$f'_y(M$

$f'_z(M$

grad

$$\frac{\partial u(M)}{\partial a} = 256 \cdot \left(-\frac{5}{13}\right) + \frac{12}{13} \cdot \frac{3}{\sqrt{2}} + 0 =$$

$$= \frac{36}{13\sqrt{2}} - \frac{1280}{13} = \frac{18\sqrt{2} - 1280}{13}$$

a) $f'_x(M) = 256$

$f'_y(M) = \frac{3}{\sqrt{2}}$

$f'_z(M) = 192$

$\text{grad}\, f(-5; 12; 6) = 256\bar{i} + \frac{3}{\sqrt{2}}\bar{j} + 192\bar{k}$

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5. $z = 5 + 12x - 8y - 3x^2 - 2y^2$

$z'_x = 12 - 6x \qquad x = 2$

$z'_y = -8 - 4y \qquad y = -2 \qquad (2;2)$

$z''_{xx} = -6 \qquad z''_{yy} = -4 \qquad z''_{xy} = 0$

$$\begin{vmatrix} -6 & 0 \\ 0 & -4 \end{vmatrix} = 24 > 0$$

$z_{min} = 5 + 12 \cdot 2 + 8 \cdot 2 - 3 \cdot 2^2 - 2 \cdot (-2)^2 = 25$

$T(2;-2)$ – локальний екстремум,

а саме локальний максимум

$3(x^2 - 4x + 4) + 2(y^2 + 4y + 4) = 25 \,|: 25$

$\frac{(x-2)}{25/3}$

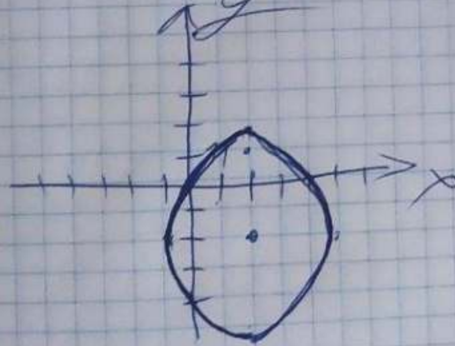
$a = \sqrt{}$

$b = \sqrt{}$

$$\frac{(x-2)^2}{25/3}+\frac{(y+2)^2}{25/2}=1 \quad y(2;-2)$$

$$a=\sqrt{\frac{25}{3}}\approx 2{,}9$$

$$b=\sqrt{\frac{25}{2}}\approx 3{,}5$$



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