Predicting the Pound: Volatility Forecasting in Foreign Exchange Markets

Comparing the forecasting performance of various volatility models on the GBP/USD and EUR/GBP currency pairs between June 2018 and June 2023.

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Abstract

In this paper, we study the volatility of the Great British Pound. Focussing on the GBP/USD and EUR/GBP currency pairs, we implement a range of models and analyse their ability to forecast the 20-day standard deviation of the pairs' daily returns using data spanning from the 15th of June 2018 to the 15th of June 2023. We specifically look at the use of EWMA, GARCH-type, and Implied Volatility models and compare the accuracy of their out-of-sample forecasts using the RMSE and MAE metrics. We delve deeper into the performance of the GARCH models, looking at their statistical properties when fitted to the in-sample data. From doing this we find evidence to suggest the presence of asymmetric returns in the EUR/GBP pair but are unable to do so for the GBP/USD pair. We also find the GARCH-type models better fit the in-sample data when assuming the residuals follow a Student's T distribution as opposed to a standard normal distribution. Additionally, we look into the best forecasting technique when using GARCH-type models, comparing the use of rolling window forecasts against expanding window forecasts but finding that there is no outright winner in the tested cases. From our experiments, we see that the best forecast of volatility for the GBP/USD currency pair comes from the use of GJR-GARCH models using a rolling window method. For the EUR/GBP pair, we see the best forecasts also come from the use of GJR-GARCH but also from the use of OLS Regression models which include the annualised implied volatility of the exchange rate as an independent variable.

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Chapter 1

Introduction

The volatility of a financial asset, as per Poon and Granger (2003) can be defined as the standard deviation of the returns on the asset over a given period of time. This value essentially describes how much the price of the asset fluctuates over time and although it does not exactly describe the risk associated with an asset, it does act as a quantitative measure of the uncertainty associated with taking a position in the asset.

The ability to forecast the volatility of exchange rates is of vital importance to international investors, whose profits can be directly exposed to exchange rate risk. When investors engage in overseas activities, their investments will often be required to be made in foreign currencies, thus requiring them to exchange their domestic currency on the foreign exchange market. If the currency they intend to buy is highly volatile, the investor will be exposed to significant risk. This risk occurs because the high fluctuations in the exchange rate can lead to losses on a profitable investment when the investor attempts to repatriate their profits into their domestic currency. Alternatively, the fluctuations could also lead to additional profit if the value of the domestic currency weakens.

Likewise, the profits and losses experienced by importers and exporters are directly affected by the volatility of the exchange rates of the currencies they trade with. When the domestic currency of an importer weakens against the domestic currency of the exporter, the cost of imports increases, potentially squeezing profit margins and leading to higher prices for domestic consumers. Conversely, when their domestic currency strengthens, importers may benefit from reduced import costs and increased purchasing power. These effects are the opposite for exporters. Periods of high volatility can lead to lots of uncertainty for importers and exporters and so accurate forecasts can be vital for driving business decisions.

Governments also need to take notice of foreign exchange volatility, particularly when their domestic currency is involved. Sudden and significant fluctuations in the exchange rate can have far-reaching implications for a country's economy. A depreciating domestic currency can lead to increased inflation, affecting the cost of goods and services for consumers and potentially putting pressure on the central bank to respond with higher interest rates. Moreover, a weakening currency may adversely impact foreign debt repayment obligations, making paying off debt more expensive.

Speculators often try to take advantage of volatility in financial markets. They seek to profit from rapid price fluctuations by buying low and selling high or vice versa, engaging in short-term trading strategies. Volatile markets offer numerous opportunities for speculators to capitalize on price movements across various assets, such as stocks, currencies, commodities, and cryptocurrencies. While volatility can present significant profit potential, it also comes with higher risks.

The importance of volatility in derivative pricing is also not to be understated. It acts as one of the key components in determining the value of call and put options, which are priced through the Black-Scholes model. Higher volatility leads to increased option premiums, reflecting greater uncertainty and potential for larger price movements. Conversely, lower volatility results in lower option premiums, indicating a calmer market with reduced potential for significant price swings. Traders and investors must closely monitor volatility levels as it directly influences their risk and reward assessments when engaging in options trading. Moreover, understanding and anticipating changes in volatility is crucial for effectively managing portfolio risks and optimising trading strategies in derivative markets.

Given the wide-reaching effects of market volatility on various industries and market participants, finding appropriate models to produce the best forecast of volatility in these markets is a necessary challenge faced by financial institutions around the world. This, however, is no straightforward task. The return on an asset is a random variable that is highly sensitive to a variety of factors, including market conditions, economic indicators, news events, and investor sentiment. As a result volatility data contains many characteristics which present significant challenges when attempting to generate forecasts.

One such characteristic is the clustering effect. This is the effect that describes how volatility tends to cluster into periods of high volatility and low volatility. Another characteristic is asymmetry, i.e. the fact that negative shocks in the market such as market downturns or negative news events may lead to larger and more rapid increases in volatility compared to positive events. This characteristic is more commonly associated with volatility data from equity markets, with the consensus being that this is caused by negative moves in a stock price increasing the leverage ratio of a firm and making it a more risky asset in the eyes of investors, thus leading to a further increase in volatility. This is known as the leverage effect (Christie 1982). Naturally, the leverage effect should not apply to foreign exchange markets as currencies are not related to a firm and as such do not have a leverage ratio. However, empirical studies show that asymmetry is still present in foreign exchange returns data, with explanations such as central bank interference and base currency effects being proposed as potential causes (Wang and Yang 2009).

In addition to these features, volatility is also not directly observable. This presents another challenge when trying to assess the accuracy of forecasts, as an appropriate proxy must be selected to represent the true volatility of the asset. There is no necessarily right or wrong proxy to use, with options such as absolute returns, squared returns, and realised volatility all making an appearance in the literature.

There are various traditional techniques that attempt to deal with these issues, such as Historic Volatility models, GARCH models and Options-Based Implied Volatility models. These techniques are all vastly different yet have been tested extensively over many decades and have been proven to be effective at tackling the challenge of volatility forecasting across a range of different assets. However, there is not a one size fits all approach to forecasting volatility, with different types of models, and different variants of these model types performing better in different markets.

Historic volatility models, such as Simple and Exponentially Weighted Moving Averages (SMA/EWMA), take historical observations of volatility and use simple functions to generate an expectation of future volatility. GARCH models also try to model the conditional variance of a time series based on historic data, treating it as a function of white noise, lagged values of variance, and lagged residual errors from estimates of returns. This variance is then used to calculate the conditional standard deviation of the asset, which is used to describe volatility. Options-Based Implied Volatility models on the other hand approach the issue by using derivative pricing models and observed derivative values to solve for Implied Volatility values, which give the market's current outlook on volatility.

The Great British Pound is a particularly interesting currency to monitor in the current financial climate. The United Kingdom is a major player on the international stage and as such the pound holds a significant role in international trade, investment, and financial markets. However, the UKs decision to leave the European Union, combined with the effects of the COVID-19 pandemic and the lack of stability at the government leadership level has led to a high level of uncertainty in the currency markets. Given its importance to international trade, any significant movements within the pound can also cause ripple effects across other financial markets.

In this project, we will examine how well a range of different models perform when attempting to forecast the volatility of exchange rates involving the pound. We focus on the daily returns of the GBP/USD and EUR/GBP currency pairs between the 15th of June 2018 and the 15th of June 2023, a time frame that encompasses the UK leaving the EU as well as the fallout from the COVID-19 pandemic. We will specifically look at the use of an Exponentially Weighted Moving Average model, four models from the GARCH family of models, namely GARCH, EGARCH, GJR-GARCH, and TGARCH, as well as the use of regression models using the Implied Volatility of the currency calculated from the Black-Scholes Options Pricing model. While the EWMA and IV-based models are relatively straightforward in their application, we will delve deeper into the GARCH models by testing various parameter sets and forecasting methods for the models to determine the best overall predictor for exchange rate volatility.

Chapter 2

Literature Review

The topic of volatility forecasting has been extensively surveyed by Poon and Granger (2003) and Poon and Granger (2005). These papers have laid the foundation for much of the research done over the last two decades. The work of the authors extensively highlights the importance of volatility forecasting, the broad array of different methods that have been utilised in the past, as well as the debate as to which methods provide the best results. Ultimately their papers conclude that volatility is indeed forecastable however there is not a one-size fits all solution to the challenge, although it appears to be agreed that Options-based ISD, Historic Volatility and GARCH models appear to be the more successful classes of models.

2.1 General Volatility Modelling

Despite being simple in nature, historic volatility models such as the Exponentially Weighted Moving Average models are considered effective and are highly regarded in the literature. Ding and Meade (2010) saw EWMA outperform more complex models such as GARCH and Stochastic Volatility (SV) when assets were in a period of medium volatility of volatility, i.e. the returns weren't exhibiting a high level of heteroscedasticity. They tested EWMA, GARCH and SV on the monthly returns on a range of assets, including the Euro/Dollar, Pound/Dollar and Yen/Dollar currency pairs, as well as various indexes, stocks and commodities. They compared the performance of the models using the RMSE, MME(O) and MME(U) metrics and consistently found EWMA to produce the most accurate forecasts over horizons between 1 to 5 months.

Ayele, Gabreyohannes, and Tesfay (2017) conducted a study to evaluate the performance of EWMA models against GARCH-type models containing explanatory variables. They tested various models on the volatility of Gold in the Ethiopian market. Their GARCH models added additional exogenous regressors, which included exchange rates and inflation rates, to further improve their accuracy and found that these models outperformed the EWMA models for forecasting when compared using the MAE, MAPE, RMSE and Theil inequality coefficient, with a GARCH-M providing the most accurate forecasting results.

The GARCH family of models originates with the GARCH model proposed by Bollerslev (1986) as an adaptation of the ARCH model proposed by Engle (1982). Since then various adaptations of the model have been proposed each attempting to tackle various challenges related to volatility trends. For example models such as GJR-GARCH (Glosten, Jagannathan, and Runkle 1993), TGARCH (Rabemananjara and Zakoian 1993; Zakoian 1994) and EGARCH (Nelson 1991) all aim to overcome the issue of asymmetric returns, whereas models such as IGARCH (Engle and Bollerslev 1986) and

FIGARCH (Baillie, Bollerslev, and Mikkelsen 1996) aim to model the persistence of conditional volatility among asset returns.

The original GARCH paper by Bollerslev (1986) did not actually focus on volatility forecasting but rather modeling the uncertainty of inflation rates on a quarterly basis. The paper contained an empirical study where the performance of the GARCH(1,1) model was compared against an ARCH(8) model (the class of model GARCH aims to improve upon) and found that it did provide a better fit and was deemed to have a more reasonable lag structure.

Various studies support the use of models in the GARCH family for the task of volatility forecasting. Akgiray (1989) finds that the ability of GARCH to model the volatility of daily movements in the price of the CRSP indices between 1963 and 1986 is consistently superior to that of Historical Estimates, EWMA and ARCH models across all used metrics (MSE, RMSE, MAE and MAPE).

Lim and Sek (2013) show evidence that the different types of GARCH models perform better in different economic environments. They tested GARCH, TGARCH and EGARCH on the malaysian stock market across the three periods before, during and after the Asian financial crisis. Their results find that TGARCH performs best during the pre and post-crisis periods whilst the original GARCH model performs best during the crisis, comparing the models in terms of MSE, RMSE and MAPE for each time period.

A study by Sahiner (2022) looked to identify which method of generating forecasts under the GARCH specification yielded the best results. Rolling forecasts use a limited number of recent observations to fit the GARCH model coefficients and generate forecasts, whereas expanding window (or recursive) forecasts fit the model to all available existing observations. In the study, they see expanding window forecasts dominate rolling window forecasts on the Asain Stock Market, when tested under the MAE, RMSE and MAPE metrics.

Bluhm and Yu (2001) test the ability of implied volatility models to predict volatility on the DAX index against EWMA, ARCH and SV models, but, their results are inconclusive as to which model provides the best overall forecasts. However, they did suggest that the most appropriate model selection varied depending on the purpose of the volatility forecasts. For forecasting volatility specifically for the purpose of monitoring Value-at-Risk, they suggested that the use of ARCH models was most appropriate. However, when focusing on options pricing, they found that implied volatility forecasts and stochastic volatility forecasts were more useful. This observation makes sense given that implied volatility forecasts are derived directly from options pricing models.

Fleming (1998) looks at the quality of implied volatility forecasts on the S&P 100 index. They see implied volatility forecasts dominate that of estimates based on historical data for out-of-sample forecasts. This is despite the fact that they see implied volatility as an upward biased forecast of volatility.

Brooks and Persand (2003) consider volatility forecasting from a Value-at-Risk perspective, they evaluate the performance of a range of models including EWMA, GARCH (assuming normal and T distributions), GJR-GARCH, EGARCH and a multivariate GARCH. Their dataset contained a range of UK based assets, namely government bonds, equities, and commodities as well as a equally weighted portfolio of the three. They test the models using traditional metrics such as MSE, MAE and proportion of over-prediction, but also using metrics that measure the effectiveness of the forecasts in a Value-at-Risk framework, such as the Time Until First Failure (TUFF). They see that the rankings of the forecasting methods vary depending on the asset and the evaluation metric used. However they do see that random walk, EGARCH and EWMA perform poorly whatever the case. In terms of statistical properties, they see GARCH(1,1) fit the data best, however, for forecasting Value-at-Risk, they claim that simple models such as long-term mean and AR(1) models work best.

Recently, there has also been notable research into the use of machine learning techniques to further refine the predictions made by GARCH models. For example, Kristjanpoller R and Hernández

P (2017) found success in using Artificial Neural Networks as an extension to GARCH by feeding in GARCH forecasts into the network along with various other explanatory variables in order to further refine the forecasts. They based their neural network on a multi-layered perceptron with two hidden layers each consisting of 5 neurons. Their study focused on predicting the volatilities of Gold, Silver and Copper. They found that use of neural networks reduced the error of the projected forecasts when compared to the standard GARCH model. Hamid and Iqbal (2004) used a neural network on its own (i.e. not hybridised with a GARCH model like in other studies) to predict the daily volatility of S&P 500 futures contracts. They input a range of daily financial data on other indices, commodities, and currencies into their model to get a forecast of volatility. They compared the performance of their model against forecasts of Implied Standard Deviation derived from the Barone-Adesi and Whaley futures options pricing model. In their results they found that their Neural Network model outperformed ISD forecasts at most horizons, adding furthering the argument for using ANNs in volatility forecasting. However, not all of this research revolves around the use of neural networks. Karasan and Gaygısız (2020) utilised a machine learning technique called Support Vector Regression (SVR) to augment the original GARCH model to create SVR-GARCH. They tested their design on a range of stocks listed in the S&P 500 index. Their results found that SVR-GARCH with a linear kernel outperformed all the other models tested including multiple versions of GARCH, GJR-GARCH, EGARCH, and FIGARCH.

2.2 Exchange Rate Specific Volatility Modelling

For specifically modeling the volatility of exchange rates, Pilbeam and Langeland (2015) show that GARCH struggles to compete with implied volatility forecasts derived from call options. Their study focused on the Euro, Pound, Japanese Yen and Swiss Franc individual exchange rates against the U.S. Dollar between 2002 and 2012. They also split the data into a period of low volatility and high volatility to compare the models forecasting ability in different market conditions. In both periods the IV forecasts proved superior.

Amin and Ng (1997) see implied volatility to dominate GARCH and GJR-GARCH when attempting to forecast the volatility of the euro-dollar futures and its associated options. They use a variety of models based on a framework proposed by Heath, Jarrow, and Morton (1992) to derive the implied volatility and find that it "explains much of the variation of realized interest rate volatility over both daily and monthly horizons".

Scott and Tucker (1989) examined the ability of Implied Standard Deviation methods to forecast the Pound, Canadian Dollar, Deutschmark, Yen, and Swiss Fanc. They use time horizons of 3,6 and 9 months, allowing them 195 contract types to use to assess volatility forecasts. They model the observed standard deviation of the asset as a simple OLS regression using the derived ISD value as the only independent variable. They see very strong performance from the options implied ISD model.

Whilst the other studies leaned towards the use of implied volatility methods over GARCH models for modeling currency volatility, a study by Hansen and Lunde (2005) found that there was potential in the use of GARCH models for this task, as they were unable to find any evidence that GARCH(1,1) could be outperformed by any other model in a pool of 330 ARCH-type models when tested on the daily exchange rates of the Deutschmark against the U.S. Dollar.

Andersen and Bollerslev (1998) also show that a GARCH(1,1) can make "strikingly accurate inter-daily forecasts" on the foreign exchange markets. Their study specifically focuses on forecasting the daily volatility of the exchange rate between the German Deutschmark and Japanese Yen against the U.S. Dollar.

Hsieh (1989) implements ARCH and GARCH models to model the volatility of the exchange rates of the Great British Pound, Canadian Dollar, the Deutschmark, the Japanese yen and the Swiss Franc all against the U.S. Dollar. They do this using daily closing prices of each currency pair during a period spanning from 1974 to 1983. They find that GARCH(1,1) and EGARCH(1,1) fit the data extremely well, with EGARCH(1,1) just coming out ahead, which they put down to the EGARCH not resulting in integrated variances. They also conclude that the residuals in the GARCH models are highly leptokurtic and that several non-normal distributions appear to result in a better fit for certain currency pairs.

Ederington and Lee (2001) examine the intraday volatility of exchange rates and interest rates, attempting to understand what data is necessary to be able to forecast this accurately. They conclude that a combination of ARCH modelling, knowledge of the time of macroeconomic announcements, and knowledge of time-of-day/day-of-week patterns should be used in conjunction to determine intraday volatility. However, they specify that of these three, knowing when macroeconomic new is going to be announced is the most important data. Another important takeaway from their study is that they do not observe any notable asymmetry in the daily timeframe when looking at Deutschmark-Dollar futures.

Naimy et al. (2021) look at the capability of a variety of GARCH-type models to forecast both real-world currencies and crypto-currencies. They look at the daily returns of the Pound, Euro, Yen, Australian & Canadian Dollars against the pound as well as a selection of crypto-currencies, sampling data from a period between October 2015 and November 2019. For the fiat currencies, they found that an IGARCH model performs best on both the in-sample and out-of-sample sets for all currencies except the Euro, where they found a CGARCH model to provide the best performance in both sets. The forecasts were evaluated under the RMSE, MAE and MAPE metrics against realised volatility.

Busch, Christensen, and Nielsen (2011) investigates the forecasting of realised volatility in multiple markets, including the foreign exchange, by using implied volatility derived from options prices. They observe that implied volatility contains incremental information about the future volatility of the markets tested and that in the foreign exchange (and stock) market, implied volatility provides an unbaised forecast.

Balaban (2004) conducts a study to compare the forecasting performance of symmetric and asymmetric GARCH-type models on the daily returns of the U.S. Dollar/Deutschmark. They find that all of the models tested (ARCH, GARCH, GJR-GARCH and EGARCH) consistently overpredict volatility. They conclude that the predictive power of a standard GARCH model is the highest for the monthly exchange rate volatility and that GJR-GARCH is the least powerful.

Jorion (1995) also finds implied volatility to provide biased forecasts, in their study on their applications to the foreign exchange market. Looking at the Deutschemark, Japanese Yen, and Swiss Franc exchange rates, they see that even when given ex-post parameter sets, statistical models (such as GARCH) are unable to outperform Implied Volatility for out-of-sample forecasting.

2.3 Data Properties

On the discussion of asymmetric returns in currency pairs, a study by Wang and Yang (2009) finds empirical evidence that asymmetry is present in exchange rate returns, finding that when traded against the U.S. Dollar, the Australian Dollar, Great British Pound and Japanese Yen exhibit an increase in volatility of 6.6%, 6.1% and 21.2% respectively following a one-standard-deviation negative return compared to a one-standard-deviation positive return. However, they find the realised volatility of the Euro against the dollar to be symmetric. They also present plausible explanations as to why this could be the case. One such explanation is the base-currency effect, which is based on the idea that some currencies (such as the US Dollar) are more economically dominant than others and so depreciation against this currency may lead to the sale of assets in the non-dominant currency thus leading to a further decrease in the exchange rate. The other explanation given is the central bank intervention effect, which describes the idea that a central bank's response to a negative movement in its domestic currency only affects one side of the market and can result in asymmetric returns.

The work of McKenzie (2002) provides support for this central bank intervention explanation by conducting a study on the daily intervention data for the Reserve Bank of Australia. Their results found that large sales of foreign reserves by the central bank was found to result in a more significant response from the market than in periods where the bank purchased foreign reserves. They conclude that governments' and central banks' attempts to settle the market are ultimately futile given that the market appears to watch and react to the sales of foreign reserves by these bodies.

More recently, Baur and Dimpfl (2018) find the opposite effect to be true for cryptocurrencies. While they find asymmetry to be present in cryptocurrency returns, they find the effect to be positive, i.e. that a significant positive return results in increased volatility. They come to this conclusion by fitting a TGARCH model to the returns data of various cryptocurrencies and finding the γ coefficient to take a negative value in all cases, which suggests that volatility generally decreases after a negative movement. They attribute this finding to the actions of uninformed irrational investors acting on the fear of missing out and the existence of pump-and-dump schemes.

One of the key challenges discussed in the literature is that of choosing an appropriate proxy for the true value of volatility to use when evaluating forecasts. Some studies make use of absolute returns, or squared returns, whereas others such as Koopman, Jungbacker, and Hol (2005) use realised volatility. Akhtar and Spence-Hilton (1984) made use of the standard deviation of returns as a proxy for uncertainty/volatility when measuring its effect exchange on German-U.S. trade.

Patton (2011), however, performs a deep analysis of this problem, evaluating the effectiveness of a range of proxies. They conclude that the use of proxies such as absolute returns and squared returns are too noisy and causes significant distortion to the rankings of forecasting methods. As a result, they suggest that the use of less noisy proxies such as realised volatility and intra-daily range may lead to less distortion.

Naimy et al. (2023) recently conducted a study of the impact of the GBP/EUR exchange rate volatility on the UK's exports to the Eurozone in a post-Brexit landscape. They model the volatility of the GBP/EUR exchange rate both before and after Brexit using EWMA, GARCH and EGARCH models. While their focus was not on the forecasting of this volatility, they did find that a GARCH(1,1) model provided the best model of volatility in both the pre and post-Brexit eras. They also found evidence of asymmetric returns, with the γ value in their EGARCH model taking a statistically significant negative value, implying that negative shocks to the pound's value against the euro lead to more volatility than a positive one.

Donaldson and Kamstra (2005) look at the importance of trading volume in the forecasting of volatility. They find that for daily and monthly data, the trading volume in the previous period affects which forecasting methodology is the most appropriate. They conclude that when trading volume is low in the previous period, ARCH modeling of volatility is at least as important as implied volatility. However, in periods of high trading volume, implied volatility is much more useful for forecasting future volatility.

* * *

Ultimately, the literature on volatility forecasting is complex and varied. The important takeaway is that volatility is indeed forecastable, however, it appears that the unique traits of data pertaining to a particular asset, time frame, and/or proxy for volatility will affect the ranking of different forecasting methods. This is also the case depending on the purpose of the forecasts and the evaluation metrics used.

Chapter 3

Methodology

3.1 Data Exploration

For the experiments in this project, data was collected from yahoo finance containing the daily market prices of each of the currency pairs. Using the `pandas` package in Python, the daily returns were obtained from this data by calculating the change in close prices across sequential days.

3.1.1 GBP/USD

The first step in the project is to explore the data to get a better understanding of any underlying patterns. To do this we first plot the daily price of the GBP/USD pair.

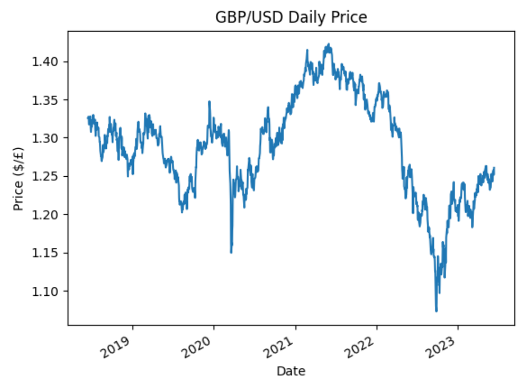

Figure 3.1: Plot of the daily price of the Pound Sterling against the US Dollar

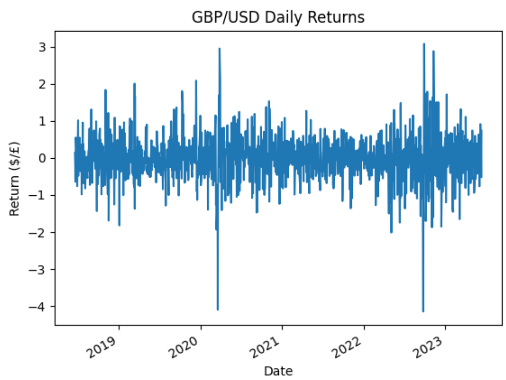

Figure 3.2: Plot of the daily returns of the Pound Sterling against the US Dollar

As can be seen from Figure 3.1 the period between June 2019 and June 2023 has been very turbulent for the pound. The high levels of volatility exhibited are to be expected given that throughout this period we have witnessed significant economic events such as the COVID-19 pandemic and a crash caused by the economic uncertainty in the fallout of the 'mini-budget' proposed by Liz Truss' government in 2022.

We can better observe the volatility of the pound by plotting the daily returns (i.e. the changes in price) of the asset across this time period. In Figure 3.2, we can see the significant spikes in daily price changes around the time of the aforementioned events more easily than in the previous graph. However, the graph does not necessarily provide too much useful statistical insight into the data other than showing that the daily price changes usually stay within a range of +/- 2% (with these economic shocks proving to be an exception).

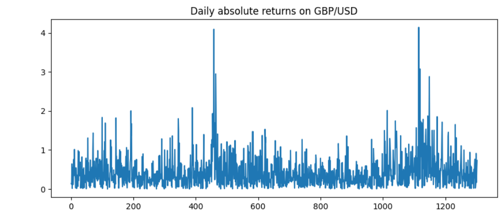

Figure 3.3: The absolute values of the daily returns of the GBP/USD pair

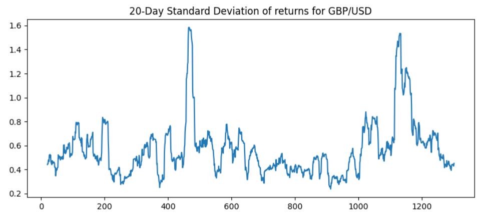

Figure 3.4: The 20-day standard deviation of the daily returns of the GBP/USD pair

As can be seen from Figure 3.3, which shows the absolute daily returns of the pair, the returns data is very noisy and as such it is difficult to identify a consistent underlying pattern in the data. Furthermore, as mentioned in 2.3, Patton (2011) show that there are negatives to using absolute returns as a volatility proxy. Therefore, in Figure 3.4, we plot the 20-day standard deviation of the returns (as per the definition of volatility in Poon and Granger (2003)) to get a clearer view of the change of volatility over time. This is also the proxy for volatility we will use to evaluate the accuracy of our forecasts. Whilst the standard deviation could be taken over any number of days, depending on the purpose of the forecasts, 20 was chosen as this roughly corresponds to one trading month, a time frame which felt appropriate for the previous examples of imports and exports and international investment. From Figure 3.4 the overall trend in volatility becomes much smoother and more meaningful, which should in turn make it easier to forecast.

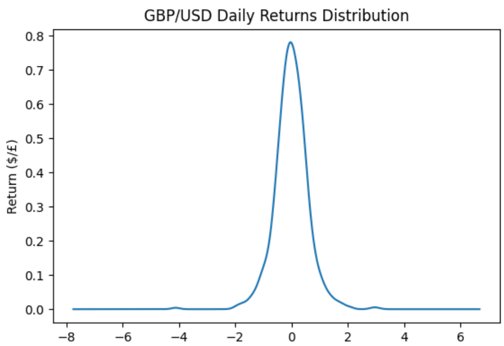

Figure 3.5: The distribution of the daily returns of the GBP/USD pair

```
count    1302.000000
mean       -0.002039
std         0.603512
skew       -0.188341
kurt        4.860408
min        -4.144021
25%        -0.329588
50%        -0.005832
75%         0.331654
max         3.077150
Name: Returns, dtype: float64
```

Figure 3.6: Python terminal output containing statistics on the distribution of GBP/USD returns

To gain further useful insight into the data, we plot the distribution of the daily returns (Figure 3.5) and use the Python `pandas` library to generate a description of its statistical properties (Figure 3.6). By combining the graph and the statistics, we can see that the returns form the bell-shaped curve associated with a normal distribution, with a slightly negative mean of -0.002039 and a standard deviation of 0.604. We can see from the graph that the is fairly symmetrical, albeit with a very slight negative skew of -0.188 suggesting that there are more (or at least more significant) negative daily returns than positive ones. We also see a very high kurtosis value (4.860) suggesting that the data follows a leptokurtic distribution, which can be observed in the graph of the distribution, characterised by its tall and slim shape, and fatter tails relative to the standard normal distribution. This suggests that outliers are more common than in normally distributed data, which seems logical for volatility data given that there can be random and significant economic events such as the outbreak of the COVID-19 pandemic. Given these observations, it could be that a Student's T-distribution may be a better model of the actual distribution of the returns data. As such, when experimenting with the GARCH models, we will run multiple tests assuming both Normal and T distributions in order to see which provides the most accurate forecasts.

3.1.2 EUR/GBP

Next, we look at the data for the GBP/EUR pair. In the same manner, as with the GBP/USD pair, we plot the daily price and the daily price change (returns) of the exchange rate.

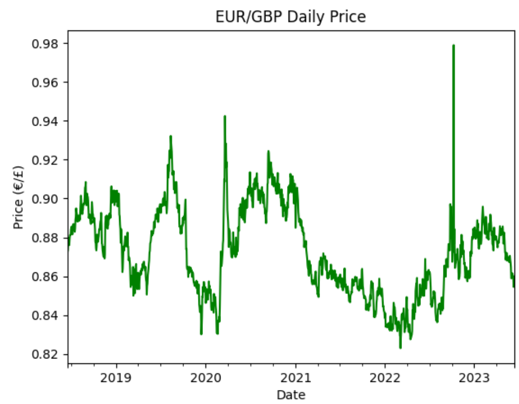

Figure 3.7: Plot of the daily price of the Euro against the Pound Sterling

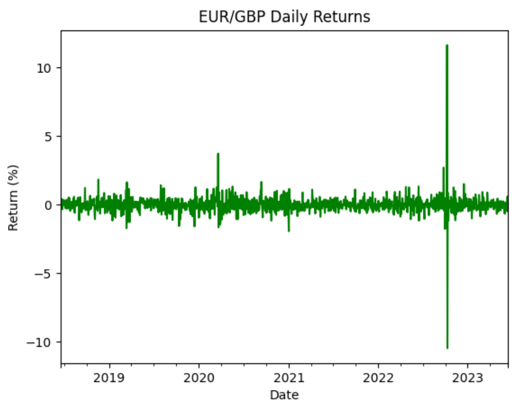

Figure 3.8: Plot of the daily returns of the Euro against the Pound Sterling

Similar to with the GBP/USD pair, we can see from Figures 3.7 and 3.8 that the period between June 2019 and June 2023 was also a very volatile time for the EUR/GBP pair. We also see similar patterns of volatility spikes around March 2020 and September 2022 in response to the COVID-19 pandemic and the pound crash. However, we also see a much more significant spike in volatility around October 2022, with the value of the euro briefly rocketing up by over 11%.

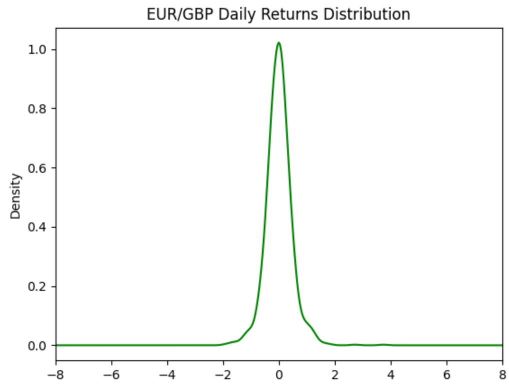

Figure 3.9: The distribution of the daily returns of the GBP/USD pair

```
count    1303.000000
mean        0.000481
std         0.630960
skew        1.539997
kurt      147.077338
min       -10.461696
25%        -0.252137
50%        -0.003372
75%         0.226179
max        11.640743
Name: Returns, dtype: float64
```

Figure 3.10: Python terminal output containing statistics on the distribution of EUR/GBP returns

From graphing the distribution of the data (Figure 3.9) and analysing the distribution statistics (Figure 3.10) we can see that this data set has a notable positive skew (1.539997) and is significantly more leptokurtic (with kurtosis of 147.00738). These values, particularly kurtosis, seem to be extreme and likely the result of an erroneous value in October 2022. Given the significant damage to the dataset caused by the data point, it was smoothed out by replacing the close price with the midpoint between the close price from the previous and following days.

Having done this, re-plotting the graphs results in much nicer-looking data with more reasonable statistical properties.

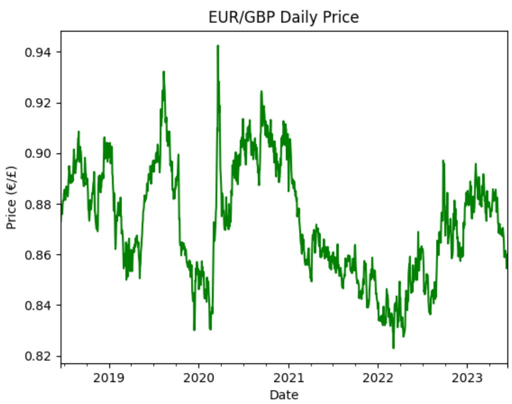

Figure 3.11: Plot of the daily price of the Euro against the Pound Sterling (after cleaning data)

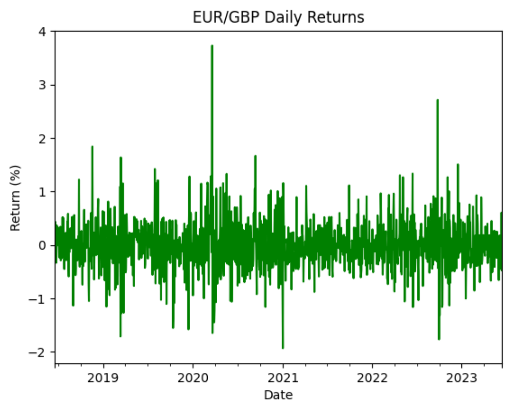

Figure 3.12: Plot of the daily returns of the Euro against the Pound Sterling (after cleaning data)

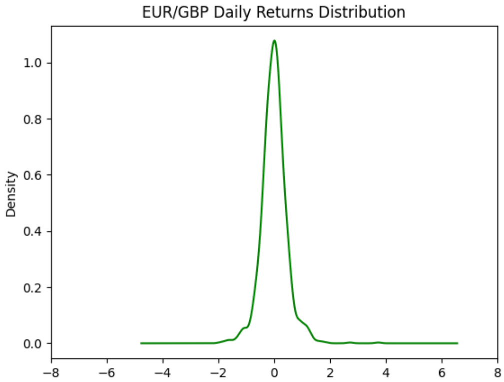

Figure 3.13: The distribution of the daily returns of the EUR/GBP pair (after cleaning data)

```
count      1303.000000
mean         -0.000454
std           0.458229
skew          0.5724768398191523
kurt          5.546744336760412
min          -1.932710
25%          -0.251678
50%          -0.003515
75%           0.225068
max           3.729873
Name: Returns, dtype: float64
```

Figure 3.14: Python terminal output containing statistics on the distribution of EUR/GBP returns (after cleaning data)

The distribution of the returns data now appears a lot more like that of the GBP/USD. This distribution now has a kurtosis of 5.5467, slightly higher than the GBP/USD but also indicating a leptokurtic distribution. The skew of the data this time is slightly positive however the skew value is still low enough for the distribution to be considered relatively unskewed. The distribution also has a very slight negative mean (-0.000454) and a standard deviation of 0.458229.

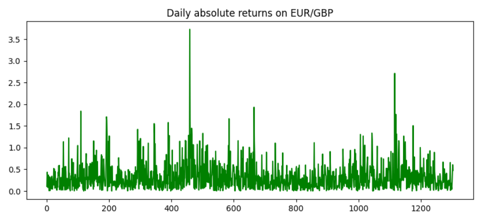

Figure 3.15: The absolute values of the daily returns of the EUR/GBP pair

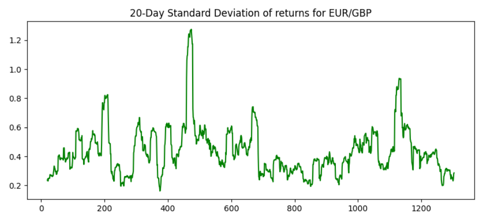

Figure 3.16: The 20-day standard deviation of the daily returns of the EUR/GBP pair

Again, in Figures 3.15 and 3.16 we see the noisy nature of absolute returns and the smooth, more forecastable nature of the 20-day standard deviation data respectively.

* * *

For both of these datasets, we split the data set into an in-sample and out-of-sample set, retaining the last 365 observations for the out-pf-sample set. The in-sample set is used to train the GARCH-type and OLS models and evaluate their goodness of fit/statistical properties, whereas, the out-of-sample set is used to test the forecasting performance of all models.

3.2 Exponentially Weighted Moving Average

The first model used in the experiments was the Exponentially Weighted Moving Average (EWMA) model. This is one of the simplest methods of forecasting volatility based on historic data, yet has proven to be effective.

$$\text{EWMA}(\sigma_t) = \lambda * \sigma_t + (1 - \lambda) * \text{EWMA}(\sigma_{t-1}) \tag{3.1}$$

The premise of the model is to take a weighted average of past volatility values and use this as an expectation of future market volatility. When taking the average, a higher weight is applied to more recent observations, under the assumption that the more recent observations have a more significant impact on future volatility. The weight assigned to each observation is controlled by a single parameter λ. As per Morgan (1996), we set the value of λ to equal 0.97, as we are looking to forecast the 20-day standard deviation of the currency pairs and the study suggests this value to be the most appropriate monthly data.

The main benefit of using such models is that it is very computationally efficient and so can be used to quickly make predictions of future volatility based on the available historic data. The downside is that the model's simplicity does not allow it to capture some of the more complicated characteristics of volatility data, such as clustering and asymmetry effects.

3.3 GARCH Models

The next four types of models that were used in the experiments all come from the GARCH family. Specifically, GARCH, EGARCH, GJR-GARCH, and TGARCH were all implemented using the `arch` package in Python.

3.3.1 GARCH

Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models are one of the most commonly used methods of volatility forecasting. Their initial proposal came from Bollerslev (1986) as an extension to the ARCH family of models, adapted to better capture large spikes in the movement of a time series variable.

Traditional ARCH models, as first described by Engle (1982) work under the assumption of conditional heteroscedasticity, i.e. the idea that the variance of the assets returns is non-constant over time. The ARCH(p) model, therefore, treats the conditional variance of the asset as a linear function of the last p squared residuals. Specifically, ARCH(p) aims to model the variance of a time series as a white noise (ω) component plus p lagged squared residuals under a regression-like framework. The conditional volatility can be solved by taking the square root of the conditional variance.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \tag{3.2}$$

The GARCH(p, q) model extends this method by incorporating q past conditional variances into the regression equation. This allows the model to better capture the clustering and persistence characteristics of time series volatility.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta \sigma_{t-j}^2 \tag{3.3}$$

The GARCH$(1,1)$ model is generally considered to be the strongest-performing parameter set for modeling asset volatility, as this has been demonstrated across numerous studies. In the GARCH$(1,1)$ model, the p and q parameters both take on the value of 1. This means that the equation above would contain only the white noise component, immediate lagged squared residual, and immediate lagged conditional variance.

Whilst based on the previous literature we expect that setting the values of p and q to 1 to result in the strongest performing GARCH model, we will iteratively test the parameters up to the 5th lag on the in-sample data to find which parameter sets perform the best. To do this, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) metrics will be used as these evaluate how well a model fits into a dataset.

3.3.2 EGARCH

The EGARCH model is an adaptation of GARCH proposed by Nelson (1991) which attempts to tackle the challenge of asymmetry inherent in volatility trends. The previous ARCH and GARCH models assume volatility effects to be symmetrical. That is, a negative shock in the market will have the same effect as a positive one (despite the existence of empirical evidence that this is not the case).

$$\log\left(\sigma_t^2\right) = \omega + \sum_{i=1}^{q}\left[\alpha_i \varepsilon_{t-i} + \gamma_i\left(|\varepsilon_{t-i}| - E\,|\varepsilon_{t-i}|\right)\right] + \sum_{j=1}^{p}\beta_j \log\left(\sigma_{t-j}^2\right) \tag{3.4}$$

The first key change EGARCH makes is that it relaxes the constraint that the parameters α and β must be non-negative, which is present in the standard GARCH specification. The second, and more notable, change it makes is that it uses (and models) the logarithm of the variance of the returns, as opposed to just the variance values as with the standard GARCH specification. The third change it makes is the addition of a second coefficient, γ, which estimates the effect of the absolute value of the residual subtracted from its expected value on the logarithm of the variance.

3.3.3 GJR-GARCH

The GJR-GARCH model introduced by Glosten, Jagannathan, and Runkle (1993), extends the basic GARCH model by incorporating some of the rationale behind EGARCH in another attempt to capture asymmetric effects in volatility. They incorporate an additional parameter that allows for different responses to positive and negative news/shocks within financial markets.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\left[\alpha_i + \gamma_i I\right]\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j \sigma_{t-j} \tag{3.5}$$

In this model, the value I_{t-1} is a binary variable that takes a value of 1 if the residual at $t-1$ is less than zero (indicating a negative shock to the market) and 0 otherwise. γ is an estimated coefficient that essentially quantifies how much of an additional effect negative shocks have on future volatility. Combining these parameters allows for negative residuals to be modeled independently to positive residuals and thus logically should improve the accuracy of the fit to the data.

3.3.4 TGARCH

The Threshold GARCH model (TGARCH) provides a similar adaptation to that described by GJR-GARCH, with it too attempting to overcome the issue of asymmetric returns. The work results from 2 studies by Rabemananjara and Zakoian (1993) and Zakoian (1994).

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i \left[(1 - \gamma_i)\, \varepsilon_{t-i}^{+} - (1 + \gamma_i)\, \varepsilon_{t-i}^{-} \right] + \sum_{j=1}^{p} \beta_j \sigma_{t-j} \tag{3.6}$$

Where GJR-GARCH regresses for conditional variances, using lagged residuals errors and lagged conditional variances, which is then used to solve for standard deviation (volatility), TGARCH regresses directly for standard deviation by replacing squared residuals with just the absolute residuals and the lagged conditional variances with lagged conditional standard deviations.

The rest of the specification is similar to that of the previous models. TGARCH also trains a γ parameter and uses it such that the coefficient of ε can be modeled differently depending on whether it takes a positive or negative value.

3.3.5 Maximum Likelihood Estimation

The coefficients used in the GARCH specification are calculated using the Maximum Likelihood Estimation algorithm. This process works by first formulating a likelihood function; a function that describes how likely the sample data is to be observed given the coefficients in the GARCH model equation. The natural logarithm of this function is then taken to give the log-likelihood. Finally, the values of the coefficients are optimised to find the values which maximise the log-likelihood. The standard MLE process works under the assumption that the residuals are normally distributed. However, as we have discussed, the returns data may better fit a Student's T distribution. As a result, when fitting a GARCH model under the assumption of a Student's T distribution, the Quasi-MLE algorithm will be used. This method relaxes the assumption of normally distributed residuals, making it more appropriate for non-normally distributed data.

3.3.6 Forecasting with GARCH models

In the Python `arch` package, there are multiple methods that can be used to generate forecasts for the GARCH-type models. The two most useful methods are rolling window forecasts and expanding window forecasts. Rolling window forecasts fit the model to the last n observations and use that to generate a forecast of the daily conditional variances of the returns for the next 20 days. The mean is taken from these values and then the square root is calculated to represent the 20-day ahead forecast of the standard deviation. Expanding window forecasts, on the other hand, perform the same task but instead use all the observed data points available to generate the forecast, all the way from when $t = 0$ to the most recent observation. There is a rationale to support the use of both methods. On one hand, it could be argued that using an expanding window is beneficial as more data is available on which to make predictions and therefore should better fit to the data. However, it could also be argued that because volatility is time-varying, an expanding window may negatively impact how well the model fits the more recent data, due to the influence of older and potentially less relevant data. In order to see which forecasting methodology is most suitable for our data, both rolling and expanding window forecasts will be implemented and tested against one another. However, as per Sahiner (2022), the expectation is that the expanding window forecasts will outperform the rolling window method.

3.4 Options Implied Standard Deviation

The method utilised by Options Implied Standard Deviation models differs significantly from that of the GARCH models. These models make use of options pricing models such as Black-Scholes and reverse engineer them using observed option prices to calculate the implied volatility of the option at that point in time. This value of implied volatility is described as the markets' outlook on the future volatility of the underlying asset.

The Black-Scholes model defines the price of a call option using the following equation:

$$C_t = N(d_1) S_t - N(d_2) K e^{-rt} \tag{3.7}$$

where

$$d_1 = \frac{\ln \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}} \tag{3.8}$$

and

$$d_2 = d_1 - \sigma \sqrt{t} \tag{3.9}$$

In the equation C_t is the Call price, S_t represents the strike price of the option, K is the underlying price of the asset, r represents the risk-free rate, σ represents implied volatility and t represents the time until maturity. Once the spot S_t and call C_t prices have been observed in the market, they can then be plugged into the equations and rearranged to solve for implied volatility σ.

The experimental procedure carried out to test how well Options ISD applied as a forecast of future volatility involved first taking the annualised implied volatility data for each asset from the Bloomberg Terminal. A common procedure is to scale this data down by a factor of $\sqrt{\frac{n}{252}}$ under the assumption of there being 252 trading days in a year, where n is the number of days over which to forecast the volatility. This value is then just used as the forecast. However, as part of their study, Scott and Tucker (1989) use two simple OLS regression models to forecast the realised volatility, one using solely the implied volatility as a parameter and another using the implied volatility and historic volatility as parameters. Following on from this idea, we craft the following regression equations to model volatility based on the annualised data collected from Bloomberg.

$$\hat{\sigma}_t = \beta_0 + \beta_1 \sigma_{\text{implied},t-1} + u_t \tag{3.10}$$

$$\hat{\sigma}_t = \beta_0 + \beta_1 \sigma_{\text{implied},t-1} + \beta_2 \sigma_{\text{actual},t-1} + u_t \tag{3.11}$$

The regression will be trained on the training subset, i.e. the initial 898 data points. Once this model is trained, it will be applied to the remaining 365 implied volatility data points (test set) to generate 20-day ahead forecasts of volatility so that the model can be compared with the others.

3.5 Evaluation Metrics

3.5.1 In-Sample

AIC & BIC

To evaluate the in-sample performance of the various GARCH and OLS models, and therefore determine the best set of model parameters, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) metrics will be used. These metrics describe how well the model fits the data whilst balancing for model complexity.

The AIC metric is defined as:

$$AIC = 2k - 2ln(L) \tag{3.12}$$

where k represents the number of parameters in the model and $ln(L)$ is the natural logarithm of the likelihood function of the model the same as that described in 3.3.5.

The BIC metric is very similar to AIC and is defined as:

$$BIC = k \times ln(n) - 2ln(L) \tag{3.13}$$

where k is the number of parameters in the model, $ln(n)$ is the natural logarithm of the sample size, and $ln(L)$ is the natural logarithm of the likelihood function of the model.

For both of these metrics, the aim of a model is to have as low a score as possible. The key difference between AIC and BIC is that BIC places a more significant penalty on the number of parameters used in the model, especially for a small sample size, thus rewarding simpler models over more complex ones.

When using these metrics to determine the best parameter set for the GARCH models the plan is to take the model(s) with the lowest AIC and BIC scores, for each model type (i.e. GARCH, GJR-GARCH and TGARCH) to compare both amongst each other and against the other forecasting methods.

3.5.2 Out-of-Sample

While understanding how the various models fit to historical data can be important for understanding some of the statistical properties of the volatility data, the main focus of this study is to see how well they can perform when predicting future volatility. Therefore, to test their ability to accurately predict future volatility in the out-of-sample set, the following metrics will be used.

RMSE

The first evaluation metric that will be used to compare the different classes of models is the Root-Mean-Squared Error metric. This metric describes how accurately the forecast generated by the model matches the actual observed values. This is calculated using the following formula.

$$RSME = \sqrt{\frac{\sum (\mathrm{y}_i - \hat{\mathrm{y}}_i)^2}{n}} \tag{3.14}$$

where y_i is the observed data value, $\hat{y}_i$ is the forecast value at time i and n is the number of observations in the test set.

The benefit of using this metric over the standard Mean-Squared-Error (MSE) is that RMSE provides a term on the same scale as the predicted and observed data, making the results more understandable.

MAE

The next evaluation metric that will be used is the Mean Absolute Error (MAE). This too is a measure of the accuracy of the forecast and is given by the following formula.

$$MAE = \frac{\sum \mathrm{y}_i - \hat{\mathrm{y}}_i}{n} \tag{3.15}$$

where y_i is the observed data value, $\hat{y}_i$ is the forecast value at time i and n is the number of observations in the test set.

Chapter 4

Results

4.1 EMWA Model

4.1.1 Forecasts

The first set of results we look at are the forecasts generated by the EWMA models. In our forecasts, we are looking last 365 days in the dataset.

Metrically, the model seems to give reasonable forecasts for both currency pairs, with an RMSE of 0.214 and an MAE of 0.136 for the GBP/USD pair, and an RMSE of 0.146 and an MAE of 0.095 for the GBP/USD pair (as shown in Figure 4.1.

	RMSE	MAE
GBP/USD	0.214	0.136
EUR/GBP	0.146	0.095

Figure 4.1: RMSE and MAE metrics for the forecasts generated by the EWMA model for the GBP/USD and EUR/GBP pairs

However, when graphing the forecasts (as done in Figures A.1 and A.2), we can see that the forecasts are not actually very useful, in a practical sense, across this type of forecasting horizon (20 days). As expected from the EWMA model, the forecasted values follow a very similar, albeit slightly smoothed, pattern to that of the actual values. However, this pattern significantly lags behind the observed standard deviations, meaning that the forecasts don't adapt very well to sharp changes in volatility, leading to predictions of high volatility when what is observed are actually low levels and vice versa. Using these forecasts to drive financial decisions would likely lead to bad outcomes. The disparity between the forecast and actual values would likely not be as severe if the predictions were used as a forecast over a shorter horizon (such as 1-day ahead), as although the model would probably not predict any sharp changes in volatility, the predictions would rectify themselves more quickly once volatility has stabilised.

4.2 GARCH-type Models

4.2.1 In-Sample fit

Our next set of results to analyse is that of the fit of different parameter sets for each of the GARCH specifications to the in-sample GBP/USD data. These were achieved by generating multiple models for each GARCH specification by iterating through the values of p and q from 1 to 5 and comparing the AIC and BIC scores of each parameter set with the others for that specification. For this part of the experiment, it is assumed that the residuals are normally distributed.

The figures A.3 to A.10 show heatmaps of the AIC and BIC scores for each parameter set tested for each model specification. From these we can see for the standard GARCH model, parameter sets of $p = 2$ and $q = 2$ return the lowest AIC score whereas $p = 1$ and $q = 1$ provide the lowest BIC score. For the EGARCH specification, we see that a parameter set where $p = 3$ and $q = 1$ provides the lowest AIC score and a set of $p = 1$ and $q = 1$ provides the lowest BIC scores. For TGARCH we see a parameter set of $p = 1$ and $q = 1$ provide the lowest scores for both metrics. For GJR-GARCH we see similar results as with the standard GARCH model, with $p = 2$ and $q = 2$ providing the best AIC score and $p = 1$ and $q = 1$ providing the lowest BIC score. Overall when comparing their ability to fit the in-sample data, we see that the best AIC score is obtained by using a GARCH(2,2) model and the best BIC score is obtained from a GARCH(1,1) model. The fact that standard GARCH models return the best in-sample fit could suggest that in this currency pair, the returns are relatively symmetrical. If this was not the case, it would make sense for at least one of EGARCH, GJR-GARCH or GJR-GARCH to improve on the standard model specification.

This process was repeated for the EUR/GBP pair. The results can be see in in Figures A.11 to A.18. For this currency pair, a parameter set of $p = 1$ and $q = 1$ returned the best AIC and BIC scores for all model specifications. The best AIC score was achieved from a TGARCH(1,1,1) model and the best BIC score came from a GARCH(1,1) model. In contrast to the GBP/USD pair, this could suggest that returns in this dataset do exhibit asymmetry, as the purpose of TGARCH is to tackle this property and it appears to be one of the best-fitting models. However, we can not confirm this theory until we analyse the t-statistics of the coefficients within the TGARCH model.

The fact that models with a lower number of lags resulted in the best AIC and BIC scores is unsurprising given that both metrics actively penalise the use of more variables. Furthermore, with BIC imposing an even heavier penalty on this than AIC, the fact that for all model specifications and on both datasets, a parameter set of $p = 1$ and $q = 1$ resulted in the best BIC score also appears to be logical.

As discussed in 3.1.1, it is possible the data might be better suited to a Student's T distribution than a Normal distribution. Whilst the best parameters for the different model specifications were determined under the assumption that the residuals followed a normal distribution, the `arch` package allows for this to be changed to a Student's T distribution. Therefore, the models with the best AIC and BIC scores were re-estimated under the assumption that the residuals followed a Student's T distribution. Figure 4.2 shows that making this change significantly reduced the AIC and BIC scores for all model specifications and parameter sets. Therefore when generating forecasts of future volatility to compare with the other volatility models, we assume the residuals to follow a Student's T distribution over the Normal distribution.

Appendix A.3 contains the summaries of the model statistics for each of the GARCH specifications when fitted to the GBP/USD and EUR/USD returns data, under the assumption of a Student's T distribution. This data was collected from the terminal output of the `arch` Python package when generating the models. By focusing on the Volatility Model sections of each summary, we can observe the statistical significance of each coefficient in the models to better understand how

Specification	Distribution	GBP/USD		EUR/GBP	
		AIC	BIC	AIC	BIC
GARCH(1,1)	Normal	1444.42	1463.78	1079.99	1099.36
	Student's T	**1409.54**	**1433.74**	**1008.65**	**1032.86**
GARCH(2,2)	Normal	1443.23	1472.27	1083.57	1112.61
	Student's T	**1411.08**	**1444.95**	**1011.57**	**1045.45**
EGARCH(1,1,1)	Normal	1455.40	1479.61	1069.71	1093.93
	Student's T	**1420.48**	**1449.54**	**1005.59**	**1034.65**
EGARCH(3,1,1)	Normal	1455.28	1489.18	1072.15	1106.05
	Student's T	**1418.94**	**1457.68**	**1005.46**	**1044.21**
GJR-GARCH(1,1,1)	Normal	1444.5	1468.7	1070	1094.2
	Student's T	**1411.14**	**1440.18**	**1005.92**	**1034.96**
GJR-GARCH(2,1,2)	Normal	1444.4	1478.28	1074	1107.89
	Student's T	**1412.39**	**1451.1**	**1009**	**1047.73**
TGARCH(1,1,1)	Normal	1453.22	1477.42	1067.79	1092
	Student's T	**1419.77**	**1448.8**	**1005.76**	**1034.8**

Figure 4.2: AIC and BIC score for each model when assuming residuals follow a Normal and Student's T distributions

the model fits the in-sample data.

For the standard GARCH(1,1) models, we see that both the α_1 and β_1 coefficients are strongly significant suggesting a strong relationship between the variance of the asset and the first lags of its variance and the squared residuals, showing that volatility is persistent over time and thus justifying the use of a GARCH(1,1) model. Looking at the GARCH(2,2) model, for the GBP/USD pair, we see that the coefficients α_1 α_2 and β_2 are all strongly significant however β_1 is not, with it's value estimated to 0. This is interesting as it further highlights the persistent nature of volatility, given that more distant lags of variance have a significant impact on future values. For the EUR/GBP pair, however, we see α_2 to be equal to 0, with α_1 being the only coefficient to be significant above the 90% confidence interval. This suggests that for this currency pair, a GARCH(2,2) model may not be a very appropriate choice.

One of the most interesting observations available from analysing this data is that, when fitting to the GBP/USD dataset, for the EGARCH, TGARCH and GJR-GARCH models, the γ component does not appear to be statistically significant. This means that we are unable to reject the null hypothesis that $\gamma = 0$ (i.e. the returns are symmetrical) and so we are unable to confirm that the daily returns of the GBP/USD pair are asymmetric. This is further supported by our previous finding that the standard GARCH specifications provided the best in-sample fit, but differs from the findings of Wang and Yang (2009) who were able to confirm the presence of asymmetry within the pairs returns, although their study focused on a different time frame to this one. Our findings do not mean that asymmetry is objectively not present in the returns of the GBP/USD, just that there is not enough evidence to confirm its presence.

However, when fitting to the EUR/GBP dataset, we can see that γ is significant at the 90% confidence interval for the EGARCH(1,1,1), GJR-GARCH(1,1,1) and GJR-GARCH(2,1,2) models and at the 95% confidence interval for the TGARCH(1,1,1) model. This allows us to reject the null hypothesis that the true value of $\gamma = 0$ for these models and thus supports our theory that the returns are asymmetrical in this dataset, made from the observation that TGARCH provided the best in-sample fit to the dataset. This again differs from the findings of Wang and Yang (2009) who found the EUR/GBP to have symmetrical returns in the past. The fact that, for the GJR-GARCH and TGARCH models, γ takes a negative value (and for EGARCH takes a positive one), implies that the pair's volatility is positively correlated with returns. This means that when the euro appreciates against the pound, volatility tends to increase. This is consistent with the findings of Naimy et al. (2023), who observe this trend by the fact that their EGARCH model which aims to fit the GBP/EUR returns (the inverse to the EUR/GBP dataset used in this study) takes a negative value for γ.

4.2.2 Forecasts

From the previous section, we take the GARCH(1,1), GARCH(2,2), EGARCH(1,1,1), EGARCH(3,1,1), GJR-GARCH(1,1,1), GJR-GARCH(2,1,2), and TGARCH(1,1,1) models and use these to generate forecasts of the volatility for the last 365 observations in the GBP/USD and EUR/GBP datasets. Given that it provided the best in-sample fit for all model types, we generate these forecasts under the assumption that the residuals follow a Student's T distribution. We first generate rolling forecasts, which fit the GARCH-type model to the last 200 observations, and use this to predict the volatility across the next 20-day period.

Figures A.19 to A.21 show the rolling forecasts of the GBP/USD pair generated by each of the specified models plotted alongside the observed 20-day standard deviation at that day. Figures A.22 to A.24 show the same but for the EUR/GBP pair. Frustratingly, it was not possible to generate rolling forecasts for the EGARCH model on either dataset due to unsolvable optimisation errors when fitting the model, despite a significant amount of time spent attempting to debug this issue.

Next, we generated expanding window forecasts which use all available observations at time t to predict the 20-day standard deviation on the next day. Figures A.25 to A.28 show the expanding window forecasts for the GBP/USD pair and Figures A.29 to A.32 show the expanding window forecasts for the EUR/GBP pair. Figure 4.3 shows the RMSE and MAE of the forecasts for each of the GARCH-type models for both currency pairs. From the table, it is not conclusive that one method is always better than the other, however, for the majority of cases, it appeared that rolling forecasts performed the best in terms of RMSE and MAE. These observations conflict with the expectation set by Sahiner (2022), which suggests expanding window forecasts to be superior. One possible explanation for this is that volatility trends in foreign exchange markets may only be short-term and that fitting the model coefficients to data from a longer period of time reduces the accuracy of the forecasts.

As can be seen, all of the GARCH model forecasts improve upon the forecasts generated by the EWMA model in terms of the RMSE metric. This is almost the case for the MAE metric as well, with the exception of the expanding window forecasts of the GARCH(2,2) and GJR-GARCH(1,1,1) models which have a slightly higher MAE for the GBP/USD dataset and the TGARCH(1,1,1) models rolling forecast which has a higher MAE for the EUR/GBP dataset.

In terms of RMSE, the most accurate forecast of the GBP/USD pair comes from the use of a GJR-GARCH(2,1,2) model, which has an RMSE score of 0.128. In terms of MAE, the best forecast comes from the GJR-GARCH(1,1,1) model, with a value of 0.100. This is surprising given that GARCH(1,1) provided the best in-sample fit to the data and that in 4.2.1 we fail to reject the hypothesis that the returns are symmetric, due to the value of γ not being significant at the 90% level.

For the EUR/GBP pair, we see that a GJR-GARCH(1,1,1) model provides the most accurate

Model	Forecasting Method	GBP/USD RMSE	GBP/USD MAE	EUR/GBP RMSE	EUR/GBP MAE
GARCH(1,1)	Rolling	0.137	0.111	0.114	0.09
	Expanding Window	0.192	0.134	0.112	0.087
GARCH(2,2)	Rolling	0.13	0.104	0.108	0.086
	Expanding Window	0.205	0.142	0.11	0.085
EGARCH(1,1,1)	Rolling	N/A	N/A	N/A	N/A
	Expanding Window	0.188	0.126	0.107	0.084
EGARCH(3,1,1)	Rolling	N/A	N/A	N/A	N/A
	Expanding Window	0.152	0.105	0.103	0.080
GJR-GARCH(1,1,1)	Rolling	0.129	**0.1**	**0.101**	**0.080**
	Expanding Window	0.195	0.135	0.113	0.089
GJR-GARCH(2,1,2)	Rolling	**0.128**	0.101	0.101	0.081
	Expanding Window	0.205	0.142	0.111	0.087
TGARCH(1,1,1)	Rolling	0.159	0.125	0.119	0.097
	Expanding Window	0.189	0.128	0.11	0.085

Figure 4.3: RMSE and MAE of forecasts generated by each of the GARCH-type models on the GBP/USD and EUR/GBP pairs

forecast, with an RMSE of 0.101 and an MAE of 0.080. This result is more in line with expectation as although 4.2.1 sees a TGARCH(1,1,1) model best fit the in-sample data, the presence of asymmetry was confirmed in the dataset, which GJR-GARCH also tries to tackle.

4.3 Implied Volatility Model

4.3.1 In-Sample Fit

Appendix A.5 shows the statistical properties of the Implied Volatility-based regression models when trained on the two different datasets. For the GBP/USD dataset, we see that the best AIC and BIC scores both come from Model 1, however for the EUR/GBP dataset we see the best BIC come from Model 2 but the best AIC score still comes from Model 1. We also see that for both datasets, the coefficient β_2 in Model 2 is much lower than that of β_1, with a large standard errors and as such is not statistically significant at the 90% confidence interval in either case. Overall this suggests that the use of Model 1 is more appropriate. This is similar to the results of Scott and Tucker (1989), who also find the coefficient of historic standard deviations to be insignificant. One possible explanation for these observations is that the information from the past 20-day standard deviation data is already somehow included within the implied volatility data.

It is worth noting that it is not possible to use these statistics to compare the goodness of the in-sample fit of these models with that of the GARCH-type models. The reason for this is that the models predict different things. The Implied Volatility regression models look to fit to the future value of the 20-day standard deviation of returns, whereas the GARCH-type models are fitting for the conditional volatility of the daily returns series.

4.3.2 Forecasts

The fitted regression model was then used to generate 20-day ahead forecasts of the 20-day standard deviations of the GBP/USD and EUR/GBP pairs. We see for the GBP/USD that both of the Implied Volatility based models fail to improve on the forecasting accuracy of the GJR-GARCH models, both achieving an RMSE of 0.175 and MAEs of 0.111 and 0.112 for Models 1 and 2 respectively, which is weaker than the GJR-GARCH(2,1,2) models RMSE score of 0.128 and the GJR-GARCH(1,1,1) models MAE of 0.100. In fact, it is outperformed by all of the GARCH variants in terms of RMSE when forecasting using a rolling window method. However, the regression model does significantly outperform the EWMA model for both the RMSE and MAE metrics and the GARCH models which make use of expanding window forecasts.

For the EUR/GBP pair, however, we see the regression models outperform all the GARCH variants and the EWMA in terms of MAE, with Models 1 and 2 scoring 0.072 and 0.073 respectively. However, the RMSE of GJR-GARCH(1,1,1) remains superior to the two Implied Volatility models.

These findings partially contradict the results of Pilbeam and Langeland (2015) and Scott and Tucker (1989), which suggest Implied Volatility models are the superior forecasting method for exchange rate volatility, given that for most metrics, GARCH actually seems to outperform the implied volatility method.

Model	GBP/USD		EUR/GBP	
	RMSE	MAE	RMSE	MAE
Model 1	0.175	0.112	0.110	0.073
Model 2	0.175	0.111	0.110	0.072

Figure 4.4: RMSE and MAE metrics for the forecasts generated by the Implied Volatility-based OLS Regression model for the GBP/USD and EUR/GBP pairs

Interestingly, despite the lack of significance in the β_2 coefficients, the accuracy of the out-of-sample forecasts is not massively affected by its inclusion, with it in fact resulting in a slight improvement in MAE on both datasets (0.001). However, as this is nearly negligible, it could be argued that the most appropriate of the two regression models is Model 1.

Chapter 5

Conclusions

In this study, we have discussed and analysed the performance of various forecasting methods for the volatility of the GBP/USD and EUR/GBP exchange rates. Specifically, we have compared the performance of an EWMA model (with $\lambda = 0.97$), the GARCH, EGARCH, GJR-GARCH and TGARCH models, and two OLS regression models which take the annualised implied volatility of the previous day as an independent variable. We calculated the 20-day standard deviations of the daily returns for each of the currency pairs and tested each of the models' ability to predict this value 20 days in advance.

We determined the most appropriate p and q parameters for the GARCH models by iterating through the values from 1 to 5 for each and comparing the AIC and BIC scores for each model to evaluate the parameter sets that provide the best overall fit to the in-sample data. We find the best fitting specifications for each model type to be; GARCH(1,1), GARCH(2,2), EGARCH(1,1,1), EGARCH(3,1,1), GJR-GARCH(1,1,1), GJR-GARCH(2,1,2) and TGARCH(1,1,1).

We then used the GARCH models to delve deeper into the statistical properties of the daily returns of each of the pairs. For the EUR/GBP exchange rate, we found evidence to support the asymmetry of daily returns, with strong positive returns on the Euro resulting in increased volatility. This was observed through the negative value of γ in the EGARCH(1,1,1) model trained to fit this data and the positive value of γ in the GJR-GARCH and TGARCH models, all of which were statistically significant at least at the 10% level. However, we were unable to confirm any asymmetry in the GBP/USD exchange rate as γ exhibited low statistical significance for all of the EGARCH, GJR-GARCH and TGARCH models.

We also found that modelling the residual values in a GARCH model as a Student's T distribution resulted in a better fit to the in-sample data when compared to a normal distribution. This was observed from the reduction in AIC and BIC scores for all of the GARCH-type models tested when using a Student's T distribution in place of a Normal distribution. We then evaluated the best forecasting technique to use for GARCH models between Rolling and Expanding Window methods but were unable to find a clear-cut winner. This is despite the initial expectation that expanding window forecasts would prove better.

We also briefly examined the statistical significance of the components of the implied volatility-based OLS regression models. We saw that the coefficients for implied volatility were always significant but that the coefficients for historical standard deviations never were. The proposed explanation for this was that implied volatility data seems to encapsulate the values of historic volatility. However, incorporating the historic volatility data did not negatively impact performance when producing an out-of-sample forecast, in fact resulting in a near negligible improvement in MAE.

By comparing the generated out-of-sample forecasts for each model by the RMSE and MAE metrics, we found that for the GBP/USD pair, the use of GJR-GARCH models (with rolling window forecasts) provided the most accurate forecasts. These results were surprising given that the in-sample returns data did not appear to exhibit asymmetry (which GJR-GARCH attempts to capture the effects of).

For the EUR/GBP model, we find that whilst the best forecast in terms of RMSE also comes from GJR-GARCH with rolling window forecasts, the OLS regression models provide the best forecasts in terms of MAE.

While from these results it appears that GJR-GARCH models are amongst the strongest models for the forecasting of the volatility of exchange rates involving the pound, it would be beneficial to carry out more research on the matter. Future work should look at the performance of these same models for forecasting the volatility of various other currency pairs involving the pound, as well as looking into their performance when forecasting over different horizons and when evaluating forecasts against different proxies. Such research would provide more results and allow for greater comparison between models.

It is also worth noting that this study is not an exhaustive test of all existing models and that there are other models out there that could prove to be better predictors of exchange rate volatility. Particularly, there have been recent adaptations to the GARCH family of models that augment them with machine learning techniques in an attempt to improve their forecasting performance. Therefore it could also be worthwhile testing these types of models on the same dataset used in this study to see if they, or any other existing/future models, can outperform GJR-GARCH or the OLS regression models.

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Appendix A

A.1 EWMA Forecasts

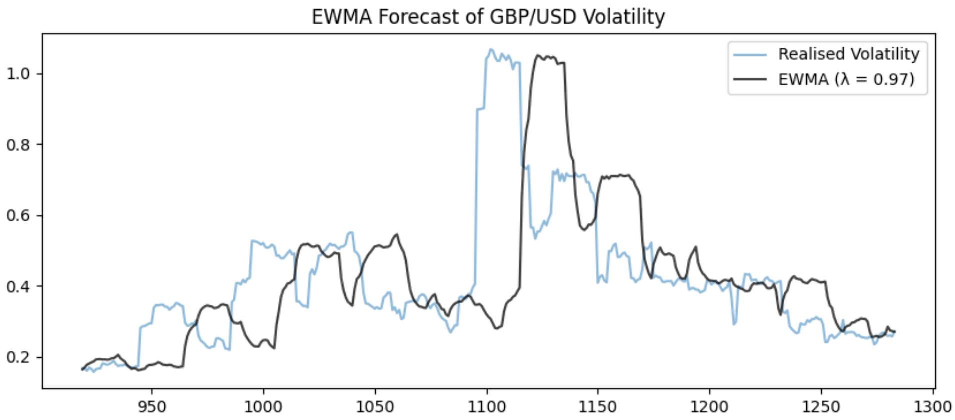

Figure A.1: EWMA forecast for the GBP/USD pair where $\gamma = 0.97$

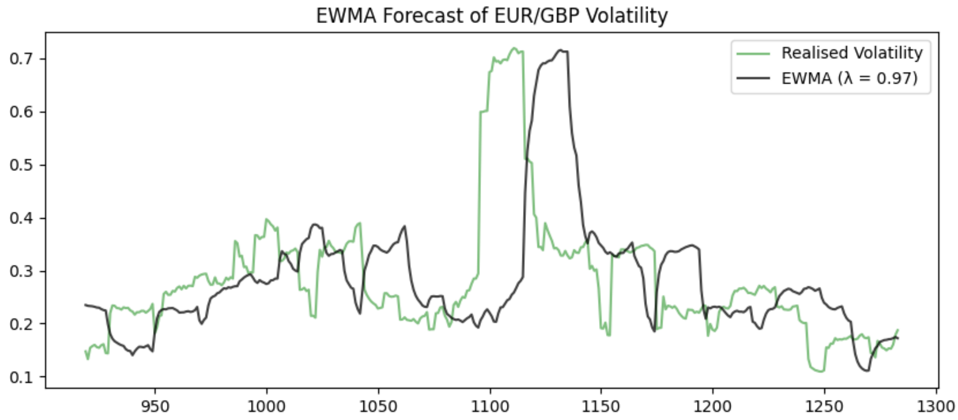

Figure A.2: EWMA forecast for the EUR/GBP pair where $\gamma = 0.97$

A.2 GARCH-type Model AIC & BIC Heat Maps

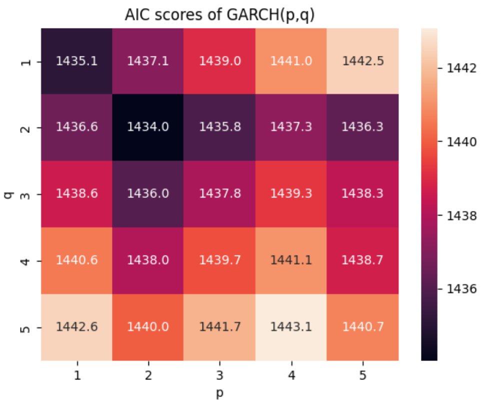

Figure A.3: Heatmap of AIC scores of GARCH model parameters fitted to the GBP/USD pair

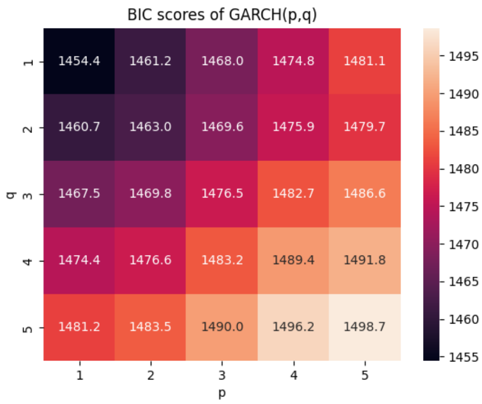

Figure A.4: Heatmap of BIC scores of GARCH model parameters fitted to the GBP/USD pair

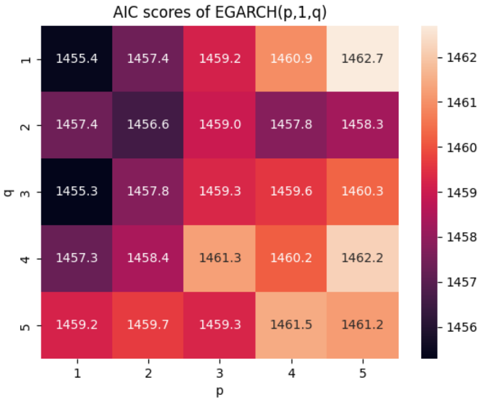

Figure A.5: Heatmap of AIC scores of EGARCH model parameters fitted to the GBP/USD pair

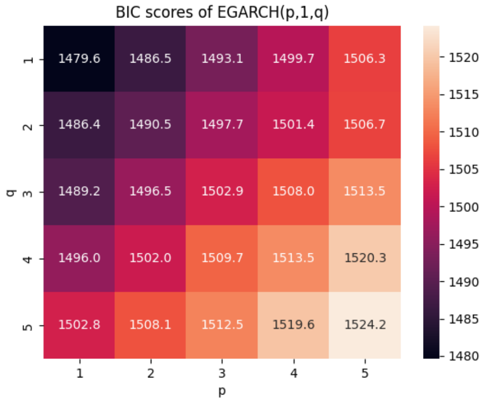

Figure A.6: Heatmap of BIC scores of EGARCH model parameters fitted to the GBP/USD pair

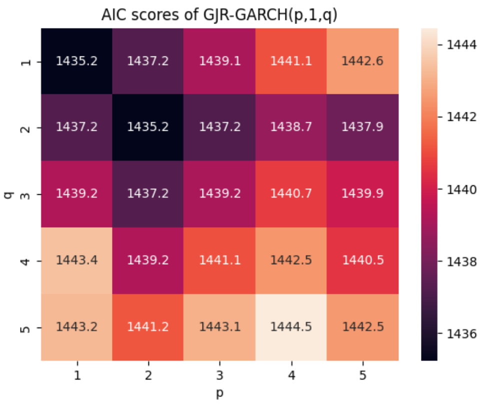

Figure A.7: Heatmap of AIC scores of GJR-GARCH model parameters fitted to the GBP/USD pair

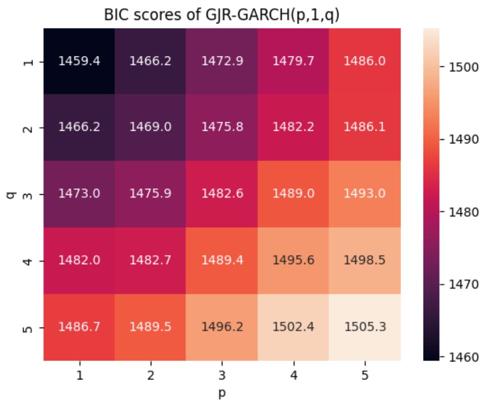

Figure A.8: Heatmap of BIC scores of GJR-GARCH model parameters fitted to the GBP/USD pair

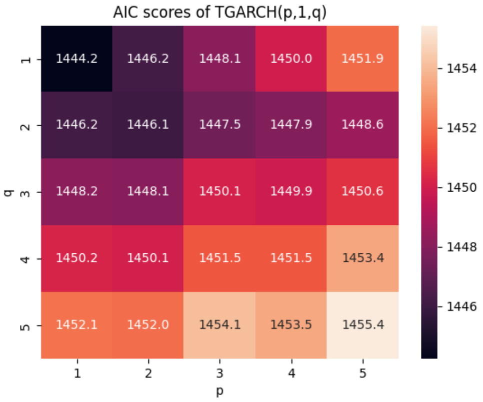

Figure A.9: Heatmap of AIC scores of TGARCH model parameters fitted to the GBP/USD pair

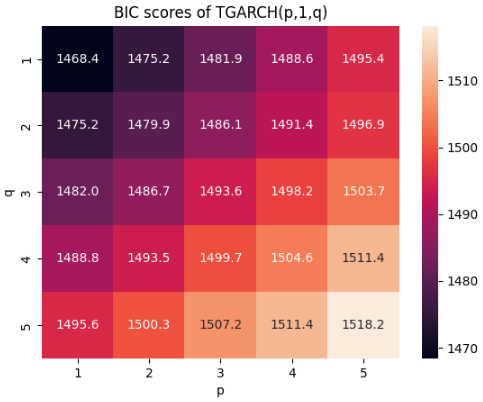

Figure A.10: Heatmap of BIC scores of TGARCH model parameters fitted to the GBP/USD pair

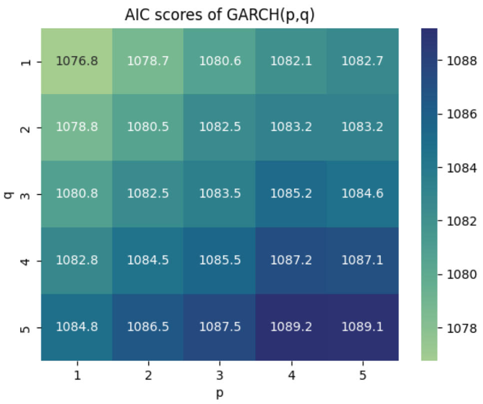

Figure A.11: Heatmap of AIC scores of GARCH model parameters fitted to the EUR/GBP pair

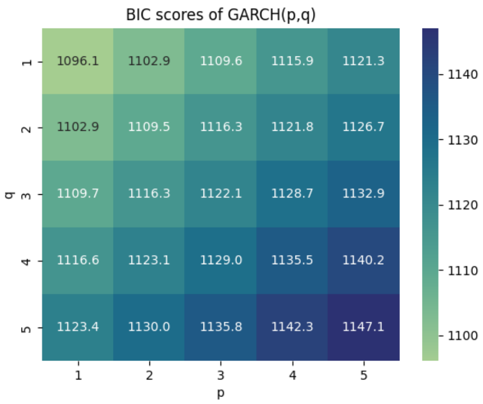

Figure A.12: Heatmap of BIC scores of GARCH model parameters fitted to the EUR/GBP pair

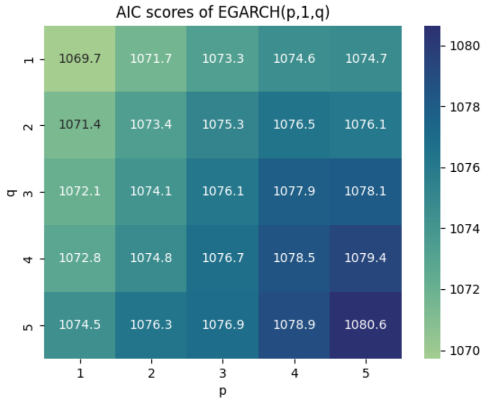

Figure A.13: Heatmap of AIC scores of EGARCH model parameters fitted to the EUR/GBP pair

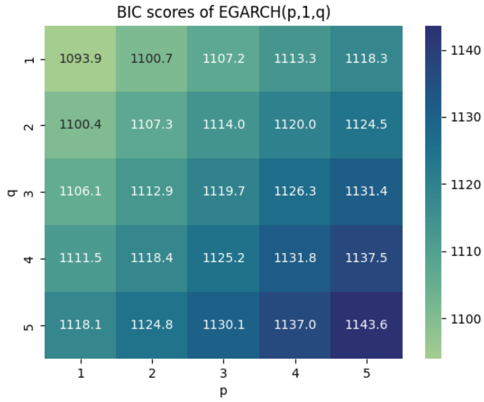

Figure A.14: Heatmap of BIC scores of EGARCH model parameters fitted to the EUR/GBP pair

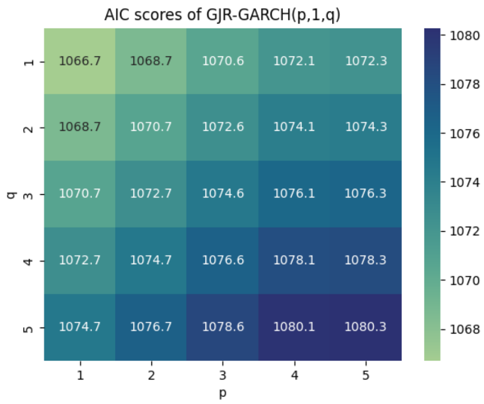

Figure A.15: Heatmap of AIC scores of GJR-GARCH model parameters fitted to the EUR/GBP pair

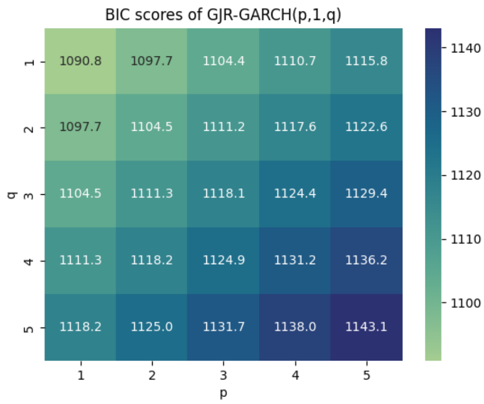

Figure A.16: Heatmap of BIC scores of GJR-GARCH model parameters fitted to the EUR/GBP pair

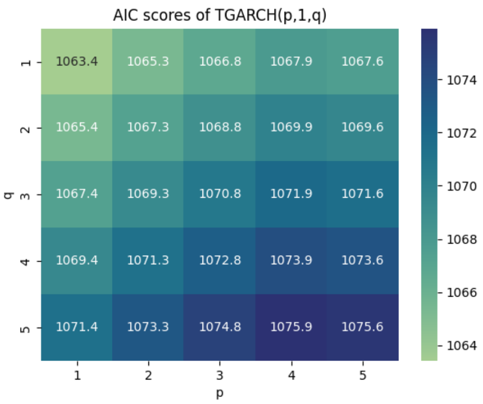

Figure A.17: Heatmap of AIC scores of TGARCH model parameters fitted to the EUR/GBP pair

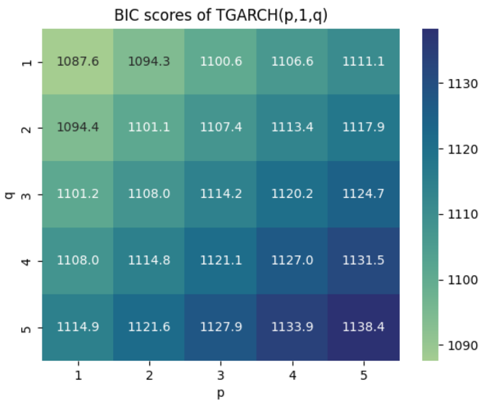

Figure A.18: Heatmap of BIC scores of TGARCH model parameters fitted to the EUR/GBP pair

A.3 GARCH-type Model Statistics (In-Sample)

A.3.1 GARCH(1,1) - GBP/USD

```
                       Constant Mean - GARCH Model Results
==================================================================================
Dep. Variable:                     returns   R-squared:                      0.000
Mean Model:                  Constant Mean   Adj. R-squared:                 0.000
Vol Model:                           GARCH   Log-Likelihood:              -699.772
Distribution:      Standardized Student's t  AIC:                          1409.54
Method:                 Maximum Likelihood   BIC:                          1433.74
                                             No. Observations:                 934
Date:                     Mon, Aug 14 2023   Df Residuals:                     933
Time:                             01:29:11   Df Model:                           1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         2.8196e-03  1.572e-02      0.179      0.858 [-2.798e-02,3.362e-02]
                               Volatility Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0375  1.579e-02      2.375  1.757e-02 [6.546e-03,6.845e-02]
alpha[1]       0.1008  2.947e-02      3.421  6.250e-04   [4.304e-02,  0.159]
beta[1]        0.7690  7.124e-02     10.795  3.653e-27     [  0.629,  0.909]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             6.8541      1.367      5.016  5.288e-07 [  4.176,  9.533]
========================================================================

Covariance estimator: robust
```

A.3.2 GARCH(2,2) - GBP/USD

```
                     Constant Mean - GARCH Model Results
==================================================================================
Dep. Variable:                    returns   R-squared:                       0.000
Mean Model:                 Constant Mean   Adj. R-squared:                  0.000
Vol Model:                          GARCH   Log-Likelihood:               -698.538
Distribution:     Standardized Student's t   AIC:                          1411.08
Method:                Maximum Likelihood   BIC:                           1444.95
                                            No. Observations:                  934
Date:                    Mon, Aug 14 2023   Df Residuals:                      933
Time:                            01:29:11   Df Model:                            1
                                  Mean Model
===========================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
---------------------------------------------------------------------------
mu         1.6368e-03  1.574e-02      0.104      0.917 [-2.922e-02,3.249e-02]
                              Volatility Model
=========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
-------------------------------------------------------------------------
omega          0.0866  5.198e-02      1.667  9.555e-02 [-1.524e-02,  0.189]
alpha[1]       0.0768  3.088e-02      2.487  1.287e-02  [1.628e-02,  0.137]
alpha[2]       0.1320  5.570e-02      2.369  1.782e-02  [2.280e-02,  0.241]
beta[1]        0.0000      0.203      0.000      1.000    [ -0.397,  0.397]
beta[2]        0.4893      0.168      2.918  3.522e-03    [  0.161,  0.818]
                              Distribution
======================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
----------------------------------------------------------------------
nu             7.0431      1.440      4.889  1.011e-06 [  4.220,  9.866]
======================================================================

Covariance estimator: robust
```

A.3.3 EGARCH(1,1,1) - GBP/USD

```
                    Constant Mean - EGARCH Model Results
============================================================================
Dep. Variable:                 returns   R-squared:                    0.000
Mean Model:              Constant Mean   Adj. R-squared:               0.000
Vol Model:                      EGARCH   Log-Likelihood:            -722.699
Distribution:                   Normal   AIC:                        1455.40
Method:             Maximum Likelihood   BIC:                        1479.61
                                         No. Observations:               937
Date:                 Tue, Aug 29 2023   Df Residuals:                   936
Time:                         16:18:06   Df Model:                         1
                                 Mean Model
==========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
--------------------------------------------------------------------------
mu         3.7901e-03  8.933e-04      4.243  2.205e-05 [2.039e-03,5.541e-03]
                               Volatility Model
===========================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
---------------------------------------------------------------------------
omega         -0.0867  5.301e-02     -1.635      0.102    [ -0.191,1.721e-02]
alpha[1]       0.1913  6.191e-02      3.089  2.006e-03    [6.992e-02,  0.313]
gamma[1]      -0.0183  3.235e-02     -0.567      0.571 [-8.174e-02,4.508e-02]
beta[1]        0.9274  4.178e-02     22.198 3.610e-109       [  0.846,  1.009]
===========================================================================

Covariance estimator: robust
```

A.3.4 EGARCH(3,1,1) - GBP/USD

```
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:               -720.642
Distribution:                  Normal   AIC:                           1455.28
Method:            Maximum Likelihood   BIC:                           1489.18
                                        No. Observations:                  937
Date:                Wed, Aug 30 2023   Df Residuals:                      936
Time:                        20:27:30   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         1.4495e-03  1.775e-02  8.168e-02      0.935 [-3.333e-02,3.623e-02]
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega         -0.0322  7.526e-02     -0.428      0.668      [ -0.180,  0.115]
alpha[1]       0.1961  7.837e-02      2.503  1.232e-02     [4.254e-02,  0.350]
alpha[2]       0.1033      0.107      0.964      0.335      [ -0.107,  0.313]
alpha[3]      -0.1837      0.163     -1.125      0.261      [ -0.504,  0.136]
gamma[1]      -0.0133  2.764e-02     -0.479      0.632 [-6.743e-02,4.092e-02]
beta[1]        0.9719  6.222e-02     15.620  5.333e-55      [  0.850,  1.094]
=============================================================================

Covariance estimator: robust
```

A.3.5 GJR-GARCH(1,1,1) - GBP/USD

```
                     Constant Mean - GJR-GARCH Model Results
====================================================================================
Dep. Variable:                      returns   R-squared:                       0.000
Mean Model:                   Constant Mean   Adj. R-squared:                  0.000
Vol Model:                        GJR-GARCH   Log-Likelihood:               -699.571
Distribution:      Standardized Student's t   AIC:                           1411.14
Method:                  Maximum Likelihood   BIC:                           1440.18
                                              No. Observations:                  934
Date:                      Mon, Aug 14 2023   Df Residuals:                      933
Time:                              01:29:11   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         8.7599e-04  1.604e-02  5.461e-02      0.956 [-3.056e-02,3.232e-02]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0373  1.618e-02      2.305  2.118e-02 [5.579e-03,6.902e-02]
alpha[1]       0.0839  3.756e-02      2.234  2.551e-02   [1.028e-02,  0.158]
gamma[1]       0.0301  4.644e-02      0.648      0.517  [-6.090e-02,  0.121]
beta[1]        0.7716  7.378e-02     10.458  1.347e-25     [  0.627,  0.916]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             6.8915      1.369      5.035  4.777e-07 [  4.209,  9.574]
========================================================================
```

```
Covariance estimator: robust
```

A.3.6 GJR-GARCH(2,1,2) - GBP/USD

```
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:                    returns   R-squared:                   0.000
Mean Model:                 Constant Mean   Adj. R-squared:              0.000
Vol Model:                      GJR-GARCH   Log-Likelihood:           -698.193
Distribution:  Standardized Student's t   AIC:                         1412.39
Method:                Maximum Likelihood   BIC:                         1451.10
                                            No. Observations:              934
Date:                    Mon, Aug 14 2023   Df Residuals:                  933
Time:                            01:29:11   Df Model:                        1
                                Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         3.1966e-03  1.588e-02      0.201      0.840 [-2.792e-02,3.431e-02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.0887  5.875e-02      1.510      0.131 [-2.646e-02,  0.204]
alpha[1]       0.0994  5.638e-02      1.762  7.801e-02 [-1.114e-02,  0.210]
alpha[2]       0.1396  6.035e-02      2.313  2.073e-02  [2.130e-02,  0.258]
gamma[1]      -0.0487  6.775e-02     -0.719      0.472  [ -0.181,8.409e-02]
beta[1]    3.5815e-16      0.245  1.459e-15      1.000     [ -0.481,  0.481]
beta[2]        0.4765      0.159      3.003  2.669e-03     [  0.166,  0.787]
                               Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             7.0933      1.475      4.808  1.522e-06 [  4.202,  9.985]
========================================================================

Covariance estimator: robust
```

A.3.7 TGARCH(1,1,1) - GBP/USD

```
                  Constant Mean - TARCH/ZARCH Model Results
==============================================================================
Dep. Variable:                    returns   R-squared:                   0.000
Mean Model:                 Constant Mean   Adj. R-squared:              0.000
Vol Model:                    TARCH/ZARCH   Log-Likelihood:           -703.883
Distribution:  Standardized Student's t   AIC:                         1419.77
Method:                Maximum Likelihood   BIC:                         1448.80
                                            No. Observations:              934
Date:                    Mon, Aug 14 2023   Df Residuals:                  933
Time:                            01:29:11   Df Model:                        1
                                Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         5.6657e-04  1.583e-02  3.580e-02      0.971 [-3.046e-02,3.159e-02]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0487  2.905e-02      1.678  9.338e-02   [-8.197e-03,  0.106]
alpha[1]       0.0830  3.628e-02      2.288  2.216e-02    [1.188e-02,  0.154]
gamma[1]       0.0196  3.318e-02      0.590      0.555 [-4.546e-02,8.459e-02]
beta[1]        0.8379  7.135e-02     11.743  7.660e-32       [  0.698,  0.978]
```

```
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             6.8265      1.362      5.013  5.365e-07 [  4.157,  9.496]
========================================================================

Covariance estimator: robust
```

A.3.8 GARCH(1,1) - EUR/GBP

```
                        Constant Mean - GARCH Model Results
====================================================================================
Dep. Variable:                     returns   R-squared:                        0.000
Mean Model:                  Constant Mean   Adj. R-squared:                   0.000
Vol Model:                           GARCH   Log-Likelihood:                -499.327
Distribution:     Standardized Student's t   AIC:                            1008.65
Method:                 Maximum Likelihood   BIC:                            1032.86
                                             No. Observations:                   935
Date:                     Mon, Aug 14 2023   Df Residuals:                       934
Time:                             16:20:27   Df Model:                             1
                                  Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu          -8.4477e-03  1.210e-02     -0.698      0.485 [-3.217e-02,1.527e-02]
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0166  1.108e-02      1.499      0.134 [-5.111e-03,3.832e-02]
alpha[1]       0.1043  4.627e-02      2.254  2.418e-02    [1.362e-02,  0.195]
beta[1]        0.8179  9.075e-02      9.013  2.003e-19       [  0.640,  0.996]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.8509      0.736      6.592  4.343e-11 [  3.409,  6.293]
========================================================================

Covariance estimator: robust
```

A.3.9 GARCH(2,2) - EUR/GBP

```
                        Constant Mean - GARCH Model Results
====================================================================================
Dep. Variable:                     returns   R-squared:                        0.000
Mean Model:                  Constant Mean   Adj. R-squared:                   0.000
Vol Model:                           GARCH   Log-Likelihood:                -498.785
Distribution:     Standardized Student's t   AIC:                            1011.57
Method:                 Maximum Likelihood   BIC:                            1045.45
                                             No. Observations:                   935
Date:                     Mon, Aug 14 2023   Df Residuals:                       934
Time:                             16:20:27   Df Model:                             1
                                  Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu          -8.4750e-03  1.212e-02     -0.699      0.484 [-3.222e-02,1.527e-02]
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
```

```
omega          0.0204  2.541e-02      0.804      0.421 [-2.937e-02,7.025e-02]
alpha[1]       0.1350  5.878e-02      2.296  2.166e-02    [1.977e-02,  0.250]
alpha[2]       0.0000      0.150      0.000      1.000     [ -0.294,  0.294]
beta[1]        0.4094      0.869      0.471      0.637     [ -1.293,  2.112]
beta[2]        0.3601      0.646      0.558      0.577     [ -0.906,  1.626]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.8713      0.759      6.420  1.367e-10 [  3.384,  6.359]
========================================================================

Covariance estimator: robust
```

A.3.10 EGARCH(1,1,1) - EUR/GBP

```
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:               -529.857
Distribution:                  Normal   AIC:                           1069.71
Method:            Maximum Likelihood   BIC:                           1093.93
                                        No. Observations:                  938
Date:                Tue, Aug 29 2023   Df Residuals:                      937
Time:                        16:16:31   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         8.1040e-03  1.376e-02      0.589      0.556 [-1.887e-02,3.508e-02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega         -0.1046  5.490e-02     -1.906  5.665e-02  [ -0.212,2.962e-03]
alpha[1]       0.2381  6.180e-02      3.853  1.166e-04    [  0.117,  0.359]
gamma[1]       0.0663  3.905e-02      1.697  8.966e-02 [-1.026e-02,  0.143]
beta[1]        0.9298  3.303e-02     28.149 2.448e-174    [  0.865,  0.995]
===========================================================================

Covariance estimator: robust
```

A.3.11 EGARCH(3,1,1) - EUR/GBP

```
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:               -529.073
Distribution:                  Normal   AIC:                           1072.15
Method:            Maximum Likelihood   BIC:                           1106.05
                                        No. Observations:                  938
Date:                Wed, Aug 30 2023   Df Residuals:                      937
Time:                        20:29:02   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         7.4890e-03  1.373e-02      0.545      0.586 [-1.943e-02,3.441e-02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
```

```
--------------------------------------------------------------------------
omega         -0.0686  6.195e-02     -1.108      0.268  [ -0.190,5.278e-02]
alpha[1]       0.2514  8.681e-02      2.896  3.777e-03  [8.127e-02,  0.422]
alpha[2]       0.0279      0.136      0.205      0.837    [ -0.238,  0.294]
alpha[3]      -0.0957      0.123     -0.780      0.435    [ -0.336,  0.145]
gamma[1]       0.0596  3.843e-02      1.550      0.121 [-1.574e-02,  0.135]
beta[1]        0.9528  3.971e-02     23.994 3.179e-127    [  0.875,  1.031]
==========================================================================

Covariance estimator: robust
```

A.3.12 GJR-GARCH(1,1,1) - EUR/GBP

```
                      Constant Mean - GJR-GARCH Model Results
==================================================================================
Dep. Variable:                      returns   R-squared:                     0.000
Mean Model:                   Constant Mean   Adj. R-squared:                0.000
Vol Model:                        GJR-GARCH   Log-Likelihood:             -496.960
Distribution:      Standardized Student's t   AIC:                         1005.92
Method:                  Maximum Likelihood   BIC:                         1034.96
                                              No. Observations:                935
Date:                      Mon, Aug 14 2023   Df Residuals:                    934
Time:                              16:20:27   Df Model:                          1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
----------------------------------------------------------------------------
mu         -4.3497e-03  1.219e-02     -0.357      0.721 [-2.824e-02,1.954e-02]
                              Volatility Model
===========================================================================
                coef    std err          t      P>|t|       95.0% Conf. Int.
---------------------------------------------------------------------------
omega         0.0158  1.039e-02      1.521      0.128 [-4.558e-03,3.617e-02]
alpha[1]      0.1502  7.025e-02      2.138  3.249e-02    [1.253e-02,  0.288]
gamma[1]     -0.0997  5.641e-02     -1.768  7.711e-02    [ -0.210,1.085e-02]
beta[1]       0.8244  9.017e-02      9.143  6.051e-20      [  0.648,  1.001]
                              Distribution
=====================================================================
                coef    std err          t      P>|t|  95.0% Conf. Int.
---------------------------------------------------------------------
nu            4.9273      0.755      6.530  6.589e-11 [  3.448,  6.406]
=====================================================================

Covariance estimator: robust
```

A.3.13 GJR-GARCH(2,1,2) - EUR/GBP

```
                      Constant Mean - GJR-GARCH Model Results
==================================================================================
Dep. Variable:                      returns   R-squared:                     0.000
Mean Model:                   Constant Mean   Adj. R-squared:                0.000
Vol Model:                        GJR-GARCH   Log-Likelihood:             -496.501
Distribution:      Standardized Student's t   AIC:                         1009.00
Method:                  Maximum Likelihood   BIC:                         1047.73
                                              No. Observations:                935
Date:                      Mon, Aug 14 2023   Df Residuals:                    934
Time:                              16:20:28   Df Model:                          1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
----------------------------------------------------------------------------
mu         -4.5215e-03  1.218e-02     -0.371      0.711 [-2.840e-02,1.936e-02]
                              Volatility Model
```

```
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega          0.0188  1.775e-02      1.059      0.290 [-1.599e-02,5.357e-02]
alpha[1]       0.1838  8.147e-02      2.256  2.406e-02    [2.413e-02,  0.343]
alpha[2]       0.0000  7.433e-02      0.000      1.000      [ -0.146,  0.146]
gamma[1]      -0.1204  7.299e-02     -1.650  9.898e-02   [ -0.263,2.264e-02]
beta[1]        0.4884      0.397      1.231      0.218      [ -0.289,  1.266]
beta[2]        0.2986      0.282      1.057      0.290      [ -0.255,  0.852]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.9222      0.769      6.402  1.535e-10 [  3.415,  6.429]
========================================================================

Covariance estimator: robust
```

A.3.14 TGARCH(1,1,1) - EUR/GBP

```
                     Constant Mean - TARCH/ZARCH Model Results
====================================================================================
Dep. Variable:                      returns   R-squared:                       0.000
Mean Model:                   Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      TARCH/ZARCH   Log-Likelihood:               -496.879
Distribution:      Standardized Student's t   AIC:                           1005.76
Method:                  Maximum Likelihood   BIC:                           1034.80
                                              No. Observations:                  935
Date:                      Mon, Aug 14 2023   Df Residuals:                      934
Time:                              16:20:28   Df Model:                            1
                                  Mean Model
===============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
-------------------------------------------------------------------------------
mu         -1.9124e-03  1.212e-02     -0.158      0.875 [-2.567e-02,2.185e-02]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega          0.0190  1.613e-02      1.178      0.239 [-1.262e-02,5.060e-02]
alpha[1]       0.1246  5.602e-02      2.225  2.611e-02    [1.482e-02,  0.234]
gamma[1]      -0.0719  3.544e-02     -2.028  4.257e-02  [ -0.141,-2.407e-03]
beta[1]        0.8924  6.742e-02     13.237  5.379e-40      [  0.760,  1.025]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.9728      0.780      6.374  1.837e-10 [  3.444,  6.502]
========================================================================

Covariance estimator: robust
```

A.4 GARCH-type Model Forecasts

A.4.1 GBP/USD Rolling Window Forecasts

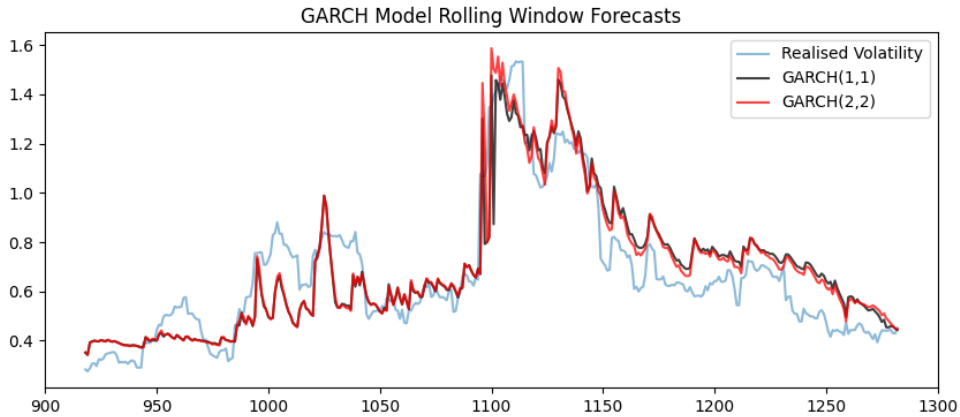

Figure A.19: Rolling forecasts of daily returns of the GBP/USD pair computed from GARCH(1,1) and GARCH(2,2)

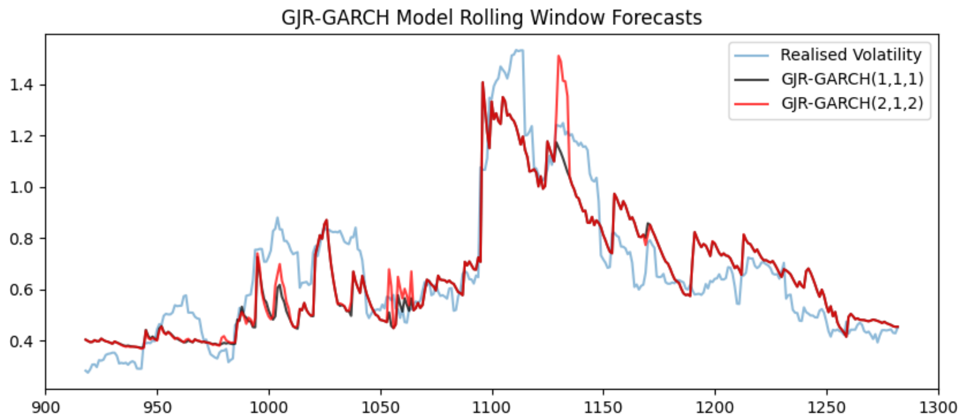

Figure A.20: Rolling forecasts of daily returns of the GBP/USD pair computed from GJR-GARCH(1,1,1) and GJR-GARCH(2,1,2)

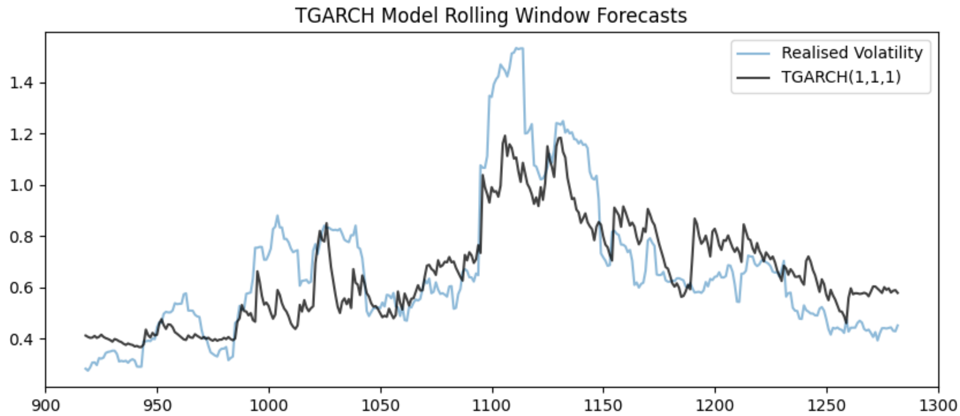

Figure A.21: Rolling forecasts of daily returns of the GBP/USD pair computed from TGARCH(1,1,1)

A.4.2 EUR/GBP Rolling Window Forecasts

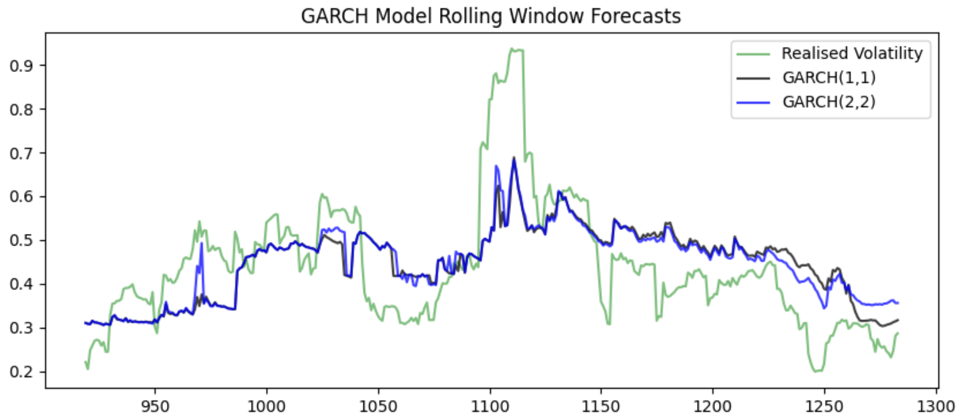

Figure A.22: Rolling forecasts of daily returns of the EUR/GBP pair computed from GARCH(1,1) and GARCH(2,2)

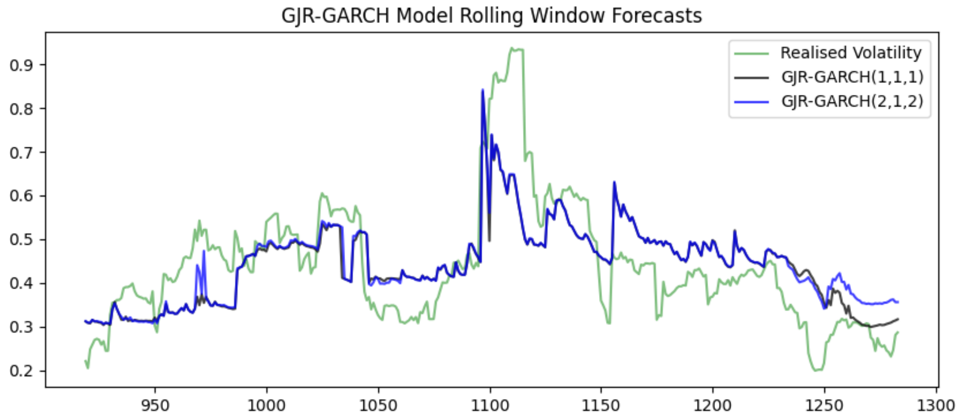

Figure A.23: Rolling forecasts of daily returns of the EUR/GBP pair computed from GJR-GARCH(1,1,1) and GJR-GARCH(2,1,2)

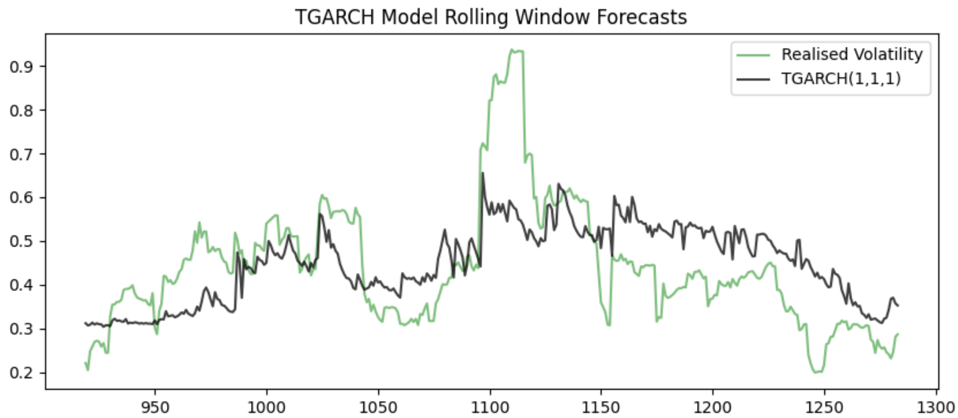

Figure A.24: Rolling forecasts of daily returns of the EUR/GBP pair computed from TGARCH(1,1,1)

A.4.3 GBP/USD Expanding Window Forecasts

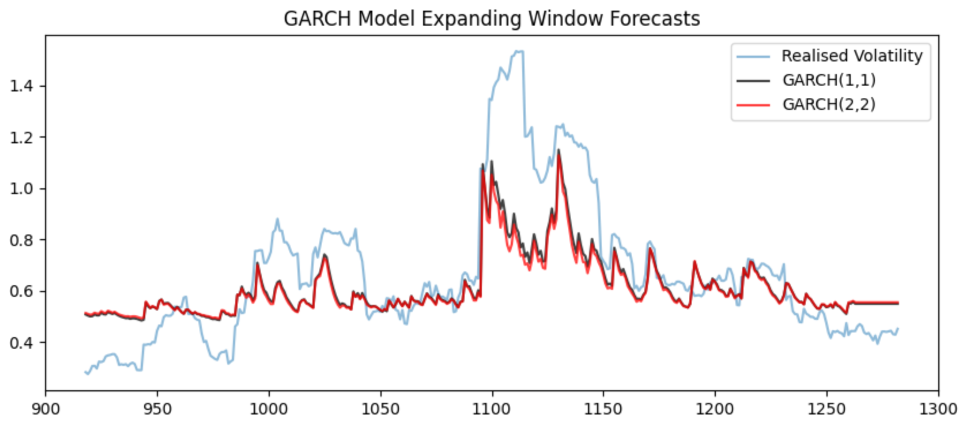

Figure A.25: Expanding window forecasts of daily returns of the GBP/USD pair computed from GARCH(1,1) and GARCH(2,2)

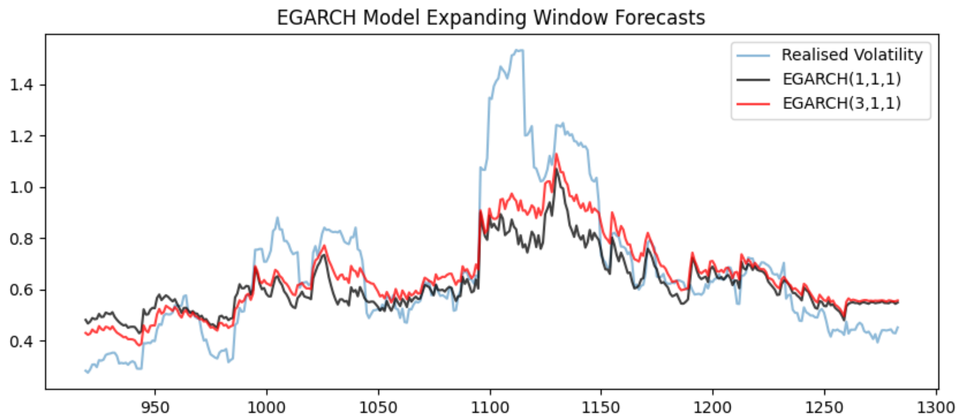

Figure A.26: Expanding window forecast of daily returns of the GBP/USD pair computed from EGARCH(1,1)

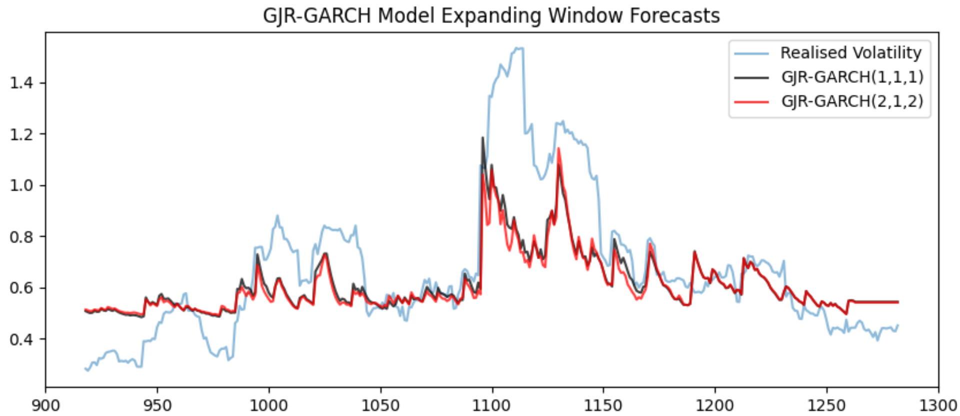

Figure A.27: Expanding window forecasts of daily returns of the GBP/USD pair computed from GJR-GARCH(1,1,1) and GJR-GARCH(2,1,2)

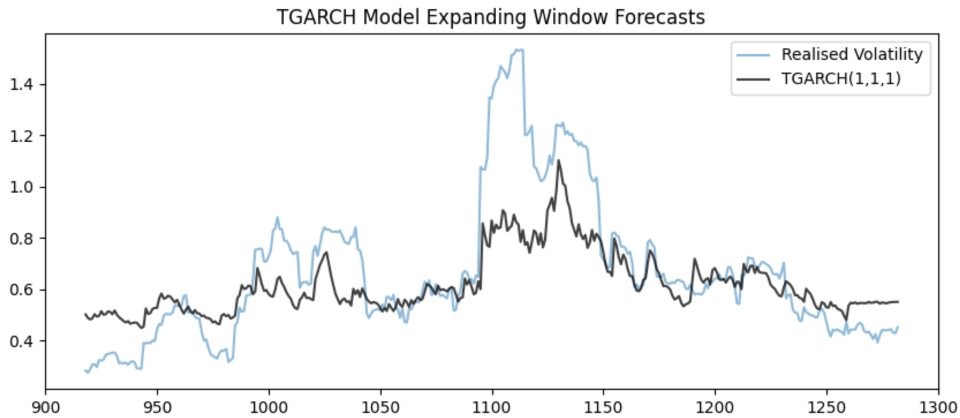

Figure A.28: Expanding window forecast of daily returns of the GBP/USD pair computed from TGARCH(1,1,1)

A.4.4 EUR/GBP Expanding Window Forecasts

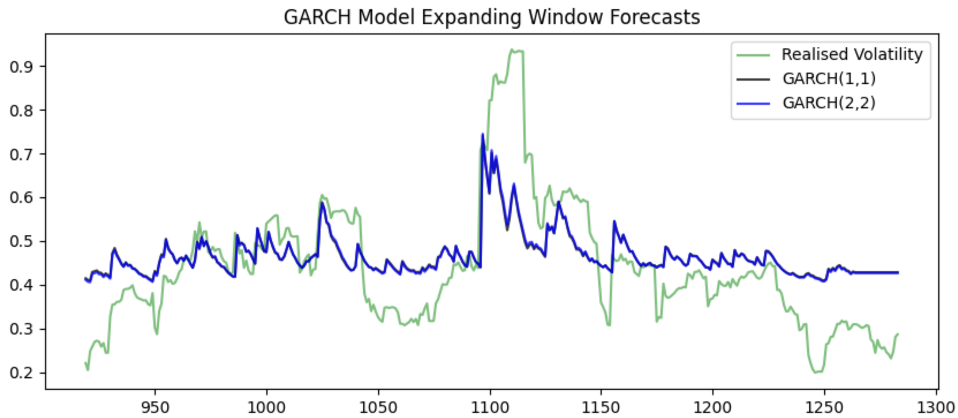

Figure A.29: Expanding window forecasts of daily returns of the EUR/GBP pair computed from GARCH(1,1) and GARCH(2,2)

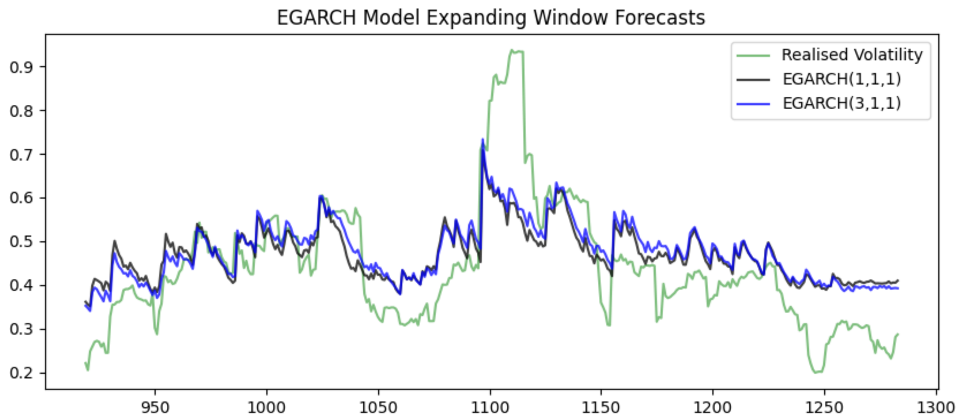

Figure A.30: Expanding window forecast of daily returns of the EUR/GBP pair computed from EGARCH(1,1)

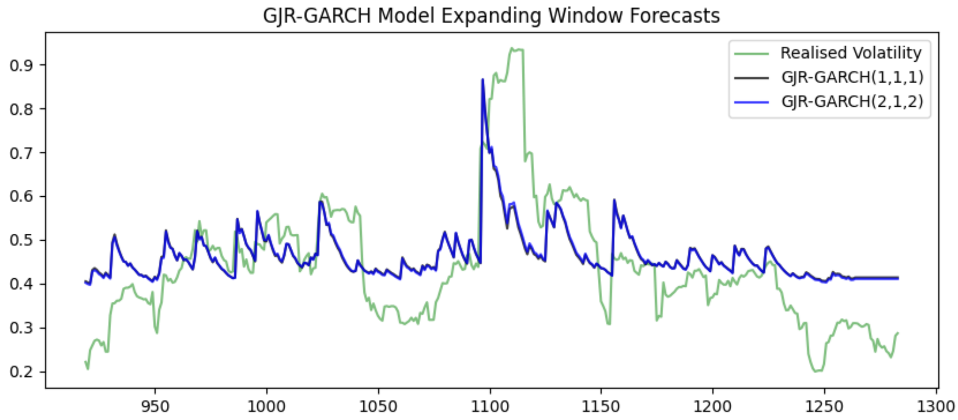

Figure A.31: Expanding window forecasts of daily returns of the EUR/GBP pair computed from GJR-GARCH(1,1,1) and GJR-GARCH(2,1,2)

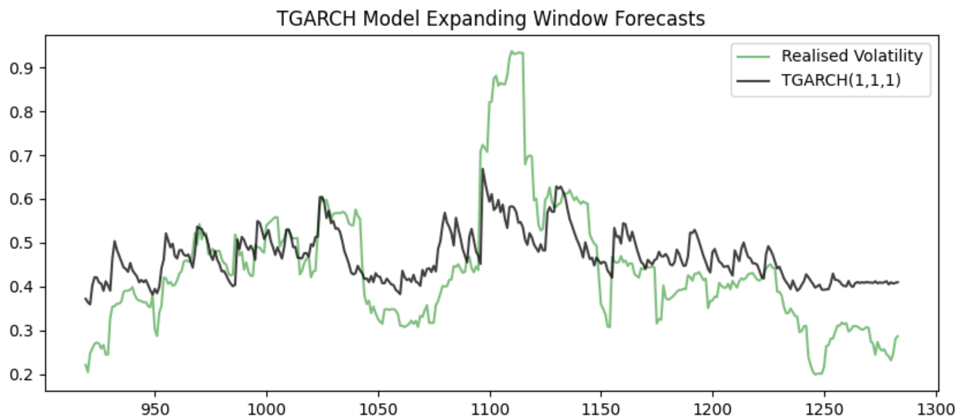

Figure A.32: Expanding window forecast of daily returns of the EUR/GBP pair computed from TGARCH(1,1,1)

A.5 IV-based OLS Regression Statistics (In-Sample)

A.5.1 GBP/EUR

Model 1

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             target_vol   R-squared:                       0.101
Model:                            OLS   Adj. R-squared:                  0.100
Method:                 Least Squares   F-statistic:                     101.3
Date:                Tue, 29 Aug 2023   Prob (F-statistic):           1.19e-22
```

```
Time:                        16:35:05   Log-Likelihood:                 541.95
No. Observations:                 899   AIC:                            -1080.
Df Residuals:                     897   BIC:                            -1070.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0260      0.031      0.850      0.395      -0.034       0.086
x1             0.1199      0.012     10.065      0.000       0.097       0.143
==============================================================================
Omnibus:                      535.050   Durbin-Watson:                   0.061
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4992.895
Skew:                           2.615   Prob(JB):                         0.00
Kurtosis:                      13.292   Cond. No.                         20.4
==============================================================================
```

Model 2

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             target_vol   R-squared:                       0.101
Model:                            OLS   Adj. R-squared:                  0.099
Method:                 Least Squares   F-statistic:                     50.61
Date:                Tue, 29 Aug 2023   Prob (F-statistic):           1.50e-21
Time:                        16:35:05   Log-Likelihood:                 541.96
No. Observations:                 899   AIC:                            -1078.
Df Residuals:                     896   BIC:                            -1064.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0264      0.031      0.862      0.389      -0.034       0.087
x1             0.1189      0.013      8.984      0.000       0.093       0.145
x2             0.0059      0.036      0.167      0.867      -0.064       0.076
==============================================================================
Omnibus:                      534.913   Durbin-Watson:                   0.061
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4989.030
Skew:                           2.615   Prob(JB):                         0.00
Kurtosis:                      13.288   Cond. No.                         23.3
==============================================================================
```

A.5.2 EUR/GBP

Model 1

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             target_vol   R-squared:                       0.072
Model:                            OLS   Adj. R-squared:                  0.071
Method:                 Least Squares   F-statistic:                     69.68
Date:                Tue, 29 Aug 2023   Prob (F-statistic):           2.62e-16
Time:                        16:55:50   Log-Likelihood:                 615.58
No. Observations:                 899   AIC:                            -1227.
Df Residuals:                     897   BIC:                            -1218.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0698      0.026      2.693      0.007       0.019       0.121
x1             0.0983      0.012      8.347      0.000       0.075       0.121
```

Omnibus:	414.689	Durbin-Watson:	0.062
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2213.700
Skew:	2.094	Prob(JB):	0.00
Kurtosis:	9.447	Cond. No.	16.8

Model 2

OLS Regression Results

Dep. Variable:	target_vol	R-squared:	0.075
Model:	OLS	Adj. R-squared:	0.073
Method:	Least Squares	F-statistic:	36.14
Date:	Tue, 29 Aug 2023	Prob (F-statistic):	8.07e-16
Time:	16:55:50	Log-Likelihood:	616.82
No. Observations:	899	AIC:	-1228.
Df Residuals:	896	BIC:	-1213.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0710	0.026	2.740	0.006	0.020	0.122
x1	0.0907	0.013	7.128	0.000	0.066	0.116
x2	0.0544	0.035	1.575	0.116	-0.013	0.122

Omnibus:	407.388	Durbin-Watson:	0.065
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2097.068
Skew:	2.065	Prob(JB):	0.00
Kurtosis:	9.239	Cond. No.	20.9

A.6 IV-based OLS Regression Forecasst

A.6.1 GBP/USD Forecast

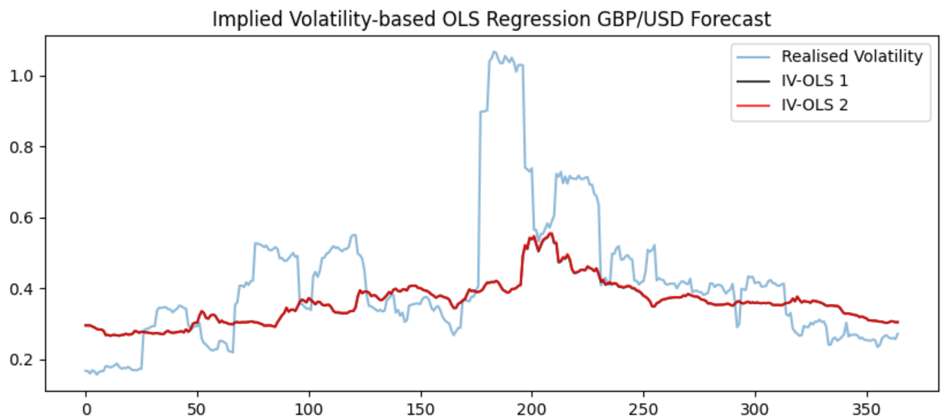

Figure A.33: Forecast of daily returns of the GBP/USD pair computed from Implied Volatility-based OLS Regression model.

A.6.2 EUR/GBP Forecast

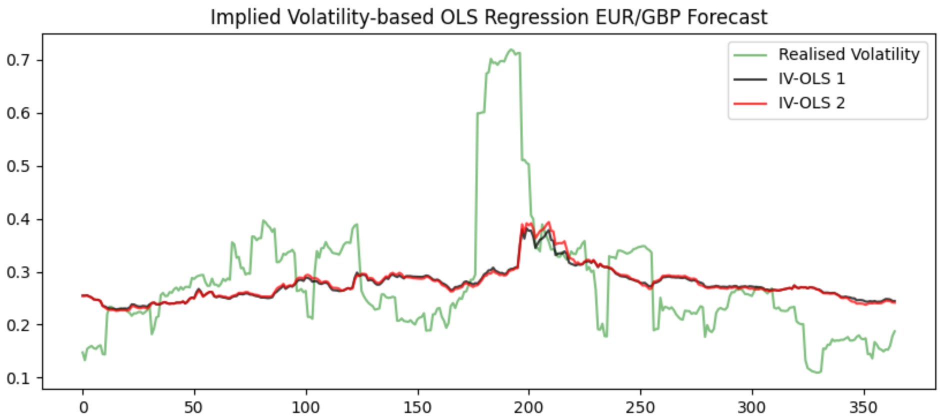

Figure A.34: Forecast of daily returns of the EUR/GBP pair computed from Implied Volatility-based OLS Regression model.