WHEEL ROBOT EQUATION

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1 Bicycle Model

1.1 Introduction

2D bicycle model can be expressed as a simplified car model. This is a classic model that does very well at capturing vehicle motion in normal driving conditions.
The bicycle model we'll develop is called the front wheel steering model, as the front wheel orientation can be controlled relative to the heading of the vehicle. The rear wheel orientation cannot be controlled. It only follows the front wheel. Our target is to compute state [x, y, θ, δ], θ is heading angle, δ is steering angle. The inputs are [v, ψ], v is velocity, ψ is steering rate.
To analyze the kinematics of the bicycle model, we must select a reference point (X, Y) on the vehicle which can be placed at the center of the rear axle, the center of the front axle, or at the center of gravity or cg.

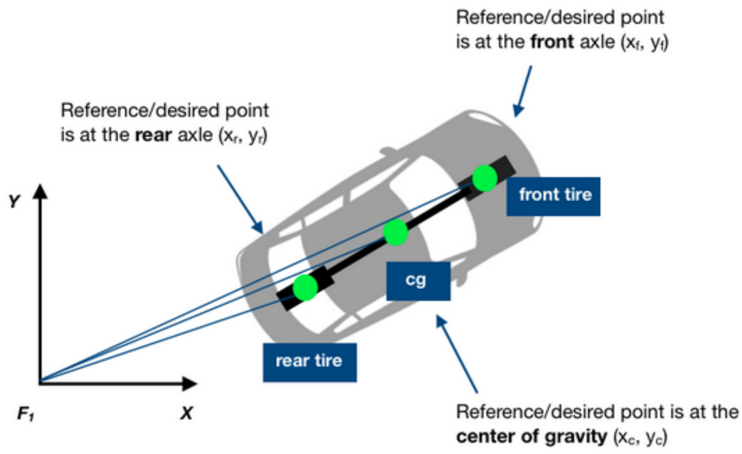

Figure 1: Bicycle Model

1.2 Model Equation

1. **If the desired point is at the center of the rear axle, we can draw below illustration picture:**

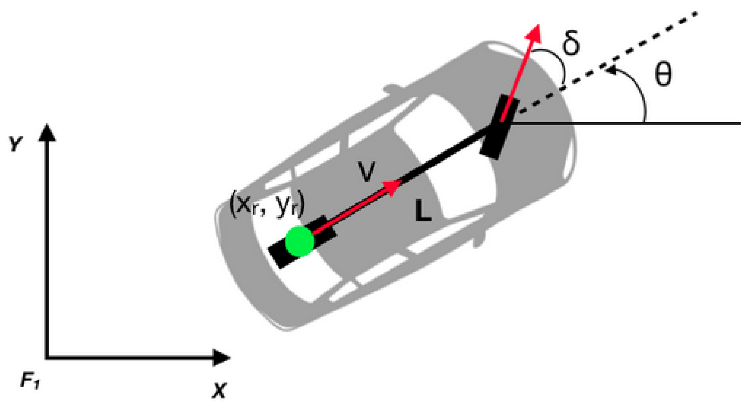

Figure 2: The Bicycle desired point is at the center of the rear axle

Next to apply the Instantaneous Center of Rotation (ICR).

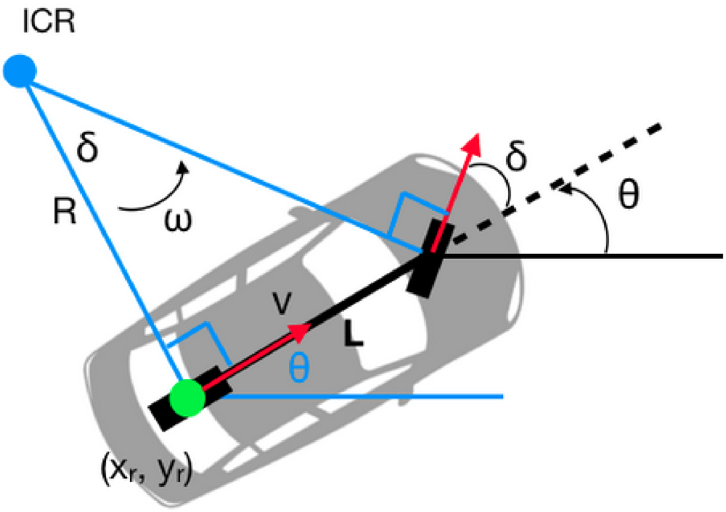

Figure 3: The Bicycle desired point is at the center of the rear axle version2

In order to get the state change [x, y, θ, δ], we need first compute the state change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}]$.

$$\dot{\boldsymbol{x}} = \boldsymbol{v} * \boldsymbol{cos}(\boldsymbol{\theta})$$
$$\dot{\boldsymbol{y}} = \boldsymbol{v} * \boldsymbol{sin}(\boldsymbol{\theta})$$
$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega}$$

Where
$\boldsymbol{\omega} = \frac{\boldsymbol{v}}{\boldsymbol{R}}$
$\boldsymbol{R} = \frac{\boldsymbol{L}}{\boldsymbol{tan}(\boldsymbol{\delta})}$ So We can write:

$$\dot{\boldsymbol{\theta}} = \frac{\boldsymbol{v} * \boldsymbol{tan}(\boldsymbol{\delta})}{\boldsymbol{L}}$$
$$\dot{\boldsymbol{\delta}} = \boldsymbol{\psi}$$

Where: ψ is the input: the rate of change of steering angle.
Summerize Equation of Model is:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{y}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\delta}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{cos}(\boldsymbol{\theta}) & \boldsymbol{0} \\ \boldsymbol{sin}(\boldsymbol{\theta}) & \boldsymbol{0} \\ \boldsymbol{tan}(\boldsymbol{\theta})/\boldsymbol{L} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\psi} \end{bmatrix}$$

2. **If the desired point is at the center of the front axle: Model analysis of desired point at front axle**

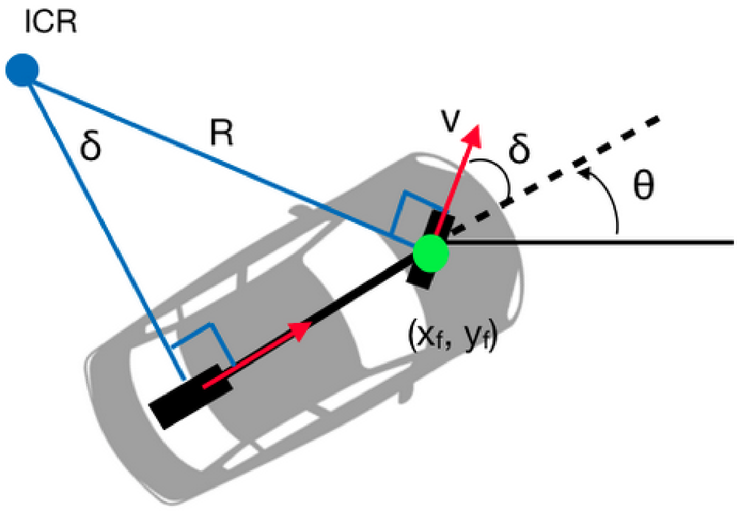

Figure 4: The Bicycle desired point is at the center of the front axle

As figure above shows, desired point is in the center of front wheel. We can arrive:

$$\boldsymbol{R = \frac{L}{sin(\delta)}}$$

So We can get the changing rate $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}]$ as following:

$$\begin{gathered}\dot{\boldsymbol{x}} = \boldsymbol{v * cos(\delta + \theta)}\\ \dot{\boldsymbol{y}} = \boldsymbol{v * sin(\delta + \theta)}\\ \dot{\boldsymbol{\theta}} = \boldsymbol{v * sin(\delta)/L}\\ \dot{\boldsymbol{\delta}} = \boldsymbol{\psi}\end{gathered}$$

3. **If the desired point is at the center of gravity or cg, we can have following figure:**

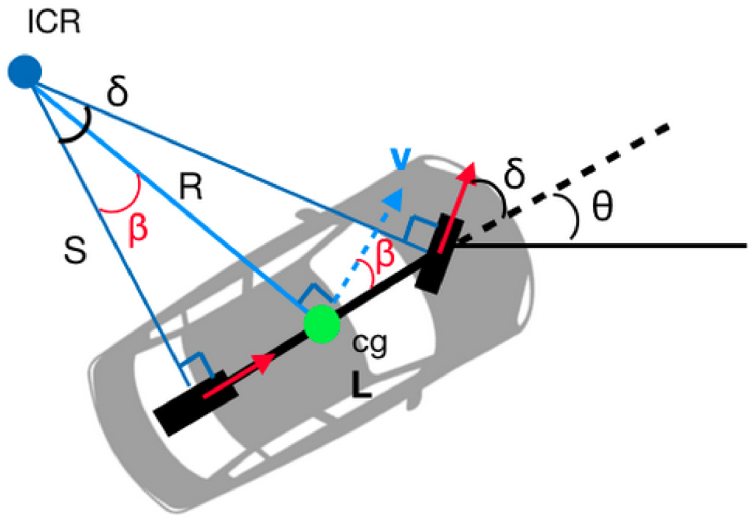

Figure 5: The Bicycle desired point is at the center of gravity or cg,

β is the slip angle.
This model is a little complicated. We need first compute R in order to get $\dot{\theta}$. As shown in figure above, we can first compute S.

$$\boldsymbol{S = \frac{L}{tan(\delta)}}$$

Then we can use S and β angle to compute R.

$$\boldsymbol{R = \frac{S}{cos(\beta)} = \frac{L}{cos(\beta) * tan(\delta)}}$$

Furthermore, if we know the distance between the rear wheel and cg denoted as l_r, we can also compute the slip angle β.

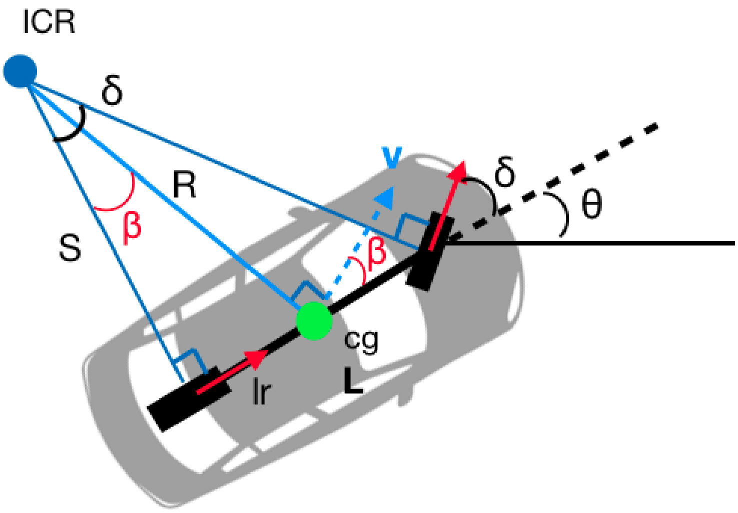

Figure 6: The Bicycle desired point is at the center of gravity or cg, version2

$$\boldsymbol{tan(\beta) = l_r/S = l_r * tan(\delta)/L}$$

So slip angle

$$\boldsymbol{\beta = arctan(\frac{l_r * tan(\delta)}{L}}$$

So the Summerize eqaution of model is :

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} cos(\beta + \delta) & 0 \\ sin(\beta + \delta) & 0 \\ tan(\delta) * cos(\beta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \psi \end{bmatrix}$$

2 DIFFERENTIAL WHEEL

2.1 What is Differential Wheel?

The differential drive is a two-wheeled drive system with independent actuators for each wheel.The name refers to the fact that the motion vector of the robot is the sum of the independent wheel motions. The drive wheels are usually placed on each side of the robot and toward the front.

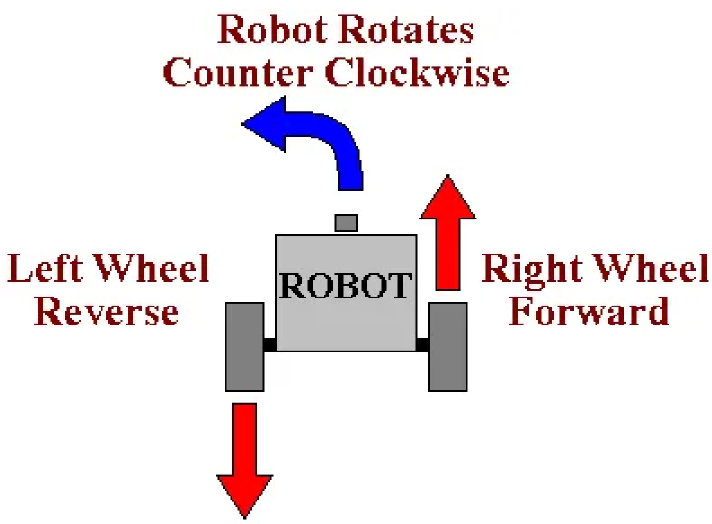

Figure 7: Differential Robot

2.2 Differential Equation

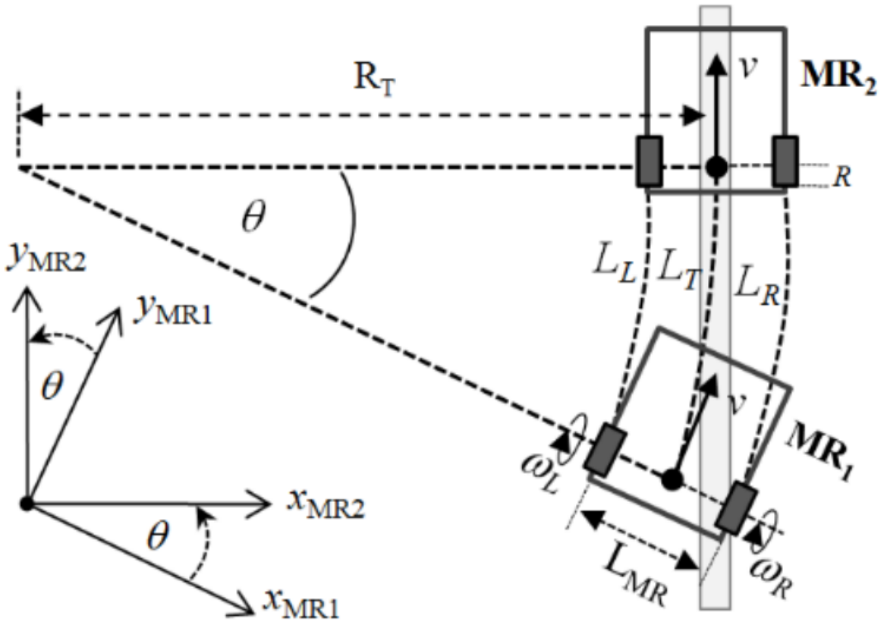

Figure 8: Differential Direction and movement

- Linear velocities both of rear driven wheels V_R and V_L : can be calculated refer to the wheel radius R and angular velocity ω each wheel and:

$$\boldsymbol{V_R = \omega_R * R} \quad \boldsymbol{V_L = \omega_L * L}$$

- A linear velocity V for the mobile robot Calculated by average of the angular velocity of right and left driven wheel:

$$\mathbf{V} = \frac{\mathbf{V_R + V_L}}{\mathbf{2}}$$

- Component of linear velocities $\dot{X}$ and $\dot{Y}$derived from the coordinate system consist of x and y position:

$$\boldsymbol{\dot{X} = V cos(\theta)}$$
$$\boldsymbol{\dot{Y} = V sin(\theta)}$$
$$\boldsymbol{\dot{\theta} = \theta}$$

- Calculate the change in global coordinates

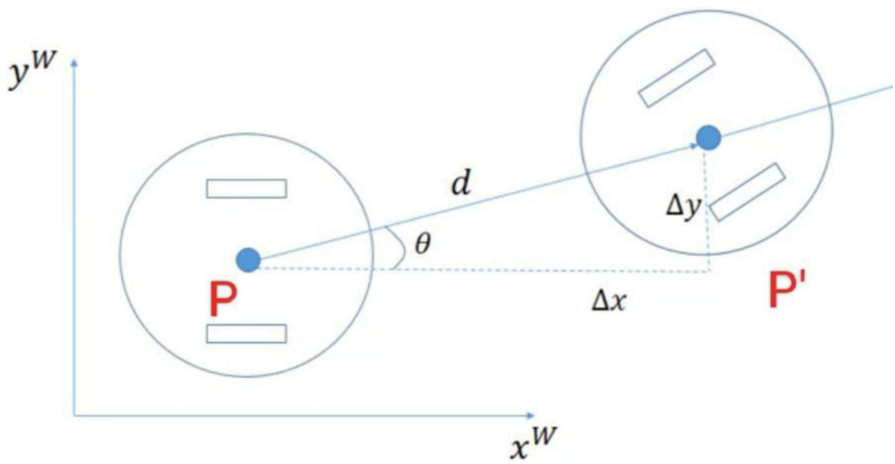

Figure 9: Differential Position

$$\boldsymbol{\Delta X = dcos(\theta)} \quad \boldsymbol{\Delta Y = dsin(\theta)}$$

$$\mathbf{P'} = \begin{bmatrix} X \\ Y \\ \theta \end{bmatrix} + \begin{bmatrix} dcos(\theta) \\ dsin(\theta) \\ \Delta\theta \end{bmatrix}$$

Where: $\Delta\theta = \frac{d_r - d_l}{2R_w}, \quad d = \frac{d_l + d_r}{2}$

- Calculate Velocity to speed control motor

$$\boldsymbol{\omega_r = \frac{2}{R}(V + L * \omega) \qquad \omega_l = \frac{2}{R}(V - L * \omega)}$$

3 MECANUM WHEEL

3.1 What is Mecanum Wheel?

The mecanum wheel is a form of tireless wheel, with a series of rubberized external rollers obliquely attached to the whole circumference of its rim. These rollers typically each have an axis of rotation at 45° to the wheel plane and at 45° to the axle line.

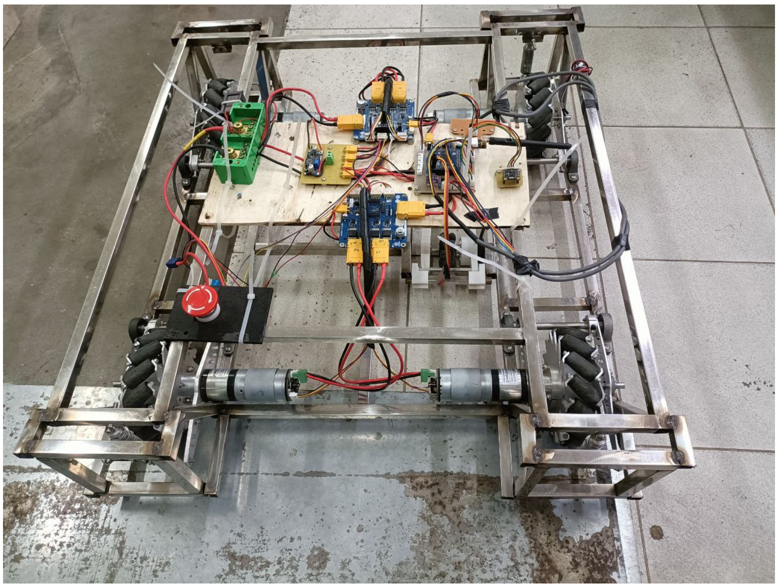

Figure 10: Mecanum Robot

3.2 Mecanum Equation

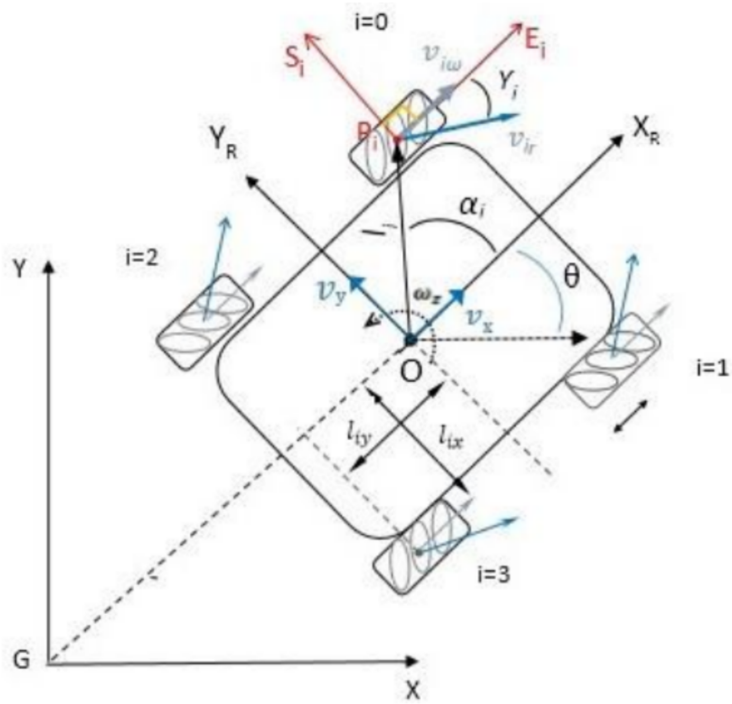

Figure 11: Movement and Deriction of mecanun wheel

_ **The configuration parameter and system velocities are defind as follows:**

- $\boldsymbol{x, y, \theta}$: Robot's position (x,y) and its orientation angle θ (The angle between X and X_R).
- **X,G,Y**: Inertial frame; x,y are the coordinates of the reference point O in the

- $\mathbf{X_R OY_R}$: Robot's base frame; Cartesian coordinate system associated with the movement of the body center.
- $\mathbf{S_i P_i E_i}$: Coordinate system of ith wheel in the wheel's center point P_i.
- $\mathbf{O}, \mathbf{Pi}$: The inertial basis of the Robot in Robot's frame and P i = X P i , Y P i the center of the rotation axis of the wheel i.
- $\tilde{\mathbf{OP_i}}$:is a vector that indicates the distance between Robot's center and the center of the wheel ith.
- $\mathbf{l_{ix}}, \mathbf{l_{iy}}$: l_{ix} :half of the distance between front wheels and l_{iy} : half of the distance between front wheel and the base (center of the robot O).
- $\mathbf{l_i}$: distance between wheels and the base (center of the robot O).
- $\mathbf{r_i}$: denotes the radius of the wheel i (Distance of the wheel's center to the roller
- $\mathbf{r_r}$: denotes the radius of the rollers on the wheel.
- $\boldsymbol{\alpha_i}$: the angel between OP_i and X_R .
- $\boldsymbol{\beta_i}$: the angle between S_i and X_R.
- $\boldsymbol{\gamma_i}$: the angle between v_{ir} and E_i.
- $\boldsymbol{\omega[rad/s]}$: wheels angular velocity;
- $\mathbf{V_{i\theta}[rad/s]}$: wheels angular velocity.
- $\mathbf{V_{ir}}$: the velocity of the passive roller in the wheel i
- $\boldsymbol{[W_s i W_E i \theta_i]^T}$: Genralizd velocity of point P_i in the frame $S_i P_i E_i$
- $\boldsymbol{[V_s i V_E i \theta_i]^T}$: Generalized velocity of point P_i in the frame $X_R OY_R$.
- $\mathbf{v_x v_y [m/s]}$: Robot linear velocity;
- $\boldsymbol{\theta(rad/s)}$: Robot angular velocity; center)

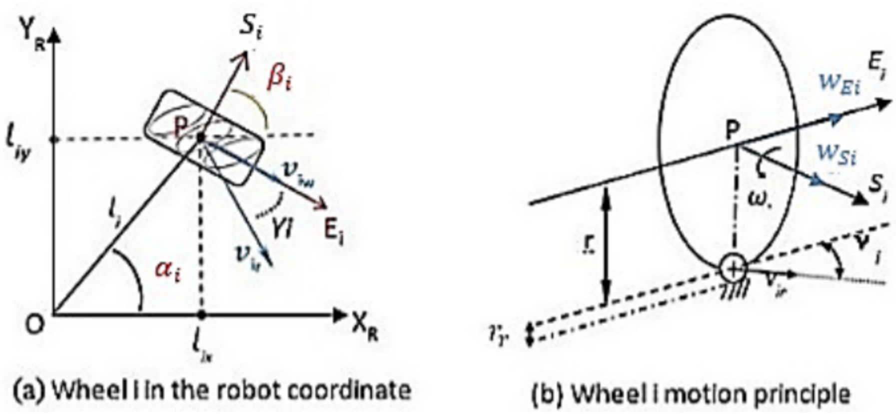

Figure 12: Coordination and motion mecanum wheel

_ **Wheel i and the tangential velocity of the free roller attaced to the wheel touching the floor:**

$$v_{ir} = \frac{1}{cos(45)} r_r \omega_i, \quad W_{Ei} = r_r \omega_i \quad (eq.1)$$

_ **The velocity of the wheel i in the frame $S_i P_i E_i$, can be derived by:**

$$\begin{bmatrix} v_{si} \\ v_{Ei} \end{bmatrix} = \begin{bmatrix} 0 & sin(\gamma_i) \\ r_i & cos(\gamma_i) \end{bmatrix} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = T_{Pi}^{W_i} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} \qquad \text{(eq.2)}$$

_ **The velocity of the wheel's center translated to the $X_R OY_R$ coordinate system can be achieved by equation 2**:

i	**Wheels**	$\boldsymbol{\alpha_i}$	$\boldsymbol{\beta_i}$	$\boldsymbol{\gamma_i}$	l_i	l_{ix}	l_{iy}
0	1sw	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$-\frac{\pi}{4}$	l	lx	ly
1	2sw	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$	$\frac{\pi}{4}$	l	lx	ly
2	3sw	$\frac{3\pi}{4}$	$\frac{\pi}{2}$	$\frac{\pi}{4}$	l	lx	ly
3	4sw	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	l	lx	ly

Table 1: Table Parameter of Mecanum Wheel

$$\begin{bmatrix} V_{iX_R} \\ V_{iY_R} \end{bmatrix} = \begin{bmatrix} cos(\beta_i) & -sin(\beta_i) \\ sin(\beta_i) & cos(\beta_i) \end{bmatrix} \begin{bmatrix} v_{si} \\ v_{Ei} \end{bmatrix} = T_{P_i}^{W} T_{R}^{P_i} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} \qquad (eq.3)$$

_ **The robot's motion is planar,we also have:**

$$\begin{bmatrix} v_{iX_R} \\ v_{iY_R} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -l_{iy} \\ 0 & 1 & l_{ix} \end{bmatrix} \begin{bmatrix} V_{X_R} \\ V_{Y_R} \\ \theta_R \end{bmatrix} = T' \begin{bmatrix} V_{X_R} \\ V_{Y_R} \\ \theta_R \end{bmatrix} \qquad \text{(eq.4)}$$

_ **The inverse kinematic model can be obtained:**

$$T_{P_i}^{W} T_{R}^{P_i} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = T' \begin{bmatrix} V_{X_R} \\ V_{Y_R} \\ \theta_R \end{bmatrix} \qquad \text{(eq.5)}$$

_ **The robot's base velocity (at point O) related to the rotational velocity of the ith wheel can be obtained from eq.5**

$$\begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = (T_{P_i}^{W})^{-1} (T_{R}^{P_i})^{-1} T' \begin{bmatrix} V_{X_R} \\ V_{Y_R} \\ \theta_R \end{bmatrix} \qquad \text{(eq.5)}$$

Let : $T = (T_{P_i}^{W})^{-1} (T_{R}^{P_i})^{-1} T'$

$$T = \begin{bmatrix} cos(\beta_i) & -sin(\beta_i) \\ sin(\beta_i) & cos(\beta_i) \end{bmatrix}^{-1} \begin{bmatrix} 0 & sin(\gamma_i) \\ r_i & cos(\gamma_i) \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -l_{iy} \\ 0 & 1 & l_{ix} \end{bmatrix}$$

Where: $l_{ix} = l_i cos(\alpha_i) and l_{iy} = l_i sin(\alpha_i)$

$$=> T = -\frac{1}{r} \begin{bmatrix} \frac{cos(\beta_i - \gamma_i)}{sin(\gamma_i)} & \frac{sin(\beta_i - \gamma_i)}{sin(\gamma_i)} & \frac{l_i sin(-\alpha_i + \beta_i - \gamma_i)}{sin(\gamma_i)} \\ -\frac{rcos(\beta_i - \gamma_i)}{sin(\gamma_i)} & -\frac{rsin(\beta_i - \gamma_i)}{sin(\gamma_i)} & -\frac{rl_i sin(-\alpha_i + \beta_i - \gamma_i)}{sin(\gamma_i)} \end{bmatrix}$$

Since there is a relation between independent variables v_{ir} and ω i in each joint and the systems angular and linear velocity, assuming that there is no wheel slipping on the ground, the system inverse kinematic can be obtained by eq.7

_ **How to calculate velocity 4 wheel:**

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = -\frac{1}{r} \begin{bmatrix} \frac{cos(\beta_1 - \gamma_1)}{sin(\gamma_1)} & \frac{sin(\beta_1 - \gamma_1)}{sin(\gamma_1)} & \frac{l_1 sin(-\alpha_1 + \beta_1 - \gamma_1)}{sin(\gamma_1)} \\ \frac{cos(\beta_2 - \gamma_2)}{sin(\gamma_2)} & \frac{sin(\beta_2 - \gamma_2)}{sin(\gamma_2)} & \frac{l_1 sin(-\alpha_2 + \beta_2 - \gamma_2)}{sin(\gamma_2)} \\ \frac{cos(\beta_3 - \gamma_3)}{sin(\gamma_3)} & \frac{sin(\beta_3 - \gamma_3)}{sin(\gamma_3)} & \frac{l_1 sin(-\alpha_3 + \beta_3 - \gamma_3)}{sin(\gamma_3)} \\ \frac{cos(\beta_4 - \gamma_4)}{sin(\gamma_4)} & \frac{sin(\beta_4 - \gamma_4)}{sin(\gamma_4)} & \frac{l_1 sin(-\alpha_4 + \beta_4 - \gamma_4)}{sin(\gamma_4)} \end{bmatrix} \begin{bmatrix} Vx \\ Vy \\ \omega_z \end{bmatrix}$$

_ **Parameter $\boldsymbol{\alpha, \beta, \gamma}$ in Mecanum Omnidirection Wheel**.

- We get Inverse Kinematics form:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 1 & -1 & -(lx + ly) \\ 1 & 1 & (lx + ly) \\ 1 & 1 & -(lx + ly) \\ 1 & -1 & (lx + ly) \end{bmatrix} \begin{bmatrix} Vx \\ Vy \\ \omega_z \end{bmatrix}$$

- Forward Kinematics form :

$$\begin{bmatrix} Vx \\ Vy \\ \omega_z \end{bmatrix} = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{lx+ly} & \frac{1}{lx+ly} & -\frac{1}{lx+ly} & \frac{1}{lx+ly} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

4 OMNI WHEEL

4.1 What is Omni Wheel?

Omni wheels or poly wheels, similar to Mecanum wheels, are wheels with small discs (called rollers) around the circumference which are perpendicular to the turning direction. The effect is that the wheel can be driven with full force, but will also slide laterally with great ease.

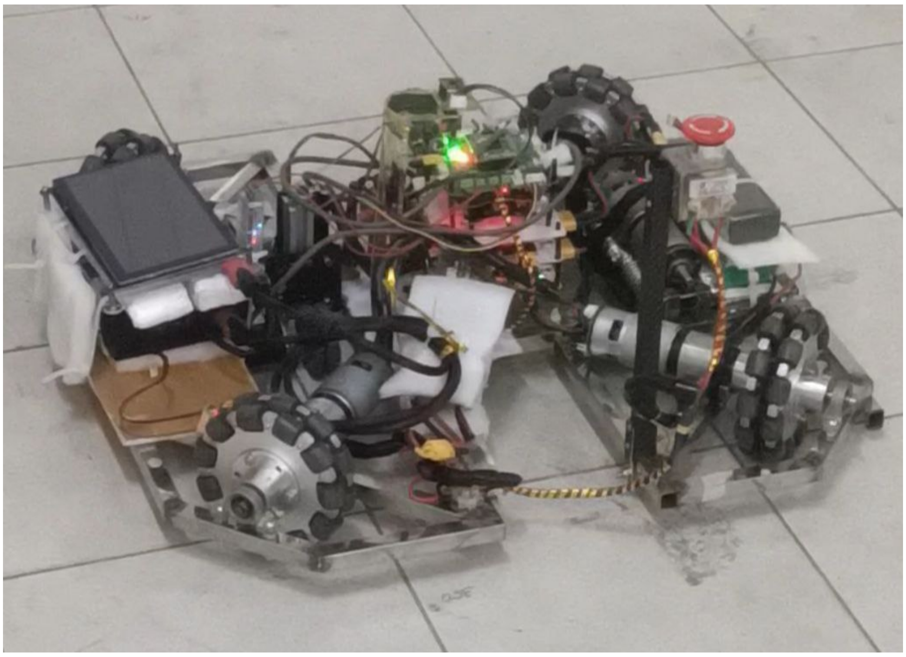

Figure 13: Omni Robot

4.2 OMNI Wheel Equation

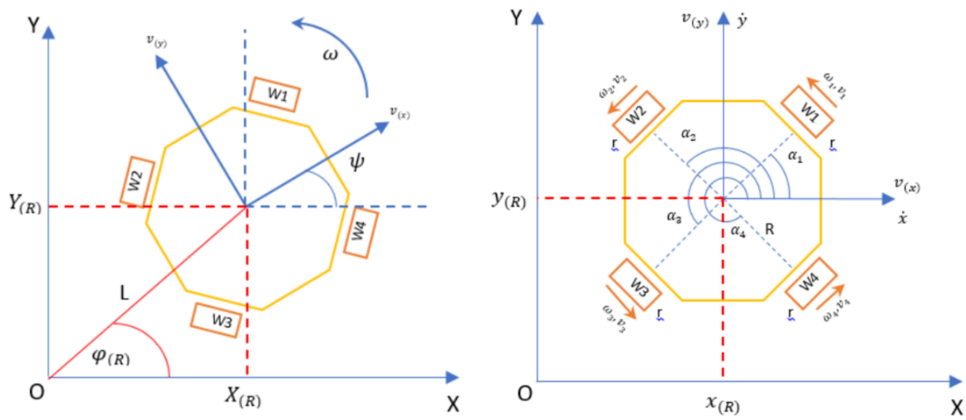

Figure 14: Movement and Direction of OMNI Wheel

_ **The configuration parameter:**

- $\boldsymbol{\alpha_1, \alpha_2, \alpha_3, \alpha_4}$ = The angle formed between the wheels and the robot reference point.
- γ = The angular from roller to wheel.
- $\boldsymbol{\psi_R}$ = is the direction of movement of the robot at global coordinates.
- $\boldsymbol{\omega_W}$ = The angular velocity of each wheel.
- $\mathbf{V_W}$ = The linear velocity of each Wheel.
- **XOY** = global coordinates are notified using $V(g) = [\dot{X}, \dot{Y}, \omega]$.

- **R** = is the distance from the Wheel to the center of the robot.
- **r** = is the radius of the omni wheel.
- $\mathbf{V_R} = [\mathbf{V_x}, \mathbf{V_y}]$ = The position of the robot's coordinate.
- $\boldsymbol{X_{(g)} = [X_R Y_R \psi]^T}$ = Orientation to global coordinates.
- $\boldsymbol{\omega}$ = is the notation of the angular velocity of the robot to the global coordinates.
- $\boldsymbol{\Phi_n}$ = is a notation of the direction of movement of the robot to global coordinates.
- **L** = is the notation of the resultant linear velocity of the robot.

_ **The OMNI Wheel equation**

$$\omega_i = h_i(\psi)\dot{q} = \begin{bmatrix} \frac{1}{r_i} & \frac{tan\gamma_i}{r_i} \end{bmatrix} \begin{bmatrix} cos\alpha_i & sin\alpha_i \\ -sin\alpha_i & cos\alpha_i \end{bmatrix} \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\psi & sin\psi \\ 0 & -sin\psi & cos\psi \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

$$h_i(\phi) = \frac{1}{r_i cos\gamma_i} \begin{bmatrix} x_i sin(\alpha_i + \gamma_i) - y_i cos(\beta_i + \gamma_i) \\ cos(\alpha_i + \gamma_i + \psi) \\ sin(\alpha_i + \gamma_i + \psi) \end{bmatrix}^T$$

Where: $\gamma = 0$ because in omni wheel is roller parallel with Wheel.

$$\omega_i = \frac{1}{r}(-sin(\psi + \alpha_i)\dot{x} + cos(\psi + \alpha_i)\dot{y} + R\dot{\psi})$$

- **4 Wheel**

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -sin(\psi + \alpha_1) & cos(\psi + \alpha_1) & R \\ -sin(\psi + \alpha_2) & cos(\psi + \alpha_2) & R \\ -sin(\psi + \alpha_3) & cos(\psi + \alpha_3) & R \\ -sin(\psi + \alpha_4) & cos(\psi + \alpha_4) & R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$$

Where: $\alpha_1 = \frac{\pi}{4}, \alpha_2 = \frac{3\pi}{4}, \alpha_3 = \frac{5\pi}{4}, \alpha_4 = \frac{7\pi}{4}$, and take $\psi = 0$ because robot start at angle 0 degree.

- **Inverse kinematic Form:**

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -sin(\frac{\pi}{4}) & cos(\frac{\pi}{4}) & R \\ -sin(\frac{\pi}{34}) & cos(\frac{3\pi}{4}) & R \\ -sin(\frac{5\pi}{4}) & cos(\frac{5\pi}{4}) & R \\ -sin(\frac{7\pi}{4}) & cos(\frac{7\pi}{4}) & R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$$

- **Forward Kinematic Form:**

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \frac{r}{4} \begin{bmatrix} -sin(\frac{\pi}{4}) & -sin(\frac{3\pi}{4}) & -sin(\frac{5\pi}{4}) & -sin(\frac{7\pi}{4}) \\ cos(\frac{\pi}{4}) & cos(\frac{3\pi}{4}) & cos(5\frac{\pi}{4}) & cos(7\frac{\pi}{4}) \\ \frac{1}{R} & \frac{1}{R} & \frac{1}{R} & \frac{1}{R} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

5 SWERVE DRIVE WHEEL

5.1 What is Swerve Drive Wheel?

Swerve Drive is a type of drive train which each wheel can point in any direction. Swerve drives use a set of independently steered wheels to manipulate the chassis. These wheels require two actuators each one to provide torque to the drive wheel and a second to turn the drive wheel assembly and direct its thrust vector where desired.

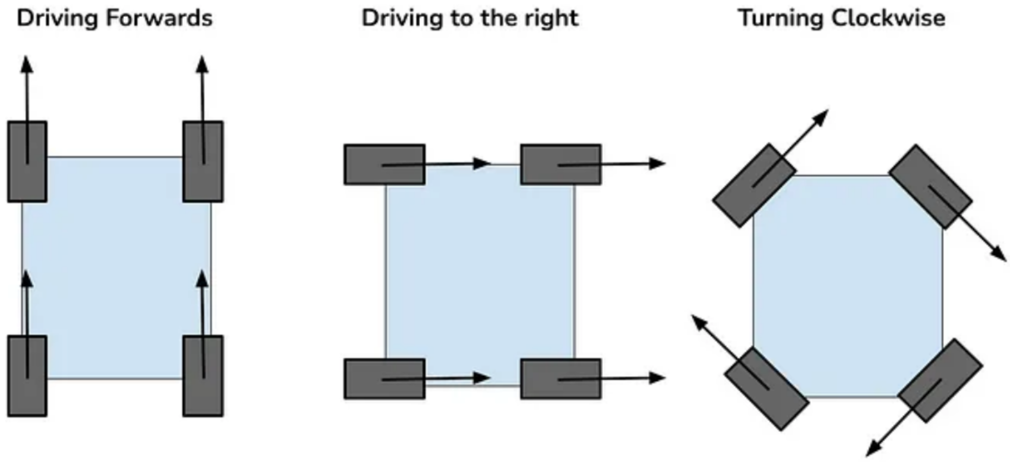

Figure 15: Movements on the swerve drive

5.2 Swerve drive Equation

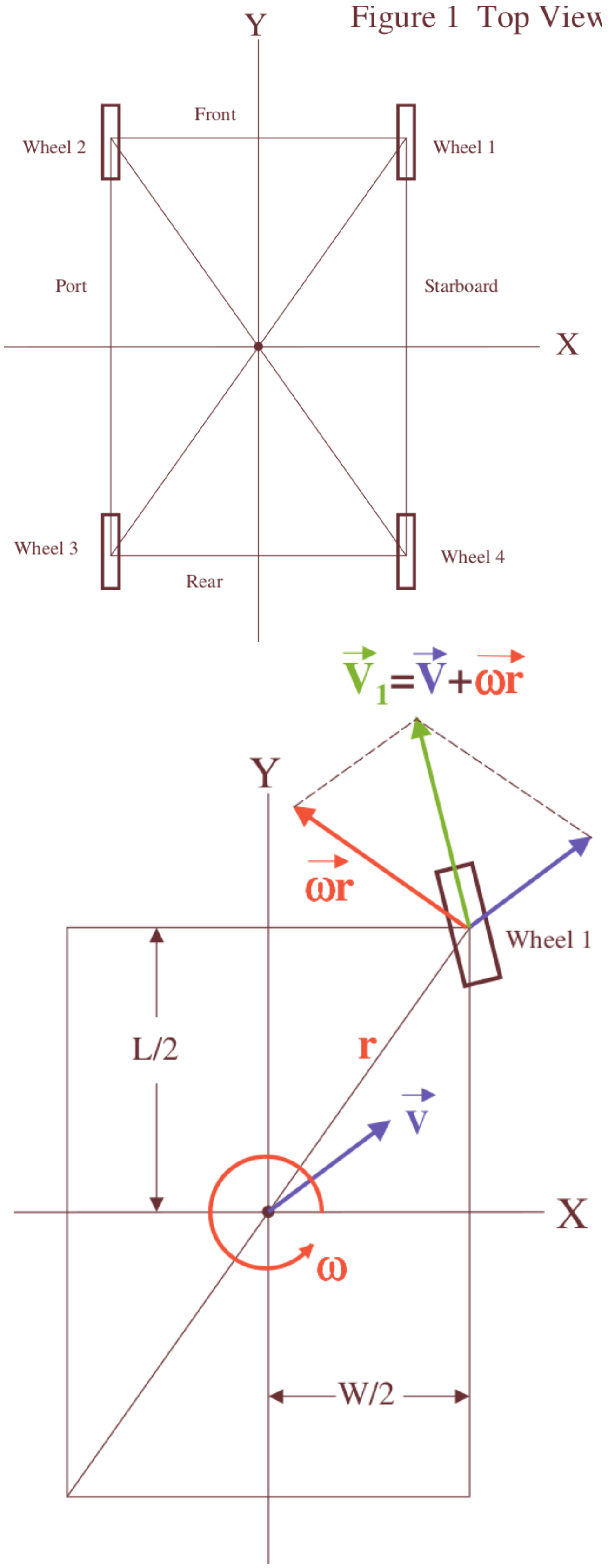

Figure 16: Motion and Direction of Swerve drive Wheel

_ **The Parameter in Swerve Drive:**

- **XOY** : is global coordination of Robot.

- $\tilde{\mathbf{V}}$: is Vehicle translation (fwd/rev plus strafe).
- $\boldsymbol{\omega}$: is angular velocity or Vehicle rotation (radians/sec and is positive clockwise).
- $\tilde{\mathbf{V_1}}$: is Wheel direction and Velocity (calculated separately for each of the 4 wheels from the vehicle translation and rotation).
- $\mathbf{V_x}, \mathbf{V_Y}(\mathbf{m/s})$: is linear velocity on the X,Y component of $\vec{V}$
- $\mathbf{L}$: is the wheelbase.
- $\mathbf{W}$: is the trackwidth.
- $\mathbf{r}$: is distance from center robot to center Wheel($\mathbf{r} = \sqrt{(\mathbf{L^2} + \mathbf{W^2})/2}$).
- $\mathbf{R}$: is Radius of Wheel.
- $\boldsymbol{\omega_i(rad/s)}$: is Linear velocity of each Wheel.
- $\boldsymbol{\theta_i(rad)}$: is angle of each wheel(Angles range from $-\pi$ to $+pi$ radian CW; zero is straight ahead).

_ **Equation of each Wheel**

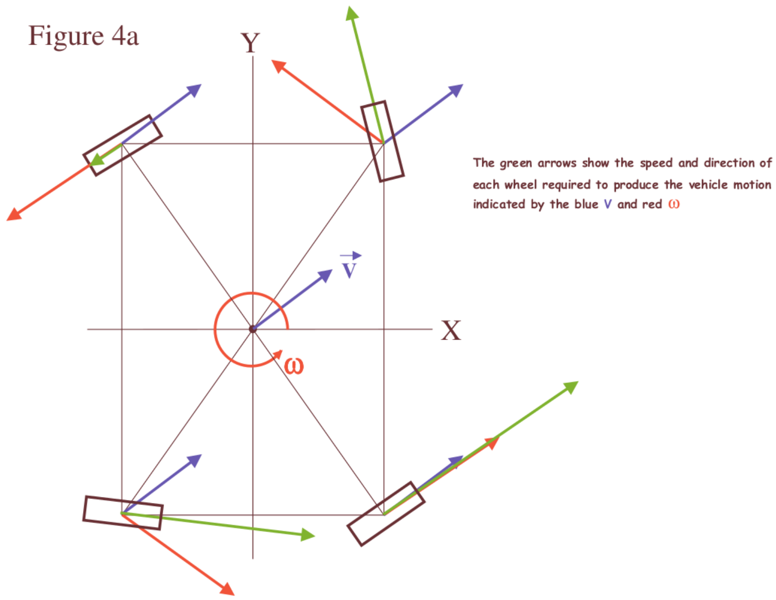

Figure 17: The speed and direction of 4wheel of Swerve drive

- **Calculate Velocity x,y from each wheel**
  - **Wheel 1:**

$$\boldsymbol{V_{1x} = V_x + (\omega r)_x = V_x + \omega L/2}$$
$$\boldsymbol{V_{1y} = V_y + (\omega r)_y = V_y - \omega W/2}$$

  - **Wheel 2:**

$$\boldsymbol{V_{2x} = V_x + (\omega r)_x = V_x + \omega L/2}$$
$$\boldsymbol{V_{2y} = V_y + (\omega r)_y = V_y + \omega W/2}$$

  - **Wheel 3:**

$$\boldsymbol{V_{3x} = V_x + (\omega r)_x = V_x - \omega L/2}$$
$$\boldsymbol{V_{3y} = V_y + (\omega r)_y = V_y + \omega W/2}$$

  - **Wheel 4:**

$$\boldsymbol{V_{4x} = V_x + (\omega r)_x = V_x - \omega L/2}$$
$$\boldsymbol{V_{4y} = V_y + (\omega r)_y = V_y - \omega W/2}$$

- **Inverse Kinematic of Swerve drive**

$$\begin{bmatrix} V_{1x} \\ V_{1y} \\ V_{2x} \\ V_{2y} \\ V_{3x} \\ V_{3y} \\ V_{4x} \\ V_{4y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & L/2 \\ 0 & 1 & -W/2 \\ 1 & 0 & L/2 \\ 0 & 1 & W/2 \\ 1 & 0 & -L/2 \\ 0 & 1 & W/2 \\ 1 & 0 & -L/2 \\ 0 & 1 & -W/2 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix}$$

- **Forward Kinematic of Swerve drive**
  $\mathbf{V} = \mathbf{AU}$ (Form Inverse kinematic)
  $\mathbf{U} = (\mathbf{A^T A})^{-1}\mathbf{A^T V}$ (Form Forward kinematic)

$$\begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \frac{2l}{(l^2+w^2)} & -\frac{2l}{(l^2+w^2)} & \frac{2l}{(l^2+w^2)} & \frac{2l}{(l^2+w^2)} & -\frac{2l}{(l^2+w^2)} & \frac{2l}{(l^2+w^2)} & -\frac{2l}{(l^2+w^2)} & -\frac{2l}{(l^2+w^2)} \end{bmatrix} \begin{bmatrix} V_{1x} \\ V_{1y} \\ V_{2x} \\ V_{2y} \\ V_{3x} \\ V_{3y} \\ V_{4x} \\ V_{4y} \end{bmatrix}$$

- **Define a,b,c,d as follow:**

$$\boldsymbol{a = V_x - \omega L/2, \quad b = V_x + \omega L/2}$$
$$\boldsymbol{c = V_y - \omega W/2, \quad d = V_y + \omega W/2}$$

OR

$$\mathbf{V_{1x} = V_{2x} = b \quad V_{1y} = V_{4y} = c}$$
$$\mathbf{V_{3x} = V_{4x} = a \quad V_{3y} = V_{2y} = d}$$

- **Calculate the speed and angle of each wheel**
  - **Wheel1**:

$$\boldsymbol{\omega_1 = \frac{\sqrt{b^2+c^2}}{R} \quad \theta_1 = atan2(b,c)}$$

  - **Wheel2**:

$$\boldsymbol{\omega_2 = \frac{\sqrt{b^2+d^2}}{R} \quad \theta_2 = atan2(b,d)}$$

  - **Wheel3**:

$$\boldsymbol{\omega_3 = \frac{\sqrt{a^2+d^2}}{R} \quad \theta_3 = atan2(a,d)}$$

  - **Wheel4**:

$$\boldsymbol{\omega_4 = \frac{\sqrt{a^2+c^2}}{R} \quad \theta_4 = atan2(a,c)}$$