Picking the correct exposure for Deep Sky

Original content by Robin Glover, here reformatted in markdown.

Formulating the Problem

Let's set out the goal to begin with, which is to answer this question:

What is the best exposure for me to use when taking Deep Sky Images?

Actually, the question is actually a good deal longer than that - something more like

Given that I'm using camera X with telescope Y and that my light pollution levels are **insert value**, and that I have a total time available of **so many** hours before dawn/clouds arrive/my target goes behind a tree/having to pack up, what exposure length should I pick for my sub-exposures to give me the best possible image quality in the final stacked image I am going to produce?

The danger here is that a new (or even an experienced) imager will follow the reasoning that if long exposures are needed to see faint things then even longer exposures must be better. This view is perpetuated to a degree by discussions of 10,15 or even 20 minute sub exposures being used with guiding and high quality mounts. These techniques of extremely long sub-exposures do produce good results (providing the mount and guiding behave well), but as we will see they are not always the only way to get good results with deep sky imaging.

In many conditions we will find that a law of diminishing returns applies as you increase exposure length, meaning that exceptionally long exposures really give no improvement in final image quality over the results that can be achieved by dividing the same total exposure time into more, but shorter, sub-exposures. Since long exposures have drawbacks of their own - guiding requirements, hot pixels, satellite, meteor and plane tracks, etc, the key is obviously to find the point at which the returns of increasing exposure diminish and choose that point as our exposure time. By the end of this article we will have seen how and why this point of diminishing returns arises and how you can find the exact exposure needed to hit it using the new Smart Histogram functionality in SharpCap 3.1.

To get started, we need to look at the causes of "noise" in a digital image and how that affects the faint detail that we hope to see.

Noise and Background

The trouble with the word "noise" (and the reason I am using it in quotes) is that it gets used to cover two different concepts in imaging that really should have different labels - these concepts are

Random Noise - this means random variations between pixel values in a single frame (or between the value at a single pixel in sequential frames). These random variations occur even when you are taking images of (for instance) a perfectly flatly illuminated grey surface - not every pixel will have the value of 50%, some

will be 49%, some 51%, a few 48% and 52%. We'll come back to the causes of these variations in a minute.

Background Signal - this means extra signal that is added to image the information that you really want. The information you really want is pictures of galaxies, nebulae, etc. The background signal comes primarily from sky glow (sometimes called sky noise) and a thermal background caused by the temperature of the camera sensor (sometimes thermal noise). We're not going to refer to either of these effects as noise here to avoid confusion, but it turns out that both of them can lead to random noise and that turns out to be critical to understanding what exposure to choose.

Ok, that will do for post one - we've introduced the problem and made sure our naming is clear, in the next section we will cover sources of random noise.

Random Noise

Let's talk for a moment about how a digital imaging sensor works...

Photons (which have been collected by your telescope or camera lens) hit one of the pixels on the sensor and each photon that hits the sensor has a chance of being detected. Being detected means that the photon has managed to transfer an electron into the charge well in the pixel and it is the number of these electrons collected in each pixel that eventually gets turned into the image that we view. The chance of a photon being detected is called the Quantum Efficiency of the sensor, and for modern camera sensors it can be 40-50% or even higher. Traditional photographic film only detected just a few percent of the photons that fell on it, which explains why digital imaging has transformed astrophotography so much!

Just to complete the picture of how we get the image out of our camera, once the exposure is finished, the camera measures the number of electrons generated each pixel, first as a voltage and then, using an analogue-to-digital-converter (ADC) as a number. Depending on the imaging sensor and its configuration the final number (which is measured in ADU) might be in the range 0..255, 0..4095 or 0..65535 or even other possible ranges.

So, let's imagine a fairly typical camera sensor (a CMOS or a CCD) and a fairly typical imaging telescope - perhaps an ED refractor or similar that we are going to use together for deep sky imaging.

Now, let's put our imaginary camera sensor onto an imaginary telescope and point it at an imaginary M31. Think about one particular pixel on our sensor maybe it's one that's collecting light from one of the spiral arms of the galaxy suppose that on average 10 photons per second from M31 are hitting that particular pixel on the sensor. If our sensor has a typical 50% quantum efficiency then that pixel will be adding an average of 5 electrons per second during the length of the exposure. If we took a 10s exposure we'd expect to collect about 50 electrons, a 20s exposure about 100 and a 60s exposure about 300.

The key thing here is that our pixel will not collect 300 electrons every time we take a 60s exposure - sometimes it will collect more, sometimes less because the rate of 10 photons per second is only an average. It's like tossing a coin - on

average if you toss a coin 10 times you get five heads, but you don't get five heads *every* time you do 10 coin tosses. This type of noise - caused by random variations in the number of electrons detected for pixels that are illuminated with the same intensity - is often called shot noise.

Now, it turns out that the random part of the number of electrons (ie by how much it is likely to be over or under the average value) can be modelled by a fairly simple and well known probability distribution called the Poisson Distribution. You can read the whole wikipedia article on it if you want, but the key fact that we need to know is that the size of the random variation is equal to the square root of the number of electrons, ie

$$\sigma_e = \sqrt{n_e}$$

where n_e is the number of electrons collected by the pixel and σ_e is the standard deviation of the number of electrons expected between pixels all exposed to the same light intensity.

To give a concrete example, in our 60s exposure case we would expect the number of electrons to be 300, so the standard deviation will be the square root of 300, or about 17. That means that if we had a lot of pixels all with the same light intensity falling on them, we'd expect about 68% of those pixels to have values between 283 and 317 (within 1 standard deviation of the mean), about 95% of them to be between 266 and 334 (within two standard deviations) and almost all of them (99.7%) to be within 3 standard deviations.

So, now we understand where random noise in images comes from, and we have probably all seen it as speckles particularly visible in the fainter portions of an image. Before moving on, we need to see how that random noise limits our ability to see faint detail in our images.

Back to our imaginary camera... We started by considering an area where the pixels were receiving an average of 10 photons per second, which gave use 300 photons on average within 60s, but with random noise meaning that actually values ranged from 266 to 334 and beyond. Now let's think about a different region of spiral arm nearby that is a bit dimmer - say 10% dimmer. That would lead to 9 photons per pixel per second on average and therefore an average of 270 photons in a 60s exposure with a standard deviation of about 16, so a typical range of 238 to 302.

You can see the sort of noise that would arise from these situations in the images below - while it's not too hard to see the differences between them when the images are 100 pixels on a side each, if the brightness variation was part of fine detail of an image it would be very easy for it to vanish completely due to the level of noise.

Ok, enough on random noise (or at least shot noise) for now. In the next part we'll see how the background signal arises and how it makes shot noise even worse. After that we'll consider the other source of random noise - read noise.

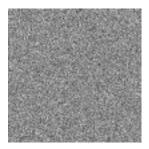


Figure 1: 50% gray noise

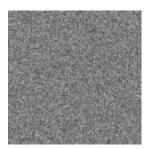


Figure 2: 45% gray noise

Background Signal

We've already mentioned that there are two main sources of background signal (that is sources of detected electrons that don't come from the faint fuzzies that we are hoping to image). Those sources are

- Light pollution background
- Thermal background

The light pollution one is fairly simple to understand - excess light from street lighting, security lights, the moon, etc is scattered by the atmosphere and some of those extra photons end up coming down our telescope and hitting our imaging sensor. There's nothing special or different about these photons (aside from the fact that we don't really want them). When they hit the pixels in the imaging sensor they have a chance of generating an electron that is given by the camera's quantum efficiency, just like any other photon.

The thermal background comes from electrons that are generated by heat in the camera sensor instead of by photons. All the atoms in the sensor are vibrating and sometimes those vibrations can knock an electron into the measurement well just like a photon does.

Now, once those background electrons have been generated, it doesn't matter which source they came from - it's impossible to tell an electron generated by thermal vibrations from one generated by sky glow from another that was generated by a photon that's spent a few million years travelling here from M31... All those electrons get collected by the pixel and eventually measured at the end of the exposure to give an ADU value for that pixel.

So, just as our imaginary pixel in the last part was receiving 10 photons per second from M31, it will also receive photons from the sky glow at some rate and electrons from thermal effects at a particular rate too. So, we can write down an equation like this

$$n_e = t(Qr_{p,target} + Qr_{p,sky} + r_{e,termal})$$

That is that the number of electrons collected by a pixel in an exposure is given by the length of the exposure multiplied by the sum of the photon rate from the target times the sensor QE, the photon rate from the sky times the sensor QE and the electron rate from thermal sources.

Using our previous example values of 60s for t, 10 for $r_{p,target}$ and 50% for Q, and picking 5 for $r_{p,sky}$ and 1 for $r_{e,thermal}$, we can calculate a total number of electrons collected in a 60s exposure including the effect of light pollution and thermal background to be 510. Remember that the number of electrons that came from our target was 300, so we have added a background level of an extra 210 electrons to this pixel, which will be reflected in higher ADU readout values.

If the only thing that sky background and thermal background did was to add a constant offset to all the pixels in the image then they wouldn't worry us much — we could just subtract dark frames and fix the problem completely in processing. Unfortunately it doesn't work like that... Now we are collecting 510 electrons in our pixel instead of 300, so we have to revisit our shot noise calculations. For 300 electrons, our shot noise standard deviation was about 17 electrons, or about 5.5% of the signal. Now we have 510 electrons, the shot noise increases to nearly 23 electrons, or about 7.7% of the original signal of 300 electrons that came from our target.

In this part we've learnt how the background from light pollution and thermal sources not only add on a constant brightness level to our images but also increase the noise levels making fine detail hard to see. We've also seen that when we are thinking from the point of view of detected electrons we don't really care whether the background was due to heat or light pollution - they both have the same effect.

In the next part we're going to talk about the final part of the noise picture - read noise - and how all the parts fit together.

Read Noise

Once the exposure is completed, the camera measures the voltage for each pixel using an ADC (analogue to digital converter). In CMOS cameras there is an ADC on each pixel, so the readout is very quick, wherease in CCD cameras the charge is moved across the rows of pixels in the camera to a single ADC, leading to slower readout times. Whichever way the readout happens, it's unfortunately not 100% accurate - this inaccuracy is referred to as Read Noise.

Read Noise means that the digital value we get out of a pixel doesn't exactly equate to the number of electrons that were collected by that pixel and that the value will vary even between pixels that have collected exactly the same number of electrons. The read noise of a camera is usually measured in terms of

the standard deviation the final image ADU values converted back from ADU measurements to electrons - that is a camera might have a read noise of 3e or 8e.

Modern CMOS cameras can have read noise values as low as 1e, although 2e-3e are more typical. Older CMOS cameras may have read noise values of up to 7-8e. Almost all CMOS cameras have a gain control that can be adjusted (the gain acts as a pre-multiplier for the pixel voltage before it is passed to the ADC, so that when the gain is increase you get a bigger increment in ADU for a single extra electron). Usually CMOS cameras show a smaller read noise as the gain is increased, although the drop in read noise usually quickly levels off. CMOS cameras typically have a very large effective read noise in 8 bit mode - this is because it may take 50 or 100 or even more extra electrons to move up to the next 8 bit ADU level.

CCD cameras usually do not have a variable gain and tend to exhibit higher read noises in the 7e to 10e range.

Read noise levels are usually unaffected by anything other than changes to the camera gain. Temperature, exposure length and other adjustments do not have a significant effect on them.

Adding Noise

This is where it starts to get interesting. Let's go back to an examples we've looked at before - our 60s exposure with sky background and thermal background from part 3 and add in the effect of read noise. We calculated a total of 510 electrons being accumulated during the 60s exposure, giving a shot noise level of $\sqrt{510}$ or about 22.6e. Let's also assume we are using a modern CMOS camera with a read noise of 2.5e. How do those two noise levels get combined to calculate the final noise level of the pixel?

You might be tempted to just add the two noise values to get 25.1e, but this isn't the right approach (and if it was then we'd come to some very different conclusions about how to take deep sky photos at the end of this series of posts...). It turns out that the right way to add two or more sources of noise is as follows

$$N_{tot} = \sqrt{N_1^2 + N_2^2 + N_3^2 + \cdots}$$

That is, each individual source of noise is squared, the squared noises are added together and then the square root of that sum gives the total noise. This is called 'adding in quadrature' and strictly speaking we need to be sure that the different sources of noise are statistically independent from each other before using this approach.

Let's see what total noise we get by adding our two noise sources (2.5e and 22.6e) in quadrature

$$\sqrt{22.6^2 + 2.5^2} = \sqrt{510 + 6.25} = \sqrt{516.25} = 22.7$$

Ok, that might be a surprising result to some of you - we added an extra noise of 2.5e onto our 22.6e shot noise and the total was 22.7e, hardly any change at all. This effect - the fact that when you add sources of noise the smaller source contributes very very little to the total noise - is the key to being able to choose a good exposure length for deep sky astrophotography, but we aren't quite ready to use it yet.

Before we move on, let's re-do our calculations for a couple more cases. First, based on the same shot noise level (ie same target brightness, sky background and thermal background), but for a CCD camera with a read noise of 8e. In this case, the total noise will be

$$\sqrt{22.6^2 + 8^2} = \sqrt{510 + 64} = \sqrt{576} = 24.0$$

In this case you can see that the camera read noise has at least had an appreciably effect on the total noise, raising it by about 6% above the shot noise level to 24.

For our last example, we will continue to use our CCD camera with 8e read noise, but will change to a 6s exposure, which will reduce the number of electrons collected from 510 to about 51, leading to a shot noise of about 7.1e.

$$\sqrt{7.1^2 + 8^2} = \sqrt{51 + 64} = \sqrt{115} = 10.7$$

The conclusion we need to take from this section is that when the read noise is much less than the shot noise then the read noise has virtually no effect on the total noise, which will be very close to the shot noise value. This is because the noise needs to be added in quadrature.

Putting it all together

Armed with our new knowledge of how to calculate image noise, let's go back to try to work out the answer to our original question from part 1 - that is what is the best sub-exposure length to use when imaging under particular conditions. Unfortunately this part is going to be rather heavy on the mathematics - if you want, you can take the maths on trust and skip on to what it all means for deep sky exposure times in part 7.

We need to set out the problem more thoroughy as follows:

Assume we are using a camera of quantum efficiency Q and read noise σ_r electrons that produces thermal background at a rate of $r_{e,thermal}$ electrons per pixel per second.

Assume also that the light pollution level leads to $r_{p,sky}$ photons per pixel per second.

We intend to image for a total time of T seconds and we will divide that time into n separate sub exposures each of duration t seconds, where T = tn.

What we need to do now is to work out the total noise level of the final stack of n images that we will have at the end of our imaging session. The lower the total noise level of that final stacked image, the better our final image will look and the more faint detail we will be able to see. In fact we will calculate the noise of the darkest parts of the image where the only signal comes from the thermal background and the sky background - this will be the lowest noise of any part of the image as brighter parts will have higher electron counts and therefore more shot noise.

To begin, let's work out the noise of a single sub-exposure. From part 3, we have

$$n_e = t(Qr_{p,target} + Qr_{p,sky} + r_{e,thermal})$$

but, in this case we are considering a dark part of the image, so there are no photons from the target. So, using the fact that the standard deviation of the shot noise is the square root of the number of electrons, we have

shot noise =
$$\sigma_e = \sqrt{t(Qr_{p,sky} + r_{e,thermal})}$$

We have assumed that we have measured the read noise of the sensor to be σ_r , so we can use the rule of adding in quadrature to get the total noise for the frame

$$\sigma_{frame} = \sqrt{\sigma_r^2 + \sigma_e^2}$$

At this point we will assume (subject to further discussion later) that the thermal noise is much smaller than the sky noise, so we can ignore the thermal noise for now. Substituting in our calculation for the shot noise, we have.

$$\sigma_{frame} = \sqrt{\sigma_r^2 + \left(\sqrt{tQr_{p,sky}}\right)^2}$$

Now, obviously we are taking the square root of the electron count from sky noise and then immediately squaring it again, getting us back where we started, which means we can simplify the frame noise value to

$$\sigma_{frame} = \sqrt{\sigma_r^2 + tQr_{p,sky}}$$

Phew - we now have a value for the noise in a single frame, let's see how we can go from there to the noise in our final stack of n frames.

We will make our final stacked image by adding all the sub frames. This gives us a clue as to how to calculate the noise - we already know how to combine noise figures together in this sort of case - we add them in quadrature. That gives us a noise figure for our final stacked frame of

$$\sigma_{stack} = \sqrt{\sigma_{frame}^2 + \sigma_{frame}^2 + \cdots}$$

The total number of frame noises that we are adding inside the square root will be equal to n, the number of sub-frames, so

$$\sigma_{stack} = \sqrt{n \cdot \sigma_{frame}^2} = \sqrt{n(\sigma_r^2 + tQr_{p,sky})}$$

Remembering that the number of sub-exposures times the sub-exposure length gives the total time T, we finally have

$$\sigma_{stack} = \sqrt{n \cdot \sigma_r^2 + TQr_{p,sky}}$$

In the next section we will at last see what this equation tells us about choosing our sub-exposure length.

What does it all mean?

So, in the last part, we at last derived a formula that gave us the total amount of noise in a stack of images

$$\sigma_{stack} = \sqrt{n \cdot \sigma_r^2 + TQr_{p,sky}}$$

Let's remind ourselves of the variables in this equation:

- T the total time we intend to image for
- n the number of sub-exposures we intend to take in our total imaging time
- σ_r the read noise of the camera typically between 2 and 10 electrons
- Q the quantum efficiency of the camera typically about 50% (ie 0.5)
- $r_{p,sky}$ the sky background (light pollution) photon rate in photons per pixel per second

We have two terms inside the square root sign in this equation. The first, $n \cdot \sigma_r^2$, represents the contribution of the read noise to the final stack noise while the second, $TQr_{p,sky}$, represents the contribution of the shot noise to the final stack noise. It's also worth noting that the second term inside the square root $TQr_{p,sky}$ - is actually equal to the number of electrons detected per pixel over the entire time T of the imaging session. This number will typically be in the 100s or 1000s.

Let's have a look at what sort of values this equation gives us.

This graph shows how the total stack noise varies with the number of sub exposures. For this I have taken a total imaging time of 1 hour (3600 seconds), a sensor QE of 50% and a sky photon rate of 1 photon per pixel per second. The red line shows the curve for a sensor with read noise of 2e and the blue line shows the curve for a read noise of 10e. Both meet at the point where we take only 1 sub-exposure of 3600s duration (where the read noise is basically negligible in both cases). It's clear from this how much the higher read noise impacts the image quality when we use shorter sub-exposures.

If we re-draw the graph so that it shows the same data but in terms of sub-exposure time on the x-axis instead of sub-frame count, it's a lot more striking

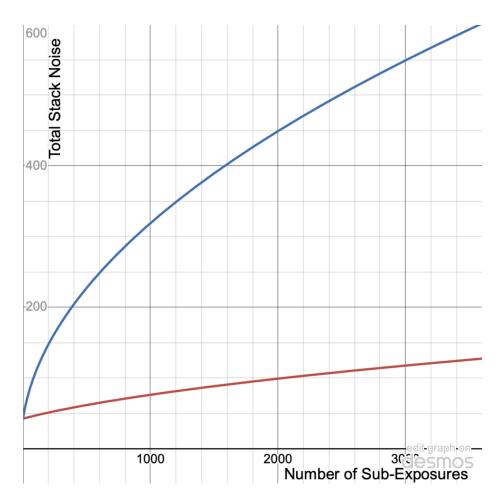


Figure 3: total stack noise vs number of sub-exposures

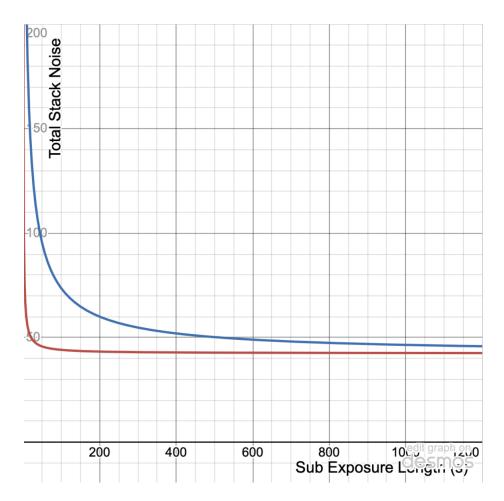


Figure 4: total stack noise vs sub-exposure length

Somewhere way out to the right hand side we can reach a total stack noise of about 42 by taking a single one hour exposure. Our 2e read noise camera can reach a stack noise of about 45 at 60s sub-exposures, but the 10e read noise camera has about double the total noise at that sub-exposure length and can't get down to a stack noise of 45 until the sub-exposure length hits 1200s (20 minutes) or more.

This last graph shows a very interesting thing - that the sub-exposure time needed for deep sky imaging depends very strongly on the read noise of the camera. High read noise cameras absolutely require very long exposures. Low read noise cameras can get the same results by stacking more, but far shorter, frames.

Ok, so now we have an outline of the answer to our question - next time we will take the maths a little further and calculate some actual numbers for optimum exposure times.

Results

If we look at the second graph in part 7, we can see three main sections to the curves

- 1. For very short exposures where the noise level rises very rapidly as the sub-exposure times become shorter and
- 2. Far out to the right hand side where the sub-exposures become long out here the noise doesn't change much,
- 3. The bit in the middle where the graph curves

Let's think about our equation for stack noise

$$\sigma_{stack} = \sqrt{n \cdot \sigma_r^2 + TQr_{p,sky}}$$

and how it applies in each of those three areas.

In area (1) where the sub-exposures are very short, n is very large, which means that the first term in the square root is much bigger than the second. In this case we have

$$\sigma_{stack} pprox \sqrt{n \cdot \sigma_r^2} = \sqrt{\frac{T}{t} \cdot \sigma_r^2}$$

So we see the stack noise rises rapidly being proportional to the square root of the number of sub-exposures in this area.

In area (2) where the sub-exposures are long, n is very small, which means that we can start to ignore the first term inside the square root as it will be much smaller than the second term. If the first term is much smaller than the second term, we have

$$\sigma_{stack} \approx \sqrt{TQr_{p,sky}}$$

which is telling us that when we get to the point where the first term can be ignored, the stack noise no longer depends on the number of sub-exposures (or the sub-exposure length) at all. (see Note 1)

This is a key result - it tells us that there is a point where making the sub-exposures longer has no further effect on improving image quality. If you are already past that point then going from a 5 minute sub exposure to a 10 or even 20 minute sub exposure will **not** improve the quality of your final stacked image at all. In fact it will probably make it worse as you are risking guiding errors and tracking errors and loosing out on dynamic range.

The final question to be answered is - what is the magic exposure length that we need to use to get good results in the final stack without entering the zone where longer exposures give us practically no improvement. We can see from the graphs above that there is always a slight extra improvement in stack noise by going to longer sub-exposures, but that the improvement quickly becomes very very slow (especially with low read noise cameras!). In order to get a number, we have to decide how much extra noise are we prepared to tolerate, over and above the minimum possible noise that we would get with very long sub-exposures.

The minimum possible noise, with no contribution from read noise, is

$$\sigma_{stack,min} = \sqrt{TQr_{p,sky}}$$

If we are prepared to accept (say) a stack noise value that is 5% higher than this, we have

$$\sigma_{stack} = \sqrt{n \cdot \sigma_r^2 + TQr_{p,sky}} = 1.05\sigma_{stack,min} = 1.05\sqrt{TQr_{p,sky}}$$

Squaring both sides, we have

$$n \cdot \sigma_r^2 + TQr_{p,sky} = 1.05^2 TQr_{p,sky}$$

so

$$n \cdot \sigma_r^2 = (1.05^2 - 1)TQr_{p.sky}$$

Now, what we are actually interested in is the exposure time t=T/n, so we can re-arrange and finally get

$$t = \frac{T}{n} = \frac{1}{1.05^2 - 1} \frac{\sigma_r^2}{Qr_{p,sky}}$$

Let's look at that equation and see what we can learn from it:

1. The optimal sub-exposure time is proportional to read noise squared! That means that a read noise 10 camera needs to use exposures 25 times longer than a read noise 2 camera to get to the same stack noise level. This explains at a stroke why extremely long sub-exposures have

become common in deep sky imaging - it's because high read noise CCD cameras need them! It also makes it clear that if you have a low read noise camera and you use those very long exposures then you are still paying the price for a problem you don't have to solve!

- 2. There is a factor at the front of the formula that depends on the amount of extra noise we were prepared to accept. For our value of 5% the factor $\frac{1}{1.05^2-1}$ works out to be about 9.75. If we were prepared to accept 10% extra noise then it's 4.76. If we were fussy and only wanted to add 1% extra noise then it's 49.75. Remember that moving from 5% extra noise to 1% isn't reducing the noise by a factor of five, it's reducing it from 105 to
 - 101. Is that worth it for the price of making your sub-exposures five times longer? Probably not!
- 3. The optimal exposure is inversely proportional to both the QE of the camera and the sky background brightness (light pollution). Worse light pollution means shorter sub exposures. We know this instinctively because if you take very long exposures with bad light pollution then your subs wash out to a sea of white you have to keep the histogram level down to a reasonable point by limiting the exposure length. In terms of the equation, more light pollution means that the number of electrons (and hence the amount of shot noise) we need to be much bigger than the read noise is reached more rapidly.

One last chart - this shows the result of our equation for the 2e read noise camera (red) and 10e camera (blue). Here we are looking at the total allowed noise level on the x axis (in %, with the minimum achievable noise being 100%) and the sub-exposure lengths needed to meet that noise level on the vertical axis.

Again, it's striking how much effect the different read noise values have on the exposure times!

To finish off, let's put some numbers into our equation to calculate optimal exposure times for our two cameras we drew graphs for earlier. Using the same QE of 50% and sky photon rate of 1 per pixel per second, and allowing a 5% increase from the minimum noise level, we obtain the following exposures:

- 10e read noise camera: 1951s (32.5 minutes)
- 2e read noise camera: 78s

If we allow a 10% increase in noise level, we get instead

- 10e read noise camera: 952s (15.8 minutes)
- 2e read noise camera: 38s

Finally, considering a more light polluted area with 5 photons per pixel per second, still at 10% noise allowance

10e read noise camera: 190s2e read noise camera: 7.6s

Of course, if you don't fancy doing these calculations in the dark and cold every time you observe, then just check out the Smart Hisogram functionality in SharpCap 3.1 - The Smart Histogram 'Brain' window will run these calculations

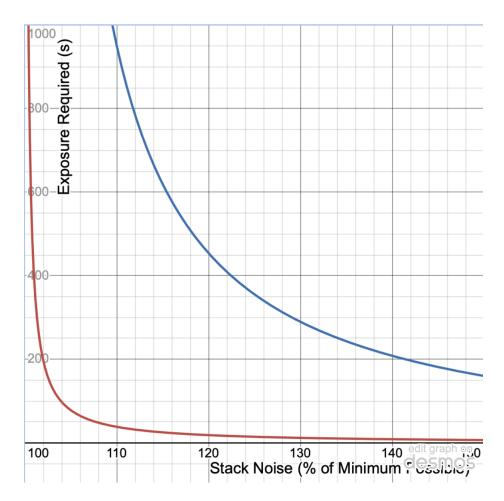


Figure 5: exposure required vs stack noise

automatically for you (and more) and suggest optimal gain and exposure values for your observiing conditions!

Note 1

If you are observant you may have noticed that the equation

$$\sigma_{stack} \approx \sqrt{TQr_{p,sky}}$$

gives a total stack noise that is proportional to the square root of the total observation time T. This means that the noise gets bigger as the total exposure time goes up, which seems wrong! However, remember that the signal we are wanting to see – the galaxy or nebula – will have a strength that is proprtional to the total observation time T itself (not its square root), so the signal will be going up even faster. In fact, the signal-to-noise ratio, which is the key factor for determining if something is visible or not, will turn out to be proportional square root of T, so will go up as the total observation time increases, which is what we would expect.