# Session Types at the Mirror

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#### Poll

x is an object with methods a and b

$$x:\{a;b\}$$

#### Behavioral operators

external choice +

internal choice  $\oplus$ 

Give a behavioral type to x

(A) 
$$x:a+b$$

(B)  $x: a \oplus b$ 

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$$x : a + b$$

(B)  $x: a \oplus b$ 

$$x: a+b$$

- the type of x tells about what x can do
- the user of x can decide which method to invoke

#### Let's think of subtyping

$$x: \{a; b\}$$
  $\{a; b\}$   $<: \{a\}$   $y: \{a\}$   $a+b \leq a$ 

How do you explain this?

$$a \leq a + b$$



$$x: a+b$$

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  $\{a; b\}$   $<: \{a\}$   $y: \{a\}$   $a+b \leq a$ 

How do you explain this?

$$a \prec a + b$$



#### Conclusion

$$\vdash P : \{c : \sigma\}$$

- **1**  $\sigma$  is not the type of c

What if we define session types like this?



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What if we define session types like this?



```
S = a?(x).x?(title:Str).x!price.x?(addr:Addr).x!date
B_1 = (\nu c)a!c.c!title.c?(price:Int).(\nu d)b!d.d!price/2.d!c
B_2 = b?(y).y?(contrib:Int).y?(z).z!address.z?(d:Date)
```

```
S = a?(x).x?(title : Str).x!price.x?(addr : Addr).x!date
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d :
                                  !Int.!\rho.1
```

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 $B_2 = b?(y).y?(contrib:Int).y?(z).z!address.z?(d:Date)$ 
 $a: ?\sigma.1 !\sigma.1$ 
 $b: !\tau.1 ?\tau.1$ 
 $c: ?Str.!Int.?Addr.!Date.1 !Str.?Int.1 !Addr.?Date.1$ 
 $d: !Int.!\rho.1 ?Int.?\rho.1$ 

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                                !Str.?Int.1
                                               !Addr.?Date.1
     ?Str.!Int.?Addr.!Date.1
```

### Session types: syntax

$$\boxed{1 \overset{\checkmark}{\longrightarrow} 1}$$

$$\frac{\sigma \xrightarrow{\checkmark} \sigma' \qquad \tau \xrightarrow{\checkmark} \tau'}{\sigma \mid \tau \xrightarrow{\checkmark} \sigma' \mid \tau'} \qquad \frac{\sigma \xrightarrow{!v} \sigma' \qquad \tau \xrightarrow{?v} \tau'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

$$\frac{\sigma \xrightarrow{!\rho} \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \preceq \rho'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

$$\underline{\sigma \mapsto \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \not\preceq \rho'}$$

$$\underline{\sigma \xrightarrow{!\rho} \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \not\preceq \rho'}$$

 $\sigma \mid \tau \longrightarrow 0$ 

$$1 \stackrel{\checkmark}{\longrightarrow} 1$$

$$\frac{\sigma \xrightarrow{!v} \sigma' \qquad \tau \xrightarrow{?v} \tau'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

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subsession

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# Characterizing complete compositions

#### Definition (completeness)

$$\sigma$$
 is *complete* if  $\sigma \Longrightarrow \sigma'$  implies  $\sigma' \stackrel{\checkmark}{\Longrightarrow}$ 

#### **Examples**

Completeness generalizes duality

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# Characterizing well-typed processes

What's the difference?

!Int.1 !Int.0

#### Definition (viability)

 $\sigma$  is *viable* if  $\sigma \mid \rho$  is complete for some  $\rho$ 

ullet viable  $\sim$  "different from empty type"

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viable ∼ "different from empty type"

# Defining type equality

#### Definition (subtyping)

 $\sigma \preceq \tau$  if  $\sigma \mid \rho$  complete implies  $\tau \mid \rho$  complete for all  $\rho$ 

$$\begin{array}{ccc} 1 \mid \sigma & \approx & \sigma \\ 0 + \sigma & \approx & \sigma \\ \alpha . \sigma + \alpha . \tau & \approx & \alpha . \sigma \oplus \alpha . \tau \end{array}$$

$$\begin{array}{ccc}
\sigma \oplus \tau & \preceq & \sigma \\
\sigma & \not\preceq & \sigma + \tau
\end{array}$$

reduce nondeterminism



# Subtyping vs refinement

#### Definition (subtyping)

 $\sigma \preceq \tau$  if  $\sigma \mid \rho$  complete implies  $\tau \mid \rho$  complete for all  $\rho$ 

 $P:\sigma$  can be replaced by  $Q:\tau$ 

(left-to-right)

$$P \mid R$$
 ok

$$\Rightarrow$$

 $Q \mid R$  o

 $u:\tau$  can be replaced by  $v:\sigma$ 

(right-to-left)

$$P : \{u : \tau\}$$

$$\Rightarrow$$

$$P\{{}^{ extsf{V}}/{}_{ extsf{U}}\}:\{{}^{ extsf{V}}: au \}$$

### Subtyping vs refinement

#### Definition (subtyping)

 $\sigma \leq \tau$  if  $\sigma \mid \rho$  complete implies  $\tau \mid \rho$  complete for all  $\rho$ 

$$P:\sigma$$
 can be replaced by  $Q:\tau$  (left-to-right) 
$$P\mid R \quad \text{ok} \qquad \Rightarrow \qquad Q\mid R \quad \text{ok}$$
  $u:\tau$  can be replaced by  $v:\sigma$  (right-to-left)

$$P: \{u: \tau\} \Rightarrow P\{v/u\}: \{v: \tau\}$$

$$P \text{ keeps behaving as } \tau$$

### $\leq$ is not a precongruence

#### $0 \approx \sigma$ if $\sigma$ is not viable

- 0 is not observable
- σ may be observable

   (a faulty process may send/receive messages)

#### Definition (strong subtyping)

Let  $\sqsubseteq$  be the largest precongruence included in  $\preceq$ 

#### Theorem

 $\sigma \preceq \tau$  if and only if either  $\sigma$  is not viable or  $\sigma \sqsubseteq \tau$ 

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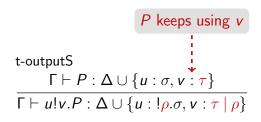
# Parallel composition

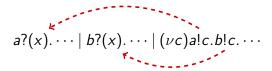
t-par
$$\frac{\Gamma \vdash P : \{u_i : \sigma_i^{i \in I}\} \qquad \Gamma \vdash Q : \{u_i : \tau_i^{i \in I}\}}{\Gamma \vdash P \mid Q : \{u_i : \sigma_i \mid \tau_i^{i \in I}\}}$$

# Parallel composition

t-par
$$\frac{\Gamma \vdash P : \{u_i : \sigma_i\}^{i \in I}\}}{\Gamma \vdash P \mid Q : \{u_i : \sigma_i \mid \tau_i\}^{i \in I}\}}$$

# Delegation





# Channel input

P not allowed to use anything else... t-inputS  $\frac{\Gamma \vdash P : \{x : \rho\}}{\Gamma \vdash u?(x).P : \{u : ?\rho.1\}}$  ... not even u

#### Restriction

$$\frac{\Gamma \vdash P : \Delta \cup \{c : \sigma\} \qquad \sigma \text{ complete}}{\Gamma \vdash (\nu c)P : \Delta}$$

## Subject reduction

#### Theorem

If 
$$\Gamma \vdash P : \Delta$$
 and  $P \longrightarrow Q$  and  $\Delta$  viable, then  $\Gamma \vdash Q : \Delta$ 

$$P \stackrel{\mathrm{def}}{=} (\nu c)a!c$$
 :  $\{a: !1.1\}$ 
 $Q \stackrel{\mathrm{def}}{=} c?(x).x!3$  :  $\{a: ?(!\mathrm{Int.1}).1\}$ 
 $|1.1|?(!\mathrm{Int.1}).1$  not viable
 $P \mid Q \longrightarrow (\nu c)(0 \mid c!3)$  OUCH!

### Type safety

#### Theorem

If  $\Gamma \vdash P : \Delta \cup \{c : \sigma\}$  and  $\sigma$  complete and  $P \downarrow c$ , then  $P \longrightarrow$ 

- $P \downarrow c$  = "whoever owns c is immediately ready to use it"
- type system does not enforce global progress

#### Summary

- Projection
- ⇒ Session types as process terms
- ⇒ Semantically grounded theory of session types
  - Completeness ∼ duality
  - $\bullet \ \ \mathsf{Viability} \sim \mathsf{well}\text{-typedness}$

#### Two questions answered

Q: What is a session type?

A: Projection of process behavior

- ccs-like formalism
- reuse known techniques: (fair) testing semantics

Q: Process refinement and subtyping?

A: Same relation

- left-to-right replacement of processes
- right-to-left replacement of channels

#### Two questions answered

Q: What is a session type?

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- ccs-like formalism
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- right-to-left replacement of channels

# Thank you.

#### Constrained delegation

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```
a!c.a!c.c?(x : Int).c?(y : Bool)
a?(x).a?(y).y!true.x!3
\cdots \{x : !Int.1, y : !Bool.1\}

projection forgets dependency
```

c?(x:Int).c?(y:Bool)



#### Constrained delegation

```
a!c.a!c.c?(x : Int).c?(y : Bool)

a?(x).a?(y).y!true.x!3 : \cdots \{x : !Int.1, y : !Bool.1\}
```

c?(x: Int).c?(y: Bool)

OUCH!

#### Replication

t-bang
$$\frac{\Gamma \vdash P : \{u_i : \sigma_i^{i \in I}\} \qquad \sigma_i \sqsubseteq \sigma_i \mid \sigma_i^{i \in I}\}}{\Gamma \vdash \star P : \{u_i : \sigma_i^{i \in I}\}}$$

$$\sigma \sqsubseteq \sigma \mid \sigma$$

- doesn't matter whether there are 1 or 2<sup>100</sup> copies of P
- $\sigma=1$  (does nothing)
- $\sigma=1\oplus\pi.\sigma$  (can do  $\pi$  at any time)

### Subsumption

t-sub
$$\frac{\Gamma \vdash P : \Delta \cup \{u : \tau\} \qquad \sigma \sqsubseteq \tau}{\Gamma \vdash P : \Delta \cup \{u : \sigma\}}$$

### Terminated process

$$\begin{tabular}{lll} $\mathsf{t}$-weak & & & & & $\mathsf{t}$-nil \\ \hline $\Gamma \vdash P : \Delta & u \not\in \mathsf{dom}(\Delta)$ & & \\ \hline $\Gamma \vdash P : \Delta \cup \{u : 1\}$ & & \\ \hline $\Gamma \vdash 0 : \emptyset$ & \\ \hline \end{tabular}$$

#### Communication

$$\frac{\Gamma, x: t \vdash P: \Delta \cup \{u: \sigma\}}{\Gamma \vdash u?(x:t).P: \Delta \cup \{u: ?t.\sigma\}}$$

#### t-output

$$\frac{\Gamma \vdash e : t \qquad \Gamma \vdash P : \Delta \cup \{u : \sigma\}}{\Gamma \vdash u!e.P : \Delta \cup \{u : !t.\sigma\}}$$

#### Choices

$$\frac{\Gamma \vdash \pi_i.P_i : \Delta \cup \{u : \sigma_i\}}{\Gamma \vdash \sum_{i \in I} \pi_i.P_i : \Delta \cup \{u : \sum_{i \in I} \sigma_i\}}$$

$$\frac{\Gamma \vdash P : \Delta \qquad \Gamma \vdash Q : \Delta}{\Gamma \vdash P \oplus Q : \Delta}$$