on sessions, linear logic and unbounded interactions

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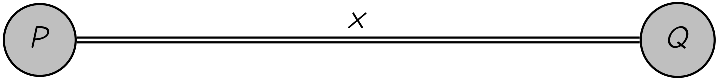
outline

outline

Q: what is a **session**?

A **session** is a private communication channel linking two processes, each using one **channel endpoint** according to a protocol specification called **session type**



Examples of session types

- First send integer, then receive string (P's viewpoint)
- First receive integer, then send string (Q's viewpoint)

Q: what is a session type system useful for?

A: enable the **compositional static analysis** of distributed programs

If P uses $\overbrace{x_1, \dots, x_n}^{\text{channels}}$ as described by $\overbrace{A_1, \dots, A_n}^{\text{protocols}}$

$$P \vdash x_1 : A_1, \dots, x_n : A_n$$

then

- exchanged messages have the **expected type** **(safety)**
- interactions occur in the **expected order** **(fidelity)**
- interactions **don't get stuck** **(deadlock freedom)**
- pending actions are completed **(livelock freedom)**
- interactions terminate **(termination)**
- ...

Q: how do we define a session type system?

A: from linear logic and its extensions

linear logic propositions	$\Longleftrightarrow$	session types
linear logic proofs	$\Longleftrightarrow$	well-typed processes
cut reduction	$\Longleftrightarrow$	communication
soundness of the logic	$\Longleftrightarrow$	soundness of typing

Q: how do we define a session type system?

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linear logic propositions	$\iff$	session types
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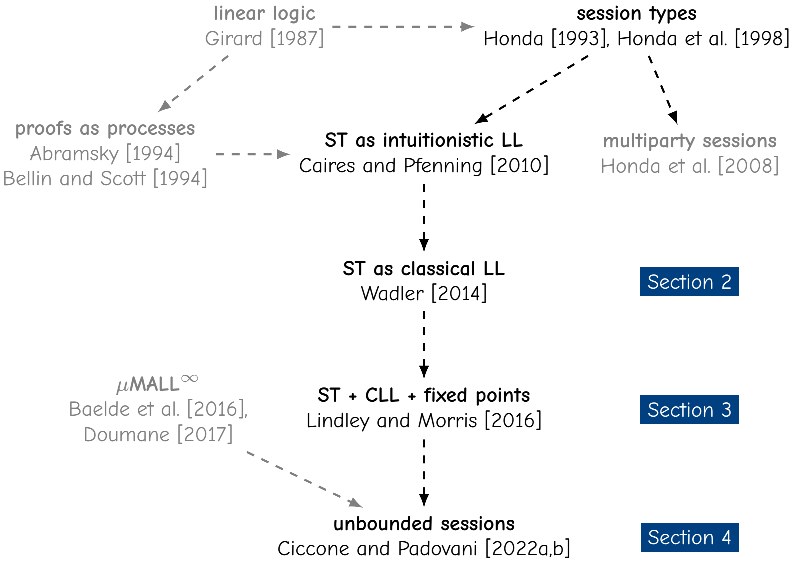
linear logic
Girard [1987]
session types
Honda [1993], Honda et al. [1998]
proofs as processes
Abramsky [1994]
Bellin and Scott [1994]
ST as intuitionistic LL
Caires and Pfenning [2010]
multiparty sessions
Honda et al. [2008]
ST as classical LL
Wadler [2014]
Section 2
μMALL$^\infty$
Baelde et al. [2016],
Doumane [2017]
ST + CLL + fixed points
Lindley and Morris [2016]
Section 3
unbounded sessions
Ciccone and Padovani [2022a,b]
Section 4

outline

MALL propositions as session types

[Wadler, 2014]

$$
\begin{array}{rcll}
A, B & ::= & \mathbf{0} & \text{unit for } \oplus \\
& | & \top & \text{unit for } \& \\
& | & \mathbf{1} & \text{send termination signal} \\
& | & \bot & \text{receive termination signal} \\
& | & A \oplus B & \text{select either } A \text{ or } B \\
& | & A \mathbin{\&} B & \text{offer choice of } A \text{ or } B \\
& | & A \otimes B & \text{output } A \text{ then behave as } B \\
& | & A \parr B & \text{input } A \text{ then behave as } B
\end{array}
$$

Example: *session types for sending/receiving a boolean*

$$
\mathbb{B} \stackrel{\text{def}}{=} \mathbf{1} \oplus \mathbf{1} \qquad\qquad \mathbb{B}^{\bot} \stackrel{\text{def}}{=} \bot \mathbin{\&} \bot
$$

MALL proof rules

no rule for 0

$$\frac{}{\vdash \Gamma, \top}$$

$$\frac{}{\vdash \mathbf{1}} \qquad \frac{\vdash \Gamma}{\vdash \Gamma, \bot}$$

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B} \qquad \frac{\vdash \Gamma, B}{\vdash \Gamma, A \oplus B} \qquad \frac{\vdash \Gamma, A \qquad \vdash \Gamma, B}{\vdash \Gamma, A \mathbin{\&} B}$$

$$\frac{\vdash \Gamma, A \qquad \vdash \Delta, B}{\vdash \Gamma, \Delta, A \otimes B} \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, A \mathbin{\wp} B}$$

$$\frac{}{\vdash A, A^{\perp}} \qquad \frac{\vdash \Gamma, A \qquad \vdash \Delta, A^{\perp}}{\vdash \Gamma, \Delta}$$

cut rule = parallel composition + restriction

proof rule
$$\frac{\vdash \Gamma, A \qquad \vdash \Delta, A^\perp}{\vdash \Gamma, \Delta}$$

typing rule
$$\frac{\vdash \Gamma, x : A \qquad \vdash \Delta, x : A^\perp}{\vdash \Gamma, \Delta}$$

cut rule = parallel composition + restriction

proof rule $$\frac{\vdash \Gamma, A \qquad \vdash \Delta, A^{\perp}}{\vdash \Gamma, \Delta}$$

typing rule $$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Delta, x : A^{\perp}}{\mathsf{new}\, x(P \mid Q) \vdash \Gamma, \Delta}$$

rules for **1** and $\perp$ = termination

$$\frac{}{\vdash x : \mathbf{1}} \qquad \frac{\vdash \Gamma}{\vdash \Gamma, x : \perp}$$

rules for $\mathbf{1}$ and $\perp$ = termination

$$\frac{}{\text{close}\, x \vdash x : \mathbf{1}}$$

$$\frac{P \vdash \Gamma}{\text{wait}\, x.P \vdash \Gamma, x : \perp}$$

rules for $\mathbf{1}$ and $\perp$ = termination

$$\frac{}{\mathsf{close}\, x \vdash x : \mathbf{1}} \qquad\qquad \frac{P \vdash \Gamma}{\mathsf{wait}\, x.P \vdash \Gamma, x : \perp}$$

$$\frac{\dfrac{}{\mathsf{close}\, x \vdash x : \mathbf{1}} \qquad \dfrac{P \vdash \Gamma}{\mathsf{wait}\, x.P \vdash \Gamma, x : \perp}}{\mathsf{new}\, x(\mathsf{close}\, x \mid \mathsf{wait}\, x.P) \vdash \Gamma} \quad \longrightarrow \quad P \vdash \Gamma$$

Note: principal cut reduction defines process reduction

rules for $\oplus$ and $\&$ = branch selection

$$\frac{\vdash \Gamma, x : A}{\vdash \Gamma, x : A \oplus B} \qquad \frac{\vdash \Gamma, x : A \qquad \vdash \Gamma, x : B}{\vdash \Gamma, x : A \mathbin{\&} B}$$

rules for $\oplus$ and $\&$ = branch selection

$$\frac{P \vdash \Gamma, x : A}{\mathsf{left}\, x.P \vdash \Gamma, x : A \oplus B}$$

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Gamma, x : B}{\mathsf{case}\, x\{P, Q\} \vdash \Gamma, x : A \,\&\, B}$$

rules for $\oplus$ and $\&$ = branch selection

$$\frac{P \vdash \Gamma, x : A}{\mathsf{left}\, x.P \vdash \Gamma, x : A \oplus B} \qquad \frac{P \vdash \Gamma, x : A \qquad Q \vdash \Gamma, x : B}{\mathsf{case}\, x\{P, Q\} \vdash \Gamma, x : A \mathbin{\&} B}$$

$$\frac{\dfrac{P \vdash \Gamma, x : A}{\mathsf{left}\, x.P \vdash \Gamma, x : A \oplus B} \qquad \dfrac{Q \vdash \Delta, x : A^\perp \qquad R \vdash \Delta, x : B^\perp}{\mathsf{case}\, x\{Q, R\} \vdash \Delta, x : A^\perp \mathbin{\&} B^\perp}}{\mathsf{new}\, x(\mathsf{left}\, x.P \mid \mathsf{case}\, x\{Q, R\}) \vdash \Gamma, \Delta}$$

$$\downarrow$$

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Delta, x : A^\perp}{\mathsf{new}\, x(P \mid Q) \vdash}$$

rules for $\otimes$ and $⅋$ = channel communication

$$\frac{P \vdash \Gamma, y : A \qquad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B} \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A ⅋ B}$$

$$\dfrac{\dfrac{P \vdash \Gamma, y : A \qquad Q \vdash \Gamma', x : B}{x[y].(P \mid Q) \vdash \Gamma, \Gamma', x : A \otimes B} \qquad \dfrac{R \vdash \Delta, y : A^\perp, x : B^\perp}{x(y).R \vdash \Delta, x : A^\perp ⅋ B^\perp}}{\mathsf{new}\, x(x[y].(P \mid Q) \mid x(y).R) \vdash \Gamma, \Gamma', \Delta}$$

$$\downarrow$$

$$\dfrac{P \vdash \Gamma, y : A \qquad \dfrac{Q \vdash \Gamma', x : B \qquad R \vdash \Delta, y : A^\perp, x : B^\perp}{\mathsf{new}\, x(Q \mid R) \vdash \Gamma', \Delta, y : A^\perp}}{\mathsf{new}\, y(P \mid \mathsf{new}\, x(Q \mid R)) \vdash \Gamma, \Gamma', \Delta}$$

example: negation of a boolean value

$$\mathsf{Neg}(x, y) \triangleq \mathsf{case}\, x\{\mathsf{wait}\, x.\mathsf{right}\, y.\mathsf{close}\, y,\ \mathsf{wait}\, x.\mathsf{left}\, y.\mathsf{close}\, y\}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathsf{close}\, y \vdash y : \mathbf{1}}}{\mathsf{right}\, y.\mathsf{close}\, y \vdash y : \mathbb{B}}}{\mathsf{wait}\, x.\mathsf{right}\, y.\mathsf{close}\, y \vdash x : \bot, y : \mathbb{B}} \quad \dfrac{\dfrac{\dfrac{}{\mathsf{close}\, y \vdash y : \mathbf{1}}}{\mathsf{left}\, y.\mathsf{close}\, y \vdash y : \mathbb{B}}}{\mathsf{wait}\, x.\mathsf{left}\, y.\mathsf{close}\, y \vdash x : \bot, y : \mathbb{B}}}{\dfrac{\mathsf{case}\, x\{\mathsf{wait}\, x.\mathsf{right}\, y.\mathsf{close}\, y, \mathsf{wait}\, x.\mathsf{left}\, y.\mathsf{close}\, y\} \vdash x : \mathbb{B}^\bot, y : \mathbb{B}}{\mathsf{Neg}\langle x, y\rangle \vdash x : \mathbb{B}^\bot, y : \mathbb{B}}}$$

properties of proofs and processes

Cut elimination

The cut rule is **admissible**

Deadlock freedom

If $P \vdash \Gamma$ then either $P \rightarrow$ or P is not a cut ☺

Successful termination

If $P \vdash x : \mathbf{1}$ then $P \Rightarrow \mathsf{close}\, x$ ☺

Livelock freedom

If $P \vdash x : \mathbf{1}$ then P is livelock free ☺

observations

Fact

MALL propositions allow us to describe **finite** protocols made of a **fixed** number of actions

Role of the exponential modalities $?A$ and $!A$

- useful to describe the behavior of **clients** and **servers**
- $!A$ = server providing A
- $?A$ = clients requesting A

Moral

The correspondence holds well, but each interaction between a client and a server is still of **fixed length**

outline

extending MALL with fixed points

[Lindley and Morris, 2016, Baelde et al., 2016, Doumane, 2017]

$$
\begin{array}{rcll}
A, B & ::= & \cdots & \text{as before} \\
& & X & \text{recursion variable} \\
& & \mu X.A & \text{least fixed point} \\
& & \nu X.A & \text{greatest fixed point}
\end{array}
$$

Example: session types for sending/receiving a natural number

$$\mathbb{N} \stackrel{\text{def}}{=} \mu X.(X \oplus \mathbf{1}) \qquad \mathbb{N}^{\perp} \stackrel{\text{def}}{=} \nu X.(X \mathbin{\&} \perp)$$

rules for fixed points in μMALL$^{\infty}$

[Baelde et al., 2016, Doumane, 2017]

$$\frac{P \vdash \Gamma, x : A\{\nu X.A/X\}}{\mathsf{corec}\, x.P \vdash \Gamma, x : \nu X.A} \qquad \frac{P \vdash \Gamma, x : A\{\mu X.A/X\}}{\mathsf{rec}\, x.P \vdash \Gamma, x : \mu X.A}$$

- there is no distinction between least and greatest fixed points in these rules (but they **must** be distinguished at some point)
- rec x and corec x are used to **unfold** the type of the channel x

example: successor of a natural number

$$\begin{aligned}\mathsf{Succ}(x,y) \triangleq \mathsf{corec}\,x.\mathsf{rec}\,y.\mathsf{case}\,x\{\ &\mathsf{left}\,y.\mathsf{Succ}\langle x,y\rangle,\\ &\mathsf{wait}\,x.\mathsf{left}\,y.\mathsf{rec}\,y.\mathsf{right}\,y.\mathsf{close}\,y\}\end{aligned}$$

go left once more before going right

example: successor of a natural number

we need an infinite proof!

$$
\dfrac{
\dfrac{\dfrac{\vdots}{\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}}{\mathsf{left}\, y.\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N} \oplus \mathbf{1}}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathsf{close}\, y \vdash y : \mathbf{1}}}{\mathsf{right}\, y.\mathsf{close}\, y \vdash y : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{rec}\, y \ldots \vdash y : \mathbb{N}}}{\mathsf{left}\, y \ldots \vdash y : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{wait}\, x \ldots \vdash x : \perp, y : \mathbb{N} \oplus \mathbf{1}}
}{
\dfrac{\mathsf{case}\, x\{\ldots, \ldots\} \vdash x : \mathbb{N}^{\perp} \,\&\, \perp, y : \mathbb{N} \oplus \mathbf{1}}{\dfrac{\mathsf{rec}\, y \ldots \vdash x : \mathbb{N}^{\perp} \,\&\, \perp, y : \mathbb{N}}{\dfrac{\mathsf{corec}\, x \ldots \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}{\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}}}
}
$$

some infinite proofs are dangerous

$$\Omega \triangleq \Omega \qquad \dfrac{\dfrac{\vdots}{\Omega \vdash \Gamma}}{\Omega \vdash \Gamma}$$

Allowing arbitrary proofs compromises the soundness of the logic

Proofs

- the cut rule is no longer admissible ☹
- the empty sequent and $\vdash \mathbf{0}$ become derivable ☹

Processes

- safety properties still hold ☺
- liveness properties are lost ☹

identifying valid proofs

[Baelde et al., 2016, Doumane, 2017]

Valid infinite branch

An infinite branch of a proof is **valid** if there is a channel x whose type is a **greatest fixed point** that is unfolded **infinitely many times**

Valid proof

A proof is **valid** if every infinite branch in it is valid

Intuition

- least fixed points **can** be unfolded finitely many times only
- greatest fixed points **must** be unfolded infinitely many times

Warning: this is an oversimplified approximation

The exact definition of valid proof is technically more involved

example: successor of a natural number

$$
\dfrac{
  \dfrac{
    \dfrac{\vdots}{\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}
  }{\mathsf{left}\, y.\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N} \oplus \mathbf{1}}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{}{\mathsf{close}\, y \vdash y : \mathbf{1}}
        }{\mathsf{right}\, y.\mathsf{close}\, y \vdash y : \mathbb{N} \oplus \mathbf{1}}
      }{\mathsf{rec}\, y \ldots \vdash y : \mathbb{N}}
    }{\mathsf{left}\, y \ldots \vdash y : \mathbb{N} \oplus \mathbf{1}}
  }{\mathsf{wait}\, x \ldots \vdash x : \perp, y : \mathbb{N} \oplus \mathbf{1}}
}{
  \dfrac{
    \dfrac{
      \mathsf{case}\, x\{\ldots, \ldots\} \vdash x : \mathbb{N}^{\perp} \,\&\, \perp, y : \mathbb{N} \oplus \mathbf{1}
    }{\mathsf{rec}\, y \ldots \vdash x : \mathbb{N}^{\perp} \,\&\, \perp, y : \mathbb{N}}
  }{
    \dfrac{\mathsf{corec}\, x \ldots \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}{\mathsf{Succ}\langle x, y\rangle \vdash x : \mathbb{N}^{\perp}, y : \mathbb{N}}
  }
}
$$

properties of valid proofs

In a valid proof **every** sequence of (principal) cut reductions is **finite** and the cut rule is **admissible** (in the limit)

Good news
All well-typed processes terminate ☺

Termination is a valuable property that entails **livelock freedom** when combined with **deadlock freedom**

Bad news
All well-typed processes terminate ☹

This is unfortunate since all (interesting) processes that engage into arbitrarily long (unbounded) interactions are ill typed

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examples

We can

Model a process that computes the successor of an arbitrary (but **specific**) natural number

We can

Model a process that buys an arbitrary (but **specific**) number of items from a store

We cannot

Model a process that computes the successor of an arbitrary (**non-deterministically chosen**) natural number

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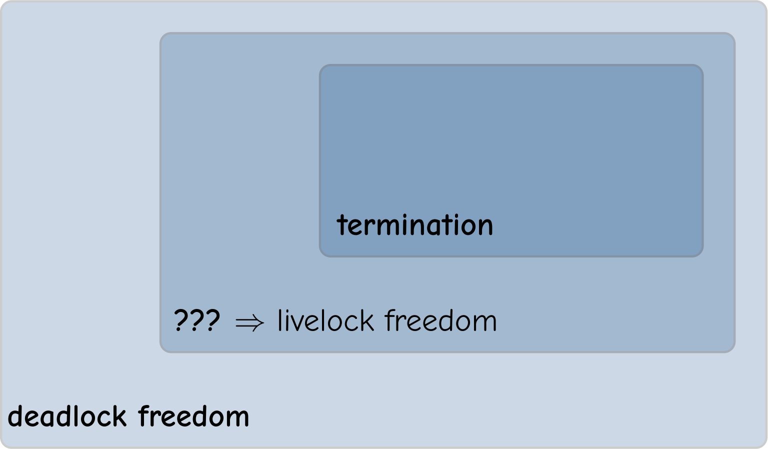
outline

objective

Relax the type system so that (some) unbounded interactions can be modeled and livelock freedom is still guaranteed



from termination to **fair termination**

run = maximal reduction sequence

Termination

A process is **terminating** of all of its runs are finite

Weak termination

A process is **weakly terminating** if at least one of its runs is finite

Fair termination [Grumberg et al., 1984, Francez, 1986]

A process is **fairly terminating** if all of its **fair runs** are finite

- **In principle**, a fairly terminating process may diverge
- **In practice**, a fairly terminating process always terminates

example: natural number generator

$$\mathsf{Nat}(x) \triangleq \mathsf{rec}\, x.(\mathsf{left}\, x.\mathsf{Nat}\langle x\rangle \oplus \mathsf{right}\, x.\mathsf{close}\, x)$$

non-deterministic choice

The run in which the process keeps going left is "unfair"

fairness assumption(s)

Dozens of fairness assumptions have been studied in the literature, see van Glabbeek and Höfner [2019] for a recent survey

In this talk
A run $P_0 \to P_1 \to P_2 \to \cdots$ is **fair** if it goes through finitely many weakly terminating processes

Intuition
In an **unfair run** the process has infinitely many opportunities to terminate, but it takes none of them

An example from real life: online shopping
The run in which you keep adding items into the cart is unfair

example: natural number generator

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N}}}{\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N} \oplus \mathbf{1}} \qquad \dfrac{\dfrac{}{\mathsf{close}\,x \vdash x : \mathbf{1}}}{\mathsf{right}\,x.\mathsf{close}\,x \vdash x : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \oplus \mathsf{right}\,x.\mathsf{close}\,x \vdash x : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{rec}\,x.(\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \oplus \mathsf{right}\,x.\mathsf{close}\,x) \vdash x : \mathbb{N}}}{\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N}}
$$

The infinite branch is **invalid**

example: natural number generator

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N}}}{\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N} \oplus \mathbf{1}} \qquad \dfrac{\dfrac{}{\mathsf{close}\,x \vdash x : \mathbf{1}}}{\mathsf{right}\,x.\mathsf{close}\,x \vdash x : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \oplus \mathsf{right}\,x.\mathsf{close}\,x \vdash x : \mathbb{N} \oplus \mathbf{1}}}{\mathsf{rec}\,x.(\mathsf{left}\,x.\mathsf{Nat}\langle x\rangle \oplus \mathsf{right}\,x.\mathsf{close}\,x) \vdash x : \mathbb{N}}}{\mathsf{Nat}\langle x\rangle \vdash x : \mathbb{N}}
$$

The infinite branch is **invalid**, but it corresponds to an unfair run

a small refinement to the notion of valid proof

[Ciccone and Padovani, 2022b]

Fair branch

A branch of a proof is **fair** if it goes through finitely many weakly terminating processes

Fairly valid proof

A proof is **fairly valid** if every **fair** infinite branch in it is valid

properties of fairly valid proofs

In a **fairly** valid proof every **fair** sequence of (principal) cut reductions is **finite** and the cut rule is **admissible** (in the limit)

Consequences

- All well-typed processes **fairly terminate** ☺
- **Fair termination** entails **livelock freedom** ☺
- At least some (interesting) processes that engage into arbitrarily long (unbounded) interactions are well typed ☺

outline

wrap up

Session type theory enjoys a natural correspondence (in the style of Curry-Howard) with linear logic

Selection of further developments

- beyond **tree-like** network topologies [Dardha and Gay, 2018]
- beyond **race-free** interactions [Balzer and Pfenning, 2017, Balzer et al., 2019, Rocha and Caires, 2021, Qian et al., 2021]
- support for **general recursion** [Ciccone and Padovani, 2022a, Ciccone et al., 2022]

wrap up

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thank you!

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