IEEE Transactions on Power Apparatus and Systems, Vol. PAS-96, no. 1, January/February 1977

OPERATING COST CALCULATION OF AN ELECTRIC POWER GENERATING SYSTEM UNDER INCREMENTAL LOADING PROCEDURE

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ABSTRACT

A phased approach to calculating the expected operating cost of a power system, under the incremental loading procedure, is presented. The units in the system are divided into subsets according to their marginal cost characteristics. The contribution of each subset to the operating costs of the system is then derived in a recursive manner, taking maintenance and forced outages into account. The procedure is also extended to yield the expected amount of energy delivered by each unit in the system as well as the expected fuel cost for each unit. Given the unit cost of fuel, the expected amount of fuel consumed by each unit is easily derived.

INTRODUCTION

The prediction of the operating and the capital costs of a power utility is one of the most important aspects of power system planning. In capacity expansion decisions, cost data are used as a major criterion to compare between various alternative investments policies, for the purpose of finding the least cost investment which meets the demand for power with a certain level of reliability. In operation planning, cost calculations are needed for financial planning, cash flow analysis, fuel budgeting etc. Cost estimations are also important elements in supporting rate structure decisions, reserve planning, sensitivity analysis, etc.

While the prediction of the capital cost of an investment policy is in itself a complicated task since it amounts to forcasting the capital cost per unit of installed capacity for each type and size of a generating unit in each of future years, the estimation of the operating cost of a power system is by far more complicated as it depends on the loading procedure, availability of units and the demand for power which are highly variable and unpredictable, especially when the calculations extend far into the future.

Various methods have been developed to predict the operating cost of a given utility. The most simplified procedures are based on the Load Duration Curve (LDC) assuming a merit order loading, with the capacities of the units reduced by a few percent to account for outages and scheduled maintenance. A linear version of this estimate has been used as the cost function in various linear programming models [1]. A more sophisticated approach which accounts for forced outages and maintenance in a more reliable way, while still assuming a merit order loading, is the probabilistic simulation method [4,5]. A greater degree of precision in caculating the operating costs can be achieved by using a simulation model as the analysis can be carried out on an hourly basis and more accurate loading procedures, specifically incremental loading, can be employed. The disadvantage of simulation models, however, is the large volume of calculation involved. Also, as the analysis extends further into the future, simulation models become ineffective since it becomes almost impossible to predict hourly data that far in advance.

The purpose of the present paper is to describe an improved approach to calculating operating cost of a power system which might serve as a substitute to cost calculation carried out by simulation. The proposed approach is virtually an extension of the probabilistic simulation approach, except that it employs the more realistic incremental loading procedure, rather than the approximate merit order loading. Depending upon the type of the cost functions of the units involved, the proposed computational algorithm is demonstrated to be very efficient while providing with very accurate results.

LOADING PROCEDURES

Given the demand for power and the availability of units, the loading procedure is the manner by which the various units are assigned to generation in order to meet the instanteous demand for power. As such, the loading procedure determines the number, type and production level of the units which are loaded to generation at any given point in time. It is therefore a most important factor in calculating the operating cost of a given unit. The major operating cost component, excluding start-up, shut-down and transmission losses, is fuel cost. It is well known that fuel consumption is a non-linear function of the unit's output; in particular, it has been empirically demonstrated [3] that the cost function of generating units in a power system, with the possible exception of hydroelectric and pumped storage units, is a non-linear, usually quadratic, increasing function, exhibiting an increasing marginal cost. Under these conditions, it can be proven [3] that the loading procedure which yields the minimum operating cost for the system is the one which equates the marginal cost of the units in the system for any demand level. This loading procedure, widely implemented by power companies, is known as the incremental loading, sometimes the economic dispatch, procedure.

It is very difficult, however, to employ the incremental loading procedure for cost calculations, as it depends on the dynamic changes in demand and/or availability of units, which can be taken into account only in an hour by hour analysis. Indeed, simulation models which are capable of an hourly analysis, have thus far provided the only means of predicting the operating cost of the system based on incremental loading.

An alternative loading procedure, which avoids

F 76 400-2. A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES Summer Meeting, Portland, OR, July 18-23, 1976. Manuscript submitted February 3, 1976; made available for printing May 4, 1976.

the difficulties presented by the incremental loading, is the merit-order loading. According to this method, units are loaded to generation in order of their average production cost which serves as an indicator to the unit's merit order. The most efficient unit, i.e. the unit with the lowest average production cost, is loaded first and operated at its rated capacity; the next most efficient unit is then loaded to generation at its rated capacity, and so on till demand is satisfied. This procedure is certainly much more convenient for cost calculation; and in conjunction with the LDC it yields a very efficient algorithm to predict the operating cost of the system. The procedure, however, yields only approximate results.

Booth [2] has suggested an improvement to the merit order loading by dividing the total capacity of a given unit into two capacity blocks, each of which with a different average production cost, which are then placed in nonadjacent positions in the merit order. This loading procedure results in greater accuracy in calculating the operating cost of the system. Obviously, the larger the number of blocks for a given unit, the more accurate are the operating cost predictions. In fact, the incremental loading is basically a further refinement of the two-block method, with the capacity of a unit divided into infinite infinitesimal blocks, the cost of each is the incremental cost of producing the energy in that block.

THE PROBABILISTIC SIMULATION

Forced outages and maintenance have a profound effect on the operation of the system, causing it to deviate from its optimal operating conditions, thus resulting in higher operating cost to meet the demand for power. Since forced outages and maintenance are so common in the power industry, there is a need to incorporate them into the calculation process in orrder to increase the reliability of the results. The probabilistic simulation method [5] is probably the first analytical model which accurately accounts for forced outages in calculating the operating cost of the system under the merit order loading. Because of its relation to our work, the procedure will be briefly reviewed.

Figure 1a describes a LDC of a given system along with the merit order loading, with unit 1 being the most efficient unit, unit 2 being the next most efficient unit, and so on. Denoting by:

$t(p_L)$ = the LDC evaluated at point p_L, interpreted as the fraction of time, in a given period, during which customers' demand equals or exceeds p_L MW,

E_i = the energy delivered by unit i,

p_i = the capacity of unit i,

q_i = the outage probability of unit i,

then, ignoring maintenance requirements, the expected energy delivered by unit 1 is given by:

$$E_1 = (1-q_1) \int_0^1 t(p_L)dp_L$$

In calculating the enrgy delivered by the second unit in the merit order, two components of costs are to be considered. When unit 1 is available, with a probability of $(1-q_1)$, unit 2 will be loaded to production between the loads p_1 and $p_1 + p_2$; however when unit 1 is out of service because of forced outage, with a probability q_1, unit 2 will occupy the first position in the merit order and will then be loaded to generation between the loads 0 and p_2, as described in Figure 1b. The expected energy delivered by unit 2 is therefore:

$$E_2 = (1-q_2)\ [(1-q_1) \int_{p_2}^{p_1+p_2} t(p_L)dp_L + q_1 \int_0^{p_2} t(p_L)dp_L]$$

Substituting

$$\int_0^{p_2} t(p_L)dp_L = \int_{p_1}^{p_1+p_2} t(p_L-p_1)dp_L$$

we have:

$$E_2 = (1-q_2)\ \{ \int_{p_1}^{p_1+p_2} [(1-q_1)t(p_L)+ q_1 t(p_L-p_1)]dp_L \}$$

Vardi, Zahavi and Avi - Itzhak [7] have shown that the expression under the integral sign is a partial Combined Load Duration Curve (CLDC) containing customers' demand and requirements for forced outages of the first unit. Following their notation we denote this expression by $\bar{t}(p_L)$ and obtain:

$$E_2 = (1-q_2) \int_{p_1}^{p_1+p_2} \bar{t}_1(p_L)dp_L$$

Continuing in this fashion it can be shown:

$$E_n = (1-q_n) \int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \bar{t}_{n-1}(p_L)dp_L$$

where $\bar{t}_n(p_L)$ is the CLDC containing the customers' demand and the demand for forced outages of the first n units.

The expected energy delivered by each unit in the system under merit order loading is therefore closely related to the CLDC. By using the recursive formula [7]:

$$\bar{t}_n(p_L) = (1-q_n)\bar{t}_{n-1}(p_L) + q_n\bar{t}_{n-1}(p_L-p_n)$$

the partial CLDC's involved in calculating the expected operating costs, can be obtained in a very efficient manner.

The total operating cost of the system is derived by multiplying the expected energy delivered by each unit by the corresponding average production cost and adding up the products for all units.

If maintenance requirements are to be taken into consideration, the period involved is to be partitioned into subperiod of constant maintenance [7]. The operating cost are then calculated for each subperiod separately, and combined to yield the total operating cost for the whole period. Provisions to incorporate hydroelectric stations and pumped storage operations into the calculation have also been developed [4,5].

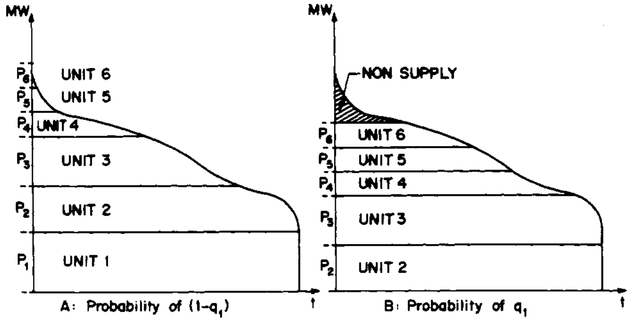

Figure 1 Merit Order Loading:
(a) Unit 1 is on; (b) Unit 1 is down

OPERATING COST CALCULATION UNDER INCREMENTAL LOADING FOR A SYSTEM WITHOUT MAINTENANCE AND FORCED OUTAGES

The probabilistic simulation model of the previous section will now be modified to yield the operating cost of a power system under an incremental loading procedure. For clarity of presentation, we will start with a system without maintenance and forced outages, postponing the analysis of the more general case to the next chapter.

The Incremental Loading Procedure

Assuming non-decreasing and differentiable operating cost function, it can be shown [3] that the incremental loading procedure results in minim operating cost for the power system. In the following we will denote the cost functions by:

$$c_i = f_i(p_{L_i}), \quad i = 1,2,\ldots,N$$

where:

p_{L_i} = the output (in MW) of generator i,

N = the number of generators in the system,

$f_i(p_{L_i})$ = a non-decreasing and differentiable function.

the marginal cost function of a unit i is derived from the cost function by differentiation:

$$c_i' = f_i'(p_{L_i}), \quad i = 1,2,\ldots,N$$

We assume for convenience that the marginal cost functions are one-to-one, so that their inverse exists and are also one-to-one. We denote the inverse functions by $f_i^{-1}(c_i')$, $i = 1,2,\ldots,N$. They clearly satisfy:

$$p_{L_i} = f_i^{-1}(c_i'), \quad i = 1,2,\ldots,N$$

The corresponding cost function and marginal cost function for the whole system will be denoted by $c = f(p_L)$ and $c' = f'(p_L)$, respectively, where p_L is the total generation in the system, satisfying:

$$p_L = \sum_{i=1}^{N} p_{L_i}$$

In the following we will ignore start-up and shut-down costs and consider fuel cost only. (Start-up and shut-down cost might be estimated quite accurately from past data and can be added to the operating cost of the system as a lump sum). The incremental loading procedure is best demonstrated by a graphical description. For simplicity we assume a 3-unit system with increasing marginal costs c_i', $i = 1,2,3$, as described in Figure 2. The marginal cost function for the system, obtained by summing up the abscissas of the individual marginal cost functions for each cost level, is also exhibited in Figure 2. Given a demand level p_L, the corresponding generation levels p_{L_i}, $i = 1,2,\ldots,N$, are determined, according to the incremental loading procedure, at the point where the marginal costs of the various generating units are equal to the marginal cost of the system. To derive these levels graphically we draw a horizontal line through $c'(p_L)$ which cuts the ordinates of the individual marginal cost curves at the points $c_1'(p_{L_1}), c_2'(p_{L_2})\ldots,c_N'(p_{L_N})$, respectively. The generation level of each unit, p_{L_i}, $i = 1,2,\ldots,N$, are then read directly off the abscissa. The procedure is demonstrated in Figure 2.

The above procedure can also be carried out mathematically. Given a demand level p_L, the marginal cost of the system, c', is obtained as $c' = f'(p_L)$. Substituting c' in the inverse marginal cost function, the generation level of each unit in the system, p_{L_i}, is easily obtained as:

$$p_{L_i} = f_i^{-1}(c'), \quad i = 1,2,\ldots,N$$

When the marginal cost of the system is higher than the maximum marginal cost of the generator (obtained by evaluating the marginal cost function at its upper capacity), the generator is scheduled to work at its maximum output; and conversely, if the marginal cost for the system is below the minimal marginal cost of the generator (obtained by evaluating the marginal cost function at the point of minimum production), the generator is scheduled to work at its minimum output.

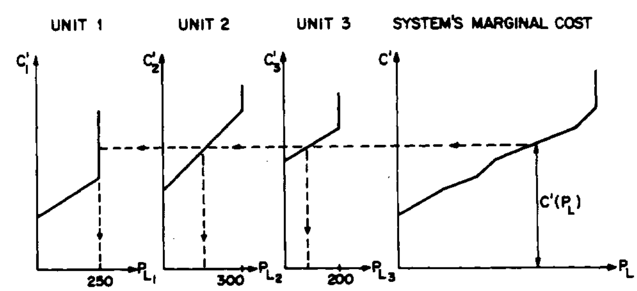

Figure 2 Incremental Loading

Calculating the Total Operating Cost of the System

The total operating cost of the power system can easily be obtained from the system's marginal cost function and the LDC. Given a demand level p_L, the incremental energy requested by the customers at that level is $t(p_L)d(p_L)$. Denoting the marginal cost of the system at demand p_L by $c'(p_L)$, the total incremental cost to deliver the extra energy is:

$$c'(p_L)t(p_L)dp_L$$

from which the total operating cost of the system is easily obtained by integration, i.e.:

$$TC = \int_0^{p_{max}} c'(p_L)t(p_L)dp_L$$

where p_{max} is the peak demand for the period.

Finding the Energy Delivered By Each Unit.

A problem of primary importance, is to find the energy delivered by each unit in the system.

Denoting the LDC for generator i by $t(p_{L_i})$, there obviously exists:

$$E_i = \int_0^{p_i} t(p_{L_i})\, dp_{L_i}$$

However $t(p_{L_i})$ is not known, as it depends on the loading pattern of the units to generation which is, in turn, a function of the demand for power. To find $t(p_{L_i})$ we will thus have to consider the contribution of generator i to generation for all levels of the demand p_L.

Multiplying and dividing by dp_L we have:

$$E_i = \int_0^{p_{max}} t(p_{L_i}) \frac{dp_{L_i}}{dp_L} dp_L$$

But at the point of equal incremental cost there exists $t(p_{L_i}) = t(p_L)$, hence:

$$E_i = \int_0^{p_{max}} t(p_L) \frac{dp_{L_i}}{dp_L} dp_L$$

To find $\frac{dp_{L_i}}{dp_L}$ we substitute:

$$\frac{dp_{L_i}}{dp_L} = \frac{\frac{dp_{L_i}}{dc'}}{\frac{dp_L}{dc'}}$$

and since $p_{L_i} = f_i^{-1}(c')$ and $p_L = \sum_{i=1}^{N} p_{L_i}$ we obtain:

$$E_i = \int_0^{p_{max}} t(p_L) \frac{(f_i^{-1}(c'))'}{\sum_{i=1}^{N} (f_i^{-1}(c'))'} dp_L$$

Taking a closer look at the latter formula we notice that the term in the numerator is the slope of the marginal cost of generator i at a demand level p_L; the corresponding term in the denominator is the slope of the marginal cost function for the system at demand p_L. The incremental demand in energy, $t(p_L)dp_L$, is thus shared by the various generators in the system according to the above ratio. Then by integrating over all possible demand level p_L, the energy delivered by each unit is easily obtained.

An extension to finding the LDC for each unit, $t(p_{L_i})$, is immediate. Denoting the previous ratio by $m(p_L)$ we obtain from the above

$$dp_{L_i} = m(p_L)dp_L$$

which upon integration yields:

$$p_{L_i} = m(p_L)p_L$$

Hence for each demand level p_L, the corresponding generation level p_{L_i} is obtained by multiplication by the ratio $m(p_L)$. Since by definition $t(p_{L_i}) = t(p_L)$, the LDC for unit i is easily obtained.

Finding Fuel Cost for Each Unit in the System.

The fuel cost (or the operating cost) for each unit in the system can be found in a similar manner. Recalling that the incremental cost for a unit i is given by:

$$dc_i = c'(p_{L_i})t(p_{L_i})dp_{L_i}$$

then:

$$c_i = \int_0^{p_i} c'(p_{L_i})t(p_{L_i})dp_{L_i}$$

Substituting $t(p_{L_i}) = t(p_L)$ and since by the incremental loading procedure $c'(p_{L_i}) = c'(p_L)$, we have:

$$c_i = \int_0^{p_{max}} c'(p_L)t(p_L)dp_{L_i}$$

Multiplying and dividing by dp_L we have, as before:

$$c_i = \int_0^{p_{max}} c'(p_L)t(p_L)m(p_L)dp_L.$$

The incremental cost $c'(p_L)t(p_L)dp_L$ is again shared by the various units according to the ratio of the slope of their marginal cost value at point p_L to the slope of the marginal cost function of the system at that point. Note that the energy delivered by each unit in the system can be obtained as a special case of cost calculations by substituting $c'(p_L) = 1$ for all levels of p_L.

Given the operating cost for unit i, and the unit cost of fuel, the amount of fuel consumed by each unit in the system can easily be determined. The capacity to accurately calculate fuel consumption is indeed very important for fuel budgeting. When implemented on a nationwide basis the total fuel consumption for power generation is obtained, a quantity of paramount importance in planning future fuel consumption and determining national energy policy.

OPERATING COST CALCULATION UNDER INCREMENTAL LOADING FOR A SYSTEM SUBJECT TO MAINTENANCE AND FORCED OUTAGES

The previous analysis will now be extended to a system which is subject to maintenance and forced outages. One possible way to calculate the operating cost of such a system, under incremental loading procedure, is to derive the marginal cost function of the system for each combination of up and down (due to forced outages) units, calculate the operating cost for each of these combinations using the previously described methods, multiply each cost by the probability of its occurrence and sum up the results to obtain the expected total operating cost of the system for the given period. This enumeration method is however impractical for most systems as it involves 2^N derivations of marginal cost functions, equal to the number of up and down combinations in an N-unit system, which increases exponentially with the number of units in the system. A more efficient method resulting in substantial savings in the amount of calculations required to derive operating cost figures for a system working under the incremental loading procedure, will now be derived.

Basic Notation

We first rearrange the units in the system in an ascending order of their minimal marginal cost, such that:

$$\min \{c'_{i+1}\} \geq \min \{c'_i\}, \quad i = 1,2,\ldots,N$$

where c'_i denotes, as before, the marginal cost function of generator i. The minimal marginal cost for a generator i is obtained by evaluating the marginal cost function at the point of minimum output.

For each unit i let Q_i denote the subset of generators (not including unit i) having at least one common marginal cost value with generator i. Under the incremental loading procedure these units will be loaded to production with generator i in at least some cases. Figure 3 demonstrates the definition of the subsets Q_i, $i = 1,2,\ldots,N$. For any given subset Q_i there exist:

$$\max \{c'_i\} \geq \min \{c'_j\}, \quad j = i+1,\ldots,k \leq N$$

For most power systems, which are composed of many units with various characteristics, the number of units in each subset Q_i is smaller than the number of generators in the system. Denoting by n_i the number of units in subset Q_i, the number of up and down (due to forced outages) units in each subset is 2^{n_i}. Since $n_i < N$, there exist $2^{n_i} \ll 2^N$. Let R_i, $i = 1,2,\ldots,N$, denote the set of combinations of up and down units in a subset Q_i, with elements r_i and occurrence probabilities P_{r_i}. Also let the marginal cost function for each combination $r_i \in R_i$ be denoted by $c'_{r_i}(p_L)$. These functions, one for each combination in R_i, are obtained by adding up the abscissas of the marginal cost functions corresponding to the working units in each combination. There are certainly 2^{n_i} marginal cost functions for each subset Q_i.

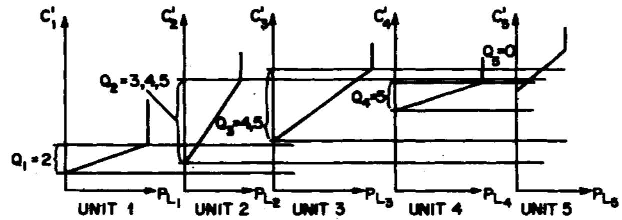

Figure 3 Definition of the subsets Q_i

Calculating the Expected Total Operating Cost of the System

Given the above definitions, a phased approach to calculating the expected operating cost of the system will be devised ignoring for the meanwhile, maintenance requirements.

When unit 1 is working, with probability $(1-q_1)$, then for any given combination $r_1 \in R_1$ (recall that by the definition of Q_1, unit 1 is not a member of the combinations in R_1), the incremental operating cost for the system is, obviously:

$$c'_{r_1}(p_L)t(p_L)d(p_L)$$

Assuming unit 1 is working, the contribution to the expected total operating cost of the units in the subset Q_1 which are loaded to generation along with the first unit in the region $0 \leq p_L \leq p_1$, is given by:

$$TC_1 = (1-q_1) \int_0^{p_1} [\sum_{r_1 \in R_1} P_{r_1} c'_{r_1}(p_L)] t(p_L) dp_L$$

The expression in the square brackets is clearly the expected marginal cost of meeting the demand by the units in subset Q_1. Denoting this expression by $\bar{c}'_{R_1}(p_L)$, we have:

$$TC_1 = (1-q_1) \int_0^{p_1} \bar{c}'_{R_1}(p_L) t(p_L) dp_L \equiv (1-q_1) \int_0^{p_1} \bar{h}_0(\bar{c}'_{R_1}(p_L)) dp_L$$

Moving on to the second unit, we can identify three components in calculating the contribution to the expected total operating cost when unit 2 is available. The first component is when the first unit is working and we are considering the region $0 \leq p_L \leq p_1$. This component has already been taken care of in the calculation of TC_1. The second component refers to the case where the first unit is available and unit 2 along with its companions in subset Q_1 are loaded to generation in the region $p_1 \leq p_L \leq p_1 + p_2$. The third component is when unit 1 is out of service because of forced outage and we are considering the region $0 \leq p_L \leq p_2$. Denoting the contribution of the latter two components to the the expected operating cost by TC_2 we have:

$$TC_2 = (1-q_2)\,[(1-q_1)\int_{p_1}^{p_1+p_2} \sum_{r_2 \varepsilon R_2} P_{r_2}\, c'_{r_2}(p_L)t(p_L)dp_L$$

$$+ q_1 \int_0^{p_2} \sum_{r_2 \varepsilon R_2} P_{r_2}\, c'_{r_2}(p_L)t(p_L)dp_L]$$

Substituting $\overline{c}'_{R_2}(p_L)$ for the expected marginal cost of meeting demand by the units in subset Q_2, rearranging terms and transforming variables we obtain:

$$TC_2 = (1-q)\int_{p_1}^{p_1+p_2} [(1-q_1)\overline{c}'_{R_2}(p_L)t(p_L)$$

$$+ q_1\overline{c}'_{R_2}(p_L-p_1)t(p_L-p_1)]dp_L$$

and by the definition of $\overline{h}_0(x)$:

$$TC_2 = (1-q_2)\int_{p_1}^{p_1+p_2}[(1-q_1)\overline{h}_0(\overline{c}'_{R_2}(p_L))$$

$$+ q_1\overline{h}_0(\overline{c}'_{R_2}(p_L-p_1))]dp_L$$

Denoting the expression in the square brackets by $\overline{h}_1(\overline{c}'_{R_2}(p_L))$ we have:

$$TC_2 = (1-q_2)\int_{p_1}^{p_1+p_2} \overline{h}_1(\overline{c}'_{R_2}(p_L))dp_L$$

Continuing in this fashion we can obtain, in general:

$$TC_n = (1-q_n)\int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \overline{h}_{n-1}(\overline{c}'_{R_n}(p_L))dp_L$$

where:

$$\overline{h}_n(\overline{c}'_{R_k}(p_L)) = (1-q_n)\overline{h}_{n-1}(\overline{c}'_{R_k}(p_L)) + q_n\overline{h}_{n-1}(\overline{c}'_{R_k}(p_L-p_n))$$

TC_n is the contribution to the expected operating cost of the system of the units in subset Q_n which have not been taken care of in previous calculations. Certainly, the expected total operating cost of the system is obtained by adding up all the contributions, i.e:

$$TC = \sum_{i=1}^{N} TC_i$$

Computationally, in order to derive TC_n we first have to generate the sequence of functions

$$\overline{h}_n(\overline{c}'_{R_i}(p_L)), \quad i = n+1,\ldots,N$$

The bulk of the computational effort is in deriving the N expected marginal cost curves $\overline{c}'_{R_i}(p_L)$ as each of which involve 2^{n_i} marginal cost functions. However recalling that $n_i << N$, then 2^{n_i} is a relatively small number, much less than the 2^N elements involved in the direct enumeration method. As an example, consider a system with $N = 20$ units for which the generating units can be divided into 5 subsets with 5 units in each subset. Then $2^5 = 32$ combinations will have to be considered to calculate the expected marginal cost function for each subset Q_i, or 160 elements in all versus the 2^{20} combinations involved in the enumeration method. A substantial saving indeed. Once the expected marginal cost functions have been derived for all subsets, the task of calculating TC_n becomes rather simple. A computer can be instructed to carry out the calculations quite efficiently.

A further simplification of the calculation process is available if instead of using the expected marginal cost functions $\overline{c}'_{R_n}(p_L)$, we consider their average value calculated by:

$$\overline{c}'_n = \int_0^{\sum_{i=1}^{n} p_i} \overline{c}'_{R_n}(p_L)f(p_L)dp_L$$

where $f(p_L)$ is the load density function of customers' demand, obtained very easily from the LDC [7].

$\overline{c}'_n$ can be interpreted to represent the average expected marginal cost of meeting demand by the units in subset Q_n. By plugging $\overline{c}'_n$ in the above formulas instead of $\overline{c}'_{R_n}(p_L)$, in can be factored out, reducing the expression for TC_n to be:

$$TC_n = (1-q_n)\overline{c}'_n \int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \overline{t}_{n-1}(p_L)dp_L$$

where $\overline{t}_n(p_L)$ is the partial CLDC containing customers' demand (both deterministic and random components) and the demand for outages of the first n units. A recursive equation to calculate $\overline{t}_n(p_L)$ has been presented in the previous chapter.

The calculation in this case, for an N - unit system, involve the derivation of N average expected marginal costs, one for each subset Q_n, calculation of N partial CLDC's and carrying out N integrations. A rather simplified procedure.

We note the similarity between the latter expression to calculating TC_n and the corresponding expression obtained when the merit order loading is used. It should be emphasized, however, that even this last approximation exhibits an improvement in calculating cost figures compared with the merit order loading, as the generating units are classified into groups according to their marginal cost functions and a different average expected marginal cost is used for each group. Of course, if exact results are required, the calculation can be carried out using the the functions $\bar{c}'_{R_n}(p_L)$, as described above, at the expense of slightly increased computational effort.

Finding the Expected Fuel Cost for Each Unit in the System

The expected fuel cost for each unit in the system can be obtained by modifying the calculation of TC_n to yield the contribution of the unit to the operating cost, and adding up the contributions for all relevant subsets.

Based on the argumentation presented in the previous chapter, the contribution of unit k to the operating cost of the system, for a given combination $r_n \in R_n$ and a given interval

$$\sum_{i=1}^{n-1} p_i \leq p_L \leq \sum_{i=1}^{n} p_i,$$

is obtained by:

$$c'_{r_n}(p_L) \frac{dp_{L_k}(r_n)}{dp_L} t(p_L)dp_L$$

where $\frac{dp_{L_k}(r_n)}{dp_L}$ is the ratio between the slope of the marginal cost function of unit k at demand level p_L and the slope of the marginal cost function of all working units in combination $r_n \in R_n$ at at the same demand level. Since the composition of working and non-working units in each combination is different, the above ratio is a function of r_n and should be recomputed for each combination. Certainly, for those combinations for which unit k is down, the above ratio is zero.

The expected contribution of unit k, $k \in Q_n$, to the operating cost at a given interval is, therefore:

$$\int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \left[\sum_{r_n \in R_n} P_{r_n} c'_{r_n}(p_L) \frac{dp_{L_k}(r_n)}{dp_L} \right] t(p_L)dp_L$$

The expression in the square brackets can be interpreted as the expected marginal cost of meeting the demand in the interval $\sum_{i=1}^{n-1} p_i \leq p_L \leq \sum_{i=1}^{n} p_i$ by unit k, $k \in Q_n$. Denoting this expression by $\bar{c}'_{R_n}(p_L,k)$, the above formula becomes:

$$\int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \bar{c}'_{R_n}(p_L,k)t(p_L)dp_L$$

To obtain the expected operating cost of unit k, it is necessary to collect its contribution to satisfying demand for all intervals in which the unit is loaded to generation. This is done by using the procedure described above for calculating the operating costs of the system, except that the computations are carried out separately for each unit k, and $\bar{c}'_{R_n}(p_L,k)$ is substituted for $\bar{c}'_{R_n}(p_L)$. Denoting the contribution of unit k to the expected operating costs incurred by the units in subset Q_n, $k \in Q_n$, which have not been accounted for in previous calculations, by $\bar{C}_k^n$, we have:

$$\bar{C}_k^n = (1-q_n) \int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \bar{h}_{n-1}(\bar{c}'_{R_n}(p_L,k))\, dp_L$$

where $\bar{h}_n(\bar{c}'_{R_n}(p_L,k))$ is computed recursively using the formula:

$$\bar{h}_n(\bar{c}'_{R_n}(p_L,k))=(1-q_n)\bar{h}_{n-1}(\bar{c}'_{R_n}(p_L,k))+q_n\bar{h}_{n-1}(\bar{c}'_{R_n}(p_L-p_n,k))$$

The expected operating cost of unit k is therefore obtained as:

$$\bar{C}_k = \sum_{n \in A} C_k^n$$

where A is the set of elements $k, k-1,\ldots,k-j$, and j is the first unit satisfying $\max\{c'_{j-1}\} < \{\min c'_k\}$.

The procedure obviously results in relatively large volume of computations. If less accuracy is permitted, the amount of calculations can be reduced in a substantial manner by approximating the functions $\bar{c}'_{R_n}(p_L,k)$ by their expected value using the formula:

$$\bar{c}'_n(k) = \int_0^{\sum_{i=1}^{n} p_i} \bar{c}'_{R_n}(p_L,k)f(p_L)dp_L$$

where $f(p_L)$ is the load density of demand. Substituting $\bar{c}'_n(k)$ for $\bar{c}'_{R_n}(p_L,k)$ in the above formulas, the expected operating cost of unit k reduces to:

$$\bar{C}_k = \sum_{n \in A} (1-q_n)\bar{c}'_n(k) \int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \bar{t}_{n-1}(p_L)\, dp_L$$

where $\bar{t}_n(p_L)$ is, as defined above, the partial CLDC containing customers' demand (both deterministic and random components) and the demand for forced outages of the first n units.

Given the unit cost of fuel for each generator, the expected amount of fuel consumed by the unit can be determined very easily.

Calculating the Expected Energy Delivered by Each Unit in the System

As mentioned above the expected energy delivered by each unit in the system can be obtained as a special case of cost calculations by substituting $c'_{r_n}(p_L) = 1$ for all levels of p_L. The incremental energy attributed to unit k, $k \in Q_n$, for a given combination $r_n \in R_n$ and a given interval $\sum_{i=1}^{n-1} p_i \leq p_L \leq \sum_{i=1}^{n} p_i$ is therefore obtained by

$$\frac{dp_{L_k}(r_n)}{dp_L} t(p_L)dp_L$$

from which the expected contribution to the energy delivered by unit k, $k \in Q_n$, in that interval is given by:

$$\int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \left[\sum_{r_n \in R_n} P_{r_n} \frac{dp_{L_k}(r_n)}{dp_L} \right] t(p_L)dp_L$$

Denoting the expression in the square brackets by $\bar{m}_{R_n}(p_L,k)$, and following the same procedure as above, we obtain:

$$\bar{E}_k^n = (1-q_n) \int_{\sum_{i=1}^{n-1} p_i}^{\sum_{i=1}^{n} p_i} \bar{h}_{n-1} (\bar{m}_{R_n} (p_L,k))$$

where $\bar{E}_k^n$ is the expected contribution of unit k to the energy delivered by the units in subset Q_n, $k \in Q_n$, which have not been accounted for in previous calculations, and where $\bar{h}_{n-1}(\bar{m}_{R_n}(p_L,k))$ is obtained using recursive equations as suggested above.

The expected energy delivered by unit k is therefore:

$$\bar{E}_k = \sum_{n \in A} \bar{E}_k^n$$

An approximate procedure to calculate $\bar{E}_k$, resulting in substantial reduction in the computational volume, can also be applied.

Incorporating Maintenance Requirements

Maintenance requirements can be incorporated in the calculations by partitioning the period involved to subperiod of constant maintenance, and carrying out the calculations for each subperiod separately, using the procedure described above. The results of the various subperiods are then combined to yield the corresponding results for the whole period.

Unit Commitment

System production costs are very sensitive to variations in unit commitment. Marsh et al [6] have suggested a procedure to account for unit commitment in cost calculations, that can also be incorporated in our case. According to their approach, each period of constant maintenance should be further divided into time zones of constant commitment by groupings hours with identical commitments and arranging the load in each grouping into a load duration curve. Then the operating cost calculations are carried out separately for each zone of constant commitment and constand maintenance using the above procedures, and summed up to yield the operating cost for the whole period. While providing us with more accurate cost measures, the method obviously results in an increased number of calculations.

CONCLUSIONS

An analytical approach to calculating the expected operating cost of a power system under an incremental loading procedure, while accounting for maintenance and forced outages, has been presented. To initiate the process, the generating units in the system are arranged in an ascending order of their minimal marginal cost. For any given unit n, an associated subset Q_n is defined, containing all the generators in the system which are loaded to production with generator n in at least some of the cases. Taking outage probabilities into consideration, the cost calculations are then performed in a phased manner by computing the contribution of each subset Q_n to the total cost in a recursive manner, and adding up the contributions to yield the expected total operating cost for the system. The calculations are shown to be a function of the Combined Load Duration Curve, a function which is obtained by convolving random deviations of customers' load, maintenance requirements and forced outage requirements with the deterministic component of customers' demand. Provided the number of units in each subset is less than the number of generators in the system, which is usually the case in realistic power systems, the procedure results in substantial savings in the volume of computations compared with simulation, let alone enumeration methods. An approximate procedure to calculating the expected operating cost, resulting in further decrease in the amount of calculations was also developed. The procedure was also extended to yield the expected amount of energy delivered by each unit in the system as well as the expected fuel cost for each unit. Given the unit cost of fuel, the expected amount of fuel consumed by each unit in the system, a quantity of paramount importance for fuel budgeting and national energy policy, is easily derived.

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