Supply Function Prediction in Electricity Auctions

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Supply Function Prediction in Electricity Auctions

Matteo Pelagatti

Abstract In the fast growing literature that addresses the problem of the optimal bidding behaviour of power generation companies that sell energy in electricity auctions it is always assumed that every firm knows the aggregate supply function of its competitors. Since this information is generally not available, real data have to be substituted by predictions. In this paper we propose two alternative approaches to the problem and apply them to the hourly prediction of the aggregate supply function of the competitors of the main Italian generation company.

1 Introduction

The last twenty years have witnessed in most European and many non European countries a radical reorganization of the electricity supply industry. Government owned monopolies have been replaced by regulated (generally pool) competitive markets, where the match between demand and supply takes place in hourly (in some cases semi-hourly) auctions. The auction mechanism is generally based on a uniform price rule, i.e. once the equilibrium price is determined, all the dispatched producers receive the same price per MWh.

The issue of determining the profit-maximising behaviour of a power company bidding in electricity auctions has been addressed by economists both from the normative (profit optimisation) and positive (market equilibrium) point of views (cf. Wolak, 2003; Hortasu and Puller, 2008; Bosco et al, 2011) and it is faced every hour by the generation companies. Regardless of the bidding strategy a firm pursues, it is necessary for her to predict the bidding behaviour of her competitors. In

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particular, each firm has to predict the aggregate quantity offered by her competitors for all possible prices before submitting her supply schedule to the market.

In this paper we propose two techniques for forecasting supply functions based on principal component analysis and reduced rank regression. The techniques are applied to the prediction of the hourly supply functions of the competitors of Enel, the main Italian generation company, as observed in two years of Italian electricity auctions.

Functional time series is a relatively new discipline in the statistical literature, even though the wider-ranging functional data analysis field has a longer history, dating back to the paper of Ramsay (1982) and the works of B.W. Silverman on density function estimation and nonparametric regression. A general framework for the problem of functional time series prediction can be found in Hyndman and Ullah (2007) and Hyndman and Shang (2009), and the first of the two algorithms proposed in this paper is a special case of the proposal in Hyndman and Ullah (2007). Our second algorithm, instead, cannot be found in the cited paper and, to the best of our knowledge, has never been explored in the statistical literature.

The paper is organised as follows: Section 2 introduces the problem of optimal bidding, Section 3 describes the Italian auction rules and the data produced by the market maker, Section 4 illustrates the two functional prediction techniques, Section 5 applies them to the Italian data and Section 6 concludes.

2 Optimal bidding behaviour

This sections introduces the problem a generation operator faces in every auction as developed in the economic literature (Wolak, 2003; Hortasu and Puller, 2008; Bosco et al, 2011). In order to have interpretable closed-form solutions, economists make a series of simplifying assumptions and approximations, that do not seem to reveal significant drawbacks when applied to real data (cf. the applications in the cited papers). However, the functional prediction techniques proposed in this paper can be used also in more involved optimisation problems in which transmission constraints and multi-period profits are taken into account.

If we assume that each firm wishes to maximise her profit in each auction independently from the other auctions (as customary in the literature), then we can summarise the optimisation problem as follows. Suppose that D is the (price-inelastic) demand for electricity, $S_{-i}(p)$ is the aggregate supply function of firm i's competitors for any given price p, $C_i(q)$ is the production cost function of firm i for any given quantity of energy q, then for those values of the residual demand $D - S_{-i}(p)$ that the production capacity of firm i can fulfill, the profit function of firm i is given by

$$\pi_i(p) = p \cdot \big(D - S_{-i}(p)\big) - C_i\big(D - S_{-i}(p)\big).$$

This profit function can be extended to include financial contracts as in Hortasu and Puller (2008) or vertical integration (i.e. the situation in which the producer is also a

retailer and plays in both sides of the auction) as in Bosco et al (2011). By assuming the continuous differentiability of S_{-i} and C_i, and the concavity of π_i, first order conditions indicate that firm i maximises her profit when she offers the quantity $D - S_{-i}(p^*)$ at the price p^* that solves

$$p^* = C_i'\big(D - S_{-i}(p^*)\big) + \frac{D - S_{-i}(p^*)}{S_{-i}'(p^*)}. \qquad (1)$$

Now, the quantity D and the supply function S_{-i} are generally unknown, but while D can be predicted using standard time series techniques (e.g. Bunn and Farmer, 1985; Harvey and Koopman, 1993; Ramanathan et al, 1997, and many aricles in the *IEEE Transactions on Power Systems*), the prediction of the function S_{-i} is more involved. The next sections illustrate two techniques for the prediction of such supply functions as observable in auction data.

3 Auction rules and data

According to the rules of the Italian electricity day-ahead market, each production unit can submit up to four "packages" of price-quantity pairs. Each pair contains the information on that quantity (in MWh) that a production unit is willing to sell and the relative unitary price (in Euro per MWh). Of course one company usually owns many production units and can, therefore, well approximate her (possibly continuous) optimal supply function using a step function with many steps. All the submitted pairs are sorted by price and the corresponding quantities are cumulated. When the cumulated offered quantity matches the total demand, the system marginal price (SMP) is determined and all the units offering energy up to that price are dispatched. If congestions in the transmission network occur the national market is split into up to seven (recently reduced to six) zonal markets and the same bids are used to determine new local equilibrium prices. In this case the optimisation problem is more involved than the one discussed in Section 2 and the solution has to be found numerically, but predictions of the competitors' zonal supply functions are still necessary.

Table 1 Relevant fields in the Italian electricity auctions database.

Producer (seller)	Retailer (buyer)
Operator name	Operator name
Plant name	Unit name
Quantity (MWh) of each offer	Quantity (MWh) of each bid
Price (Euro/MWh) of each offer	Price (Euro/MWh) of each bid
Awarded quantity (MWh) for each offer	Awarded quantity (MWh) for each bid
Awarded price (Euro/MWh) for each offer	Awarded price (Euro/MWh) for each bid
Zone of each offer (plant)	Zone of each bid (unit)
Status of the offer: accepted vs. rejected	Status of the bid: accepted vs. rejected

Each record of the Italian auction results database[1] (cf. Table 1) contains the price-quantity pair, the name of the offering production unit and the name of the owner of that unit. This allows the construction of the supply function of any firm bidding in the auctions or aggregations thereof (e.g. step function in Figure 1).
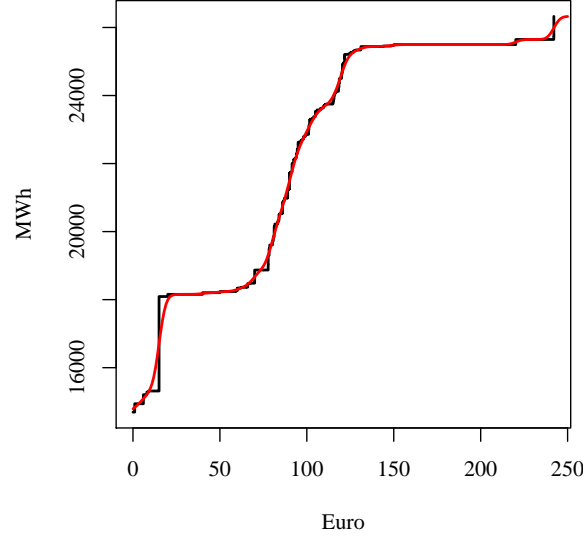

Fig. 1 Supply function of Enel's competitors on 3.12.2008 at 10am and kernel approximation.

From the above reasoning it is clear that real supply schedules are step functions and, thus, the optimal bidding theory discussed in the previous section is not directly applicable. This issue is generally dealt with by approximating the step functions with continuously differentiable functions obtained though kernel smoothing[2]. Smoothing is also necessary for regularising functions before applying canonical correlation techniques such as reduced rank regression (cf. Sec. 11.5 of Ramsay and Silverman, 2005). Since supply functions are nondecreasing in price, we use the kernel

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[1] It can be downloaded (one file per day) from the market operator web site `www.mercatoelettrico.org`

[2] Kastl (2011) solves the problem of optimal bidding when supply functions are step functions with a given number of steps. However, even in this case the optimal predictions of these step functions need not be step functions, as the prices at which the steps take place may be absolutely continuous random variables and this condition makes the expectation of any random step function a continuous function.

$$S(p) = \sum_{k=1}^{K} q_k \Phi\left(\frac{p - p_k}{h}\right),$$

where Φ is the standard normal cumulative probability function, h is the bandwidth parameter and (q_k, p_k) are the observed quantity-price pairs. Notice that the total number of offers K may change in each auction. The derivative of the smoothed function needed in equation (1) is given by

$$S'(p) = \sum_{k=1}^{K} q_k \frac{1}{h} \phi\left(\frac{p - p_k}{h}\right),$$

with ϕ standard normal density. Figure 1 depicts the actual supply function of Enel's competitors on 3 December 2008 at 10am and the kernel approximation thereof ($h = 3$ Euro).

4 Supply functions prediction

Both prediction techniques proposed in this paper entail some common steps.

The first step consists in sampling the kernel-smoothed function on a grid of abscissa points. This is necessary as the price set on which the function can be evaluated changes in every auction. Since the function can be approximated more accurately where bid pairs are more dense, we sample more frequently in these intervals by using quantiles. In particular, we used 50-iles of unique prices submitted over the entire sample (2007-2008). The forty-nine 50-iles are supplemented with the minimum (0) and the theoretical maximum (500) due to the price capping rule of the Italian market, obtaining 51 time series of ordinate points (quantities). Figure 2 displays one week of Enel's competitors aggregate supply functions sampled at 50-iles. The within-day periodicity and the lower level and slightly different shape of the curve in the weekend are evident from the plot.

The second common step consists in transforming the original ordinate points in a way that preserves the two features of positivity and non-decreasing monotonicity of the original functions also in their predictions. If we denote with $\{p_0, p_1, \ldots, p_{50}\}$ the points in the price grid and with $S_t(p_i)$ the smoothed supply function at time t for price p_i, then we transform the time series as

$$q_{i,t} := \begin{cases} \log S_t(p_i), & \text{for } i = 0; \\ \log\left(S_t(p_i) - S_t(p_{i-1}) + c\right), & \text{for } i = 1, \ldots, 50, \end{cases}$$

where c is a small positive constant that guarantees the existence of the logarithm also in constant tracts of $S_t(p)$ (in our application we set $c = 1$). If we assume that the prediction of $q_{i,t}$, say $\hat{q}_{i,t}$, is unbiased, and the prediction error is approximately normal with standard error $s_{i,t}$, unbiased forecasts of the original function can be recovered as
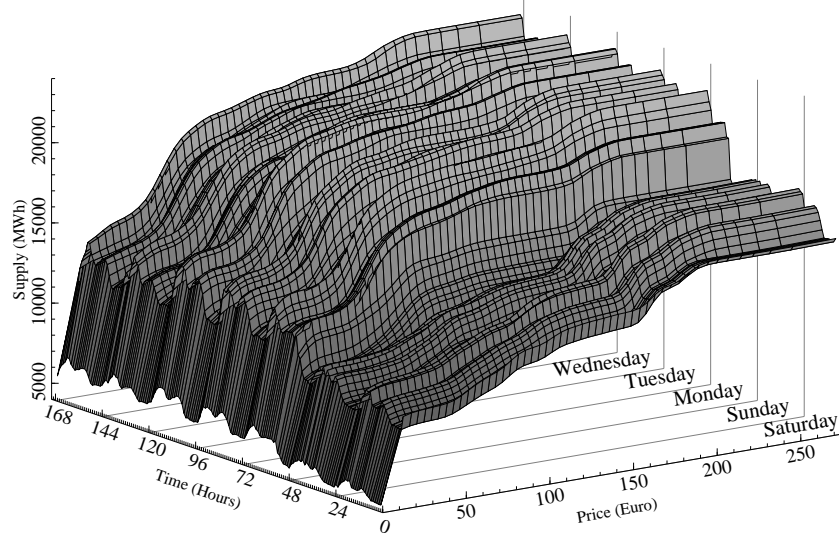
Fig. 2 The supply function sampled at 50-iles over one week.

$$\hat{S}_t(p_i) = \begin{cases} \exp\left(\hat{q}_{i,t} + s_{i,t}^2/2\right), & \text{for } i = 0; \\ \exp\left(\hat{q}_{i,t} + s_{i,t}^2/2\right) + \hat{S}_t(p_{i-1}) - c, & \text{for } i = 1\ldots,50. \end{cases} \tag{2}$$

Now, since we expect the 51 time series to share some information, it is natural to seek some form of dimension reduction. The two alternative algorithms we propose are based on principal component (PC) analysis and reduced rank regression (RRR). We base the choice of dimension reduction on one month of k-steps-ahead out-of-sample predictions, where $k = 1$ for one-hour-ahead predictions and $k = 24$ for one-day-ahead predictions. In particular, the model is fit to the hourly observations of the years 2007-2008, while the dimension reduction assessment is based on Jan-2009 prediction mean square errors (MSE).

In describing the two algorithms we collect the 51 transformed time series in the vector $\mathbf{q}_t$ and the original supply function ordinate-points in the vector $\mathbf{S_t}$. The predictions are based on lagged responses and deterministic regressors.

Algorithm 1 (Principal component analysis based) *For $r = \{51, 50, \ldots, 1\}$ iterate through the following steps.*

1. *Take the first r PCs of $\mathbf{q}_t$ (supply function log increments) based on its in-sample covariance matrix, and name the scores $\mathbf{y}_t$.*
2. *Regress each score $y_{i,t}$ on its lags $\mathbf{x}_{i,t}$ and deterministic regressors $\mathbf{z}_t$ and compute predictions $\hat{\mathbf{y}}_t$.*
3. *Regress the vector $\mathbf{q}_t$ on the predicted scores $\hat{\mathbf{y}}_t$ and a constant.*

4. *Compute the out-of-sample predictions of the supply function* $\mathbf{S}_t$ *as in (2) and the relative MSE.*

Pick the rank r that minimises the out-of-sample MSE.

In the PC approach, the time series are first reduced in number by taking the best linear approximation to the original data, and then these are predicted using standard time series models. The main advantage of this approach is the freedom left to the analyst to choose the time series model to predict the PC scores. The main drawback is that rank-reduction is not obtained directly for the prediction of future values.

The second approach is based on reduced rank regression. Since this technique is less popular than principal component analysis, we briefly survey its main features. Consider the linear model

$$\underset{n\times 1}{\mathbf{y}_t} = \underset{m\times 1}{\mathbf{C}\ \mathbf{x}_t} + \underset{p\times 1}{\mathbf{D}\ \mathbf{z}_t} + \underset{n\times 1}{\varepsilon_t}$$

where $\mathbf{x}_t$ and $\mathbf{z}_t$ are regressors, $\mathbf{D}$ is a full-rank $n \times p$ coefficient matrix, $\mathbf{C}$ is a $n \times m$ reduced-rank coefficient matrix and ε_t is a sequence of zero-mean random errors uncorrelated with all the regressors. The fact that $\mathbf{C}$ is reduced-rank means that few linear combinations of the regressors $\mathbf{x}_t$ are sufficient to take account of all the variability of $\mathbf{y}_t$ due to $\mathbf{x}_t$. Now, suppose that the rank of $\mathbf{C}$ is $r < \min(m, n)$, then $\mathbf{C}$ can be factorised as $\mathbf{C} = \mathbf{AB}^\top$, with $\mathbf{A}$ $n \times r$ and $\mathbf{B}$ $m \times r$ matrices. The matrices $\mathbf{A}$ and $\mathbf{B}$ are not uniquely identified, but if one restricts the r column vectors forming $\mathbf{B}$ to be orthonormal, then a least squares solution for $\mathbf{B}$ is found by solving the following eigenvalue problem:

$$\mathbf{S}_{xx|z}\mathbf{V}\Lambda = \mathbf{S}_{xy|z}\mathbf{S}_{yy|x}^{-1}\mathbf{S}_{yx|z}\mathbf{V},$$

where $\mathbf{S}_{ab|c}$ indicates the partial product-moment matrix of $\mathbf{a}$ and $\mathbf{b}$ given $\mathbf{c}$, $\mathbf{V}$ is an orthonormal matrix and Λ is a diagonal matrix. The first r columns of $\mathbf{V}$ provide least square estimates of $\mathbf{B}$. Least squares estimates of $\mathbf{A}$ and $\mathbf{D}$ are found by regressing $\mathbf{y}_t$ simultaneously on $\mathbf{w}_t := \mathbf{B}^\top\mathbf{x}_t$ and $\mathbf{z}_t$. For details on RRR refer to the excellent monograph by Reinsel and Velu (1998).

Algorithm 2 (Reduce rank regression based) *For* $r = \{51, 50, \ldots, 1\}$ *iterate through the following steps.*

1. *Regress the vector* $\mathbf{y}_t = \mathbf{q}_t$ *on its lags* $\mathbf{x}_t$*, imposing rank r to the reduced-rank coefficient matrix* $\mathbf{C}$*, and on the deterministic regressors* $\mathbf{z}_t$ *without any rank restrictions on* $\mathbf{D}$.
2. *Compute the out-of-sample predictions* $\hat{\mathbf{S}}_t$ *of the supply function* $\mathbf{S}_t$ *as in (2) and the relative MSE.*

Pick the rank r that minimises the out-of-sample MSE.

The main advantage of the RRR-based algorithm is that rank-reduction is obtained though the minimisation of the prediction MSE. The drawback is that only (vector) autoregressive models with exogenous variables are allowed.

5 Application to the Italian electricity auctions

The two algorithms are applied to the hourly Italian electricity auction results for the years 2007-2008 (17544 auctions); Jan-2009 (744 auctions) is used for determining the rank r as explained in the previous section.

We build models for predicting one-hour-ahead and models for forecasting one-day-ahead. As for the deterministic regressors ($\mathbf{z}_t$) we implement the following three increasing set of variables.

1. Linear trend, $\cos(\omega_j t)$, $\sin(\omega_j t)$, with $\omega_j = 2\pi j/(24 \cdot 365)$ and $j = 1, \ldots, 20$.
2. Regressors at point 1. plus dummies for Saturday, Sunday and Monday.
3. Regressors at point 2. plus $\cos(\lambda_i t)$, $\sin(\lambda_i t)$, with $\lambda_i = 2\pi i/24$ and $i = 1, \ldots, 6$.

Notice that the sinusoids at point 1. take care of the within-year seasonality, while those at point 3. model the within-day seasonality. These latter sinusoids are also supplemented with 24h-lagged prices (see below), that also help modelling the within-day seasonality.

Both vector autoregressive models and error correction mechanisms are explored. In particular, we regress:

Level 1-step: $\mathbf{y}_t$ on $\mathbf{y}_{t-1}, \mathbf{y}_{t-24}, \mathbf{y}_{t-168}, \mathbf{z}_t$;
Diff 1-step: $\Delta\mathbf{y}_t$ on $\mathbf{y}_{t-1}, \Delta\mathbf{y}_{t-1}, \Delta\mathbf{y}_{t-24}, \Delta\mathbf{y}_{t-168}, \Delta\mathbf{z}_t$;
Level 24-step: $\mathbf{y}_t$ on $\mathbf{y}_{t-24}, \mathbf{y}_{t-168}, \mathbf{z}_t$;
Diff 24-step: $\Delta_{24}\mathbf{y}_t$ on $\mathbf{y}_{t-24}, \Delta_{24}\mathbf{y}_{t-24}, \Delta_{24}\mathbf{y}_{t-168}, \Delta_{24}\mathbf{z}_t$.

The chosen rank and the actual root MSE (RMSE) are summarized in Table 2. Three features appear evident from these figures: i) the optimal rank of both PC

Table 2 Out-of-sample root mean square error for the two algorithms and 12 models.

	one-hour-ahead						one-day-ahead					
	Reg. 1.		Reg. 2.		Reg. 3.		Reg. 1.		Reg. 2.		Reg. 3.	
	rank	RMSE	rank	RMSE	rank	RMSE	rank	RMSE	rank	RMSE	rank	RMSE
RRR-Level	50	76.5	50	76.5	50	**67.1**	44	216.1	44	216.1	44	215.5
RRR-Diff	51	*73.7*	51	*73.7*	51	73.7	51	198.7	51	198.7	51	198.7
PC-level	50	80.9	50	80.9	50	72.7	41	*191.1*	41	*191.1*	41	**188.8**
PC-Diff	37	*73.7*	47	*73.7*	37	73.7	51	198.8	51	198.8	51	198.8

and RRR models is very close to the full rank (51), indicating that almost all the information that the time series carry is relevant for forecasting; ii) there is no clear indication about the choice of the algorithm, as the best algorithm for one-hour-ahead predictions is RRR while that for one-day-ahead predictions is PC; iii) a large number of deterministic regressors is better than a small one.

Of course these regressors could have been supplemented with variables such as (lagged) oil prices, weather forecasts, holidays dummies, etc., that would certainly improve the in- and out-of-sample fit, but the main objective of this paper is proposing feasible techniques for forecasting this type of functional time series and testing

them on real electricity auction data. The main features of the data are well-captured by these regressors and lagged supply functions, and at this stage the fine tuning of the models is not necessarily interesting.
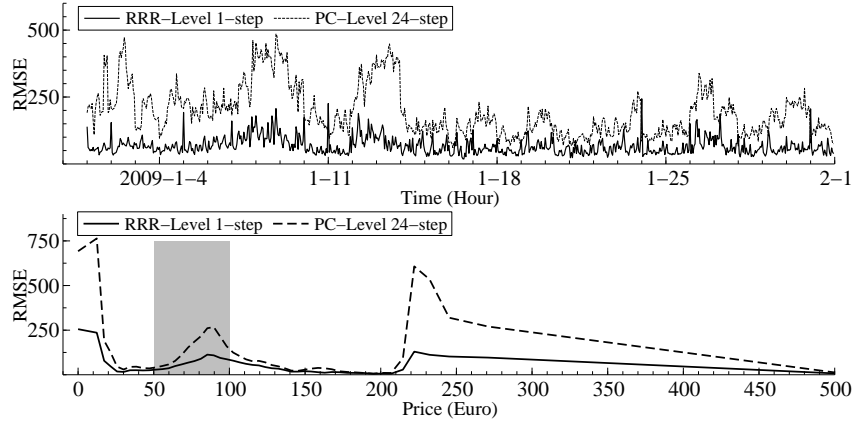

Fig. 3 Root mean square error of prediction as functions of time and price.

Figure 3 depicts the out-of-sample RMSE as a function of time (first panel) and of price (second panel). It appears clear that the precision of the predictions vary over time and price. In particular, the first half of Jan-2009 seems to be harder to predict then the following part of that month. Indeed, those days are characterised by holidays and school vacations that were not explicitly modelled. As for the precision of the prediction at different points of the supply function, the hardest quantities to predict are those in correspondence of the price intervals $[0, 20]$ and $[220, 250]$. Most observed SMPs are in the range $[50, 100]$, and so this interval is the most interesting to predict. The RMSE in that interval is not particularly large: it peaks at some 90 MWh for one-hour-ahead predictions and 250 MWh for one-day-ahead predictions, respectively, some 0.5% and 1.3% relative RMSE.

Figure 4 depicts the one-hour- and one-day-ahead functional prediction for an arbitrary auction chosen in the out-of-sample period, just to give a visual idea of the outcomes of the proposed algorithms. Table 3 reports the RMSE computed for each day of the week. It reveals that supply functions are easiest to predict on weekends, and hardest to forecast on Mondays. The same table shows also that for one-hour-ahead predictions the RRR model on levels is the best choice (almost) uniformly in the seven days, while for one-day-ahead forecasts one should change the model according to the day of interest.
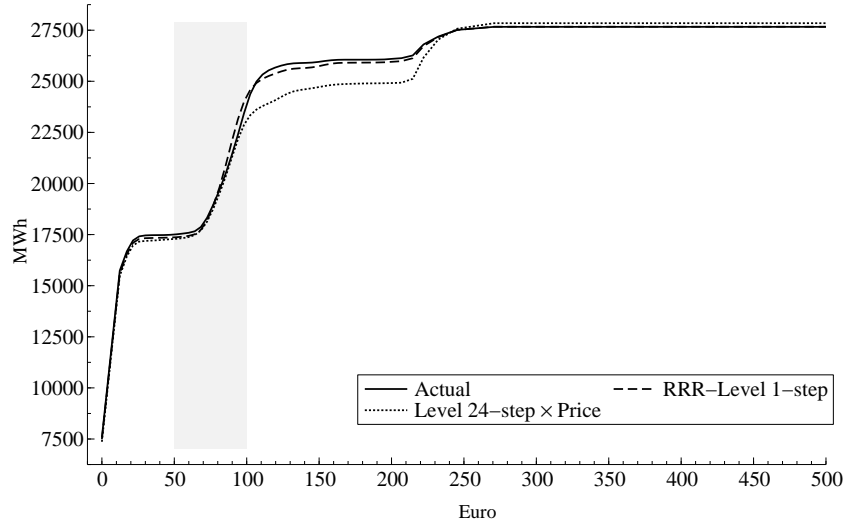
Fig. 4 Predicted and actual supply functions of Enel's competitors on Wed 14.01.2009 at 10am.

Table 3 Out-of-sample root mean square errors computed for each day of the week.

1-step	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Total
RRR-level	**75.9**	**62.8**	**71.6**	**72.7**	**68.5**	**61.3**	54.0	**67.0**
RRR-diff	80.1	64.8	83.9	81.6	72.7	72.9	**53.9**	73.5
PC-level	90.9	77.8	74.2	78.0	71.8	65.4	56.3	74.2
PC-diff	80.1	64.8	83.9	81.6	72.7	72.9	53.9	73.5

24-step	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Total
Level 24-step	241.4	250.6	**177.7**	243.2	249.6	158.2	155.4	214.9
Diff 24-step	275.2	**146.3**	198.3	**171.4**	**144.0**	240.1	186.8	199.7
PC-level	**235.9**	219.3	179.0	199.5	181.9	**151.3**	151.4	**190.7**
PC-diff	275.3	146.4	198.3	**171.4**	144.0	240.1	186.9	199.7

6 Conclusion

We have introduced two different approaches to forecasting supply functions in electricity auctions. Accurate approximations of actual competitors' supply functions are indeed needed by all the generation companies bidding in the hourly (or semi-hourly) uniform-price auctions that characterise most electricity markets around the world. The two techniques are easy to implement and assure that the predictions

share the same characteristics as the actual supply functions (i.e. positivity and non-decreasing monotonicity).

The application of the two techniques to the aggregate supply functions bid in the Italian day-ahead-market by Enel's competitors reveals that the predictions turn out to be quite accurate. The proposed prediction algorithms, possibly supplemented with other relevant regressors, seem to represent a valuable tool for helping generation companies to design their bidding strategies in a more profitable way.

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