Strategic Bidding in Extended Locational Marginal Price Scheme

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***Abstract*—The locational marginal price scheme, though widely adopted, fails to reflect the startup costs for many generators in the real-time market. To solve this issue, the extended locational marginal price (eLMP in short, a.k.a. convex hull price) has been proposed. The key idea is to convexify the non-convex generation cost functions due to the startup costs, and then design the uplift payment whenever necessary. While eLMP partially solves the incentive issue, there are still chances for generators to manipulate the market prices (as well as the uplift payments) by strategic bidding. In this letter, we first propose a profit decomposition method to evaluate different bidding strategies' impacts on individual payoffs. This decomposition allows us to better identify the potential strategic generators and investigate their best strategies. We further propose to use the maximal markup as an index for eLMP scheme to quantify market power in the whole system. Numerical studies further highlight the existence of market power in practice.**

***Index Terms*—Smart grid, optimization, power systems.**

I. Introduction

AN EFFECTIVE pricing scheme is vital to the success of any market. However, different from the market for conventional commodities, the pricing scheme design is rather challenging for the electricity market. The difficulties mostly come from the complex physical nature of the power flows. To tackle the challenge, the locational marginal pricing (LMP) scheme has been proposed by Schweppe *et al.*, dating back to the year of 1988 [1].

However, LMP is still not sufficient to guarantee an effective market. One major issue is that the LMP scheme fails to compensate startup costs for many generators (e.g., large coal and nuclear power plants) in the market, and these costs could be major. To solve this issue, Gribik *et al.* propose the notion of extended LMP (eLMP in short, a.k.a., convex hull price) in [2], which first convexifies the economic dispatch problem and then offers uplift payments to improve market efficiency.

We submit that in the eLMP scheme, uplift payments cannot guarantee truthful bidding. Both theoretical analysis and numerical investigation indicate the existence of market power via strategic bidding.

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A. Related Works

To tackle the challenges brought by the complex physical nature in the electricity market operation, various pricing schemes have been proposed, including mixed-integer linear programming based uniform price [3], non-linear price for price discrimination [4], semi-Lagrangian based price for individual rationality [5], and Vickrey-Clarke-Groves auction based scheme [6]. However, these pricing schemes often suffer from computational challenges largely due to the NP-hardness of unit commitment [7]. Noticing this deficiency, the eLMP scheme is believed to be a promising alternative. In [2], Gribik *et al.* overview the pricing and uplift payment in the eLMP scheme. In [8], Schiro *et al.* further comprehensively clarify the formulation and implementation procedure.

Market power analysis is crucial to every market and so it is for the electricity market. In [9], Borenstein clarifies the definition of market power in the electricity sector and reveals its effects and implications. In [10], David and Wen focus on studying power suppliers' ability to affect the equilibrium price in an electricity pool model. Bunn and Oliveira use game theoretic framework to model and simulate market participants' potential market power in [11]. The generators could adopt various strategic behaviors to exercise market power. In [12], Cardell *et al.* illustrate that they can increase their production quantities to block transmission lines in the network and raise LMP. In [13], Anderson and Cau present that a duopoly can operate implicit collusion to gain market power. Specifically, we follow the literature and concentrate on a special form of market power manipulation, strategic bidding (see [14] for a comprehensive survey).

B. Our Contributions

Towards effectively identifying and quantifying the market power issues in the eLMP scheme, our principal contributions can be summarized as follows:

- *Profits Decomposition:* By exploiting the structure of generators' profits in the eLMP scheme, we propose a profit decomposition method to characterize the impact of strategic behaviors on individual generator's profits.
- *Strategic Behavior Characterization:* We focus on understanding the potential impacts of strategic bidding on market efficiency. Such analysis allows us to exploit the

structure of strategic bidding in various market conditions, which helps us ultimately characterize the maximal markup through strategic bidding.

- *Market Power Quantification:* We propose to use the maximal markup index to quantify market power in eLMP scheme. Both theoretical analysis and numerical studies highlight the deficiencies of eLMP scheme.

In the rest of this letter, we sketch all the necessary proofs in the Appendix.

II. The eLMP Scheme:Basics

In this letter, theoretically, we focus on the stylized electricity pool model to highlight the existence of market power in the eLMP scheme. As illustrated by the numerical studies, the network constraints will create more loopholes (especially the locational market power) in the system.

In the electricity pool model, the ISO seeks to minimize the total generation costs at each time slot:

$$\begin{aligned} c(y) := \min_{\mathbf{g}} \quad & \sum_{i=1}^{n} f_i(g_i) \\ s.t. \quad & \sum_{i=1}^{n} g_i = y, \\ & 0 \le g_i \le G_i, \end{aligned} \tag{1}$$

where g_i denotes the output of generator i, G_i denotes the capacity of generator i, $f_i(g_i)$ denotes the generation cost for generator i, vector $\mathbf{g}$ is $[g_1, \ldots, g_n]$, and y is the total load in the system. The optimal solution $\mathbf{g}^*(y)$ and the optimal objective value $c(y)$ are both functions of the total load y.

If $f_i(g_i)$ is linear, then the operator could conduct the economic dispatch according to the merit order of marginal generation costs and set the price as the marginal cost.

However, in practice, there are startup costs associated with generation. Hence, for generator i, $f_i(g_i)$ is often of the following form:

$$f_i(g_i) = s_i \cdot I(g_i^0 = 0, g_i > 0) + v_i \cdot g_i, 0 \le g_i \le G_i \tag{2}$$

where g_i^0 is i's previous generation profile; $I(\cdot)$ is the indicator function; s_i is the startup cost; and v_i is the variable cost. In other words, generator i only needs to bear the startup cost when it was off-line at the previous time slot and is dispatched at the current time slot. Actually, this indicator function can be simplified by making the startup cost time-varying. That is, when $g_i^0 > 0$, we can simply set s_i to be 0. Such a cost structure challenges the conventional scheme in terms of dispatch profile, price design, and incentive analysis.

The solution proposed by eLMP is to use the uplift payment to incentivize the generators to follow the system operator's dispatch profile.

For each generator i, given price p, its desired generation level $g_i^d(p)$ is associated with its maximal profits, and can be different from the dispatch profile g_i^*:

$$g_i^d(p) = \sup\left\{\arg \max_{z\in[0,G_i]} \{p \cdot z - f_i(z)\}\right\}. \tag{3}$$

The desired generation levels may not be unique, and hence we use supremum to select the maximal desired generation level. This generation level leads to the maximal profits given p, denoted by $\pi_i^*(p)$, i.e.,

$$\pi_i^*(p) = p \cdot g_i^d(p) - f_i(g_i^d(p)). \tag{4}$$

The difference in the profits between generating $g_i^d(p)$ and g_i^* needs to be compensated by uplift payment:

$$\text{Uplift}_i(p, y) = \pi_i^*(p) - [p \cdot g_i^* - f_i(g_i^*)]. \tag{5}$$

And the eLMP scheme is proved to guarantee the optimal system price p^* in terms of minimizing the total uplift payment [2].

The optimal price p^* in eLMP can be expressed as follows:

$$p^* = \inf\left\{p \Big| \sum_{i=1}^{n} g_i^d(p) \ge y\right\}. \tag{6}$$

This expression holds under mild conditions. Generally speaking, it requires the generation cost function $f_i(\cdot)$ to be piece-wise linear. See our preliminary research [15] for more detailed discussion.

The optimal system price p^* is a function of the system load y and all the reported cost functions, which allows us to better exploit the structures of the eLMP scheme. Without loss of generality, assume the generators have been sorted according to their average generation costs, i.e.,

$$\frac{f_1(G_1)}{G_1} \le \frac{f_2(G_2)}{G_2} \le \cdots \le \frac{f_n(G_n)}{G_n}. \tag{7}$$

Then, we can define the following auxiliary function $h(y)$ to characterize the eLMP for some given demand y:

$$h(y) = \sum_{i=1}^{k-1} f_i(G_i) + \frac{f_k(G_k)}{G_k} \cdot \left(y - \sum_{i=1}^{k-1} G_i\right), \tag{8}$$

where k is the unique solution to the following inequality:

$$\sum_{i=1}^{k-1} G_i < y \le \sum_{i=1}^{k} G_i. \tag{9}$$

Note that, by the definition of summation, $\sum_{i=1}^{0} G_i = 0$.

This auxiliary function characterizes the eLMP scheme by providing the closed form expression for the convex hull of the optimization problem (1):

Lemma 1: Denote $\bar{c}(y)$ the convex hull function of $c(y)$, defined in (1). For any $y \in [0, \sum_{i=1}^{n} G_i]$, we have

$$\bar{c}(y) = h(y). \tag{10}$$

Combining Lemma 1 and Eq. (6) yields the following eLMP characterization corollary.

Corollary 1: For any $y \in (\sum_{i=1}^{k-1} G_i, \sum_{i=1}^{k} G_i]$, we know

$$p^*(y) = \frac{f_k(G_k)}{G_k}. \tag{11}$$

Remark: While eLMP guarantees individual rationality (as suggested by the uplift payment in (5)), it implicitly assumes that the generators will truthfully bid to the ISO. However, this assumption may not be practical, which leaves the market loophole for strategic generators to exploit.

III. Strategic Behavior Analysis

In this section, we first explore the impact of strategic bidding on the eLMP scheme and the dispatch outcome, and then investigate the potential profits by strategic bidding. Such analysis allows us to identify maximal markups through strategic bidding for different kinds of generators.

A. Profits Decomposition

For each generator, its total profits are gained from the uplift payment as well as selling electricity according to the ISO's dispatch profile. We want to emphasize that the second component is not necessarily positive, which further implies the importance of uplift payment.

In practice, the generators are allowed to bid their cost functions as well as their available capacities to the ISO. In this letter, we assume the generators are not allowed to withhold their capacities. Hence, it can only strategically bid its cost function which can be determined by the startup cost s_i and variable cost v_i.

Mathematically, if generator i truthfully bids its (s_i, v_i), given a demand of y, we denote its profits P_i as benchmark:

$$P_i = \pi_i(p^*(s_i, v_i)) = \{p^* \cdot G_i - f_i(G_i)\}^+, \tag{12}$$

where $\{x\}^+ := \max\{0, x\}$.

If the generator strategically reports its generation cost as:

$$\tilde{f}_i(g_i) = \tilde{s} \cdot I(g_i > 0) + \tilde{v} \cdot g_i, \tag{13}$$

which may lead to a potentially different $\tilde{p}^*$ given by eLMP, and a potentially different dispatch profile $\tilde{\mathbf{g}}^*$. Then, generator i's total profits $\tilde{P}_i(\tilde{s}, \tilde{v})$ via strategic bidding can be decomposed into two parts. For linear generation functions, the decomposition can be characterized as follows:

$$\tilde{P}_i(\tilde{s}, \tilde{v}) = \underbrace{\left\{\tilde{p}^* \cdot G_i - \tilde{f}_i(G_i)\right\}^+}_{\text{Profits by Strategic Bidding}} + \underbrace{\left(\tilde{f}_i(\tilde{g}_i^*) - f_i(\tilde{g}_i^*)\right)}_{\text{Profits in Generation Cost}}, \tag{14}$$

where $(\tilde{s}, \tilde{v}) \in \mathbb{R}^+ \times \mathbb{R}^+$.

Note that, according to (14), the strategic bidding could manipulate the market by affecting the price, the dispatch profile, or both. Hence, in the subsequent analysis, we will investigate the manipulation on eLMP and the dispatch profile, sequentially.

B. Manipulation on eLMP

When analyzing strategic behaviors, the common approach is to assume only a particular generator is strategic while everyone else is truthful. We adopt this approach in our analysis. This motivates us to define an auxiliary price function $p^{\{i\}}(y)$, which indicates the eLMP in a system without generator i, given a total load of y.

To characterize how generator i's strategic bidding affects the eLMP scheme, it suffices to identify two critical points:

$$p_1 := p^{\{i\}}(y - G_i), \tag{15}$$

$$p_2 := p^{\{i\}}(y). \tag{16}$$

These two critical points allow us to analyze the impact of strategic bidding on eLMP, following the similar routine of VCG auction [16].

Specifically, combining these two points with Eq. (6), we can further characterize the eLMP with single generator strategic bidding.

Lemma 2: In a system with total load of y, if a single generator i strategically bids its cost function as in Eq. (13), then, the eLMP $\tilde{p}^*(y|\tilde{s}, \tilde{v})$ can be calculated as follows:

$$\tilde{p}^*(y|\tilde{s}, \tilde{v}) = \begin{cases} p_1, & \text{if } \tilde{f}_i(G_i)/G_i \le p_1, \\ \tilde{f}_i(G_i)/G_i, & \text{if } p_1 < \tilde{f}_i(G_i)/G_i \le p_2, \\ p_2, & \text{otherwise.} \end{cases} \tag{17}$$

This indicates that the system price is only influenced by i's average cost, i.e., $\tilde{f}_i(G_i)/G_i$. Strategies which share the same average cost will result in the same system price.

One immediate application of Lemma 2 is to simplify the total profits for generator i through strategic bidding, as shown in Eq. (14). Lemma 2 states that, for any strategic bidding $(\tilde{s}, \tilde{v})$, the following condition holds,

$$\tilde{p}^* \le \frac{\tilde{f}_i(G_i)}{G_i} \text{ when } p_1 < \frac{\tilde{f}_i(G_i)}{G_i}. \tag{18}$$

Hence, we can safely replace $\tilde{p}^*$ with p_1 in Eq.(14), i.e.,

$$\tilde{P}_i(\tilde{s}, \tilde{v}) = \left\{p_1 \cdot G_i - \tilde{f}_i(G_i)\right\}^+ + \left(\tilde{f}_i(\tilde{g}_i^*) - f_i(\tilde{g}_i^*)\right). \tag{19}$$

C. Manipulation on Dispatch Profile

The system operator decides the dispatch profile according to the reported cost functions to minimize the total (reported) generation cost. Hence, strategic bidding will inevitably affect the optimal dispatch profile. This calls for characterizing the dispatch profile as a function of strategic bidding $(\tilde{s}, \tilde{v})$.

Inspired by dynamic programming, we can decompose the economic dispatch problem (1) as follows:

$$c(y|\tilde{s}, \tilde{v}) = \min_{u \in [y - G_i, y]} \{c^{\{i\}}(u) + \tilde{f}_i(y - u)\}, \tag{20}$$

where

$$\begin{aligned} c^{\{i\}}(u) := \min_{\mathbf{g}} \ & \sum_{j \ne i} f_j(g_j) \\ s.t. \ & \sum_{j \ne i} g_j = u, \\ & 0 \le g_j \le G_j \quad \forall j \ne i, \end{aligned} \tag{21}$$

and

$$\tilde{f}_i(y - u) = \tilde{s} \cdot I(u < y) + \tilde{v} \cdot (y - u). \tag{22}$$

Note that, such decomposition decouples the strategic bidding's impact on the economic dispatch problem. Specifically, for any given u, $c^{\{i\}}(u)$ will not be influenced by $(\tilde{s}, \tilde{v})$. All the strategic bidding's impacts are restricted to generator i's reported cost function, i.e., the second component in (20), $\tilde{f}_i(y - u)$.

Note that $c^{\{i\}}(u)$, just as the economic dispatch problem (1), is non-convex. Hence, it is not easy to solve the dynamic programming formulation (20) directly. Fortunately, with the help of convex hull of $c^{\{i\}}(u)$, it is possible to calculate $c(y|\tilde{s}, \tilde{v})$ effectively. Denote the convex hull of $c^{\{i\}}(u)$ over $[y - G_i, y]$ by $\bar{c}^{\{i\}}(u)$. Then, we can show that:

Lemma 3: The function $c(y|\tilde{s}, \tilde{v})$ can be effectively estimated with the knowledge on $c^{\{i\}}(y)$, and the convex hull of $c^{\{i\}}(u)$ over $[y - G_i, y]$. Specifically,

$$c(y|\tilde{s}, \tilde{v}) = \min\left\{c^{\{i\}}(y), \tilde{s} + \tilde{v} \cdot y + \theta_i(y)\right\}, \tag{23}$$

where

$$\theta_i(y) = \min_{u\in[y-G_i,y]} \bar{c}^{\{i\}}(u) - \tilde{v}\cdot u. \tag{24}$$

Lemma 3 suggests that, to understand how strategic bidding affects the dispatch profile, it suffices to understand the structure of the convex hull of $c^{\{i\}}(u)$. In general, for function $f(u)$ over domain $[\mathbb{L}, \mathbb{R}]$, we can make the following observations on the properties of its convex hull $\bar{f}(u)$. Mathematically,

Proposition 1: For any piece-wise linear, left continuous and non-decreasing function $f(u)$ over $[\mathbb{L}, \mathbb{R}]$, define its convex hull $\bar{f}(u)$ over $[L, R] \subset [\mathbb{L}, \mathbb{R}]$. Then, $\bar{f}(u)$ can be uniquely determined by a subset of the breaking points of $f(u)$ over $[L, R]$. Mathematically, denote the sequence $\{b_i\}_{i=0}^M$ the breaking points of $f(u)$ over $[L, R]$, satisfying

(a) $\forall b_j, j \in \{1, \dots, M-1\}$ is a breaking point of $f(u)$;

(b) $b_0 = L < b_1 < \dots < b_M = R$;

(c) $f(b_i) = \bar{f}(b_i)$.

Then, $\bar{f}(u)$ over $[L, R]$ can be characterized as follows:

$$\bar{f}(u) = \begin{cases} f(u), & u = b_i,\\ f(b_i) + l_{i+1}\cdot(u - b_i), & b_i < u < b_{i+1}, \end{cases} \tag{25}$$

where

$$l_{i+1} = \frac{f(b_{i+1}) - f(b_i)}{b_{i+1} - b_i}. \tag{26}$$

Since $c^{\{i\}}(u)$ satisfies all the conditions in Proposition 1, we can make use of the structure of its convex hull to uncover the impact of strategic bidding on the dispatch profile.

Mathematically, suppose there are M breaking points of $\bar{c}^{\{i\}}(u)$, the convex hull of $c^{\{i\}}(u)$ over $[y - G_i, y]$. For the j^{th} linear segment of the convex hull, we denote its associated right end point by b_j, and its slope by l_j, both according to Proposition 1. Then, we can prove that

Theorem 1: Given strategic bidding $(\tilde{s}, \tilde{v})$, the dispatched generation function can be characterized as follows:

$$\tilde{g}_i^*(\tilde{s}, \tilde{v}) = y - b_j, \tag{27}$$

where $j \in \{0, \dots, M-1\}$, such that

$$l_j \le \tilde{v} < l_{j+1}, \tag{28}$$

$$c^{\{i\}}(y) > c^{\{i\}}(b_j) + \tilde{s} + \tilde{v}\cdot(y - b_j). \tag{29}$$

Remark: Note that Theorem 1 is an immediate result of Lemma 3 and Proposition 1. It not only describes the dynamics of dispatched generation function in strategic bidding but also clarifies how to effectively construct the strategy to achieve the maximal markup. This serves as the basis for our definition of market power for each generator.

IV. Numerical Studies

We first use seven types of generators from the IEEE RTS 96 node system [17] to examine the market power issues in the electricity pool model. Table I summarizes the generation information. We propose to use the maximal markup index (MMI) to measure the market power. Mathematically, for each generator i, its MMI_i is defined as follows:

$$\text{MMI}_i := \sup_{(\tilde{s},\tilde{v})\in\mathbb{R}^+\times\mathbb{R}^+} \{\tilde{P}_i(\tilde{s}, \tilde{v})\} - P_i, \tag{30}$$

TABLE I
Generator Information in the System

Gen Type	Fuel	Quantity	G_i	s_i	v_i	$\bar{v}_i$
U12	oil	5	12	367.84	109.18	139.83
U20	oil	4	20	48.4	104.02	106.44
U76	coal	4	76	1227.76	22.73	38.89
U100	oil	4	100	2420	87.13	111.33
U155	coal	4	155	535.6	17.89	21.34
U197	oil	3	197	4288.24	86.14	107.91
U350	coal	1	350	3944.9	18.39	29.66

[1] $\bar{v}_i$ is the average cost.
[2] The price information comes from [18].

where $\tilde{P}_i(\tilde{s}, \tilde{v})$ is defined in Eq. (14), and P_i is defined in Eq. (12).

Obviously, this index is a function of the total load in the system. We demonstrate, for the seven types of generators in our system, how their MMIs vary with the total system load in Fig. 1.

We classify generators into 2 groups according to their maximal MMI. Fig. 1a shows the trend of market power in load for the group with smaller MMI, and Fig. 1b shows that for the counterpart. It is evident that market power arises and disappears over different levels of loads. This highlights the non-convexity in our problem. The general trend is that lower startup cost implies more fluctuations in MMI. The sudden drop in Fig. 1b is due to the limited market power for a single generator when the total load is high. In such cases, eLMP will not be significantly affected by single generator strategic bidding. Fig. 1c further plots the distribution of MMIs for the seven types and indicates that lower average cost often leads to higher maximal MMI.

We also examine how often the generators change the dispatch profiles while exercising market power. It turns out that in general, generators with high startup costs and low variable costs have lower chances to manipulate the dispatch profile. The frequencies of U76 and U350 to change the dispatch profile are both nearly 0. This also corresponds to our intuition since higher startup cost implies a lower chance to get dispatched. On the other hand, the generator with the highest chance to manipulate the dispatch profile is U100: with a frequency of 11.35%. Note that this observation is made in the electricity pool model without any network congestion, which highlights the risk that the eLMP may pose in the electricity market.

Another interesting study is to evaluate how single generator strategic bidding may affect other players in the system. We use p^* ($\tilde{p}^*$) to represent the eLMP price without (with) generator i's strategic bidding. The change in total profit for all other generators, ΔU_{-i}, can be expressed as follows:

$$\Delta U_{-i} := \sum_{j\neq i}\left(\{p^*\cdot G_j - f_j(G_j)\}^+ - \{\tilde{p}^*\cdot G_j - f_j(G_j)\}^+\right).$$

Fig. 2 shows that the generators with low average cost (U76/U155/U350) tend to lower their average cost for more profits, which leads to a lower market price. This implies that the strategic bidding in eLMP scheme is usually not a zero-sum game, since other generators may benefit from some generator's strategic bidding. This is strong evidence for potential collusion in the market, which highlights the severity of market power issues in the eLMP scheme.

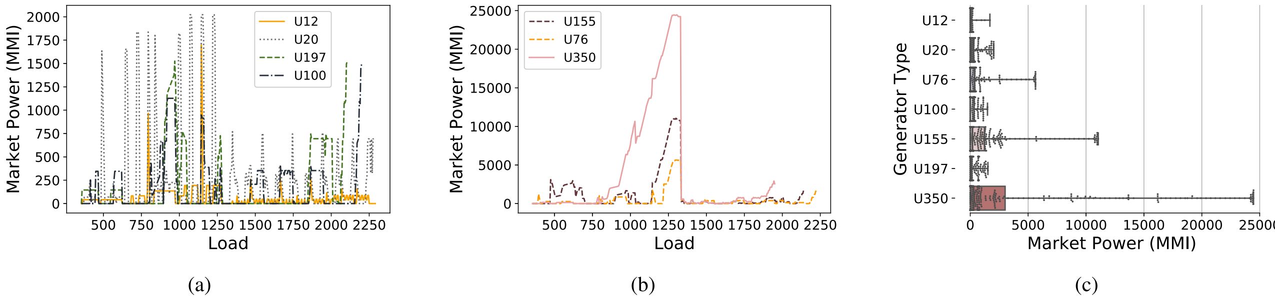

Fig. 1. Market Power Evaluation.

TABLE II
MARKET POWER ANALYSIS OF THE IIT 6-BUS SYSTEM

Scenario	Demand at Bus [3,4,5]	Congestion	Price under Truthful Bidding	Dishonest Generator	Price after Strategical Bidding	Uniform Price[1]
1	[54,108,108]	[line 3]	[13.95,42.00,46.16]	Gen 1	[4999,4999,4999]	17.70
				Gen 2	[13.95,4999,4999]	17.70
				Gen 6	[13.95,42.00,46.17]	42.00
2	[135,27,108]	[line 2]	[13.95,13.21,17.70]	Gen 1	[4999,42.00,4999]	17.70
				Gen 2	[13.95,13.21,17.70]	17.70
				Gen 6	[13.95,13.20,17.76]	42.00
3	[135,0,135]	[line 2]	[13.95,42.00,71.96]	Gen 1	[4999,42.00,4999]	17.70
				Gen 2	[13.95,4999,4999]	17.70
				Gen 6	[13.95,42.00,4999]	42.00

[1] Uniform price under enough transmission capabilities

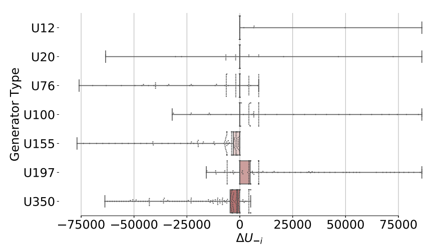

Fig. 2. Impacts on the Total Profits.

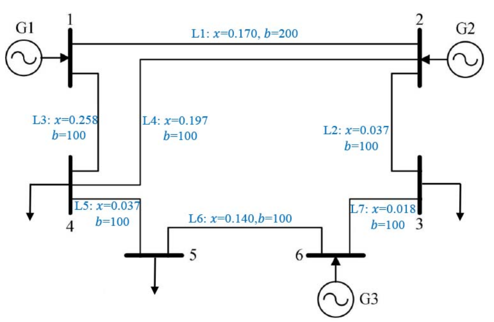

Fig. 3. IIT 6-Bus system: all quantities are measured in per units (x denotes impedance and b denotes line capacity [19]).

Next, we use the IIT (Illinois Institute of Technology) 6-Bus system [19] with 3 generators and 3 loads as shown in Fig. 3 to investigate how the network constraints further complicate the eLMP scheme. We adopt \$4999/MWh as the electricity price cap in the system. Table II illustrates how the generators could strategically manipulate the prices in three carefully selected scenarios: all the three scenarios suffer from congestion. It is interesting to observe that only generator 1 has significant market power (in terms of achieving the price cap) in scenario 2 while the market powers of all generators in scenario 3 are significant. It is surprising to note the shift in market power merit order in scenario 2. When there are no network constraints, generator 6 owns more market power than the other two rivals. However, with network constraints, generator 1 owns the most market power. In fact, the network constraints enable generator 1 to manipulate all the prices to price cap in scenario 1. All these observations illustrate the additional complex risk that network constraints pose on the eLMP scheme.

V. CONCLUSION

In this letter, we examine the strategic behaviors in the eLMP scheme, and systematically characterize how different types of manipulation may lead to a higher markup for generators. By defining the maximal markup as the index of market power, we observe that the chances that generator can manipulate in the system can be as high as 11%.

This letter can be extended in many interesting ways. For instance, it will be interesting to theoretically understand how the network constraints affect generators' bidding strategies.

APPENDIX

A. Proof Sketch of Lemma 1

It suffices to show that $h(\cdot)$ is the lower convex envelope of $c(\cdot)$. Clearly, by construction, $h(\cdot)$ is convex. Hence the remaining hurdles are to show (1) $h(u) \leq c(u), \forall u \in [0, \sum_{i=1}^{n} G_i]$; and (2) $h(\cdot)$ is the envelope.

The first part can be proved by contradiction. The key step is to observe the following inequalities:

$$\frac{f_i(G_i) - f_i(g_i)}{G_i - g_i} \leq \frac{f_k(G_k)}{G_k} \leq \frac{f_j(g_j)}{g_j}, \forall i < k < j. \quad (31)$$

The next part can be proved by construction. Define

$$\mathbf{H} := \{(u, h(u)) | u = \sum_{i=1}^{k} G_i, k = \{0, \ldots, n\}\}, \quad (32)$$

$$\mathbf{C} := \{(u, c(u)) | u \in [0, \sum_{i=1}^{n} G_i]\}. \quad (33)$$

We can show that $h(\cdot)$ is the convex hull of $\mathbf{H}$ and $\bar{c}(\cdot)$ is the convex hull of $\mathbf{C}$. Moreover, by the definition of $c(\cdot)$, we know that

$$c(\sum_{i=1}^{k} G_i) = \sum_{i=1}^{k} f_i(G_i). \quad (34)$$

Hence, $\mathbf{H} \subset \mathbf{C}$. This implies that $\bar{c}(u) \leq h(u), \forall u \in [0, \sum_{i=1}^{n} G_i]$, which completes our proof. ■

B. Proof Sketch of Lemma 2

We prove Lemma 2 case by case. Since the proof techniques in all the three cases are similar, we take $\tilde{f}_i(G_i)/G_i \leq p_1$ as an illustrating example. The key tool is to use Lemma 1 in [15], which suggests:

$$\sum_{j \neq i} g_j^d(p) < y - G_i, \quad \forall p < p_1, \quad (35)$$

$$\sum_{j \neq i} g_j^d(p) \geq y - G_i, \quad \forall p \geq p_1. \quad (36)$$

Since $\tilde{f}_i(G_i)/G_i \leq p_1$, for any price $p \geq p_1$, generator i would rather generate at its maximal capacity, i.e., $\tilde{g}_i^d(p) = G_i$. This implies that

$$\sum_{j \neq i} g_i^d(p) + \tilde{g}_i^d(p) \geq y - G_i + G_i = y. \quad (37)$$

On the other hand, for any price $p < p_1$, Eq.(35) implies that the total desired generation is smaller than y. Hence, the eLMP in the system has to be larger than p. Together, they imply that $\tilde{p}^*(y|\tilde{s}, \tilde{v}) = p_1$ in this case. ■

C. Proof Sketch of Lemma 3

Note that simple mathematical manipulation yields that

$$\begin{aligned} c(y|\tilde{s}, \tilde{v}) &= \min_{u \in [y - G_i, y]} \left\{ c^{\{i\}}(u) + \tilde{f}_i(y - u) \right\} \\ &= \min \left\{ c^{\{i\}}(y), \tilde{s} + \tilde{v} \cdot y + \min_{u \in [y - G_i, y)} c^{\{i\}}(u) - \tilde{v} \cdot u \right\} \\ &= \min \left\{ c^{\{i\}}(y), \tilde{s} + \tilde{v} \cdot y + \min_{u \in [y - G_i, y]} c^{\{i\}}(u) - \tilde{v} \cdot u \right\} \quad (38) \end{aligned}$$

Comparing Eq.(38) and Lemma 3, it suffices to show the following holds:

$$\min_{u \in [y - G_i, y]} c^{\{i\}}(u) - \tilde{v} \cdot u = \min_{u \in [y - G_i, y]} \bar{c}^{\{i\}}(u) - \tilde{v} \cdot u, \quad (39)$$

which is the direct result by the definition of convex hull. ■

D. Proof of Proposition 1

Define an auxiliary function $g(u)$:

$$g(u) = f(b_i) + l_{i+1} \cdot (u - b_i), b_i < u \leq b_{i+1} \quad (40)$$

where b_i, l_i are defined as in Proposition 1. Since $g(\cdot)$ is convex by its construction, $\bar{g}(u) = g(u)$.

On the one hand, since $f(u)$ is left continuous and non-decreasing, $f(u) \geq g(u)$. This leads to that

$$\bar{f}(u) \geq \bar{g}(u) = g(u), \forall u \in [L, R]. \quad (41)$$

On the other hand, for $u \in (b_i, b_{i+1}]$, due to the convexity of $\bar{f}(\cdot)$, we know

$$\bar{f}(u) \leq f(b_i) + l_{i+1}(u - b_i) = g(u). \quad (42)$$

Together, we can show $\bar{f}(u) = g(u)$. This completes our proof for Proposition 1. ■

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