Estimating Cost Function in Power Markets Under Pay-As-Bid Pricing Rules Using Observed Bid Data

E. Maghool[1], A. Naghavi[*1], S. F. Ghaderi[1]

***Abstract*—Estimating cost function of firms is very important in competitive electricity market. Complete information on the factors which can be used for estimating cost function is not definite and also is not accessible. The importance of cost function estimation is appeared when the Independent System Operator (ISO) decide to measure the market power which can be exerted by a market participant. In this paper a new method is applied for estimating cost function using observed bid information, weighted average prices and quantities in pay as bid auction in which each firm submit its prices and quantities. Wolak's cost function estimation model which is in uniform pricing auction, is applied to Iran's Electricity Market (IEM), where each genset submit its prices in ten increments which can be changed hourly, and quantities for producing in each load period. In this market, firm's cost function is recovered by optimizing behavior for each firm. After defining the profit of the firm and using the first order conditions for expected profit maximization, the generalized method of moment (GMM) is used for estimating cost function. In estimation procedure, the firm's cost function is obtained by using the weighted average prices, for computing the total revenue of each firm and the maximum accepted price, instead of market clearing price.**

Index Terms —cost function estimation, electricity markets, pay-As-bib auction, weighted average price

I. INTRODUCTION

ESTIMATING cost function for electricity generation is very important in a competitive electricity market. There are different applications for this work, the major use of this work is the measurment of market power. In the competitive electricity markets, designers try to limit the ability of generation unit to exercise market power. The main purpuse of monitoring the electricity market within the Independent system operator (ISO), is to study all aspects of market performance to detect the firms which exercise market power and try to degrade market performance only by using bids, inaddition to market clearing price and quantities. The Independent system operator (ISO), will be able to devise some rules to limit the works of firms which exercise market power, if there is enough information about their costs. When the information of estimating cost function is incomplete, a new different method is applied for estimating cost function by using only bid data and quantities. An assumption of maximizing profit or minimizing cost function is used as the first order condition for estimating cost function.

In previous works, Rosse [1], used a sample of monopoly local newspapers and estimated marginal cost function of the monopolist by assumption of profit maximization. By defining the profit function, and using inverse demand function p(q,w,θ,ε) and total cost function c(q,z,θ,η), in which w,z are demand and cost function shifters, and applying the first order condition, the procedure for estimating is completed. θ is a vector of parameters to be estimated, and ε,η are unobserved stochastic shocks.

The stages of estimation procedure are:

1.definition the profit function of monopolist.

$$\Pi(q) = p(q,w,\theta,\varepsilon)q - c(q,z,\theta,\eta) \quad (1)$$

2. applying the first order condition for profit equation.

$$\Pi'(q) = p'(q,w,\theta,\varepsilon)q + p(q,w,\theta,\varepsilon) - c'(q,z.\theta,\eta) = 0 \quad (2)$$

3. considering a functional form for the inverse demand function.

$$p(q,w,\theta,\varepsilon) = a + bq + cw + \varepsilon \quad (3)$$

where a,b, and c are elements of θ.

After estimating these parameters, for achieving marginal cost function, the q^E, the equilibrium quantity is used in the model and the market clearing price p^E is determined by substituting q^E in to the inverse function. Then the value of marginal cost is achieved by using the first order condition.

$$c'(q^E,z,\theta,\eta) = p'(q^E,w,\theta,\varepsilon) + p(q^E,w,\theta,\varepsilon) = bq^E + p^E \quad (4)$$

By defining a functional form for marginal cost, we can estimate its parameters.

Rosse, estimated the parameters of cost function and inverse demand by using the assumption of profit maximization. Porter [2], considered a functional form of cost for each firm, and the aggregate supply function for the industry is obtained by assumption of cartel or perfect competition among firms. Porter applied a swiching regression model for estimating cost function.

Bresnaham [3], considered a discret-choice demand structure for purchasing an automobile. He defined an aggregate demand, and estimated the parameters of this aggregate model by using the first order conditions which is related to profit maximization.

Berry et al [4], applied a multinomial logit discrete-choice demand system. They applied an instrumental variables estimation technique to estimate the demand system and marginal cost function under the assumption of Nash-Bertrand competition amonge automobile producers.

Coldberg [5], considered a general discrete choice model for automobile purchase at the household level. She used weights in aggregate demand functions for representing the ratio of these households in the population of U.S. by assumption of Nash Bertrand competition amonge automobile producers and estimate a marginal cost function for each

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automobile model. In all previous work, researchers considered a functional form for marginal cost, and applied different method for estimating the parameters of cost function by assumption of profit maximizing. Furthermore, Jean-Pierre and Florens [6],used the non-parametric and semiparametric estimation method.

Wolak [10], presented e new technique for recovering firm's cost function estimates in electricity market. Wolak, considered two models for recovering cost function by using bid data, and quantities and market clearing price in Australian National Electricity Market. In the first model, it is assumed that each firm is able to choose the market clearing price that maximizes its profit given the bids submitted by its competitors, but in the second model each firms bids in the rule base market. In this situation, each firm can influance the market clearing price by submitting its bids.

In order to apply Wolak's model for estimating cost function in Iran's electricity market (IEM), we adjusted Wolak's model with this market. This market is designed in the base of pay as bid rules, where each firm is able to submit its prices in k increments, and its quantities which can be produced by each genset of this firm. The submitted prices can be changed hourly. The submitted quantity by each genset could not be larger than its capacity of outputs. The accepted quantity for producing of a specified firm, is computed by using the aggregate supply function and the total market demand. By subtracting the aggregate supply function of all firms except the specified firm (i.e. firm A), from the total market demand, the resedual demand of this firm is computed.

In pay as bid auction, the payment for each genset is the base of its bids, therefore each firm tries to bid closer to market clearing price in order to increase its profit. The value of total revenue for each genset is computed with multiplying the bid prices by accepted quantity of this genset. But in uniform pricing auction, the payment of each genset is computed with multiplying the market clearing price by the accepted quantity for producing of this genset. In figure 2 and figure 3, the highlighted areas show the total revenue of a genset in these markets.

The remainder of this paper is organized as follows. In section 2, the market structure and market rules in Iran's Electricity Market is summarized. In section 3, the proposed model by Wolak [10], for estimating cost function in Australian Electricity Market is explained. The adjusted model for Iran's Electricity Market is defined in section 4, and the generalized method of moments is explained in section 5.

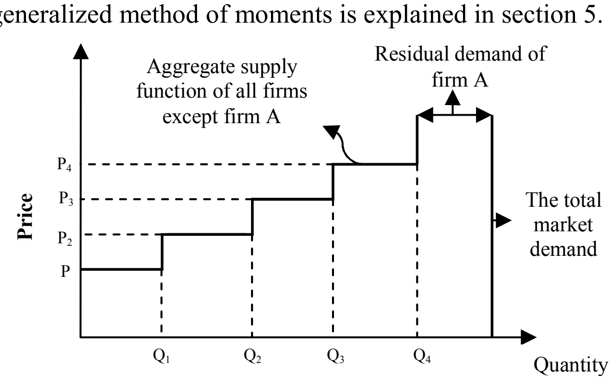

Fig. 1. Sample of computing the residual demand of firm A by using aggregate supply function

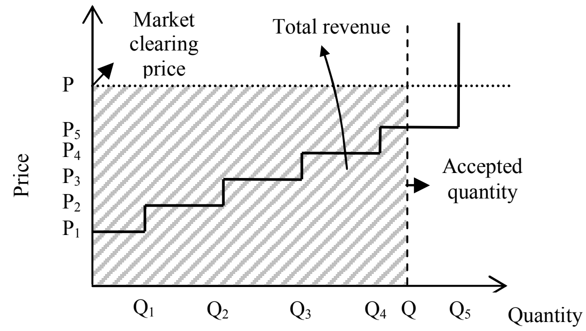

Fig. 2. Computing the total revenue in uniform pricing market

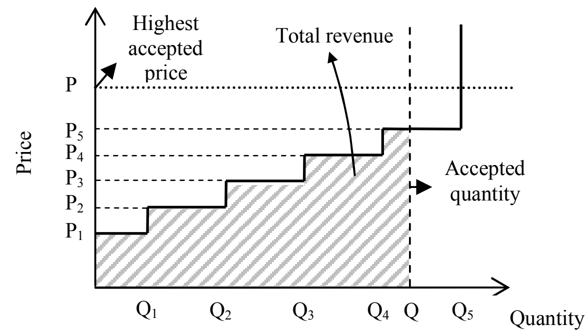

Fig. 3. Computing the total revenue in pay- as- bid market

II. MARKET STRUCTURE AND MARKET RULES IN IRANIAN ELECTRICITY MARKET

This market is under a pay as bid auction design. The pay as bid mechanism in Iranian day- ahead Electricity Market works as follows:

All firms should submit their offers which include prices and quantities in maximum ten increments for each genset, for each hour of day-ahead. Each increment is determined by its (MW) and its offered price. A security constraint unit commitment (SCUS) determines the market arrangement, in three steps:

In 1st step, the entire system is considered as a single zone (congestion free). So the highest accepted offer price in this step is computed only by using aggregated supply function and inelastic market demand.

In 2nd and 3rd steps, the transmission system constraint and generators constraints are applied respectively, and the highest accepted offer price is re-calculated by considering these constraints.

All offer prices which are bellow the 1st step highest accepted offer price, and those which are added in 2nd and 3rd steps to satisfy the constraints, are accepted. All firms whit these offered prices are succeeded to sell their offered quantities, and are paid exactly what they bid. Those offer which are accepted in 1st step, and cancelled due to transmission system or generators constraints, are paid by their associated opportunity cost which is calculated by subtracting the offered price with average variable cost (AVC). It should be mentioned that AVC is supplied by the generator to the ISO.

In this market, the highest accepted offer price may be treated as the market-clearing price in uniform pricing market. The weighted average price is used for calculating the total revenue of each firm in this market.

The proposed model by wolak [10] for estimating cost function is adjusted for Iran's Electricity Market under pay as bid pricing. Some new changes are essential for applying the wolak's model for IEM. There are some difference in calculating the total revenue of each firm in the Wolak's model and the adjusted model for Iran's electricity market. The first difference is in computing the total revenue of each firm. The weighted average price is computed for each firm and it is substituted with market clearing price in the Wolak's model to calculate the total revenue of each firm. By considering the bid function of each genset and using the accepted quantity of producing, the weighted average price for this genset is calculated such as follows:

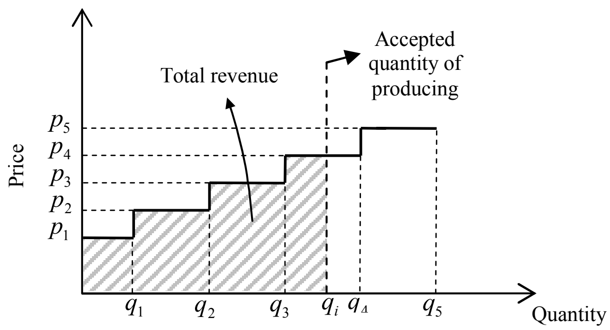

Fig. 4. The method of computing the average weighted price

Total revenue of genset i in load period t:

$$TR_i = p_1.q_1 + p_2.(q_2 - q_1) + p_3.(q_3 - q_2) + p_4.(q_i - q_3) \qquad (5)$$

$$TR_i = \left(\frac{p_1.q_1 + p_2.(q_2 - q_1) + p_3.(q_3 - q_2) + p_4.(q_i - q_3)}{q_i} \right) q_i \qquad (6)$$

The weighted average price of genset i in load period t:

$$p_{i,t} = \left(\frac{\sum_{j=1}^{k} p_{i,t,j}.q_{i,t,j}}{q_{i,t}} \right) \qquad (7)$$

Where, $q_{i,t}$ is the accepted quantity for producing of genset i in load period t.

Another difference between these markets is changes of bid prices. In Iran's electricity market, each genset bids its quantity and prices in each bid increment, for a period of 24 hours. In Wolak's model, for Australian Electricity Market, the daily bid prices and half hourly bid quantities are considered but in the adjusted model for IEM the hourly bid prices and the hourly bid quantities are used. There is no hedge contract in Iran's electricity market. After exerting these conditions on Wolak's model, we adjusted this model for Iran's electricity market.

In the next section, Wolak's model is introduced for estimating cost function in Australian Electricity Market under uniform pricing condition, and by considering this model and exerting some changes adjusted model for IEM under pay as bid condition is obtained.

III. WOLAK'S MODEL FORMULATION

For estimating cost function, Wolak, considered two models of optimizing behavior by the firm that recover an estimate of the firm's marginal cost function. In the first model he assumed that the firm is able to choose the market clearing price which maximize its profits given the bids submitted by its competitors. In the second model, he assumed that all of the constraints implied by the market rules on the bids used by the firm to set the market clearing price is imposed. In this model, the parameters are defined for estimating cost function for a specified firm, like firm A as follow:

QC_{id}	contact quantity for load period i of day d for firm A
PC_{id}	quantity-weighted average (over all hedge contracts signed for that load period and day) contract price for load period i of day d for firm A
$\pi_{id}(p)$	variable profits to firm A at price p, in load period i of day d
MC	marginal cost of producing a megawatt hour by firm A
$SA_{id}(p)$	bid function of firm A for load period i of day d giving the amount it is willing to supply as a function of the price p

The magnitudes QC_i and PC_i are usually set far in advance of the actual day-ahead bidding process. Generators sign hedge contracts with electricity supplier or large consumer for a pattern of prices throughout the day, week, and month, for an entire year or for a number of years.

Wolak defined profit function which showed the variable profits (profit excluding fixed costs) earned by firm A for load period i during day d at price p as

$$\pi_{id}(p) = DR_{id}(p)(p - MC) - (p - PC_{id})QC_{id} \qquad (8)$$

Where, $\pi_{id}(p)$ is the profit for firm A in load period i of day d. The first term is the variable profits from selling electricity in the spot market and the second term is the pay offs from buying and selling hedge contracts. By assuming $QC_{id}>0$, if $p>PC_{id}$ the second term is the total payments made to purchase of hedge contracts by firm A and if $p<PC_{id}$, the second term is the total payments made by purchasers of hedge contracts to firm A.

The market clearing price (p) is the smallest price which satisfy this equation

$$SA_{id}(p) = DR_{id}(p) \qquad (9)$$

There are different ways for computing the residual demand. One way is computing the aggregate demand function, D(p) where DR(p)=D(p)-q_c in which q_c is the quantity made available by the firm's competitor. In the competitive electricity market for achieving DR(p), the estimating of D(p) is required but in the bid-based market the residual demand function can be constructed by using bid data, there is no need to make a functional form assumption for the demand because the price elasticity of the residual demand curve show the extent of market power. The market designer tries to face all bidders with a price-elastic residual demand function.

There are two approaches for achieving cost function from bid data which is related to stochastic demand shocks to the price-setting process. One of them is a case which the firm is able to achieve prices that maximize profits given the realization of this uncertainty or achieve only market prices that maximize expected profits taken with respect to the distribution of this uncertainty. Before describing each model, we define some expressions which are used in these models.

Let ε_i is the demand shock of firm A in load period i (i=1,…, 48)

$DR_i(p,\varepsilon_i)$ The residual demand in load period i

$$\Theta = (p_{11},\ldots,p_{JK}, q_{1,11},\ldots,q_{1,JK}, q_{2,11},\ldots,q_{2,JK},\ldots,q_{48,11},\ldots,q_{48,JK})$$

Θ is the vector of daily bid prices and quantities submitted by firm A.

There are K increments for each of the J gensets, owned by firm A.

p_{kj}	The price which is set for each of the K bid increments for each of the j=1,…,J gensets owned by firm A for the entire day;
q_{ijk}	The quantity which is made available to produce electricity in load period i from each of the j=1,…,J gensets owned by firm A;
$SA_i(p,\Theta)$	Firm A's bid function in load period i which is parameterized by Θ;
$p_i(\varepsilon_i,\Theta)$	The market clearing price for load period i, the residual demand shock realization ε_i, and daily bid vector Θ.

Where, p_i is obtained from the equation (9).

$f(\varepsilon)$ Probability density function of $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_{48})$

As the expected profits to firm A for the daily bid vector is:

$$E(\Pi(\Theta)) = \int_0^{\infty}\ldots\int_0^{\infty} \sum_{i=1}^{48}[DR_i(P_i(\varepsilon_i,\Theta)(P_i(\varepsilon_i,\Theta)-MC) \\ -(P_i(\varepsilon_i,\Theta)-PC_i)QC_i]f(\varepsilon)d\varepsilon_1\ldots d\varepsilon_{48} \tag{10}$$

By this assumption, the firm A's best-reply bidding strategy is the solution of following optimization problem

$$\max_{\Theta} E(\Pi(\Theta)) \\ s.t.\quad b_U >= R\Theta >= b_L \tag{11}$$

A. Wolak's Model for Recovering Cost Function Estimates from Best- Response Prices

In this approach, Wolak assumed that no restriction imposed by the market rules and the firm is able to observe the market demand and the bids which are submitted by all other market participants. Therefore, we can obtain the value of the residual demand function and these bids and select the profit-maximizing price that is related with this residual demand given the firm's hedge contract position and marginal cost function.

The procedure for estimating cost function is as follows:

$$\Pi(p) = DR(p,\varepsilon)p - c(DR(p,\varepsilon)) - (p-pc)QC \tag{12}$$

Where *c(q)* is the total variable cost associated with producing output level q.

The best-reply price is conducted by differentiating from the equation (12).

$$\Pi'(p) = DR'(p,\varepsilon)(p - c'(DR(p,\varepsilon))) \\ +(DR(p,\varepsilon)-QC) = 0 \tag{13}$$

$$c'(DR(p^E,\varepsilon)) = p^E - (QC - DR(p^E,\varepsilon))/DR'(p^E,\varepsilon) \tag{14}$$

Where, p^E is Market clearing price;

$$DR(p,\varepsilon) = Q_d(\varepsilon) - SO_h(p,\varepsilon) \tag{15}$$

$$SO_h(p,\varepsilon) = \sum_{n=1}^{N}\sum_{k=1}^{10} qo_{nk}\,\phi((p - po_{nk})/h) \tag{16}$$

SO_h	The aggregate bid supply function of all other market participants besides firm A;
qo_{nk}	k th bid increment of genset n;
po_{nk}	Bid price for increment k of genset n;
N	Total number of gensets in the market which are owned by firm A;
$\Phi(t)$	Standard normal cumulative distribution function;
h	User selected smoothing parameter (this parameter smooth the corners on the aggregate supply bid function of all other market participants besides firm A. smaller value of h introduce less smoothing at the cost of a value for $DR(P^E, \varepsilon)$ that may be at one of the smoothed corners;

$$DR'_h(p,\varepsilon) = -1/h\sum_{n=1}^{N}\sum_{k=1}^{10} qo_{nk}\,\varphi((p - po_{nk})/h) \tag{17}$$

$\varphi(t)$ Standard normal density function.

This approach is valid only if the best-reply prices can be obtained for all realization of ε_i. Wolak showed that this is not possible because market rules imply restrictions on generators to set best reply prices.

B. Wolak's Model for Recovering Cost Function Estimates from Best-Response Bidding

For achieving cost function estimation, by assumption of best-response bidding in which market rules are applied on feasible bid function for estimating cost function. Wolak applied a generalized method of moments estimation technique (GMM) for recovering cost function by using bid data.

In this model, the parameters are defined such as follow:

$SA_{ij}(p,\Theta)$	The amount bid by genset j at price p during load period i;
$c_j(q,\beta_j)$	The variable cost of producing output q from genset j;
β_j	The vector of parameters of the cost functions for genset j.
$SA_i(p,\Theta) = \sum_{j=1}^{J} SA_{ij}(p,\Theta)$	The total amount bid by firm A at price p during load period i .

Variable profit for firm A during day d is as follows:

$$\Pi_d(\Theta,\varepsilon) = \sum_{i=1}^{48}[DR_i(p_i(\varepsilon_i,\Theta),\varepsilon_i)p_i(\varepsilon_i,\Theta)$$
$$-\sum_{j=1}^{J} C_j(SA_{ij}(p_i(\varepsilon_i,\Theta),\Theta),\beta_j) \qquad (18)$$
$$-(p_i(\varepsilon_i,\Theta)-PC_i)QC_i]$$

Where, ε is the vector of realizations of ε_i for i=1,…,48 and $p_i(\varepsilon_i,\Theta)$ is the market clearing price for load period i given the residual demand shock realization, ε_i and the daily bid vector Θ.

Wolak considered maximizing the expected value of $\Pi_d(\Theta,\varepsilon)$ with respect to Θ, subject to the constraints which the bid quantity increments q_{jik} is less than the capacity that gensets can be produced (CAP_j) and is greater than zero. Also there are upper and lower bound for daily bid prices.

Wolak considered the first order condition for profit maximization in order to apply GMM technique. The first order condition is defined such as follows:

$$E_\varepsilon(\partial\Pi_d(\Theta_d,\varepsilon)/\partial p_{km})=0 \qquad (19)$$

For all gensets, m, and bid increments, k. The index, d for Θ shows the different value for Θ for each day. The above restriction shows that there are J×K moment restrictions for estimating the parameters of cost function.

The sample analog of this moment restriction is as follows:

$$\partial\Pi_d(\Theta_d,\varepsilon)/\partial p_{km} = \sum_{i=1}^{48}[(DR'_i(p_i(\varepsilon_i,\Theta),\varepsilon_i)p_i(\varepsilon_i,\Theta)$$
$$+(DR_i(p_i(\varepsilon_i,\Theta),\varepsilon_i)-QC_i)$$
$$-\sum_{j=1}^{J} C'_j(SA_{ij}(p_i(\varepsilon_i,\Theta)),\beta_j)(\partial SA_{ij}/\partial p_i))\partial p_i/\partial p_{km} \qquad (20)$$
$$-\sum_{j=1}^{J} C'_j(SA_{ij}(p_i(\varepsilon_i,\Theta)),\beta_j)\partial SA_{ij}/\partial p_{km}]$$

Where p_i is the shorthand for the market-clearing price in load period i.

The values of partial derivative are computed as follow:

$$SA_{ij}^h(P,\Theta) = \sum_{k=1}^{10} q_{ikj}\Phi((P-P_{ij})/h) \qquad (21)$$

$$SA_i^h(P,\Theta) = \sum_{j=1}^{J}\sum_{k=1}^{10} q_{ikj}\Phi((P-P_{ij})/h) \qquad (22)$$

$$\frac{\partial SA_{ij}}{\partial P_i} = \frac{1}{h}\sum_{k=1}^{10} q_{ikj}\phi((P-P_{ij})/h) \qquad (23)$$

$$\frac{\partial SA_{ij}}{\partial P_{kj}} = -\frac{1}{h} q_{ikj}\phi((P-P_{ij})/h) \qquad (24)$$

$$\frac{\partial P_i}{\partial P_{kj}} = \frac{\dfrac{\partial SA_i(P_i(\varepsilon_i,\Theta),\Theta)}{\partial P_{kj}}}{DR'_i(P_i(\varepsilon_i,\Theta),\varepsilon_i)-SA'_i(P_i(\varepsilon_i,\Theta),\Theta)} \qquad (25)$$

$$DR(P,\varepsilon) = Q_d(\varepsilon) - SO_h(P,\varepsilon) \qquad (26)$$

$$SO_h(P,\varepsilon) = \sum_{n=1}^{N}\sum_{k=1}^{10} qo_{nk}\Phi((P-Po_{nk})/h) \qquad (27)$$

Then, a functional form for $C_j(q,\beta_j)$ is required for estimating cost function, Wolak considered this parametric functional forms for two unit-level marginal cost function:

$$C'_1(q,\beta_1) = \beta_{10} + \beta_{11}(q-200) + \beta_{12}(q-200)^2 \qquad (28)$$

$$C'_2(q,\beta_1) = \beta_{20} + \beta_{21}(q-180) + \beta_{22}(q-180)^2 \qquad (29)$$

He considered two plant with different capacities which their capacities are 200 and 180 and $\beta=(\beta_{10},\beta_{11},\beta_{12},\beta_{20},\beta_{21},\beta_{22})$ and q is the actual level of output produced by that unit during the half-hour period under consideration. If the functional form of $C_j(q,\beta_j)$ is correct, the first order conditions for expected profit maximization which is related to daily bid prices imply that $E(l_d(\beta^0))=0$, where β^0 is the true value of β. Solving the value of b that minimizes

$$\left[\frac{1}{D}\sum_{d=1}^{D} l_d(b)\right]'\left[\frac{1}{D}\sum_{d=1}^{D} l_d(b)\right] \qquad (30)$$

Will yield a consistent estimate of β. b(I) denotes the consistent estimate of β, where I is the identity matrix which is used as the GMM weighting matrix. Wolak constructed a consistent estimate of the optimal GMM weighting matrix using the consistent estimate of β as follows:

$$V_D(b(I)) = \frac{1}{D}\sum_{d=1}^{D} l_d(b(I))l_d(b(I))' \qquad (31)$$

The optimal GMM estimator finds the value of b that minimizes

$$[\frac{1}{D}\sum l_d(b)]'V_D(b(I)^{-1}[\frac{1}{D}\sum_{d=1}^{D} l_d(b)] \qquad (32)$$

IV. THE ADJUSTED MODEL FOR IRAN'S ELECTRICITY MARKET

Now in this section, the adjusted model for pay as bid auction is introduced and essential assumption is defined for recovering firm's cost function in IEM.

A. Parameters Definition

The daily bid prices and quantities submitted by firm A is:

$$\Theta = (p_{1,11},\dots p_{1,KJ}, p_{2,11},\dots, p_{2,KJ},\dots, p_{24,11},\dots, p_{24,KJ}$$
$$,q_{1,11},\dots, q_{1,KJ}, q_{2,11},\dots, q_{2,KJ}, q_{24,11},\dots, q_{24,KJ})$$

$SA_{ij}(p,\Theta)$	The amount bid by genset j at price p during load period i;
$c_j(q,\beta_j)$	The variable cost of producing output q from genset j;
β_j	The vector of parameters of the cost function for genset j;

$SA_i(p,\Theta)=\sum_{j=1}^{J} SA_{ij}(p,\Theta)$	The total amount bid by firm A at price p during load period i;
$p_{\max i}$	The highest accepted price in electricity market;
p_{ji}	The weighted average price for genset j in load period i;
p_{jik}	The submitted price by genset j in load period i of increment k;
q_{jik}	The submitted quantity by genset j in load period i of increment k.

Variable profit of firm A in load period is:

$$\Pi_d(\Theta,\varepsilon)=\sum_{i=1}^{24}[DR_i(p_{\max i}(\varepsilon_i,\Theta))p_i^{wap} - \sum_{j=1}^{J} C_j(SA_{ij}(p_{\max i}(\varepsilon_i,\Theta),\Theta),\beta_j)] \tag{33}$$

ε	The vector of realizations of ε_i for i=1,…,24;
p_i^{wap}	The weighted average price of firm A in load period i.

The first order condition for estimating cost function is:

$$E_\varepsilon(\partial\Pi_d(\Theta_d,\varepsilon)/\partial p_{kmt})=0 \tag{34}$$

For all gensets, m, and bid increments, k and load periods, t.

Equation (33), show that there are K×J×I moment restriction for estimating cost function.

$$\begin{aligned}\partial\Pi_d(\Theta_d,\varepsilon)/\partial p_{kmt} &= \sum_{i=1}^{24}[(DR'_i(p_{\max i}(\varepsilon_i,\Theta),\varepsilon_i)p_i^{wap} \\ &+ DR_i(p_{\max i}(\varepsilon_i,\Theta),\varepsilon_i) \\ &- \sum_{j=1}^{J} c'_j(SA_{ij}(p_{\max i}(\varepsilon_i,\Theta)),\beta_j)(\partial SA_{ij}/\partial p_{\max i})) \\ &\partial p_{\max i}/\partial p_{kmt} \\ &- \sum_{j=1}^{J} c'_j(SA_{ij}(p_{\max i}(\varepsilon_i,\Theta)),\beta_j)\partial SA_{ij}/\partial p_{kmt}]\end{aligned} \tag{35}$$

$$SA_{ij}^h(P_{\max i},\Theta)=\sum_{k=1}^{10} q_{jik}\Phi((P_{\max i}-P_{jik})/h) \tag{36}$$

$$SA_i^h(P_{\max i},\Theta)=\sum_{j=1}^{J}\sum_{k=1}^{10} q_{jik}\Phi((P_{\max i}-P_{jik})/h) \tag{37}$$

$$\frac{\partial SA_{ij}}{\partial p_{\max i}}=\frac{1}{h}\sum_{k=1}^{10} q_{jik}\phi((P_{\max i}-P_{jik})/h) \tag{38}$$

$$\frac{\partial SA_{ij}}{\partial P_{jik}}=-\frac{1}{h}q_{jik}\phi((P_{\max i}-P_{jik})/h) \tag{39}$$

$$\frac{\partial P_{\max i}}{\partial P_{jik}}=\frac{\dfrac{\partial SA_i(P_{\max i}(\varepsilon_i,\Theta),\Theta)}{\partial P_{jik}}}{DR_i'(P_{\max i}(\varepsilon_i,\Theta),\varepsilon_i)-SA_i'(P_{\max i}(\varepsilon_i,\Theta),\Theta)} \tag{40}$$

$$DR(P_{\max i},\varepsilon)=Q_d(\varepsilon)-SO_h(P_{\max i},\varepsilon) \tag{41}$$

$$SO_h(P_{\max i},\varepsilon)=\sum_{n=1}^{N}\sum_{k=1}^{10} qo_{nk}\Phi((P_{\max i}-Po_{nk})/h) \tag{42}$$

The wolak's model is adjusted for IEM by substituting the market clearing price with weighted average price to compute the total revenue of each firm. After defining a functional form for cost function, and omitting the term which is related to hedge contract the profit function is defined. Hence, the first order condition is obtained and the GMM method is applied for estimating cost function. The procedure of this method is described in the next section.

V. THE GENERALIZED METHOD OF MOMENTS ESTIMATION

The generalized method of moments, as the name suggest, can be thought of just as a generalization of the classical MM. A key condition is a set of population moment conditions that are derived from the assumption of the econometric model. For example in classical linear regression the moment condition is obtained as follow:

$$\begin{aligned} y_t &= x'_t\beta+e_t \\ x_t &= (x_{1t},\dots,x_{mt})\end{aligned} \tag{43}$$

where

x_t	A m-vector of regression coefficient and
e_t	An error term

The moment conditions are:

$$1.\ \mathrm{var}[e_t]=\sigma^2 \tag{44}$$

Where, σ^2 is a constant for all t.

$$2.\ E[(y_t-x_t'\beta)x_t]=E[e_t x_t]=0 \tag{45}$$

For all t

$$3.\ E[e_t e_u]=0 \tag{46}$$

For all t ≠ u.

In which 2 is the key condition for estimating β.

Given data on the observable variables the GMM finds values for the model parameters such that corresponding sample moment conditions are satisfied as closely as possible.

A. *The Two Step Procedure in GMM Method*

Step1: set W=I, the identity matrix and solve the (nonlinear) least squares problem

$$\hat{\theta}^{(1)}=\arg\min_{\theta} g_T(\theta)'g_T(\theta) \tag{47}$$

Step 2: compute

$$\hat{u}_t=u(x_t;\hat{\theta}^{(1)}) \tag{48}$$

And estimate S_j as

$$\hat{S}_j=1/T\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t+j}, \quad j=0,1,\dots,l \tag{49}$$

Estimate S by

$$\hat{S} = \hat{S}_0 + \sum_{j=1}^{l} w_j (\hat{S}_j + \hat{S}'_j) \tag{50}$$

Select W=S^{-1} and obtain the second step estimate

$$\hat{\theta}^{(2)} = \arg\min_{\theta} g_T(\theta)' g_T(\theta) \tag{51}$$

VI. CONCLUSION

In this paper, estimating cost function of generation unit in power market under pay as bid pricing is considered. The proposed model by Wolak for estimating cost function in Australian Electricity Market is adjusted with Iranian Electricity Market which is designed under pay as bid rules. The essential changes are applied to adjust Wolak's model with IEM. The profit function is formulated and the first order condition is applied for estimating cost function in this market. The generalized method of moment can be used for estimating the parameters of marginal cost function.

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