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$$T(n)=\begin{cases}0 & n=1\\ n-1+T(n-1) & n>1\end{cases}$$

$T(n) = n-1+T(n-1)$ ①

$= n-1+(n-2+T(n-2))$

$= (n-1)+(n-2)+T(n-2)$ ②

$= (n-1)+(n-2)+(n-3)+T(n-3)$ ③

$\vdots$

i-th row $= (n-1)+(n-2)+\cdots+(n-i)+T(n-i)$ ⓘ i-th row.

$= i\cdot n-(1+2+3+\cdots+i)+T(n-i)$

$= i\cdot n-\frac{(1+i)i}{2}+T(n-i)$

row r ~~we conclude~~ $= rn-\frac{(1+r)r}{2}+T(n-r)$ $n-r=1$ $r=n-1$ 匡

$= (n-1)n-\frac{(1+n-1)(n-1)}{2}+0$

$= n(n-1)-\frac{n(n-1)}{2} = n(n-1)(1-\frac{1}{2}) = \frac{n(n-1)}{2}$

Guess: $T(n)=\frac{n(n-1)}{2}$

proof by induction:

Base case: $n=1$ $T(1)=\frac{1\times 0}{2}=0$ → true.

Inductive step: Assume that $T(k)=\frac{k(k-1)}{2}$

By recurrence relation:

$T(k+1)=k+T(k)=k+\frac{k(k-1)}{2}$

$=\frac{2k+k^2-k}{2}=\frac{k^2+k}{2}=\frac{k(k+1)}{2}$

$=\frac{(k+1)((k+1)-1)}{2}$

Thus, it also holds for $k+1$

Therefore, $T(n)=\frac{n(n-1)}{2}$ holds for any $n\geq 1$

Namely, $T(n)=\frac{n(n-1)}{2}\in\Theta(n^2)$