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WQ#6.

Proof: Given f, g, h, we know that

$\exists c_1 > 0$, $n_1 \in \mathbb{N}$ such that for any $n \geq n_1$, we have $f(n) \leq c_1 h(n)$.

$\exists c_2 > 0$, $n_2 \in \mathbb{N}$ such that for any $n \geq n_2$, we have $g(n) \leq c_2 h(n)$.

Therefore, for any $n \geq \max\{n_1, n_2\}$ both upper-bounds apply.

Which means that for any $n \geq \max\{n_1, n_2\}$ we have that

$$f(n)g(n) \leq (c_1 h(n))\, g(n) \leq (c_1 h(n))(c_2 h(n))$$

$$\leq (c_1 \cdot c_2)\, h(n) \cdot h(n) = (c_1 \cdot c_2)\, h^2(n)$$

$$f(n) \cdot g(n) \in O(h^2(n))$$

So, It's false, $f(n) \cdot g(n) \notin O(h(n))$.