HW#5.

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Problem 1

(a) procedure Shortest_path(s, f, t)

** s, f are points containing their coordinate and path (which is a queue)
** s.x ← i s.y ← j f.x ← i' f.y ← j' for s(i, j) f(i', j')
** t is the size of chess board t×t — also the # of points
** Q is a queue store points.

```
dx ← {2, 2, -2, -2, 1, 1, -1, -1}
dy ← {1, -1, 1, -1, 2, -2, 2, -2}
s.path.enqueue((s.x, s.y))
Q.enqueue(s)
for (i from 1 to t)
    for (j from 1 to t)
        visit[i][j] ← false
visit[s.x][s.y] ← true
while (Q ≠ φ) then
    t ← Q.dequeue()
    if (t.x = f.x and t.y = f.y) then
        return f.path
    for (k from 1 to 8)
        x ← t.x + dx[k]
        y ← t.y + dy[k]
        If (IsInside(x, y, t) and !visit[x][y]) then
            v.x ← x
            v.y ← y
            path ← t.path
            v.path ← path.enqueue((x, y))
            visit[x][y] ← true
            Q.enqueue(v)
```

procedure IsInside(x, y, n)

```
return x>0 and x<=n and y>0 and y<=n
```

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Procedure PrintPath (Q)

```
while ( Q ≠ ∅ ) then
    t ← Q.dequeue ( )
    print ( t )
```

For this case, we use BFS to ~~st~~ solve the problem. Each point (or ~~pts~~ position) have 8 possible positions that the knight may move to. So we traverse them one by one. if the possible position is in the t×t chesseboard and hasn't been visited before then add them into the path Queue and keep traversing its next step. Until we reach the finish position f (i', j'), we get the shortest path that stored in f.path (is a queue).

This is the shortest steps path → has been proved in BFS.

In the worst case, we need to traversing all t×t positions :

- we need to Initilize visit[x][y] take $O(t^2)$ time
- We need to dequeue t^2 points which take $O(t^2)$ time
- In each iteration we will take a constant time (since we only have 8 possible points each time)

Overall, the runtime is $O(t^2)$

(b) In this case, we still use BFS to find the shortest path. Since there will be 12 different possible operations to these bottles. (① Fill bottle 1 ; ② Fill bottle 2 ; ③ Fill bottle 3; ④ Empty bottle 1 ; ⑤ Empty bottle 2 ; ⑥ Empty bottle 3 ; ⑦ pour bottle 1 to bottle 2 (1→2); ⑧ Pour bottle 1 to bottle 3 (1→3) ; ⑨ Pour bottle 2 to bottle 1 (2→1) ; ⑩ Pour bottle 2 to bottle 3 (2→3) ; ⑪ Pour bottle 3 to bottle 1 (3→1) ; ⑫ Pour bottle 3 to bottle 2 (3→2))

Traverse all possible operations until we get to the ~~final~~ end with (a, b, c)

Then we would get the shortest sequence from (0, 0, 0) → to (a, b, c)

for (bottle 1 (k), bottle 2 (l), bottle 3 (m)) k, l, m are all integers.

procedure FindBottle (a, b, c , k, l, m)

** Each Node will have x, y, z (the water in bottle1 = x, bottle 2 = y, bottle 3 = z)

~~other~~ and the path (a queue) to ~~get~~ reach this Node

** a, b, c will be integers bottles end with (bottle1 = a, bottle 2 = b, bottle 3 = c)

** bottle1. capacity = k , bottle2. capacity = l , bottle3 . capacity = m

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```
start . x ← 0
start . y ← 0
start . z ← 0
start . path. enqueue ({0,0,0})
Q . enqueue ( start )
Visited [ start ] = true
while ( Q ≠ ∅ ) do
    u ← Q. dequeue ()
    if ( u.x = a and u.y = b and u.z = c ) the
        return u.path
 ①  if ( u.x < k ) then
        new.x ← k
        new.y ← u.y
        new.z ← u.z    path ← u.path
        new.path ← path. enqueue ( {new.x, new.y, new.z} )
        if ( Visited[ new ] = false ) then
            Q. enqueue ( new )
            Visited [ new ] ← true
 ②  if ( u.y < l ) then
        new.x ← u.x
        new.y ← l
        new.z ← u.y    path ← u.path
        new.path ← path. enqueue ( {new.x, new.y, new.z} )
        if ( Visited[ new ] == false ) then
            Q. enqueue ( new )
            Visited[ new ] ← true
 ③. if ( u.z < m ) then
        new.x ← u.x
        new.y ← u.y
        new.z ← m    path ← u.path
        new.path ← path. enqueue ( {new.x, new.y, new.z} )
        if ( Visited[ new ] = false ) then
            Q. enqueue ( new )
            Visited [ new ] ← true
 ④. if ( u.x > 0 ) then
        new.x ← 0
        new.y ← u.y
        new.z ← u.z    path ← u.path
        new.path ← path. enqueue ( {new.x, new.y, new.z} )
        if ( Visited[ new ] = false ) then
            Q. enqueue ( new )
            Visited [ new ] ← true
```

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⑤ If (u.y > 0) then
new.x ← u.x
new.y ← 0
new.z ← u.z
path ← u.path
new.path ← path.enquene ([new.x, new.y, new.z])
If (Visited [new] == false) then
Q.enquene (new)
Visited [new] ← true

⑥ If (u.z > 0) then
new.x ← u.x
new.y ← u.y
new.z ← 0
path ← u.path
new.path ← path.enquene ([new.x, new.y, new.z])
If (Visited [new] = false) then
Q.enquene (new)
Visited [new] ← true

⑦ If (u.x > 0 and u.y < l) then
1->2
new.x ← max (0, u.x + u.y − l)
new.y ← min (u.x + u.y , l)
new.z ← u.z
path ← u.path
new.path ← path.enquene ([new.x, new.y, new.z])
If (Visited [new] = false) then
Q.enquene (new)
Visited [new] ← true

⑧ If (u.x > 0 and u.z < m) then
1->3
new.x ← max (0, u.x + u.z − m)
new.y ← u.y
new.z ← min (u.x + u.z, m)
path ← u.path
new.path ← path.enquene ([new.x, new.y, new.y])
If (Visited [new] == false) then
Q.enquene (new)
Visited [new] ← true

⑨ If (u.y > 0 and u.x < k) then
2->1
new.x ← min (u.x + u.y, k)
new.y ← max (0, u.x + u.y − k)
new.z ← u.z
path ← u.path
new.path ← path.enquene
If (Visited [new] = false) then
Q.enquene (new)
Visited [new] ← true

⑩ If (u.y > 0 and u.z < m) then
2->3
new.x ← u.x
new.y ← max (0, u.y + u.z − m)
new.z ← min (u.y + u.z , m)
path ← u.path
new.path ← ~~u.path~~ path.enquene ([new.x, new.y, new.z])
If (Visited [new] = false) then
Q.enquene (new)
Visited [new] ← true.

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```
⑪  If ( u.z > 0 and u.x < k ) then
3->1      new.x ← min( u.x+u.z , k )
          new.y ← u.y
          new.z ← max ( 0, u.x+u.z -k )
          path ← u.path
          new.path ← path.enquene ( {new.x, new.y, new.z} )
          If ( Visited[new] == false ) then
                Q.enquene( new )
                Visited [new] ← true
⑫  If ( u.z>0 and u.y < l ) then
3->2      new.x ← u.x
          new.y ← min( u.y+ u.z, l )
          new.z ← max( 0, u.y+u.z - l )
          path ← u.path
          new.path ← path.enquene ( {new.x, new.y, new.z} )
          If ( Visited [new] = false ) then
                Q.enquene ( new )
                Visited [new] ← true
end while

Print (" NOT Find ")
return 0
```

~~procedure InVisit(Visit, new)~~
~~while (Visit ≠ ∅) then~~
~~t ← Visit.dequene ()~~
~~If (t.x = new.x and t.y = new.y and t.z = new.z) then~~
~~return true~~

return false.

Runtime:

In the worst case,

- Each bottle have 4 cases (① Fill ② Empty ③ pour to another bottle ④ pour to another bottle)
  So, we have total 12 cases.
- We have $k \times l \times m$ posible nodes intotal (k, l, m are all integers)
- So we will dequene $O(k \times l \times m)$ times

Overall It takes $O(k \times l \times m)$ time.

Problem 2

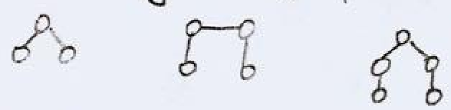
(a). A node is an articulation point iff removing it (and edges through it) disconnect the graph.

For any $n \geq 3$, there will be a connected graph with n edges where all nodes are not an articulation. This is true only when n nodes connected to a circle with n edges.

eg.

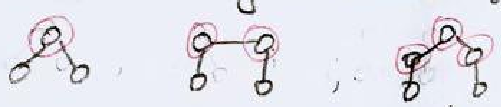
$n=3$ $n=4$ ...

when we removing 1 edge from the graph, all nodes are still connected with n-1 edges



...

However, when we trying to remove one more edge, there may be a disconnected graph



So when all connected graphs with $\leq n-1$ edges must have at least 1 articulation point to make ~~all not all nodes are~~ nodes disconnected

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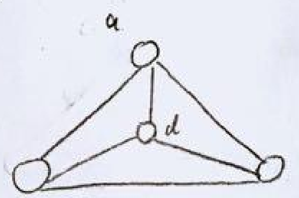
(b).

$n = 4$ $m = 6 \in \Theta(n)$



$a \to b \to d \to a$
$a \to c \to d \to a$
$a \to b \to c \to a$
$a \to b \to c \to d \to a$
$a \to c \to b \to d \to a$
$a \to b \to d \to c \to a$
$b \to d \to c \to b$

are 7 cycles.

$$7 > 2^{\sqrt{4}} = 2^2 = 4$$

$\therefore$ The number of cycles $\in \Theta(2^{\sqrt{n}}) = \Theta(2^{n^{\frac{1}{2}}})$

Such graph has ~~the~~ a number of cycles is exponential in n $\in \Theta(2^{n^c})$

6

(c)

For any undirected graph $G = (V, E)$, if we ~~a~~ have a cycle in this ~~Graph~~ graph G with edges $e_1, \cdots, e_k$ for k connected nodes, $(V_1 \to V_2 \to \cdots V_k)$ we can just put the directed edges of graph G'' like $V_1 \to V_2 \to \cdots V_{k-1}$ but put the last edge $V_{k-1} \leftarrow V_k$ which ~~n~~ can ~~not~~ make $(V_1, \cdots V_k)$ not a cycle.

So for each cycle in graph G we have a non-cycle version in G'', for any undirected graph G, there exists an orientation G'' which is acyclic.

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Problem 3

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(a) Yes.

This is actually a greedy ~~pro~~ algorithm.

Optimal substructure: an optimal solution to the ~~orith~~ original problem contains within it optimal solutions to subproblems.

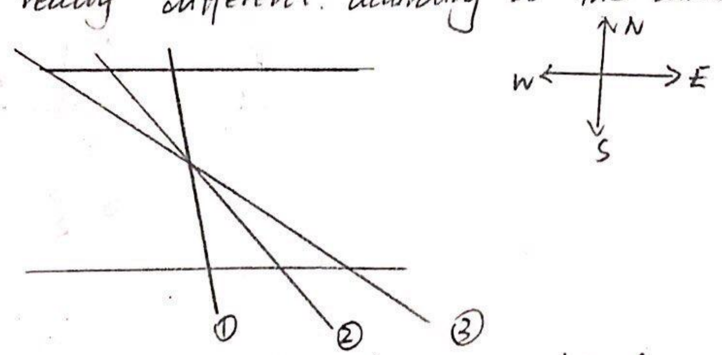
For this case, driving along the only high way ~~accor~~ arc crossing the Australian outlook, try to go as far as I can before refueling.

Each time, I ~~chose~~ make the best choice which can be locally minimum drive as far as I can. Then overall, I will have the minimum stops ~~as well~~. by union all local minimums.

(b). No.

Since, I start a direction between south and east, so the total distance I traveling can be really different. according to the direction I choose.

For example,



In this case ①, ②, ③ are all directions between south and east, however, the distance is ① < ② < ③ which implies that ② and ③ ~~can use more~~ will need more stops or gases to travel through.

So For this can, I can chose the shortest distance that connects Morocco and Sudan, and follow along that direction to stop at furthest possible gas station which is still within distance $\le d$ from the last stop.

Problem 4

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(a). Procedure CheckNest (B_1, B_2, d)

** . $B_1 = (l_1, l_2, \cdots l_d)$, $B_2 = (m_1, m_2, \cdots, m_d)$

** d is the dimentional of boxes.

Sort both B_1 and B_2 in a non-decreasing order.

IsNest ← true

for(i from 1 to d)

    if($B_1[i] > B_2[i]$)

        IsNest ← false

return IsNest.

First sort B_1 and B_2 in a non-decreasing order. Then traversing their length one by one, if there exist length $l_i > m_i$ then, the box $B_1(l_1, l_2, \ldots l_d)$ can not be nested into the box $B_2(m_1, \ldots, m_d)$. B_1 can be nested into B_2 only when all length $l_i < m_i$.

(b). procedure DFS (B, n, d)

** $B = \{B_1, B_2, \cdots, B_n\}$. n is the # of box in B, d is the dimention of boxes.

for (i from 1 to n)

    DFS-visit (B[i], d, n, Q_i)

    Q. enquene (Q_i)

procedure DFS-visit (B_i, d, n, Q_i)

Q_i. enquene (B_i)

for(j from 1 to n)

    if (CheckNest (B_i, B[j], d) = true) then

        DFS-Visit (B[j], d, n, Q_i)

For this problem, we make use of DFS to find the longest sequence of each B_i in $B = \{B_1, B_2, \cdots, B_n\}$. So that we can tell their relations easily and store them nicely in cupboard.

We will traverse each B_i in B and then for each B_i try to find the longest sequence Q_i, store all Q_i in Q. Then we can tell their relations by getting Q.

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Problem 5

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For this claim in the ~~pr~~ problem:

We get T_i after iteration i and $t_{u,v}(i)$ donate the weight of the heaviest edge on T_i.

Then we will do the next iteration ($i+1$):

We will find the heavist edge in T_i $e' = (u,v)$

Case I: If $w(e') \leq w(e)$ then we will do nothing. T_{i+1} is the same as T_i

So $t_{u,v}(i+1) = t_{u,v}(i)$

Case II: If $w(e') > w(e)$ the we ~~re~~ remove the edge e' in T_i and ~~add~~

union the edge e with T_i: $w(e) - w(e') < 0$.

$T_{i+1} = T_i - w(e') + w(e) = T_i + (w(e) - w(e')) < T_i$

So the heavest edge $e' = (u,v)$ has been removed from T_i.

The heavest edge $t_{u,v}(i+1)$ now is less than $t_{u,v}(i) = e' = (u,v)$

$\therefore$ $t_{u,v}(i+1) < t_{u,v}(i)$

So the claim is true.

According to the claim above, we can tell this algorithm return a MST.

Since each time we are removing the heavist edge connected u and v, we are actually reducing its weight, until we get the minimum weight of the edge. For each subproblem we will get its ~~s~~ minimum weight, then after union all subproblems, we will get the MST.

Runtime:

Runtime of kruskal's algorithm, including sorting m edges and $O(m)$ calls to findCC()

$O(m\log(m)) + O(m\log(n)) = O(m\log(m)) = O(m\log(n))$ for $n-1 \leq m \leq \binom{n}{2}$

- b. In Some Tree foreach all back-edges B, ~~and~~ $b = |B|$

Find the ~~largest~~ heavest edge each time takes $O(\log b)$ and we need foreach b edges which will take $O(b\log b)$ time.

Some Tree would be faster only when $b \in \Omega(n)$, we have at most $b-(n-1)$ updates as we will remove at most $b-(n-1)$ edges, then we will get MST with $n-1$ ~~e~~ edges.