HW1.

Q1

① Since there are $(0)+(0)+2(0)=0$ and $(0)-(0)=0$, $\underline{0}$ is in W.

② Next, take two vectors

$\underline{u}=\begin{bmatrix}a_1\\b_1\\c_1\\d_1\end{bmatrix}$ $\underline{v}=\begin{bmatrix}a_2\\b_2\\c_2\\d_2\end{bmatrix}$ in W, which means that

$\underline{u}$ in W $\begin{cases} a_1+b_1+2c_1=0 \text{ and } b_1-d_1=0 \\ a_2+b_2+2c_2=0 \text{ and } b_2-d_2=0 \end{cases}$ If t is any scalar, then

$\underline{v}$ in W

$$t\underline{u}+\underline{v} = t\begin{bmatrix}a_1\\b_1\\c_1\\d_1\end{bmatrix}+\begin{bmatrix}a_2\\b_2\\c_2\\d_2\end{bmatrix} = \begin{bmatrix}ta_1+a_2\\tb_1+b_2\\tc_1+c_2\\td_1+d_2\end{bmatrix}$$

check: $(ta_1+a_2)+(tb_1+b_2)+2(tc_1+c_2)$

$= t(a_1+b_1+2c_1)+(a_2+b_2+2c_2) = 0+0=0$

and $(tb_1+b_2)-(td_1+d_2)$

$= t(b_1-d_1)+(b_2-d_2) = 0+0=0$

So, $t\underline{u}+\underline{v}$ is always in W, so W is a subspace of V.

Q2 It's not true.

when $\underline{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\underline{u}$ is in W. -1 is a scalar.

$(-1)\underline{u} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $-1 < 0$ so, $(-1)\underline{u}$ is not in W.

$\therefore$ It's not true.

Q_3 = ① $\because$ $O^T = O = (-1)O$ $\therefore$ the zero matrix O in W.

② Let A, B be in W and let t be a scalar

So we know that $A^T = -A$ $B^T = -B$. $X = tA + B$.

$X^T = (tA + B)^T = (tA)^T + B^T = tA^T + B^T$

$= t(-A) + (-B) = -tA - B = -(tA + B) = -X$

So, W is a subspace.

Q4. (a) Suppose there are 4 scalars $c_1\underline{u_1} + c_2\underline{u_2} + c_3\underline{u_3} + c_4\underline{u_4} = 0$.

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ -1 & 2 & 1 & -3 & 0 \\ 1 & 1 & 2 & 0 & 0 \end{array}\right] \xrightarrow[R_4-R_1]{R_3+R_1} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & -2 & 0 \\ 0 & 1 & 1 & -1 & 0 \end{array}\right] \xrightarrow[R_3-2R_2]{R_4-R_2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\underline{u_1}, \underline{u_2}$, are linearly independent, $\underline{u_3}, \underline{u_4}$ are combinations of $\underline{u_1}, \underline{u_2}$.

$\underline{u_3} = \underline{u_1} + \underline{u_2}$; $\underline{u_4} = \underline{u_1} - \underline{u_2}$. So, $\underline{u_3}, \underline{u_4}$ can be dropped from the span,

$W = \text{span}(\underline{u_1}, \underline{u_2})$

(b) $W = \text{span}(\underline{u_1}, \underline{u_2}) \neq R^4$. Because there's not a leading entry in every row.

Q5. Yes. Since $k(x) = -x + x^2 = -f(x) + 0 \cdot g(x) + h(x)$

$= -(1 + x + x^3) + 0 + (1 + x^2 + x^3)$

$= -x + x^2$

k is an element of W.