Math 334 Section A1 Homework #1

Due on Friday, Sept. 11, 2020, by 5:00 pm, Edmonton time, in Assign2.

I. Evaluate the following integrals

1. $\displaystyle\int \frac{\cos t}{(1+\sin t)^{\frac{1}{2}}}\,dt$

2. $\displaystyle\int \frac{dx}{x\ln x}$

3. $\displaystyle\int \frac{dy}{y^2-4y+8}$

4. $\displaystyle\int \frac{dx}{(x-1)\sqrt{x^2-x}}$

5. $\displaystyle\int \sin^2 x\,dx$

6. $\displaystyle\int x^2\ln x\,dx.$

7. $\displaystyle\int e^{2x}\sin x\,dx$

8. $\displaystyle\int \frac{3x^2+4x+4}{x^3+x}\,dx$

9. $\displaystyle\int \sin^3 x\cos^4 x\,dx$

10. $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}.$

II. Find all the eigenvalues and eigenvectors of following matrices. Determine if they are diagonalizable.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}.$$

III. Questions 11-16 on pages 8-9 of the textbook.

Problems

In each of Problems 1 through 4, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.

G 1. $y' = 3 - 2y$

G 2. $y' = 2y - 3$

G 3. $y' = -1 - 2y$

G 4. $y' = 1 + 2y$

In each of Problems 5 and 6, write down a differential equation of the form $dy/dt = ay + b$ whose solutions have the required behavior as $t \to \infty$.

5. All solutions approach $y = 2/3$.

6. All other solutions diverge from $y = 2$.

In each of Problems 7 through 10, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that in these problems the equations are not of the form $y' = ay + b$, and the behavior of their solutions is somewhat more complicated than for the equations in the text.

G 7. $y' = y(4 - y)$

G 8. $y' = -y(5 - y)$

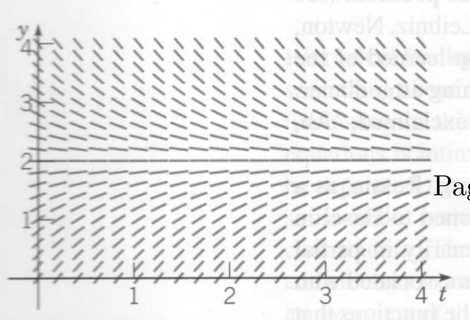
G 9. $y' = y^2$

G 10. $y' = y(y - 2)^2$

Consider the following list of differential equations, some of which produced the direction fields shown in Figures 1.1.5 through 1.1.10. In each of Problems 11 through 16, identify the differential equation that corresponds to the given direction field.

a. $y' = 2y - 1$
b. $y' = 2 + y$
c. $y' = y - 2$
d. $y' = y(y + 3)$
e. $y' = y(y - 3)$
f. $y' = 1 + 2y$
g. $y' = -2 - y$
h. $y' = y(3 - y)$
i. $y' = 1 - 2y$
j. $y' = 2 - y$

11. The direction field of Figure 1.1.5.



12. The direction field of Figure 1.1.6.

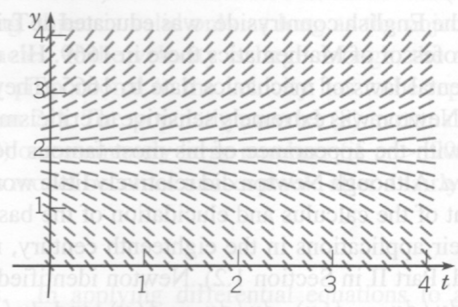


FIGURE 1.1.6 Problem 12.

13. The direction field of Figure 1.1.7.

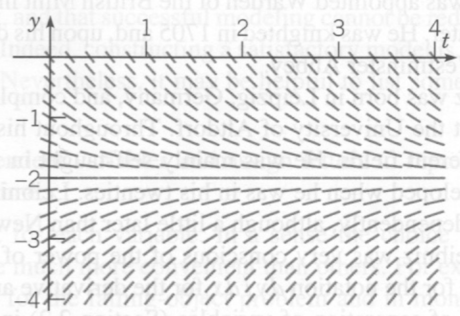


FIGURE 1.1.7 Problem 13.

14. The direction field of Figure 1.1.8.

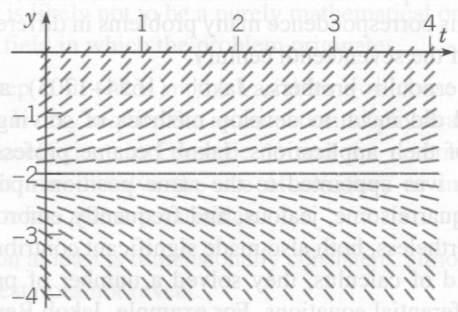
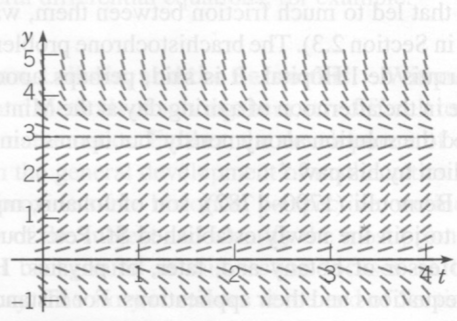


FIGURE 1.1.8 Problem 14.

15. The direction field of Figure 1.1.9.



16. The direction field of Figure 1.1.10.

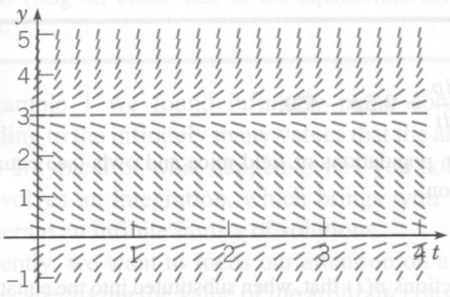


FIGURE 1.1.10 Problem 16.

17. A pond initially contains 1,000,000 gal of water and an unknown amount of an undesirable chemical. Water containing 0.01 grams of this chemical per gallon flows into the pond at a rate of 300 gal/h. The mixture flows out at the same rate, so the amount of water in the pond remains constant. Assume that the chemical is uniformly distributed throughout the pond.

a. Write a differential equation for the amount of chemical in the pond at any time.

b. How much of the chemical will be in the pond after a very long time? Does this limiting amount depend on the amount that was present initially?

c. Write a differential equation for the concentration of the chemical in the pond at time t. *Hint:* The concentration is $c = a/v = a(t)/10^6$.

18. A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time.

19. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings (the ambient air temperature in most cases). Suppose that the ambient temperature is 70°F and that the rate constant is $0.05\ (\text{min})^{-1}$. Write a differential equation for the temperature of the object at any time. Note that the differential equation is the same whether the temperature of the object is above or below the ambient temperature.

20. A certain drug is being administered intravenous patient. Fluid containing 5 mg/cm^3 of the drug ente bloodstream at a rate of 100 cm^3/h. The drug is abs tissues or otherwise leaves the bloodstream at a rate the amount present, with a rate constant of 0.4/h.

a. Assuming that the drug is always uniforn throughout the bloodstream, write a differential e amount of the drug that is present in the bloodstre

b. How much of the drug is present in the bloo long time?

N 21. For small, slowly falling objects, the assur the text that the drag force is proportional to the velocit For larger, more rapidly falling objects, it is more accu that the drag force is proportional to the square of the

a. Write a differential equation for the veloci object of mass m if the magnitude of the proportional to the square of the velocity and opposite to that of the velocity.

b. Determine the limiting velocity after a long ti

c. If $m = 10$ kg, find the drag coefficient so th velocity is 49 m/s.

N d. Using the data in part c, draw a direction fie it with Figure 1.1.3.

In each of Problems 22 through 25, draw a directio given differential equation. Based on the direction fiel behavior of y as $t \to \infty$. If this behavior depends on of y at $t = 0$, describe this dependency. Note that the r of these equations depend on t as well as y; therefore, can exhibit more complicated behavior than those in t

G 22. $y' = -2 + t - y$

G 23. $y' = e^{-t} + y$

G 24. $y' = 3\sin t + 1 + y$

G 25. $y' = -\dfrac{2t + y}{2y}$

[2]See Lyle N. Long and Howard Weiss, "The Velocity Aerodynamic Drag: A Primer for Mathematicians," *Americ Monthly 106* (1999), 2, pp. 127–135.

1.2 Solutions of Some Differential Equations

In the preceding section we derived the differential equations

$$m\frac{dv}{dt} = mg - \gamma v \tag{1}$$

and

$$\frac{dp}{dt} = rp - k. \tag{2}$$

Equation (1) models a falling object, and equation (2) models a population of field mice preyed on by owls. Both of these equations are of the general form

$$\frac{dy}{dt} = ay - b, \tag{3}$$

where a and b are given constants. We were able to draw some important qualitative conclusions about the behavior of solutions of equations (1) and (2) by considering the