A1

Math 334 Q1 Solution to Homework # 1 Dr. Michael Li

Part I.

1.
$$\int \frac{\cos t}{(1+\sin t)^{\frac{1}{2}}}dt \overset{u=1+\sin t}{=} \int \frac{du}{u^{\frac{1}{2}}} = 2u^{\frac{1}{2}} + c \quad = 2\sqrt{1+\sin t} + c.$$

2.
$$\int \frac{dx}{x\ln x} \overset{u=\ln x}{=} \int \frac{du}{u} = \ln|u| + c = \ln|\ln x| + c.$$

3.
$$\begin{aligned}\int \frac{dy}{y^2-4y+8} &= \int \frac{dy}{(y-2)^2+4} \overset{u=y-2}{=} \int \frac{du}{u^2+4} \\ &= \frac{1}{2}\tan^{-1}\frac{u}{2} + c = \frac{1}{2}\tan^{-1}\frac{y-2}{2} + c.\end{aligned}$$

4.
$$\begin{aligned}\int \frac{dx}{(x-1)\sqrt{x^2-x}} &= 2\int \frac{du}{(u^2-1)^{\frac{3}{2}}} \quad (x=u^2) \\ &= 2\int \frac{\sec t}{\tan^2 t}dt \quad (u=\sec t,\ du=\sec t\tan t dt,\ u^2-1=\tan^2 t) \\ &= 2\int \frac{\cos t}{\sin^2 t}dt \overset{v=\sin t}{=} 2\int \frac{dv}{v^2} = -\frac{2}{v} + c \\ &= -\frac{2}{\sin t} + c = -\frac{2}{\sqrt{1-(1/u)^2}} + c = -\frac{2u}{\sqrt{u^2-1}} + c \\ &= -\frac{2\sqrt{x}}{\sqrt{x-1}} + c.\end{aligned}$$

5.
$$\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos(2x))\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x) + c.$$

6.
$$\begin{aligned}\int x^2\ln x\,dx &= \frac{1}{3}\int \ln(x)\,d(x^3) = \frac{1}{3}[x^3\ln x - \int x^3\,d(\ln x)] \\ &= \frac{1}{3}[x^3\ln x - \int x^2\,dx] = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + c\end{aligned}$$

7.
$$\begin{aligned}\int e^{2x}\sin x\,dx &= -\int e^{2x}\,d(\cos x) = -[e^{2x}\cos x - \int \cos x\,d(e^{2x})]\\ &= -e^{2x}\cos x + 2\int e^{2x}\cos x\,dx = -e^{2x}\cos x + 2\int e^{2x}\,d(\sin x)\\ &= -e^{2x}\cos x + 2e^{2x}\sin x - 2\int \sin x\,d(e^{2x})\\ &= -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx.\end{aligned}$$

Namely
$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx.$$

Solving for $\int e^{2x}\sin x\,dx$ we obtain
$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x + c,$$

and thus
$$\int e^{2x}\sin x\,dx = -\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x + c.$$

8. Using partial fractions we obtain
$$\frac{3x^2+4x+4}{x^3+x} = \frac{3x^2+4x+4}{(x^2+1)x} = \frac{4}{x} + \frac{-x+4}{x^2+1}.$$

Therefore
$$\begin{aligned}\int \frac{3x^2+4x+4}{x^3+x}\,dx &= \int \frac{4}{x}\,dx + \int \frac{-x+4}{x^2+1}\,dx\\ &= 4\ln|x| - \int \frac{x}{x^2+1}dx + 4\int \frac{dx}{x^2+1}\\ &= 4\ln|x| - \frac{1}{2}\ln(x^2+1) + 4\tan^{-1}x + c\end{aligned}$$

9.
$$\begin{aligned}\int \sin^3 x\,\cos^4 x\,dx &= \int \sin^2 x\,\cos^4 x\,\sin x\,dx = -\int (1-\cos^2 x)\cos^4 x\,d(\cos x)\\ &= -\int (1-u^2)u^4 du\ (u=\cos x)\ = \int (u^6-u^4)\,du = \frac{u^7}{7} - \frac{u^5}{5} + c\\ &= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c\end{aligned}$$

10. Set $u = e^x$ or $x = \ln u$. Then $dx = du/u$. Therefore

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{du}{u\sqrt{u^2-1}} \quad (u = \sec t,\ du = \sec t \tan t\, dt,\ \sqrt{u^2-1} = \tan t)$$
$$= \int 1\, dt = t + c = \sec^{-1} u + c = \sec^{-1}(e^x) + c.$$

Part II.

Eigenvalues of

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

are $\lambda_1 = 2 + i$ and $\lambda_2 = 2 - i$. Corresponding eigenvectors are

$$u_1 = \begin{bmatrix} i \\ 1 \end{bmatrix} \quad \text{and} \quad u_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix},$$

respectively.

Eigenvalues of

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

are $\lambda_1 = \lambda_2 = \lambda_3 = -1$. To find eigenvectors we solve $(\lambda_1 I - B)x = 0$ for nontrivial solutions x.

$$\begin{bmatrix} 0 & 0 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$-2x_1 + x_3 = 0,$$

which gives

$$x_1 = \frac{1}{2}x_3.$$

Set $x_2 = s,\ x_3 = t$. Then $x_1 = \frac{1}{2}t$, and thus

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}t \\ 0 \\ t \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}.$$

Therefore, the eigenspace of $\lambda = -1$ has a basis

$$u_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad u_2 = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}.$$

Since B only has two linearly independent eigenvectors, it is not diagonalizable.

III. Questions 15-20 on pages 8-9 of the textbook.

- Figure 1.1.5 for equation (j)
- Figure 1.1.6 for equation (c)
- Figure 1.1.7 for equation (g)
- Figure 1.1.8 for equation (b)
- Figure 1.1.9 for equation (h)
- Figure 1.1.10 for equation (e)