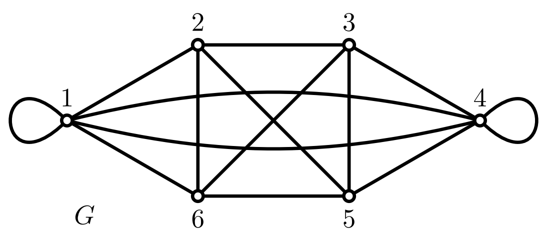
Graph Theory (MATH 322): Assignment 1

There are seven questions; answer them all. The assignment is due by 23:58 MDT on Wednesday 15th May.

1. Consider the graph G below, having vertices $1, \ldots, 6$.



(a) (*i*) How many edges are incident with vertex 1?

(*ii*) Write down all vertices adjacent to vertex 1.

(*iii*) What is the degree of vertex 1?

(b) Repeat part (a) with vertex 2 instead of vertex 1.

(c) (*i*) How many subgraphs of G are isomorphic to K_3?

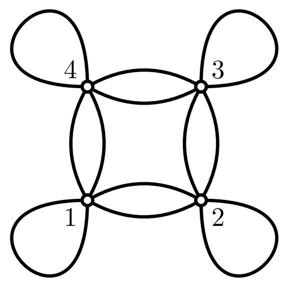
(*ii*) How many subgraphs of G are isomorphic to K_4?

2. For each of the degree sequences below, decide whether it is graphic. If it is, draw a simple graph with the degree sequence. If it is not graphic, explain why not and instead draw a non-simple graph with that degree sequence.

(a) $(6, 6, 5, 3, 2, 2, 2)$.

(b) $(6, 6, 3, 3, 2, 2, 2)$.

3. (a) Write down the adjacency matrix of the following graph with respect to the given numbering of the vertices:

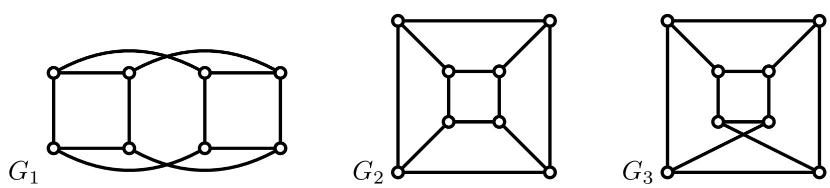


(b) Let

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

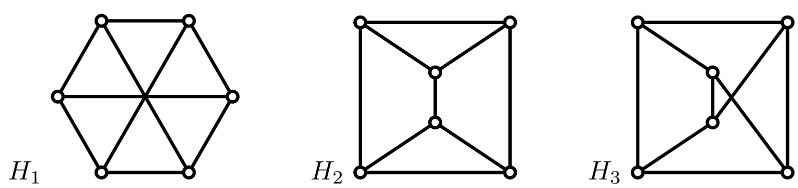
Draw the graph on vertices $1, 2, 3, 4, 5$ that has A as its adjacency matrix. Make sure you label the vertices in your graph.

4. (a) Consider the graphs G_1, G_2, and G_3 below.



Which pairs G_i, G_j are isomorphic? For each pair of isomorphic graphs, provide an explicit isomorphism by labelling the vertices. For each pair of non-isomorphic graphs, explain why they are not isomorphic.

(b) Repeat part (a) with the following three graphs instead.



5. Let G be a simple graph with at least two vertices. Show that there are two vertices in G of the same degree.

6. (a) Show that the 3-cube is bipartite.

   (b) More generally, show that the k-cube is bipartite for all $k \geq 2$. (If you provide a proof of this generalization, you may omit part (a).)

7. Let G be a bipartite graph with bipartition (A, B). Show that if G is k-regular with $k \geq 1$, then $\#A = \#B$.