# Statistics 252 – Midterm Exam A

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Instructor: Paul Cartledge

*Instructions:*

1. *Read all the instructions carefully.*
2. *This is a closed book exam.*
3. *You may use the formula sheets and the tables provided and a calculator only.*
4. *You have* ***50*** *minutes to complete the exam.*
5. *The exam is out of a total of* ***42*** *marks.*
6. *Show your work in all sections to receive full credit. Final numerical answers should have* ***THREE*** *significant decimal places.*
7. *Use the backs of the pages for scrap work.*
8. *Make sure your name and signature are on the front and that your ID number is on the top of page two.*
9. *When referring to "log", I am always referring to the natural log.*
10. *If no significance level is given, use the "judgment approach".*
11. *When asked for a "confidence interval", state the estimate, the standard error, and the critical value. Then, calculate and interpret the interval.*
12. *When asked to "carry out a full analysis in detail", set up the hypotheses, calculate the test statistic, state the distribution of the test statistic (such as* $t_9$ *or* $F_{3,10}$*), approximate the p-value, and state your conclusion in plain English.*

**Name (last name, first name): SOLUTION**

Signature: ______________________________________________

| Component | Notes | Worth | Mark |
|---|---|---|---|
| | | | |
| **Short Answer** | 6 questions | 11 | |
| **Long Answer** | | | |
| Question 7 | 3 parts | 21 | |
| Question 8 | 4 parts | 10 | |
| | | | |
| **Total** | | 42 | |

Statistics 252 – Midterm Exam – Paul Cartledge – Winter 2016

**ID:**______________________________

**Question 1 (2 marks)** Starting their midterm, a student sees a test statistic of 2.21 in a test for any differences between several means. From a data structure where the groups have respective sample sizes of 5, 4, 5, 6, and 4, what is the distribution of the test statistic?

With more than 2 groups, this should be an $F$-distribution.
$I = 5$, $N = 5 + 4 + 5 + 6 + 4 = 24$; thus the $F$-distribution is $F_{4, 19}$.

**Question 2 (2 marks)** Using information from Question 1 and a $p$-value of 0.107, apply the "judgment approach" and make a decision *and* conclusion about the rejection (OR non-rejection) of the null hypothesis that claims equality among the several means.

The given $p$-value provides weak evidence against $H_0$. Thus, it is best to not reject the null hypothesis, so there is insufficient evidence of a difference between the means.

**Question 3 (2 marks)** Is the following statement true or false? Defend your answer either way in one or two sentences. Simply an answer of "true" or "false" will not receive any credit. "If random sampling is present, it is possible to make causal inferences."

The statement is "false" because random assignment needs to be present to make causal inferences possible.

**Question 4 (1 mark)** What is the advantage of Scheffe multiple comparisons?

Most conservative.

**Question 5 (2 marks)** Bono attempts to analyse two independent random samples. Putting the first sample first in the order, some skewness in the sample distributions requires a log transformation of the data. Thus, he reports a 99% confidence interval of the ratio of the medians on the original scale to be (0.522, 0.864). What can you say about the possible rejection of the null hypothesis testing to see if the medians on the original scale are the same? Explain your answer in one or two sentences. No calculation required.

Since the value of 1 is not within the 99% confidence interval for the ratio of medians, the corresponding null hypothesis will be rejected. Thus, the medians are different at $\alpha = 0.01$.

**Question 6 (2 marks)** Using the confidence interval from Question 5, find the corresponding significance level to test if the second median is larger than the first median. Also, how would the $p$-value compare to this significance level?

Since $1 - \alpha = 0.99$, then $\alpha = 0.01$ and for a one-tailed test, the appropriate $\alpha$ would be 0.005.

Since the interval above does not contain 1, then $p$-value $< \alpha$.

**Question 7 (21 marks total)** At Greendale Community College, an instructor is getting concerned about student performance in the night classes. Professor P. Professorson decided to accumulate information on the following classes. The tables below summarize: summary statistics of final exam percentage (measured in decimals); the ANOVA output; selected linear combinations. Assume all assumptions hold.

| Group | Class | $n_i$ | Sample Mean | Sample S.D. |
|---|---|---|---|---|
| 1 | History of Something | 10 | 0.847 | 0.0224 |
| 2 | Introduction to Basics | 20 | 0.851 | 0.0265 |
| 3 | Studyology | 10 | 0.905 | 0.0310 |
| 4 | Theoretical Physical Education | 10 | 0.792 | 0.0309 |
| 5 | Math 1-2-3 | 20 | 0.742 | 0.0542 |
| 6 | Class 101 | 20 | 0.881 | 0.0515 |

| Source of Variation | Sum of Squares | df | Mean Square | F-Statistic | *p*-value |
|---|---|---|---|---|---|
| Between (Extra) | 0.289 | 5 | 0.0578 | 34.372 | 0.000 |
| Within (Full) | 0.141 | 84 | 0.00168 | | |
| Total (Reduced) | 0.430 | 89 | | | |

**a) (6 marks)**

**i) (1 mark)** If you had to fully analyse "unplanned comparisons" upon the groups, how many unique pairings of the groups will you need?

**ii) (1 mark)** Using the Bonferroni method, if the experiment-wise confidence level is 94%, what are the corresponding individual confidence levels?

**iii) (4 marks)** Using the Bonferroni method and the table below that summarizes the *margin of error* between each pair (for example, the margin of error for group 1 vs. group 2 is 0.0505), construct a visual diagram below the table that joins groups together that are **not** different.

$$\boxed{m = \frac{I(I-1)}{2} = \frac{6(5)}{2} = 15}$$

$$\boxed{1 - \frac{\alpha_E}{m} = 1 - \frac{0.06}{15} = 0.996 \rightarrow 99.6\%}$$

| Row vs. Column | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 0.0505 | | | | |
| 3 | 0.0583 | **0.0505** | | | |
| 4 | 0.0583 | **0.0505** | **0.0583** | | |
| 5 | **0.0505** | **0.0413** | **0.0505** | 0.0505 | |
| 6 | 0.0505 | 0.0413 | 0.0505 | **0.0505** | **0.0413** |

(Note: The table above identifies differences larger than their margin of error **in this form**.)

| 0.742 | 0.792 | 0.847 | 0.851 | 0.881 | 0.905 |
|---|---|---|---|---|---|
| 5 | 4 | 1 | 2 | 6 | 3 |

The ANOVA output below compares average final exam percentage by grouping the above groups into two categories: classes that average over 80% and those that average less.

**ANOVA**

| | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | .245 | 1 | .245 | 116.628 | .000 |
| Within Groups | .185 | 88 | .00210 | | |
| Total | .430 | 89 | | | |

The ANOVA output below compares average final exam percentage by grouping the above groups into three categories: classes that average < 80%, between 80 – 90%, and > 90%.

**ANOVA**

| | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | .261 | 2 | .130 | 66.837 | .000 |
| Within Groups | .170 | 87 | .00195 | | |
| Total | .430 | 89 | | | |

**b) (8 marks)** Test if the model where all classes have potentially different mean percentages is better than the model that groups classes by comparing to an 80% boundary. Carry out a full analysis in detail, identifying the SSR and df for the respective models. (Hint: You can assume the *p*-value will be less than 0.01.)

$H_0$: (2-mean model) $\mu_4 = \mu_5$, $\mu_1 = \mu_2 = \mu_3 = \mu_6$
$H_A$: (6-mean model) $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$

SSR($r$) = 0.185 SSR($f$) = 0.141
df($r$) = 88 df($f$) = 84

$$\boxed{\begin{aligned} F_0 &= \frac{(SSR(r) - SSR(f))/(df(r) - df(f))}{SSR(f)/df(f)} = \frac{(0.185 - 0.141)/(88-84)}{0.141/84} \\ F_0 &= \frac{(0.044)/(4)}{0.141/84} = 6.553 \sim F_{df(f)}^{df(r)-df(f)} = F_{84}^{4} \end{aligned}}$$

Told that the *p*-value is less than 0.01.

JA: Strong to convincing evidence against $H_0$. Reject $H_0$. Thus, the six-mean model is better than the two-mean model.

**Contrast Coefficients**

| Contrast | Type 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | -1 | -1 | -1 |
| 2 | 1 | -1 | 1 | 1 | -1 | -1 |
| 3 | 1 | 0 | -.5 | -.5 | 0 | 0 |
| 4 | 1 | -.5 | 1 | 1 | -.5 | -.5 |
| 5 | 0 | 1 | 0 | 0 | -.5 | -.5 |

**Contrast Tests**

| | | Contrast | Value of Contrast | Std. Error | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|
| Gross | Assume equal variances | 1 | .188 | .028 | 6.837 | 84 | .000 |
| | | 2 | .069 | .028 | 2.509 | 84 | .014 |
| | | 3 | -.002 | .016 | -.126 | 84 | .900 |
| | | 4 | 1.306 | .024 | 54.842 | 84 | .000 |
| | | 5 | .040 | .011 | 3.536 | 84 | .001 |

**c) (7 marks) Do the smaller classes have <u>different</u> final exam percentages than the larger classes? Construct a confidence interval to answer this question.**

i) **(2 marks)** First, define a linear combination of means contrasting the average percentage for the smaller classes vs. the larger classes. Fill in the blanks below for the coefficients in your contrast.

$$\gamma = \boxed{\left(\frac{1}{3}\right)} \mu_1 + \boxed{\left(-\frac{1}{3}\right)} \mu_2 + \boxed{\left(\frac{1}{3}\right)} \mu_3 + \boxed{\left(\frac{1}{3}\right)} \mu_4 + \boxed{\left(-\frac{1}{3}\right)} \mu_5 + \boxed{\left(-\frac{1}{3}\right)} \mu_6$$

ii) **(2 marks)** Give the estimate and standard error for this contrast.
See contrast 2 for $3g$.

$g = 0.069/3 = 0.0230$ $S.E.(g) = 0.028/3 = 0.00933$

iii) **(3 marks)** Calculate a 90% <u>confidence interval</u> for the contrast.

$g \pm t_{N-I,\alpha/2} \times S.E.(g) \rightarrow 0.0230 \pm (1.664)(0.00933) \rightarrow 0.0230 \pm 0.0155 \rightarrow (0.00747, 0.0385)$

With 90% confidence, the contrast is between 0.00747 and 0.0385.

**Question 9 (10 marks)** Recent events in the past year have got people wondering about the marketability of double-decker couches in certain locations. Investigating the business, a marketing expert takes a random sample of 45 department stores in Bricksburgh and a sample of the same size in Clown Town. He measures the monthly income of each store's furniture department to obtain the following summary statistics (units are in thousands of $US).

| **Summary statistic** | Bricksburgh | Clown Town | Difference |
|---|---|---|---|
| Average | 193.40 | 172.51 | 20.89 |
| Standard Deviation | 39.80 | 20.14 | 44.60 |

NOTE: Please note that the third column summarize the differences from the original observations. By choosing a test, you will be using certain columns of the above table, but not all of them.

Based on statistical evidence, is Bricksburgh selling more double-decker couches?

**a) (3 marks)** Is the above situation two independent samples or a paired sample?

Two independent samples

**b) (2 marks)** Write the appropriate null and alternative hypotheses.

$H_0$: $\mu_B - \mu_C \leq 0$ $H_A$: $\mu_B - \mu_C > 0$

**c) (3 marks)** Suppose the test statistic is $t_0 = 3.142$. State the distribution of the test statistic and determine the range of the $p$-value.

$$t_0 = t_{n_1+n_2-2} = t_{45+45-2} = t_{88} \approx t_{80}$$

$$2.887 < t_0 = 3.142 < 3.195$$
$$0.0025 < p\text{-value} < 0.001$$

**d) (2 marks)** Make a decision and state a conclusion in plain English.

Strong to convincing evidence against $H_0$. → Reject $H_0$.
→ Bricksburgh sells more double-decker couches, so everything is awesome.