Ch. 12 – From Randomness to Probability

Def'n: An experiment is a process that, when performed, results in one and only one of many observations (or outcomes).

Probability is a numerical measure of likelihood that a specific outcome occurs.

3 Conceptual Approaches to Probability:

1) Classical probability

- equally likely outcomes exist when two or more outcomes have the same probability of occurrence
- *classical probability rule*:

  $P(A)$ = (# of outcomes favourable to A) / (total # of outcomes for experiment)

2) Relative frequency concept of probability

- experiment repeated n times to simulate probability
- relative frequencies are NOT probabilities, they only approximate them.
- *Law of Large Numbers*: If an experiment is repeated again and again, the prob. of an event obtained from the relative frequency approaches the actual or theoretical prob.

3) Personal (or subjective) probability

- personal probability is the degree of belief that an outcome will occur, based on the available information

Calculating Probability

Def'n: A sample space (S) is the set of all *elementary* outcomes of an experiment.

An event (A) is a set of some of the elementary outcomes; $A \subset S$.

→ $P(A)$ = probability that A occurs

- A *union* of 2 events (A, B, **or** both happen) is denoted by A or B (or $A \cup B$).
- An *intersection* of 2 events (A **and** B happen together) is by A and B (or $A \cap B$).
- A *complement* of an event (event does **not** happen) is denoted by A^C.

A Venn diagram is a picture that depicts S (events above drawn in class).

Experiment	**Outcomes**	**Sample Space**
Toss a coin		
Toss 2-headed coin		
Toss a $5 bill		
Pick a suit		

Properties for calculating probabilities:
1. $0 \leq P(A) \leq 1$
2. $P(A)$ is the sum of probabilities of all elementary outcomes comprising A.
3. $P(S) = 1$

Ch. 13 – Probability Rules!

Basic Rules for Finding the Probability of a Pair of Events:

Table 13X0 – 2-way table of responses

	Like Hockey (A)	Indifferent (B)	Dislike Hockey (C)	**Total**
Male (M)				
Female (F)				
Total				

Def'n: Marginal probability is the probability of a single event without consideration of any other event.

Ex13.1) $P(M) =$ $P(F) =$
$P(A) =$ $P(B) =$ $P(C) =$

Conditional probability is the probability that an event will occur given that another event has already occurred. If A and B are 2 events, then the conditional probability of A given B is written as $P(A \mid B)$. Keywords: **given**, **if**, **of**

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

such that $P(A) \neq 0$ and $P(B) \neq 0$.

Ex13.2) a) If you are male in this class, what is the probability that you like hockey?

b) What is the probability of being female in this class, given that you are indifferent to hockey?

Two events are independent if the occurrence of one does not affect the probability of the occurrence of the other. In other words,

$P(A \mid B) = P(A)$ OR $P(B \mid A) = P(B)$

Ex13.3) From Table 13X0, $P(F) =$ $P(F \mid B) =$

Ex13.4) deck of cards: $P(Black) =$ $P(Black \mid Face) =$

Disjoint (or mutually exclusive) events are events that cannot occur together.

Ex13.5) deck of cards

R = get red suit →

B = get black suit →

F = get face card →

Which pairs are disjoint?

Ex13.6) a single die

E = even =

O = odd =

Pr = prime =

Note: Two events are either disjoint or independent, but not both (unless one has zero probability). How to differentiate between disjoint, independent, and dependent events?

Complement Rule: $P(A) + P(A^C) = 1$, so

$P(A) = 1 - P(A^C)$ and $P(A^C) = 1 - P(A)$

Ex13.7) From Table 13X0, $P(Female^C) = P(F^C) = 1 - P(F) =$

Ex13.8) deck of cards: $P(Face^C) = P(F^C) = 1 - P(F) =$

Note: $P(A^C \mid B) = 1 - P(A \mid B)$ Does $P(A \mid B^C) = 1 - P(A \mid B)$? Not necessarily.

Ex13.9) deck of cards:

$P(Face \mid Black) =$

$P(Face^C \mid Black) =$

$P(Face \mid Black^C) =$

Ex13.10) deck of cards: $P(Heart \mid Red) =$

Multiplication Rule: $P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)$

- If A and B are two *independent* events, $P(A \cap B) = P(A) \times P(B)$.
- If A and B are two *disjoint* events, $P(A \cap B) = 0$.

Ex13.11) From Table 13X0, what is the probability of being male and liking hockey? Being indifferent to hockey and female?

Ex13.12) deck of cards: What is the probability of drawing a black face card? Black and red card off single draw?

Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If A and B are two *disjoint* events, $P(A \cup B) = P(A) + P(B)$.

Ex13.13) From Table 13X0, what is the probability of being male or liking hockey? Being indifferent to hockey or female?

Ex13.14) deck of cards: What is the probability of drawing a black card or a red card? Face card or ace? Black card or face card?

With 3 or more independent events, the multiplication rule becomes

$$P\left(A_1 \cap A_2 \cap \ldots \cap A_k\right) = P(A_1) \times P(A_2) \times \ldots \times P(A_k)$$

With 3 or more disjoint events, the addition rule becomes

$$P\left(A_1 \cup A_2 \cup \ldots \cup A_k\right) = P(A_1) + P(A_2) + \ldots + P(A_k)$$

Total Probability Rule (for two events): → diagram drawn in class

$P(A) = P\left(A \cap B\right) + P\left(A \cap B^C\right)$ OR $P(B) = P\left(A \cap B\right) + P\left(A^C \cap B\right)$

Overall examples:

Ex13.15) Suppose the probability of liking Gretzky is 0.86, the probability of liking Crosby is 0.79, and the probability of liking both is 0.71.

a) What is the probability of liking neither Gretzky nor Crosby? The probability of liking Gretzky but not Crosby?

b) What is the probability of liking Gretzky or Crosby?

c) What is the probability of liking Gretzky or not liking Crosby?

d) What is the probability of liking Crosby, given you like Gretzky?

Ex13.16) Suppose 30% of calls to an Oilers ticket phone line result in a sale being made. Assume all calls are independent. Suppose an operator handles 10 calls.

a) What is the probability that none of the 10 calls results in a sale?

b) What is the probability that at least one call results in a sale being made?

Ex13.17) Three friends play tennis (call them A, B, and C). The probability that A beats B is 0.7, the probability that A beats C is 0.8 and the probability that B beats C is 0.6. Assume all events are independent and that each player plays another at most once.

a) What is the probability that A wins both of its games?

b) What is the probability that A loses both of its games?

c) What is the probability that everyone wins a game?

Ex13.18) Assume that 70% of students who take the midterm next month have studied for the test. Of those who study for the midterm, 95% pass; of those who do not study for the test, 60% pass. What is the probability that a student did not study for the midterm, given that they pass the midterm?

Ex13.19) Bob and Mark regularly play a simple darts game. Each play consists of one throw each at a target. A player wins if they hit the target and the other doesn't. Each time they play, they flip a coin to determine who throws first. History has shown that Bob hits the target 30% of the time and Mark hits the target 37% of the time. Also, there is a 9% chance that both will hit the target on any random play.

(a) Are Bob and Mark's throws independent?

(b) On a single play, what is the probability that neither hits the target?

(c) Suppose that on a single play, somebody wins. What is the probability that it was Bob?