STAT 252 Homework 1 (79 marks) – Due Friday, February 2 by 5 pm

For questions that state "SHOW ALL STEPS", write all the steps of a hypothesis test or confidence interval as indicated below. For other questions that say do "NOT" show all steps, read the question carefully and follow the exact instructions regarding what is required.

Whenever you are asked to "carry out the most appropriate test" and "SHOW ALL STEPS":

i) Select the most appropriate hypothesis test and define the parameter(s) of interest.
ii) State clearly the null and alternative hypothesis in terms of the parameter(s).
iii) Calculate the test statistic, being sure to state its components (estimate and standard error).
iv) Calculate *df*. Determine the *P*-value for the test AND state the strength of the evidence against H_0. State whether *P* is less than or greater than alpha and, based on this comparison, decide whether to reject or not reject H_0. If the exact *P*-value is given in output, then report it as is. If not, then you must estimate the *P*-value (within a range of values) using the appropriate statistical table.
v) Based on the research problem and referring to the significance level given, write a conclusion in words.

Whenever you are asked to calculate a "confidence interval" and "SHOW ALL STEPS":

i) State the critical value of the test statistic.
ii) Calculate the confidence interval, stating its components (estimate and standard error).
iii) Interpret the interval.

Other: If you need to use the t-table and the t-distribution you need is NOT on the table, round your degrees of freedom DOWN to the nearest one.

1. **(Two parts; 4 marks in total)** Suppose $m = c\frac{s}{\sqrt{n}}$

   (a) (2 marks) Solve for *c* (as a function of *m*, *n,* and *s*).

$$\Rightarrow m\sqrt{n} = cs$$

$$\Rightarrow c = \frac{m\sqrt{n}}{s}$$

   (b) (2 marks) Solve for *n* (as a function of *c*, *m,* and *s*).

$$\Rightarrow m\sqrt{n} = cs$$

$$\Rightarrow \sqrt{n} = \frac{cs}{m}$$

$$\Rightarrow n = \left(\frac{cs}{m}\right)^2$$

2. **(Three parts; 7 marks in total)** A company that is setting up a new factory is interested in comparing the efficiency of two types of machines (Model A and Model B) in assembling their product. They randomly selected 8 workers employed by the company. Subsequently, they randomly assigned each worker to operate one machine of Model A and one machine of Model B in a random order. The table below shows the time (in minutes) taken for each worker to assemble one unit of the product on each type of machine.

Worker	**Time (in min)**	
	Model A	**Model B**
Fred	23	21
Stanley	26	24
Jessica	19	23
Ali	24	25
Lorraine	27	24
Chen	22	28
Fatuma	20	24
Victoria	18	23

(a) **(1 mark)** What is the population of interest?

The population of interest is all the products assembled by the two types of machines.

(b) **(3 marks)** Is this an observational study or an experiment? Very briefly explain your answer.

This is an experimental study since there is an explanatory variable that is manipulated, that is, testing machine Model A versus Model B. Moreover, the workers are randomly assigned to machines of each type and they are assigned in a random order.

(c) **(3 marks)** Based on this study, will it be appropriate to make population inferences, causal inferences, or both? Briefly explain your answer.

[Either answer given below can be acceptable as long as logical reasoning is provided.]
It is appropriate to make population inferences since 8 workers were randomly sampled from the workers in the company (the target population). It will also be appropriate to make causal inferences since the workers were randomly assigned to the two types of machines in a random order, that is, we can conclude that any differences in assembling time were caused by the model of the machine.

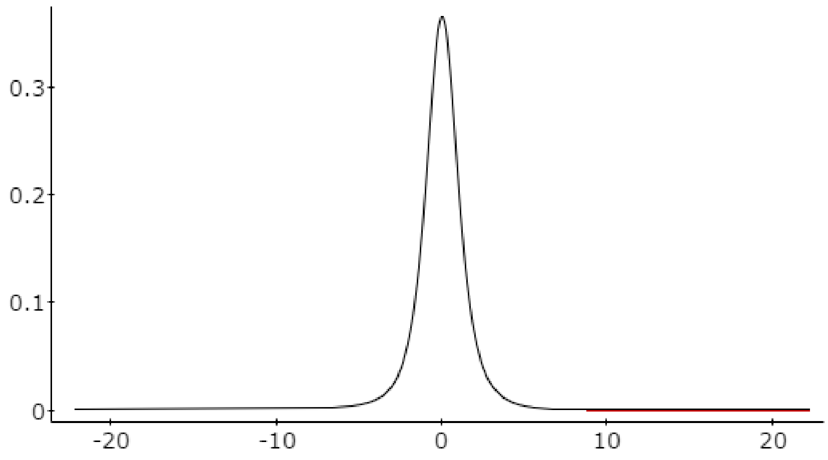
OR

Though the workers are randomly sampled, they are not the population of interest. Since the products are not randomly sampled, it is not appropriate to make population inferences to the population of interest. It will be appropriate, however, to make causal inferences since the workers were randomly assigned to the two types of machines in a random order, that is, we can conclude that any differences in assembling time were caused by the model of the machine.

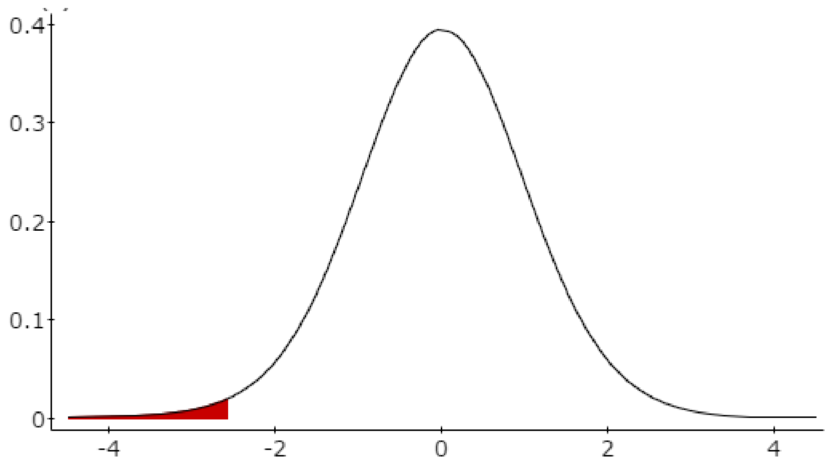
3. (Two parts; 2 marks in total) Calculate the following probabilities using the *t*-table (posted on eClass) from the course website (find the NARROWEST range possible):

As mentioned in class, when using the table, the answers should be estimated within a range of values as shown below. The diagrams are not necessary in your solution.

(a) $P(t_3 > 8.722) \underset{between}{\in} (0.001, 0.0025)$ or $0.001 < P < 0.0025$



(b) $P(t_{21} < -2.586) = P(t_{21} > 2.586) \in (0.005, 0.01)$ or $0.005 < P < 0.01$



4. (Three parts; 10 marks in total) Conservation biologists stationed in St. John's, Newfoundland have been researching the lifespan of local avian wildlife. Specifically, they have recently focused on puffins. A recent random sample of 8 puffins gave a standard deviation of 1.715 and a confidence interval of (6.826, 8.542). Assume that the assumptions are met for the required analysis.

(a) (4 marks) Determine the confidence level of this interval.

Margin of Error (ME) = $m = \dfrac{[8.542 - 6.826]}{2} = 0.858$ **(0.5 marks for ME)**

For a one-mean t-interval: $Estimate \ \pm CV \times SE(Estimate)$

$$\bar{y} \pm t_{n-1,\alpha/2} \times \frac{s}{\sqrt{n}} \quad \Rightarrow \quad t_{n-1,\alpha/2} \times \frac{s}{\sqrt{n}} = m$$

$$t_{n-1,\alpha/2} = m \times \frac{\sqrt{n}}{s}$$

$$t_{n-1,\alpha/2} = 0.858 \times \frac{\sqrt{8}}{1.715} = 1.415$$ **(1.5 marks for $t_{\alpha/2}$)**

$$t_{7,\alpha/2} = 1.415 = t_{7,0.10} \quad \text{(By examining the t-table)}$$

$t_{7,\alpha/2} = t_{7,0.10}$, so $\alpha / 2 = 0.10 \quad \Rightarrow \alpha = (0.10)2 = 0.20$

Thus, the confidence level $= 1 - 0.20 = 0.80 \Rightarrow 80\%$

The confidence level for this reported interval is 80%.

(2 marks for confidence level and interpretation)

(b) (4 marks) Find a 98% confidence interval for the mean lifespan of all puffins in St. John's (SHOW ALL STEPS).

Parameter: μ =mean lifespan

Estimate: $\bar{y} = \dfrac{[6.826 + 8.542]}{2} = 7.684$ **(0.5 marks for Estimate)**

Standard error of the estimate of the mean:

$$SE(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{1.715}{\sqrt{8}} = 0.6063$$ **(0.5 marks for SE)**

For a 98% confidence interval, α = 1 – 0.98 = 0.02 and the critical value (CV) is:

$$CV = t_{n-1,\alpha/2} = t_{8-1,\alpha/2} = t_{7,0.02/2} = t_{7,0.01} = 2.998$$ **(1 mark for CV)**

Calculation of the confidence interval:

$$Estimate \ \pm CV \times SE(Estimate)$$
$$\bar{y} \pm t_{n-1,\alpha/2} \times SE(\bar{y})$$
$$7.684 \pm 2.998 \times 0.6063$$
$$7.684 \pm 1.8177$$
$$(5.866, 9.502)$$ **(1.5 marks for CI)**

It is estimated with 98% confidence that the mean lifespan of puffins is between 5.866 and 9.502 years.

(0.5 marks for interpretation)

(c) (2 marks) A previous research study claims that the average lifespan of puffins is 6.5 years. Based on the confidence interval given in the question (6.826, 8.542), evaluate whether this claim is true. Also, based on the 98% confidence interval you calculated in part (b), evaluate whether this claim is true. Did these two confidence intervals give you the same evaluation? Explain why or why not.

No, these two confidence intervals did not give the same evaluation. Based on the 80% confidence interval given in the question, we would conclude that the average lifespan of puffins is not 6.5, since the confidence interval does not contain 6.5. However, based on the 98% confidence interval calculated in part (b), we would conclude that the average lifespan of puffins can 6.5, because the confidence interval does contain 6.5. This difference in conclusions is because the 80% confidence interval given in the question (6.826, 8.542), is narrower and more precise than the 98% confidence interval calculated in part (b), (5.866, 9.502).

5. (Seven parts; 34 marks in total) Experiments on Teaching Styles

Experiment 1

Education students wanted to conduct a research project on the effectiveness of a new teaching style. Out of a population of two thousand first-year university students, they randomly sampled 40 students and then randomly assigned them to two groups (20 students to each group). The two groups were offered a short course on the same topic; one group was taught using the new teaching style and the other group was taught with a standard method. At the end of the course, students in both groups were given the same exam, which was marked on a continuous, quantitative scale from 0 – 50. The student researchers didn't quite know what the correct analysis would be, so they first calculated the following statistics:

Difference between means: $(\bar{y}_{\text{New}} - \bar{y}_{\text{Standard}}) = 39.4000 - 35.5500 = 3.8500$

Pooled standard deviation: $s_p = 7.5564$

Mean difference: $\bar{d} = \frac{\sum d}{n} = \frac{77}{20} = 3.8500$

Standard deviation of the differences: $s_d = 11.9176$

(a) (4 marks) Help out the student researchers. Based on the research design described in Experiment 1, what would be the most appropriate hypothesis test to determine if there is a difference in the effectiveness of the two teaching styles (new style versus standard style), measured in terms of average marks? Explain why you have chosen this test. Assuming that all the assumptions for the test you have selected are met, state what these assumptions are. (If you think the samples are independent, then assume equal variances.)

The most appropriate test would be the two-sample (or two-mean) *t*-test for independent samples, specifically, the pooled *t*-test. This test should be chosen because 40 randomly selected students were randomly assigned to two groups of 20 students each. The 20 students who were taught using the new teaching style were different students from the 20 who were taught using the standard method and there was no link or connection between the students in the two groups. The assumptions of this test are that there must be two independent random samples. The two distributions are either normal or both samples sizes are at least 30. (Since the sample sizes are less than 30, then the distributions must be normal.) Since the samples are independent, then as stated in the question, equal variances are assumed.

(b) (6 marks) Carry out the most appropriate test (at the 5% significance level), which you selected in part (a), in order to determine if there is a difference in the effectiveness of the two teaching styles (new and standard styles) , measured in terms of average marks. SHOW ALL STEPS of the hypothesis test. Note: you may not need to use all of the statistics (shown above) that the education students calculated.

$H_0 : \mu_{\text{New}} = \mu_{\text{Standard}}$ (There is no difference in the effectiveness of the two teaching styles.)

$H_a : \mu_{\text{New}} \neq \mu_{\text{Standard}}$ (There is a difference in the effectiveness of the two teaching styles.)

Parameter: $\mu_{\text{New}} - \mu_{\text{Standard}}$ **(1 mark for hypotheses)**

Estimate: $(\bar{y}_{\text{New}} - \bar{y}_{\text{Standard}}) = 39.4000 - 35.5500 = 3.8500$

Standard error of the estimate of the difference between two means:

$$SE(\bar{y}_1 - \bar{y}_2) = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 7.5564\sqrt{\frac{1}{20} + \frac{1}{20}} = 2.3895$$ **(1 mark for components)**

$$t = \frac{Estimate - H_0 \text{ value}}{SE(Estimate)} = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{SE(\bar{y}_1 - \bar{y}_2)} = \frac{3.8500}{2.3895} = 1.611$$ **(1 mark for calculating *t*)**

$df = n_1 + n_2 - 2 = 20 + 20 - 2 = 38 \approx 30$

P-value: (0.05) x 2 < P < (0.10) x 2 = 0.10 < P < 0.20. There is weak evidence against H_0.

Since P-value > α (0.05), do not reject H_0.

(2 marks for *df, P*-value, evidence, and decision)

Conclusion**:** At the 5% significance level, there is insufficient evidence to conclude that there is a difference in the effectiveness of the two teaching styles, measured in terms of average marks.

(1 mark for conclusion)

(c) (5 marks) Calculate a 99% confidence interval for the difference in effectiveness between the two teaching styles (SHOW ALL STEPS), measured in terms of average marks. Based only on this confidence interval, conclude whether there is a difference in the effectiveness of the two teaching styles and explain the logic of your answer.

For a 99% confidence interval, α = 1 – 0.99 = 0.01 and the critical value (CV) is:

$CV = t_{n_1+n_2-2,\alpha/2} = t_{20+20-2,0.01/2} = t_{38,0.005} \approx t_{30,0.005} = 2.750$ **(1 mark for CV)**

Parameter: $\mu_{\text{New}} - \mu_{\text{Standard}}$

Estimate: $(\bar{y}_{\text{New}} - \bar{y}_{\text{Standard}}) = 39.4000 - 35.5500 = 3.8500$

Standard error of the estimate of the difference between two means:

$SE(\bar{y}_1 - \bar{y}_2) = 2.3895$ (calculated in part (b)) **(0.5 marks for restating components)**

Calculation of the confidence interval:

$$Estimate \pm CV \times SE(Estimate)$$
$$(\bar{y}_1 - \bar{y}_2) \pm t_{n_1+n_2-2,\alpha/2} \times SE(\bar{y}_1 - \bar{y}_2)$$
$$3.85 \pm 2.750 \times 2.3895$$
$$3.8500 \pm 6.5711$$
$$(-2.721, 10.421)$$ **(1 mark for calculation of CI)**

It is estimated with 99% confidence that the difference in effectiveness between the two teaching styles, measured in terms of average marks, is between -2.721 and 10.421.

OR

It is estimated with 99% confidence that the average mark attained by students taught using the new method is between 2.721 marks lower and 10.421 marks higher than the average mark attained by students taught using the standard method.

(**1 mark for interpretation)**

Since this confidence interval contains 0, we can be 99% confident that there is insufficient evidence to conclude that there is a difference in the effectiveness of the two teaching styles.

(1.5 marks for stating no difference and the logic)

NOTE: If students chose to analyze Experiment 1 (parts (a), (b), and (c)) based on paired samples, mark the question using the same marking guide as described above with calculated results below. Then dock 5 marks from the total for these 3 parts. For example, if everything was done correctly, but based on independent samples, they should be awarded 10/15.

(a) If the t-test for paired samples is selected, then the assumptions of this test are that there must be paired random samples and that the differences between the paired measurements must be normally distributed or the paired sample size must be large, that is, at least 30.

(b) Estimate: $\bar{d} = \dfrac{\sum d}{n} = \dfrac{77}{20} = 3.8500$

SE(Estimate): $SE(\bar{d}) = s_d \big/ \sqrt{n} = 11.9176 \big/ \sqrt{20} = 2.6649$

$$t = \frac{Estimate - H_0 \text{ value}}{SE(Estimate)} = \frac{\bar{d} - 0}{SE(\bar{d})} = \frac{3.8500 - 0}{2.6649} = 1.445$$

$df = n - 1 = 20 - 1 = 19$

P-value: (0.05) x 2 < P < (0.10) x 2 = 0.10 < P < 0.20. There is weak evidence against H_0.

P-value > α (0.01), therefore do not reject H_0.

Conclusion: At the 5% significance level, there is insufficient evidence to conclude that there is a there is a difference in the effectiveness of the two teaching styles.

(c) $CV = t_{n-1,\alpha/2} = t_{19,0.005} = 2.861$

99% C.I. $\Rightarrow 3.8500 \pm 2.861 \times 2.6649 \Rightarrow$ (-3.774, 11.474)

Experiment 2

The student researchers were not convinced that the results of their first experiment were reliable. Therefore, from the same population, they took a second random sample of 20 students. However, this time, they provided each student with two short courses on two separate topics. Each student was taught one topic using the new teaching style and the other topic using the standard method in a random order. At the end of each course, students did the same exam for the given topic, which was again marked on a continuous, quantitative scale from 0 – 50. To account for any possible difference in difficulty levels between the two topics, all scores were standardized. Again, the student researchers didn't quite know what the correct analysis would be so they first calculated the following statistics:

Difference between means: $(\bar{y}_{\text{New}} - \bar{y}_{\text{Standard}}) = 38.5500 - 35.1500 = 3.4000$

Pooled standard deviation: $s_p = 8.3153$

Mean difference: $\bar{d} = \dfrac{\sum d}{n} = \dfrac{68}{20} = 3.4000$

Standard deviation of the differences: $s_d = 5.2154$

(d) (4 marks) Again, help the student researchers out. Based on the research design described in Experiment 2, what would be the most appropriate hypothesis test to determine if there is a difference in the effectiveness of the two teaching styles (new and standard styles), that is, based on differences between marks that each student obtained with the new teaching style (on one topic) and with the standard method (on the other topic)? Explain why you have chosen this test. Assuming that all the assumptions for the test you have selected are met, state what these assumptions are. (If you think the samples are independent, then assume equal variances.)

The most appropriate test would be the paired-sample t-test. This test should be chosen because this experiment is a paired design since it tests for the differences between marks that each student obtained with the new teaching style and the standard method, which means the marks are paired on the same subjects, the students. The assumptions of this test are that there must be random paired samples. Although the paired observations are dependent, there is independent simple random sampling from one pair to the other. Also, the differences between the paired measurements must be normally distributed or the paired sample size must be large, that is, at least 30. (Since the paired sample sizes is less than 30, then the paired differences must be normal.)

(e) (6 marks) Carry out the most appropriate test (at the 5% significance level), which you selected in part (d), in order to determine if there is a difference in the effectiveness of the two teaching styles, based on differences between marks that each student obtained with the new teaching style (on one topic) and with the standard method (on the other topic). SHOW ALL STEPS of the hypothesis test. Note: you may not need to use all of the statistics (shown above) that the education students calculated.

$H_0 : \mu_d = 0$ (There is no difference in the effectiveness of the two teaching styles.)

$H_a : \mu_d \neq 0$ (There is a difference in the effectiveness of the two teaching styles.)

Parameter: The mean difference, $\mu_d = \mu_{\text{New}} - \mu_{\text{Standard}}$

(1 mark for hypotheses)

Estimate: $\bar{d} = \dfrac{\sum d}{n} = \dfrac{68}{20} = 3.4000$

Standard error of the estimate of the mean difference:

$$SE(Estimate) = SE(\bar{d}) = {s_d}/{\sqrt{n}} = {5.2154}/{\sqrt{20}} = 1.1662$$ **(1 mark for components)**

$$t = \frac{Estimate - H_0 \text{ value}}{SE(Estimate)} = \frac{\bar{d} - 0}{SE(\bar{d})} = \frac{3.4000 - 0}{1.1662} = 2.915$$ **(1 mark for calculating *t*)**

$$df = n - 1 = 20 - 1 = 19$$

P-value: (0.0025) x 2 < P < (0.005) x 2 = 0.005 < P < 0.01. There is very strong evidence against H_0.
Since P-value < α (0.05), reject H_0.

(2 marks for *df, P*-value, evidence, and decision)

Conclusion: At the 5% significance level, there is sufficient evidence to conclude that there is a difference in the effectiveness of the two teaching styles.

(1 mark for conclusion)

(f) (5 marks) Calculate a 95% confidence interval for the difference in effectiveness between the two teaching styles (SHOW ALL STEPS). Based only on this confidence interval, conclude whether there is a difference in the effectiveness of the two teaching styles and explain the logic of your answer.

For a 95% confidence interval, α = 1 – 0.95 = 0.05 and the critical value (CV) is:

$CV = t_{n-1,\alpha/2} = t_{20-1,0.05/2} = t_{19,0.025} = 2.093$ **(1 mark for CV)**

Parameter: The mean difference, $\mu_d = \mu_{\text{New}} - \mu_{\text{Standard}}$

Estimate: $\bar{d} = 3.4000$

Standard error of the estimate of the mean difference: $SE(\bar{d}) = 1.1662$

(calculated in part (e)) **(0.5 marks for restating components)**

Calculation of the confidence interval:

$$Estimate \pm CV \times SE(Estimate)$$

$$\bar{d} \pm t_{n-1,\alpha/2} \times SE(\bar{d})$$

$$3.4000 \pm 2.093 \times 1.1662$$

$$3.4000 \pm 2.4409$$

$$(0.959, 5.841)$$

(1 mark for calculation of CI)

It is estimated with 95% confidence that the difference in effectiveness between the two teaching styles is between 0.959 and 5.841.

OR

It is estimated with 95% confidence that the average mark attained by students taught using the new method is between 0.959 and 5.841 marks higher than the average mark attained by students taught using the standard method. **(1 mark for interpretation)**

Since this confidence interval does NOT contain 0, we can be 95% confident that there is a difference in the effectiveness of the two teaching styles.

(1.5 marks for stating a difference and the logic)

NOTE: If students chose to analyze Experiment 2 (parts (d), (e), and (f)) based on independent samples, mark the question using the same marking guide as described above with calculated results below. Then dock 5 marks from the total for these 3 parts. For example, if everything was done correctly, but based on independent samples, they should be awarded 10/15.

(d) If the *t*-test for independent samples is selected (specifically the pooled *t*-test), then the assumptions are that there are two independent random samples, both samples come from normally distributed populations or both sample sizes are large (at least 30), and, since the samples are independent, then as stated in the question, equal variances are assumed.

(e) Estimate: $(\bar{y}_{\text{New}} - \bar{y}_{\text{Standard}}) = 38.5500 - 35.1500 = 3.4000$

SE(Estimate): $SE(\bar{y}_1 - \bar{y}_2) = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 8.3153\sqrt{\frac{1}{20}+\frac{1}{20}} = 2.6295$

$$t = \frac{Estimate - H_0 \text{ value}}{SE(Estimate)} = \frac{3.4000 - 0}{2.6295} = 1.293$$

$$df = n_1 + n_2 - 2 = 20 + 20 - 2 = 38 \approx 30$$

P-value: (0.10) x 2 < P < (0.15) x 2 = 0.20 < P < 0.30. There is weak evidence against H_0.

P-value > α (0.05), therefore do not reject H_0.

Conclusion: At the 5% significance level, the data do not provide sufficient evidence to conclude that there is a difference in the effectiveness of the two teaching styles.

(f) $CV = t_{n_1+n_2-2,\alpha/2} = t_{38,0.025} \approx t_{30,0.025} = 2.042$

$95\%\text{ C.I.} \Rightarrow 3.4000 \pm 2.042 \times 2.6295 \Rightarrow (-1.969,\ 8.769)$

(g) (4 marks) Did you get the same or different conclusions from the hypotheses tests you conducted in part (b), based on Experiment 1, and part (e), based on Experiment 2. Provide possible reasons why you either got the same conclusions or different conclusions.

Different conclusions are obtained in parts (b) and (e). One possible reason could be that these were two separate experiments based on two different random samples.

However, another possible reason is that, in studies that differentiate by experiment design (independent vs. paired), the paired-sample design is often more powerful in rejecting the null hypothesis than a design based on two independent samples. In this study, it is likely that the variation in performance between students, in terms of intelligence, effort, and ability, was greater than the difference in performance of students taught using the two methods. The paired design tests for the difference between paired observations and thus reduces the effect of variation between individuals.

6. **(Three parts; 12 marks in total)** An experiment was conducted to test the effectiveness of a new drug for the treatment of high blood pressure. Forty people with high blood pressure were randomly allocated to two groups (20 per group), a control group receiving no drug (just a placebo) and a treatment group receiving the new drug. The systolic blood pressure (SBP) of each person was recorded after six weeks. Incomplete SPSS output of the analysis is shown below. The data are normally distributed.

Group Statistics

	Drug	N	Mean	Std. Deviation	Std. Error Mean
SBP	No	20	149.000	9.188	2.055
	New	20	141.500	9.384	2.098

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	*99% Confidence Interval of the Difference*	
									Lower	Upper
SBP	Equal variances assumed	.052	.820	2.554	38	.015	7.500	2.937	-.463	15.463
	Equal variances not assumed			2.554	37.983	.015	7.500	2.937	-.463	15.463

Note: The numbers highlighted in yellow in the tables above were not given in the question.

(a) (4 marks) Suppose you want to test the hypothesis that there is a difference in SBP between the control group and the treatment group, what would be the most appropriate hypothesis test t perform? Describe and check the assumptions of the test you have selected.

The pooled two-sample t-test would be the appropriate test to perform. Since 20 people are randomly assigned to the treatment group and 20 different people are randomly assigned to the control group, the assumptions of independence and randomness are met. The assumption of normally distributed data is met, as stated in the question. Since Levene's test gives a large P-value (P = 0.820), there is no *significant* difference in the standard deviations of the two groups, therefore the assumption of equal standard deviations (equal variances) is met.

(b) (5 marks) Since this is a medical issue, which is crucial, test the hypothesis stated in part (a) at the 1% significance level. Carry out the most appropriate test, which you selected in part (a), in order to determine if there is a difference in SBP between the control and treatment groups. SHOW ALL STEPS of the hypothesis test; however, for the calculation step, make use of the SPSS output.

$H_0 : \mu_C = \mu_T$ (There is no difference in mean SBP between the control and treatment groups.)

$H_a : \mu_C \neq \mu_T$ (There is a difference in mean SBP between the control and treatment groups.)

Parameter: $\mu_C - \mu_T = \mu_{Control} - \mu_{Treatment}$ **(1 mark for hypotheses)**

Estimate = $\bar{y}_1 - \bar{y}_2 = 149.000 - 141.500 = 7.500$

Standard error of the estimate of the difference between two means:

$SE(\bar{y}_1 - \bar{y}_2) = 2.937$ **(0.5 marks for components)**

$$t = \frac{Estimate - H_0 \text{ value}}{SE(Estimate)} = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{SE(\bar{y}_1 - \bar{y}_2)} = \frac{7.500}{2.937} = 2.554$$ **(1 mark for calculating *t*)**

$df = n_1 + n_2 - 2 = 20 + 20 - 2 = 38 \approx 30$ (also given in the SPSS output)

P-value: (0.005) x 2 < P < (0.01) x 2 = 0.01 < P < 0.02. There is strong evidence against H_0.
However, P-value > α (0.01), therefore do not reject H_0.

(1.5 marks for *df, P*-value, evidence, and decision)

Conclusion: At the 1% significance level, the data do not provide sufficient evidence to conclude that there is a difference in mean SBP between the control and treatment groups.

(1 mark for conclusion)

(c) (3 marks) The researcher was certain that it is impossible that the new drug could increase SBP, but that there is a possibility that it could reduce SBP. Therefore, at the 1% significance level, test the hypothesis that mean SBP in the treatment group was lower than in the control group. You do NOT need to perform all steps. Only rewrite the hypotheses and then state the P-value, the strength of the evidence against H_0, and the conclusion.

$H_0 : \mu_C = \mu_T$ (Mean SBP of the treatment group was not different from that of the control group.)

$H_a : \mu_C > \mu_T (Or, \mu_T < \mu_C)$ (Mean SBP of the treatment group was lower than in the control group.)

(1 mark for hypotheses)

P-value = 0.01 > P > 0.005. There is very strong evidence against H_0.

(1 mark for *P*-value and decision)

Conclusion: At the 1% significance level, the data provide sufficient evidence to conclude that mean SBP in the treatment group was lower than in the control group.

(1 mark for conclusion)

7. (Four parts; 10 marks in total) A business survey randomly sampled 20 different hotels and 20 different log cabins in British Columbia and recorded each of their nightly rates. The survey provided the following summary statistics regarding nightly rates in Canadian dollars.

Summary statistic	Hotel rates	Log cabin rates	Differences
Average	102.05	290.28	-188.23
Standard deviation	15.50	52.05	48.96

Note: Please note that the third column summarizes the differences between the original observations. By choosing a test, you will be using certain columns of the above table, but not all of them.

(a) (1 mark) Are the two samples independent of each other or paired? Why?

The two samples are independent because the observations are not paired and/or linked to each other in any way.

(b) (2 marks) Would it be appropriate to carry out a test using the summary statistics presented in the table above? Explain your answer. What options could you consider?

Since the standard deviations for the two samples are 15.50 and 52.05, it would be inappropriate to use the t-tools assuming equal variances to analyze this data since 52.05/15.5 > 2. We can consider a transformation. We could also carry out a t-test not assuming equal variability (the nonpooled t-test).

(c) (5 marks) Suppose that it was decided that a natural log transformation was needed in order for the data to satisfy the assumptions for the t-tools. The table below is the summary statistics for the natural logs of nightly rates. Calculate the 90% confidence interval for the difference in mean logged nightly rates between hotels and log cabins (SHOW ALL STEPS). Back transform this confidence interval to the original scale and interpret this interval on the original scale.

LnNightlyRate

Type	Mean	N	Std. Deviation
Hotel rates	4.6152	20	.14510
Log cabin rates	5.6555	20	.18008
Total	5.1353	40	.55099

Let group 1 be log cabins and group 2 be hotels.

For a 90% confidence interval, α = 1 – 0.90 = 0.10 and the critical value (CV) is:

$$CV = t_{n_1+n_2-2,\alpha/2} = t_{20+20-2,0.10/2} = t_{38,0.05} \approx t_{30,0.05} = 1.697$$

(1 mark for CV)

Parameter: $\mu_{\ln Y_1} - \mu_{\ln Y_2}$

Estimate: $\overline{LnY}_1 - \overline{LnY}_2 = 5.6555 - 4.6152 = 1.0403$

Standard error of the estimate of the difference between two means:

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(19)(0.18008^2) + (19)(0.14510^2)}{19+19-2}} = 0.16353$$

$$SE(\overline{LnY}_1 - \overline{LnY}_2) = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 0.16353\sqrt{\frac{1}{20}+\frac{1}{20}} = 0.05171$$

(1 mark for components)

Calculation of the confidence interval:

$$Estimate\ \pm CV \times SE(Estimate)$$
$$(\bar{y}_1 - \bar{y}_2) \pm t_{n_1+n_2-2,\alpha/2} \times SE(\bar{y}_1 - \bar{y}_2)$$
$$1.0403 \pm 1.697 \times 0.05171$$
$$1.0403 \pm 0.08775$$
$$(0.953, 1.128)$$

(1 mark for calculation of CI)

Back transformation: $(e^{0.953}, e^{1.128}) = (2.592, 3.090)$ as a 90% confidence interval on the original scale.

(1 mark for back transformation)

Interpretation on the original scale: We can be 90% confidence that the median nightly rate for log cabins is between 2.592 and 3.090 times the median nightly rate for hotels (in dollars).

(1 mark for interpretation)

Alternatively, if the parameter was chosen to be $\mu_{\ln Y_2} - \mu_{\ln Y_1}$:

For a 90% confidence interval, α = 1 – 0.90 = 0.10 and the critical value (CV) is:

$$CV = t_{n_1+n_2-2,\alpha/2} = t_{20+20-2,0.10/2} = t_{38,0.05} \approx t_{30,0.05} = 1.697$$ **(1 mark for CV)**

Estimate: $\overline{LnY}_2 - \overline{LnY}_1 = 4.6152 - 5.6555 = -1.0403$

$SE(\overline{LnY}_2 - \overline{LnY}_1) = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 0.05171$ (calculated as above) **(1 mark for components)**

Calculation of the confidence interval:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{n_1+n_2-2,\alpha/2} \times SE(\bar{y}_1 - \bar{y}_2)$$
$$-1.0403 \pm 1.697 \times 0.05171$$
$$-1.0403 \pm 0.08775$$
$$(-1.128, -0.953)$$

(1 mark for calculation of CI)

Back transformation: $(e^{-1.128}, e^{-0.953}) = (0.324, 0.386)$ as a 90% confidence interval on the original scale.

(1 mark for back transformation)

Interpretation on the original scale: We can be 90% confidence that the median nightly rate for hotels is between 0.324 and 0.386 times the median nightly rate for log cabins (in dollars).

(1 mark for interpretation)

(d) (2 mark) Based on the confidence interval you found in part (c), after back transformation, would you conclude that there is a difference between the median rates for hotels and log cabins (with 90% confidence)? Explain the logic of your answer.

After back transformation to the original scale, since the confidence interval, which is (2.592, 3.090) or (0.324, 0.386), does not contain 1, that indicates we can be 90% confident that there is a difference between the median rates for log cabins and hotels.