

Statistics 252 – Final Exam – Version A

Instructor: Paul Cartledge

Instructions:

1. Read all the instructions **CAREFULLY**.
2. This is a closed book exam.
3. You may only use the formula sheets, the output provided, and a non-programmable calculator.
4. You have 3 hours to complete the exam.
5. The exam is out of a total of 80 marks and has 14 pages (in two parts).
6. Show your work in all sections to receive full credit.
7. Use the reverse side of pages for scrap work.
8. Make sure your name and signature are on the front and that your ID number is on the top of page two.
9. All biological examples approved by a quasi-medical professional.
10. When referring to “log”, I am always referring to the natural log.
11. Unless instructed otherwise, give a range for the p -value. Also, use the “judgment approach” to help state your conclusion in plain English.
12. When asked for a “confidence interval”, state the estimate, the standard error, and the critical value. Then, calculate and interpret the interval.
13. When asked to “carry out a full analysis in detail”, set up the hypotheses, calculate the test statistic, state the distribution of the test statistic (i.e. t_9 or $F_{3,10}$), find the p -value (or its range), and state your conclusion in plain English.

Name: _____

Signature: _____

Component	Notes	Worth	Mark
Short Answer	3 questions	8	
What test?	2 questions	6	
Case Study 1	7 parts	31	
Case Study 2	7 parts	35	
Total		80	

ID: _____

Short Answer Problems (8 marks)

Question 1 (2 marks) In testing for five equal means, you set up the following set of hypotheses. Is there something wrong? If so, correct the mistake by simply re-writing the hypotheses. If not, say why there is nothing wrong.

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = 0$$

$$H_A: \text{at least one } \mu_i \neq 0 \quad (i = 1, \dots, 5)$$

Question 2 (3 marks) Suppose Harry Potter determines a simple linear relationship between his grade average (in %) each year at Hogwarts School of Witchcraft and Wizardry and how many hours he spends studying per week. Checking assumptions, the response variable required a log-transformation. The estimated regression line is $\hat{\mu}(\ln(\text{grade}) | \text{study}) = 4.587 - 0.0300\text{study}$. Estimate the change in grade associated with a change of 3 to 5 hours of studying per week. Give a statement relating this change to the given variables. Also, predict the grade for first value.

Question 3 (3 marks) Suppose an ANOVA table for simple linear regression gave a test statistic of 8.423. If the standard deviation of the errors is 2.52 and there are 50 observations, what is the coefficient of determination? [Hint: $R^2 = SS(\text{Extra})/SS(\text{Total})$]

What test would you use? (6 marks)

In each scenario, identify the appropriate procedure needed to answer the question. Be as descriptive as possible.

Choose from the following:

- i) One-Sample t-test for a single population mean,
- ii) Paired t-test for the difference between two population means,
- iii) Two Independent Sample t-test for the difference between two population means,
- iv) One-Factor ANOVA F-test for any difference among I population means,
- v) A t-test for a linear combination of means,
- vi) Some Extra-Sum-of-Squares F-test comparing two models for the I population means,
- vii) An ANOVA F-test for any regression model effects,
- viii) A t-test for a single regression coefficient,
- ix) Some Extra-Sum-of-Squares F-test comparing two regression models (testing a subset of coefficients),
- x) An F-test for any factor effects (main OR interaction) in Two-Factor ANOVA,
- xi) The F-test for additivity in a Two-Factor ANOVA.

Question 4 (3 marks) An animated biologist in need of money is trying to see if he can get frogs to sing. Randomly sampling 20 frogs in a nearby swamp, 20 frogs in a swamp 2 km away, and 20 frogs in a swamp 20 km away, he measures each frog's pitch level. In testing to see if the closest frogs have a different average pitch level than the frogs further away, what test would you use? What is the distribution of the test statistic?

Question 5 (3 marks) A local irate samurai is known to businesses as an expert in "fixing" aggravating photocopiers. For future potential customers, he decides to prove his worth by randomly sampling 60 photocopiers and measuring their performances, before and after he has "fixed" them. What test would you use to assist this samurai? What is the distribution of the test statistic?

Case Study 1 – Music written by that guy... (25 marks)

When needed, use the output on pages 11 and 12 to answer the following questions.

In class, we determined that everyone’s movie knowledge could be more awesome, especially when it comes to film music composer, John Williams. Suppose an observational study was done to investigate 58 random films where John Williams composed the music, where the response variable is the film’s rating at IMDB.com (*Rating*, measured on a scale of 0 to 10, with 10 being the highest). The film’s rating, the film’s length (*Runtime*, measured in minutes), and budget (measured in millions of US\$, but requiring log transformation) are modeled as continuous (numerical) variables. “Director credit” is categorized into 3 levels: Steven Spielberg, George Lucas, and Other. Each film was also categorized (*Nom*) by whether or not it received an Academy Award nomination for Best Music Score.

To fit an MLR model, the categorical variable *Dir* uses the first two levels listed to correspond to indicator variables. Use the following “original model” to answer questions.

$$\begin{aligned} \mu(Rating \mid Runtime, \ln(Budget), Dir, Nom) = & \beta_0 + \beta_1 Runtime + \beta_2 \ln(Budget) \\ & + \beta_3 Dir1 + \beta_4 Dir2 + \beta_5 Nom + \beta_6 Runtime \times \ln(Budget) + \beta_7 Runtime \times Dir1 \\ & + \beta_8 Runtime \times Dir2 + \beta_9 Runtime \times Dir1 \times Nom + \beta_{10} Runtime \times Dir2 \times Nom \end{aligned}$$

a) (3 marks) What is the effect of runtime on mean rating, after accounting for director credit, Academy Award nomination, and $\ln(\text{budget})$?

b) (4 marks) What is the effect of director credit on mean rating for each listed pair of levels below, after accounting for runtime, Academy Award nomination, and $\ln(\text{budget})$? (Hint: If you need more room, please direct me to where you did your work...perhaps the back of page 3?)

Level 1	Level 2	Effect of director credit on mean rating
Steven Spielberg	George Lucas	
Steven Spielberg	Other	

c) (3 marks) Using the original model, state the null and alternative hypothesis to test whether Academy Award nomination has any effect on mean rating, after accounting for runtime, director credit, and $\ln(\text{budget})$. What is the distribution of the test statistic under the null hypothesis?

Note: For parts d) – g), remove all interaction terms from the “original model”.

d) (4 marks) Calculate a 90% confidence interval for the effect of changing budget from 50 to 100 million \$US on mean rating.

e) (5 marks) Carry out a full analysis in detail to determine if $\ln(\text{budget})$ has a negative association with mean rating, after accounting for runtime. You must provide the *exact* p-value for this analysis.

f) (4 marks) Calculate a 98% prediction interval for the rating of a film directed by Oliver Stone that has a runtime of 150 minutes, a budget of \$US 30 million, and had an award nomination.

g) (8 marks) Carry out a full analysis in detail to determine if mean rating depends on award nomination or director credit, after accounting for $\ln(\text{budget})$ and runtime. You must also clearly label the sum-of-squares residuals for the models under the null and alternative hypotheses.

Case Study 2 – Airline food: SURELY it's good for you? (25 marks)

When needed, use additional output on page 13 to answer the following questions.

Dr. Rumack can't tell if people are getting sick via airline food. Randomly sampling months of the year, he observes how many passengers are getting sick for each airline, also differentiating by another factor: what type of food was ordered by at least 50% of the passengers. The table below summarizes the results.

Group	Airline	Food	n	Sample Mean	Sample S.D.
1	Oceanic	Steak	10	30.18	2.72
2	Oceanic	Fish	10	33.23	2.58
3	Qantas	Steak	10	30.44	4.47
4	Qantas	Fish	10	31.37	3.68
5	Northwest	Steak	10	27.54	3.53
6	Northwest	Lasagna	10	22.68	4.11
7	Trans American	Steak	10	33.44	3.74
8	Trans American	Fish	10	48.17	4.98

The following table is the ANOVA output.

ANOVA					
NumPass					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3778.180				
Within Groups	1040.736				
Total	4818.916				

a) (4 marks) Is there any evidence of a difference in the average number of sick passengers among the eight different groups? State the sum-of-squares residuals for the models under the alternative hypothesis, the test statistic, and the distribution of the test statistic (you do not have to find the p -value or answer the question). Also, estimate the common standard deviation.

SSR for the model under H_A : _____

Test Statistic: _____

Distribution of test statistic: _____

Common Standard Deviation: _____

There is a treatment contrast that might be of interest for estimating the main effects of the two factors on the mean number of sick passengers. Let $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7$ and μ_8 correspond to the population mean responses for groups 1, 2, 3, 4, 5, 6, 7 and 8, respectively.

b) (6 marks) Does the average number of sick passengers decrease if the type of food ordered by at least 50% of the passengers is steak instead of fish?

i. (2 marks) First, define the treatment contrast (i.e. fill in the blanks with the appropriate contrast coefficients) that will define the effect described in the above question.

$$\gamma = \underline{\hspace{1cm}}\mu_1 + \underline{\hspace{1cm}}\mu_2 + \underline{\hspace{1cm}}\mu_3 + \underline{\hspace{1cm}}\mu_4 \\ + \underline{\hspace{1cm}}\mu_5 + \underline{\hspace{1cm}}\mu_6 + \underline{\hspace{1cm}}\mu_7 + \underline{\hspace{1cm}}\mu_8$$

ii. (2 marks) Next, define the null and alternative hypotheses for this contrast.

iii. (2 marks) Then, determine the test statistic and the *exact* p -value.

Dr. Rumack decided to be safe and eat lasagna on a Northwest flight, but his lasagna was a bit fishy, so he got sick. Enlisting the help of research assistants Shirley and Leslie, the lasagna was determined to have large concentrations of fish, so group 6 was then re-categorized as Fish. NOTE: This re-categorization does NOT affect how you answered parts a) and b).

Nevertheless, the new data structure allowed another approach to test the effects of the two factors, as well as their interaction: Two-Way ANOVA with two factors.

Factor A – Airline (Northwest, Oceanic, Qantas, Trans American)

Factor B – Food (Fish, Steak)

c) (8 marks) Use the following incomplete Two-Way ANOVA table.

Tests of Between-Subjects Effects

Dependent Variable: NumPass

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3778.180 ^a				.000
A	2525.281		841.760		.000
B	239.339		239.339		.000
A * B	1013.559				.000
Error	1040.736				
Corrected Total	4818.916				

a R Squared = .784 (Adjusted R Squared = .763)

i. (6 marks) Carry out a full analysis in detail to determine if Airline or Food has any effect on the mean number of sick passengers.

ii. (2 marks) Calculate the sum-of-squares residuals for the following models:

1. Two-way ANOVA for Airline and the interaction of Airline and Food.
2. One-way ANOVA for only Food.

Delirious from the sickness, Dr. Rumack demanded the removal of the interaction term.

d) (7 marks) The Two-Way ANOVA table is given below (Additive Fit)
Tests of Between-Subjects Effects

Dependent Variable: NumPass

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2764.620 ^a	4			.000
A	2525.281	3			.000
B	239.339	1			.004
Error	2054.296	75	27.391		
Corrected Total	4818.916	79			

a R Squared = .574 (Adjusted R Squared = .551)

i. (4 marks) Determine if Airline has an effect on the mean number of sick passengers, after accounting for Food. Only provide the null and alternative hypotheses, the test statistic, and the *p*-value.

ii. (3 marks) Determine if Food has an effect on the mean number of sick passengers, after accounting for Airline. Only provide the test statistic, its distribution, and the *p*-value.

Yet another approach to test the effects of the two factors is to model the data as a multiple linear regression model using indicator variables:

Let Airline be represented by three indicator variables (Q , NW , and TA) that will indicate Qantas, Northwest, and Trans American, respectively; Oceanic is the “default”. Let Food be represented by one indicator variable ($Steak$); Fish is the “default”.

The corresponding regression model is:

$$\mu(\text{NumPass} \mid \text{Airline}, \text{Food}) = \beta_0 + \beta_1 Q + \beta_2 NW + \beta_3 TA + \beta_4 \text{Steak} + \beta_5 Q \times \text{Steak} \\ + \beta_6 NW \times \text{Steak} + \beta_7 TA \times \text{Steak}$$

e) (4 marks) In terms of the coefficients, what is the effect of Airline on the mean number of sick passengers, after accounting for Food for each pair of levels? Show your work.

Fill in the table:

Level 1	Level 2	Effect of Airline on mean response
Qantas	Trans American	
Oceanic	Northwest	

f) (4 marks): Determine if the average number of sick passengers for flights from the *airline* in the first column, where the type of food ordered by at least 50% of the passengers is the *food* in the first column, is **lower** than the average number of sick passengers for flights from the *airline* in the second column, where the type of food ordered by at least 50% of the passengers is the *food* in the second column. Only provide the appropriate test statistic and *exact* p -value.

		Test statistic	p -value
Qantas & fish	Oceanic & fish		
Oceanic & steak	Oceanic & fish		

g) (2 marks): Rewrite the regression model above with Trans American and Fish as the respective default levels.

VERSION A SPSS OUTPUT

Output for Case Study 1:

Regression on lnBudget

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.372	1	.372	.427	.516 ^a
	Residual	48.758	56	.871		
	Total	49.130	57			

a Predictors: (Constant), lnBudget

b Dependent Variable: Rating

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	6.868	.372		18.443	.000
	lnBudget	.068	.104		.654	.516

a Dependent Variable: Rating

Regression on lnBudget and Runtime

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	8.631	2	4.315	5.861	.005 ^a
	Residual	40.499	55	.736		
	Total	49.130	57			

a Predictors: (Constant), lnBudget, Runtime

b Dependent Variable: Rating

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.904	.679		7.219	.000
	lnBudget	-.121	.111		-1.088	.281
	Runtime	.019	.006		3.349	.001

a Dependent Variable: Rating

Regression on InBudget, Runtime, Nomination, and Director Credit

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	25.303	5	5.061	11.044	.000 ^a
	Residual	23.827	52	.458		
	Total	49.130	57			

a Predictors: (Constant), InBudget, Runtime, Nom, Dir1, Dir2

b Dependent Variable: Rating

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.116	.550		9.301	.000
	InBudget	-.153	.096		-1.598	.116
	Runtime	.012	.005		2.426	.019
	Nom	.646	.195		3.318	.002
	Dir1	1.089	.378		2.881	.006
	Dir2	.942	.198		4.750	.000

a Dependent Variable: Rating

Regression on Nomination and Director Credit

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	22.560	3	7.520	15.284	.000 ^a
	Residual	26.569	54	.492		
	Total	49.130	57			

a Predictors: (Constant), Nom, Dir1, Dir2

b Dependent Variable: Rating

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	6.177	.167		37.071	.000
	Nom	.573	.193		2.970	.004
	Dir1	.990	.378		2.619	.011
	Dir2	1.134	.188		6.040	.000

a Dependent Variable: Rating

Output for Case Study 2:

Linear Combinations of Means

Contrast Coefficients

Contrast	Group							
	1	2	3	4	5	6	7	8
1	1	1	1	1	-1	-1	-1	-1
2	3	-4	3	-4	3	0	3	-4
3	1	0	1	0	1	-4	1	0
4	3	-5	3	-5	3	3	3	-5
5	0	-1	0	-1	0	3	0	-1
6	1	-1	1	-1	1	-1	1	-1

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
NumPass	Assume equal Variances	1	-6.614	3.401	-1.945	72	.056
		2	-86.250	11.019	-7.827	72	.000
		3	30.898	5.377	5.746	72	.000
		4	-130.986	13.170	-9.945	72	.000
		5	-44.736	4.165	-10.741	72	.000
		6	-13.838	3.401	-4.069	72	.000

Regression on Airline, Food, and their interaction terms

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	33.226	1.202		27.636	.000
	Q	-1.857	1.700		-1.092	.278
	NW	-10.551	1.700		-6.205	.000
	TA	14.941	1.700		8.787	.000
	Steak	-3.051	1.700		-1.794	.077
	QxSteak	2.122	2.405		.883	.380
	NWxSteak	7.915	2.405		3.292	.002
	TAxSteak	-11.672	2.405		-4.854	.000

a. Dependent Variable: NumPass