Distributed Slack Bus

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I. Introduction

Load flow is the most important element for the power system. By solving load flow equations in a network under the steady date, each bus, i.e, each node's state inside the grid can be determined. The load flow study solves the node's voltage and phase angles. The conventional load flow studies are based on the power-balance equations. However, the conventional power flow analysis has two significantly drawbacks because of the single slack bus algorithm used in modeling. The first negative aspect is that it is unrealistic for the power system to put all loses on one generator. The second negative aspect is for the Economic Load Dispatch problems only. The economic load dispatch schema solves for a dispatch scheduler in which all generators have the same incremental cost, but due to the existence of a slack bus, one generator's generation is determined after the calculation. In the past studies, researchers identifies this challenge as the slack bus burden. Up to now, there exists some studies to solve this burden and using the technique on other applications. One way is through the Automatic Generation Control(AGC) as shown in [2], [1], and [8], and the second way is through including the loss term in the Newton-Raphson power flow equations. In this report, both the AGC and the new NR power flow techniques will be explained with modified distributed slack bus algorithms; Advantages and drawbacks are compared between conventional single slack bus algorithm and the modified algorithm in each case study categorised by different participation factor definitions. and discussed with case studies' results. The report will also state the importance of using distributed slack bus models in certain special situations.

II. The Slack Bus

In the load flow equations, normally there are three type of buses. The three types of nodes are generator loads, bus loads and slack buses. The existence of the slack bus is for two purposes. First, as the grid is interconnected, one critical factor is the phase differences between different buses. One bus is needed to be specified as the zero angle. Second, since load-flow equations are power-balanced, the system's losses will need to be dumped into some buses. As there are real and reactive losses in the system, the slack bus, where both PQ are not specified, is chosen. The conventional single slack bus method is leaving one generator's P unspecified and δ specified, and then all degree of freedom for load flow equations are met and hence the equations are solvable.

III. Economic Load Dispatch With the Loss term

A. Problem Model

1) Forming and Solving ELD: Economic Load Dispatch(ELD) problems, is the study that searching for the most economical solution among all feasible load flow setups. Most economical solution refers to the scheme where the generation cost is minimized and all constraints are satisfied.

The conventional economic load dispatch problem is as follows:

$$\begin{aligned} &\min \sum_{i=1}^{\mathrm{NG}} C_i\left(\mathrm{PG}_i\right) \\ &\text{s.t.} \quad \sum_{i=1}^{\mathrm{NG}} \mathrm{PG}_i - \mathrm{PD}_{\mathrm{TOTAL}} = 0 \end{aligned} \tag{1}$$

In equation 1, PG_i is the amount of generation at the ith generator, C_i is the cost function used to describe the ith generator. PD_{TOTAL} is the summation of all the loads. The cost function is expressed as a quadratic function which

$$C_i = \alpha_i PG_i^2 + \beta_i PG_i + c_i \tag{2}$$

B. Distributed Slack Bus Algorithm

1) Algorithm: As stated in section II, the slack bus is introduced for mathematical purposes only. The ELD problem has some aspects in physical architectures that can not be gave up for mathematical convenience. To modify the conventional ELD problem with distributed slack bus taking into consideration, a new term ΔP_{IB} is added to equation 1. With this new term, equation 1 now becomes:

$$\begin{aligned} &\min \sum_{i=1}^{\mathrm{NG}} C_i\left(\mathrm{PG}_i\right) \\ &\text{s.t.} \quad \sum_{i=1}^{\mathrm{NG}} \mathrm{PG}_i - \mathrm{PD}_{\mathrm{TOTAL}} - \Delta P_{IB} = 0 \end{aligned} \tag{3}$$

The new PG_i is defined in equation 4 and the power flow equation for the active power at the ith generator is rewritten in equation 5.

$$PG_i = PG_{i_{convl}} + pf_i \Delta P_{IB} \tag{4}$$

$$P_i = \sum_{j=1}^{n} |V_i\|V_j\|Y_{ij}| \cos(\delta_i - \delta j) + pf_i \Delta P_{IB} \tag{5}$$

The new term will remove the burden on the slack bus, but the system will still need a node as the reference angle. This node will only have mathematical meanings. Because the new load imbalance term mainly effects the real power, the reactive power terms in the Jacobian matrix will remain unchanged. From equation 5 ,The new real power NR matrix is shown in the equation 6

$$\begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_{(n-1)}} & \frac{\partial P_1}{\partial \delta \Delta P_{IB}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_n}{\partial \delta_1} & \cdots & \frac{\partial P_n}{\partial \delta_{(n-1)}} & \frac{\partial P_n}{\partial \delta \Delta P_{IB}} \end{bmatrix} \begin{bmatrix} \Delta \delta_1 \\ \vdots \\ \Delta \delta_{n-1} \\ \Delta \delta \Delta P_{IB} \end{bmatrix} \tag{6}$$

Pointing back to equation 4, The term $PG_{i_{convl}}$ is the result of power generated in the conventional ELD algorithm, the pf_i is the participation factor. In the modified algorithm, the $PG_{i_{convl}}$ will be calculated first by conventional power flow algorithm. $\Delta P_{IB}$is the losses coming from load power imbalance calculated after the δ, V are known from the conventional algorithm. With the participation factor introduced, PG_i is finally calculated according to equation 4.

The ΔP_{IB},in other words, is the burden on the slack buses. Participation factor refers to how the system is redistributing the burden for every generator participated in the system. It is just a weighted algebraic ratio. It can be defined in different ways to best match the physical system architectures. But it must be summed up to 1 since redistributing the losses can not reduce the losses. Also, this means that only generators have participation factors, so that the bus not participates in the system will have a zero participation factor. One important usage of distributed slack bus power flow is to remove the burden on single slack bus, and this will be illustrated in part 2 below.

2) Burden On Slack Bus: To maintain the "equal incremental cost" after the power flow calculation, it is impossible if there is a slack bus. In conventional power flow calculations, the loss put on the slack bus can be calculated as the follow:

$$P_{\text{slack}} = P_{\text{loss}} - \left(\sum_{i \neq \text{ slack}} P_i + \sum_{i \in D} P_i \right) \tag{7}$$

D is the set of the load buses, and the previous term is the sum of loss for all the generator except the slack bus. But by using the distributed slack bus algorithm, the equation 7 is revised into:

$$(P_{\text{slack sch}} + \Delta P_{\text{IB}}) + \sum_{i \neq \text{ slack}} P_i + \sum_{i \in D} P_i = P_{\text{loss}} \tag{8}$$

where $P_{slack} = P_{slack_{sch}} + \Delta P_{IB}$ and $P_{slack_{sch}}$ is the generation in the conventional ELD step. Now substitute P_i according to equation 4, equation 8 will become:

$$\sum_{i \in G} (P_i + \text{pf}_i \Delta P_{\text{IB}}) + \sum_{i \in D} P_i = P_{\text{loss}} \tag{9}$$

Where G is the set of generator buses. Now the loss term is entirely based on each generator and load, no additional loss term is given for the slack bus. Hence the burden is removed.

3) Participation Factor:

a) Based on The Distance: For physical system with consideration on transmission losses, the distance based participation factor can be used to improve efficiency. In a real system, it is impossible for all the generators be presented at an equal distance hence even though each generator will have same incremental fuel production cost, the cost at the demand point will be different. To improve this, a participation factor based on distance is used as follow in [6]:

$$pf_i = \frac{L_i P_{Gi}^{\text{Load}}}{\sum_{j=1}^{m} L_i P_{Gi}^{\text{load}}} \tag{10}$$

Here L_i is the penalty factor of the bus, this participation factor is simply stating that the generator with closer distance to the demand point will allocate more losses since their transmission loss cost is lower. A case study using this participation factor is performed in [6] to compare the improvements from the conventional power flow. The total generation cost before and after using the new algorithm is compared in the table below:

	Total Gen. Cost ($/MWh)	Total Gen. (MW)
Before	1631.23	292.64
New	1416.48	291.8

TABLE I
COMPARISON IN TOTAL INCREMENTAL COST BEFORE AND AFTER THE NEW POWER FLOW

The results show that there is a saving in the total generation cost. But as mentioned in [6], this improvement requires a way more complicated bidding process. It is a doubt that whether the benefit gained can cover the cost of communication and extra process modification costs. More comprehensive tests are needed.

b) Based on The Cost Function: When a cost function is approximated as equation 2, it is understandable that the generator with less cost will allocate more losses. In [8], the participation factor is defined as:

$$pf_i = \frac{1}{2\alpha_i \sum_{i=1}^{m} \frac{1}{2\alpha_i}} \tag{11}$$

In [3], a case study is performed to study the differences before and after using equation 11. The factor of number of generators is also studied for the new power flow. The results are shown in table II. There are minor improvements, but not significant.

	14 bus total cost($)	39 bus total cost($)
Before	2477.62	288597.62
New	2475.62	288506.44

TABLE II
COMPARISON IN COST MINIMIZATION FOR CONVENTIONAL AND NEW POWER FLOW

It is understandable since cost function is also considered in the single slack bus power flow.Also, the result still shows a pattern that the saving increases with the number of generators. The most important outcome of the study in [3] should be that the real equal incremental cost is achieved. As shown in table

3 in [3], generator bus 1 and 3 have the same cost function, but in conventional power flow algorithm, bus 1 is the slack bus so the generation output is higher than bus 3. The transmission loss is not considered in this scenery, so it makes no economic sense for . After using the distributed slack bus, the generators with same cost functions will have same generation output for sure.

c) Based on The Generator Power of The Buses: By using this participation factor, the single slack bus can be considered as a special case in distributed slack bus algorithm which the participation factor for the slack bus is set to 1. It is understandable that the larger the generator, the more efficient it becomes for a constant loss. With the increase in size, cost of production decreases due to the marginal effects. In fact, this is how most power systems in real world solve the problem. After solving the load flow, all the losses will be allocated on the generator with the largest generation output. But this is not optimal, for example, as shown in [4], if the system take generator power adjustment(GPA) variation into consideration the dispatch can be more economical.

The GPA is very straight-forward, as in [4], it is defined as:

$$pf_i = \frac{P_{Gi}}{\sum_{i=1}^{m} P_{Gi}} \tag{12}$$

Where $P_{Gi}$is the generation output level for bus i under the single slack bus load flow algorithm. Some case studies are performed in [5] to compare the conventional method and the new power flow. The results are shown in TABLE III. In the conventional power flow, plant 1 is the slack bus. It has a generation capacity way higher than the other two plants. This is the case where the participation factor is close to 1. There is about 0.2% changes in the total generation cost. This is happened because of the participation factor introduced.

	Before	New
Plant 1 output (MW)	23.649	23.646
Plant 2 output (MW)	69.518	69.52
Plant 3 output (MW)	58.99	58.992
System loss (MW)	2.1569	2.1568
Total Generation cost ($)	1596.96	1593.12

TABLE III
RESULT COMPARING FOR A 5 BUS SYSTEM

4) Importance of distributed slack bus algorithm on Economic Optimal Dispatch Scenario: With the discussion in section 3, it can be seen that the involvement of participation factor in the power flow equations does not improve the dispatching results comparing to conventional single slack bus power flow. But the Economic Load Dispatch doe not only care about generation outputs, it also cares sensitivity, and sensitivity even means more in nowadays with more resources and money invested in the electricity market. Consider the problem of the Economic Optimal Dispatch. If a system operator knows where generation changes are to take place, then a heavily weighted participation factor can readily be defined on these locations and an optimized λ for the entire system can be defined. But if the locations of changes are not specifically known, for example, the operator only knows that a unit change for the entire system is required, then equation 12, might be a better choice of participation factors. A case study is performed in [4] and the results are shown in Table 1 in [4]. In the case study, the operator knows that the change will occur at the slack bus, generator 2. So the system-λ is even smaller than the GPA method. But if the operator does not know the specific location, for example, if the operator assigned generator 1 to take the system change, then the system-λ will dramatically change to about 30 % higher than GPA's λ.

The system-λ represents the minimal incremental cost for each generator under the situation of system demand changes, and this can not be calculated through single slack bus algorithm unless the system change occurs on the slack bus, which does not happen every time.

IV. Automatic Generation Control

A. Problem Model

On the electric grid, it can difficult to match the supply of electric power to the minute to minute change in demand for electric power. To facilitate this balance, the independent system operator (ISO) will purchase automatic generation control (AGC), spinning reserves, and non-spinning reserves from the ancillary services [2]. AGC is portion of the generation capacity of an machine or set of machines belonging to the ancillary services that is reserved for receiving secondary frequency regulation from ISO in order to compensate for those rapid load changes. In a traditional single slack bus model of electric power transmission there are two methods of compensating for the mismatch between scheduled power supply and actual power demand: a single machine per control area can compensate for that power imbalance [3] or all the the machines in an area can have their power output ramped to drive the area control error to "cross zero" periodically [2]. This continuously controls the amount of power entered or exiting a zone to the quantity agreed up by service providers.

B. Participation Factor

In [3] and [2], it is proposed that a distributed slack bus model be used to distributed power imbalance to all machines in the system based up their participation factor. This participation factor would be the "weighted average of AGC accepted quantities in the ancillary services market" [2]. Using this participation factor is impartial and unambiguous due to being agreed upon by the service providers and optimized during the economic dispatch calculation. The power dispatch equations is:

$$PG_i = PG_i^{\circ} + \pi_i \psi \tag{13}$$

such that

$$\sum_{i=1}^{NG} \pi_i = 1 \tag{14}$$

where PG_i is the actual power dispatched, PG_i° is the scheduled power from the economic load dispatch problem,

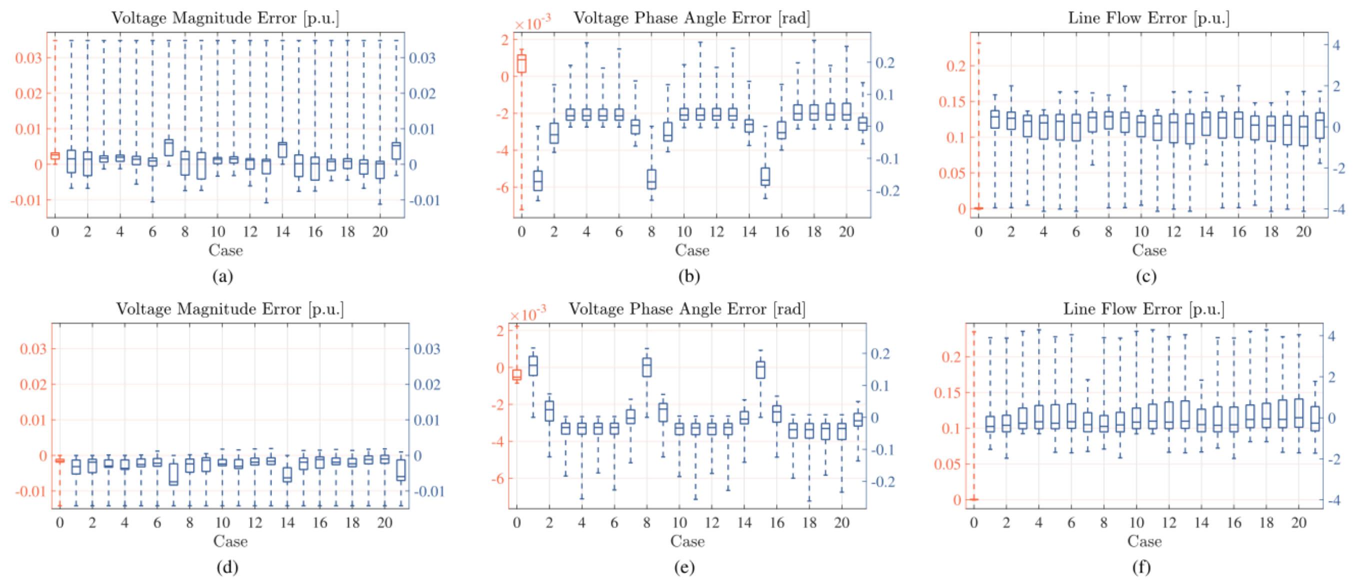

Fig. 1. "Voltage Magnituted and Phase Angle error. Red plot and left axis is for the distributed slack bus model and blue plot and right axis is for an exhastive list of possible single slack busses."

π_i is the participation factor, ψ is the net-load imbalance for the control area, and NG is the number of generators in the control area.

C. *Case Study and Results*

While single slack bus models have be reliable and robust in providing automatic generation control, the distributed slack bus models show a number of benefits. [3] used the New England 39-bus, 10-machine test system divided into two control areas. A uniform 10% increase and decrease in system loads was simulated and the voltage magnitude and phase angle of the distributed slack bus model and an exhaustive list of the possible single slack busses was compared to the differential algebraic equation (DAE) model. The distributed slack bus model showed an order of magnitude reduction in the voltage and phase angle error (Fig.1).

[2] used the IEEE-30 bus test system with a 1% increase in system loads. The total real load of the system was 286.23 MW after uniform increase in load. The real power produced by all machines was 289.18 MW and 289.15 MW in the single slack bus model and distributed slack bus model, respectively. This means the distributed slack bus model was able to reduce the real power loss from 2.95 MW to 2.92 MW, approximately a 1% reduction real power loss.

Compared to the single slack bus model with ramped control, the distributed slack bus model shows 1% less real power loss for a 1% increase in system load [2]. Compared to the single slack bus model with single machine area control, the distributed slack bus model shows an order of magnitude improvement in frequency and voltage control deviation for 10% change in system load [3].

V. Congestion Management

A. *Problem Model*

In order to operate the electric grid in a optimized economic manner, it may be necessary to schedule the power transmission along a line to very near line limit. Even a small unexpected power exchange somewhere in the system could cause a transmission line to exceed its limit. In order to prevent damage to the transmission lines, it may be necessary for the system operator (SO) to modify the grid in a process called congestion management (CM). There are many ways to perform congestion management, but the system operators generally prefer to reschedule generators, because it does not alter system topology [5]. In a traditional single slack bus model of electric power transmission changes in generation and load incurred by performing congestion management are compensated for by changes in the slack bus power generation [5].

B. *Participation Factor*

In [5], it is proposed that that the excess burden produced while alleviating the congestion be distributed to all generators based on the their participation factor. This participation factor, called the Power Distribution Factor (PDF) in [5], would be the proportional to the maximum rated power of the participating machines. Is used in conjunction with a Generator Sensitivity Factor (GSF), which identifies the most sensitive buses for the rescheduling process, to transfer load from the generators most sensitive to congestion to generators less sensitive to congestion. The power dispatch equations is same as equation 13, such that

$$\pi_i = \frac{p_i^{max}}{\sum_{i=1}^{NG} p_i^{max}} \tag{15}$$

where p_i is the maximum rated power of the generators.

The system operator does not normally expect congestion in the transmission system. Normally, the market clearing mechanism schedules the generators to not overload any generators. If transmission line congestion does occur, the system operator will formulated “a nonlinear optimization problem with the objective of minimizing the congestion management cost using generator rescheduling with time frame of one hour” [5].

C. Case Study and Results

Congestion management can be a difficult problem under the best of conditions. Several artificial intelligence (AI) techniques are be being explored to solve congestion management problems in restructuring power systems. [5] uses IEEE 30 bus and IEEE 108-bus test systems to compare the cost savings of the following artificial intelligence techniques: genetic algorithms (GA), particle swarm optimization (PSO), differential evolution (DE), improved differential evolution (IDE), teaching-learning based optimization (TLBO), and an improved teaching-learning based optimization (ITLBO) that they develop for their paper. Using the single slack bus model, the congestion management cost (CMC), the addition of operating generators in a manner that does not overload any transmission lines, was between 1,561.63 and 1,486.20 dollars per hour, with genetic algorithm having the highest cost and improved teaching-learning based optimization having the lowest cost. Using the distributed slack bus model, the congestion management cost was reduced to between 1,380.44 and 1,170.71 dollars per hour, again with genetic algorithm having the highest cost and improved teaching-learning based optimization having the lowest cost. This is a reduction of congestion management cost by 10-20% depending on the choice of algorithm. The choice of slack bus model did not change the calculation time of any algorithms.

The 108-bus system showed even greater cost saving (Table IV). Using the improved teaching-learning based optimization algorithm, the congestion management cost was reduced from 7,874.19 to 3,344.58 dollars per hour, over a 50% reduction in cost. The other algorithms showed similar reductions in cost, between 4,059.80 and 3,344.58 dollars per hour.

Algorithm	CMC ($/hr)
Genetic Algorithm	4,059.80
Particle Swarm Optimization	3,653.43
Differential Evolution	3,925.81
Improved Differential Evolution	3,694.60
Teaching Learning Based Optimization	3,505.70
Improved Teaching Learning Based Optimization	3,344.58

TABLE IV

COMPARISON OF CONGESTION MANAGEMENT COSTS (CMC) WITH OTHER ALGORITHMS WITH DSB FOR IEEE 118-BUS TEST SYSTEM.

VI. CONCLUSION

It would be desirable to use the distributed slack bus model as it more closely represents how generators are controlled on the actual power grid. The papers reference all show that the distributed slack bus model provides improvements over the single slack bus, but all of these gains are relatively small.

There are cases that distributed slack bus algorithm plays an important role, for example, in the Economic Optimal Dispatch problem mentioned in section III part 4. But all the reference papers point to a crucial fact that determine sensitivity is more important than remove the slack bus, and the sensitivities in distributed slack bus algorithm are defined by participation factors. By saying this, there does not exist a universal way to modify the power flow equations for every system since each system will have different sensitivity for different factors. For example, in [7], the author conclude that for the same distance participation factor used, average sensitivity and maximum sensitivity will give totally different dispatch schedules. In the future with autonomous grid and machine learning, there might be some applications to associate the sensitivity with participation factors, but consider the cost of modifying and the gain, it does not attract big interest in the industry.

When applied to automatic generation controls problems the system losses and voltage stability improved. The electric grid is already very efficient regarding transmission losses and the improvements were very small on the already very small transmission losses. The voltage magnitude and phase angle errors were more significant, but the single slack bus model produces satisfactory results and the gains produced by the distributed slack bus model would have very little effect on real world performance. The congestion management problem produced some the most significant results, but a most of those improvements were from the improved artificial intelligence algorithm, rather than the distributed slack bus model. System operators are hesitant to implement artificial intelligence algorithms, because the ”black box” behavior is un-intuitive for operators and unpredictable for investors. Further, the performance improvements shown in these problems do not justify the vast increases in communication costs that system operators would need to invest in to implement a distributed slack bus model on the their grid.

Overall, For these reasons we do not put confidence on distributed slack models being used practically in the near future.

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