114-1 電工實驗（通信專題）

Multi-Antenna Systems

Chia-Yi Yeh (葉佳宜)

ycyyeh@ntu.edu.tw

Department of Electrical Engineering

National Taiwan University

Review

CFO

Antenna Array
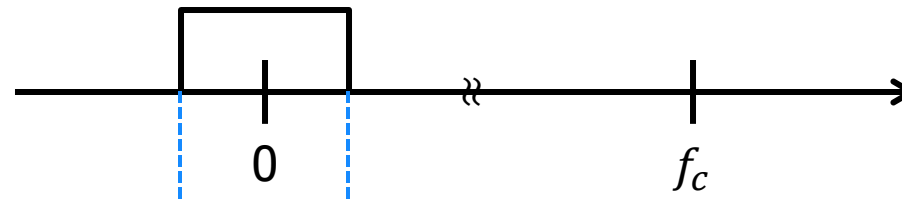
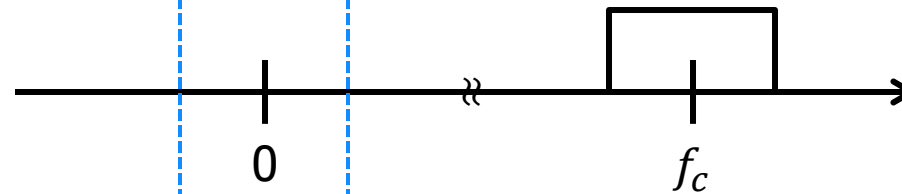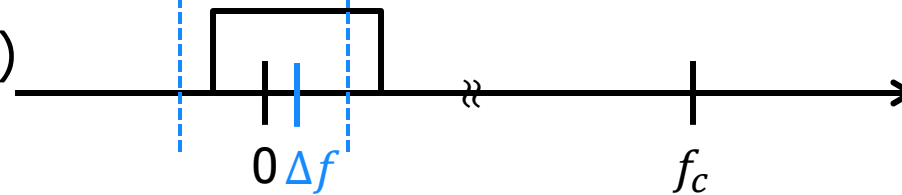# Carrier Frequency Offset (CFO)

Baseband

0   f_c

Upconverted (f_c)

(at TX)

0   f_c

Downconverted ($f_c - \Delta f$)

(at RX)

0 Δf f_c

TX & RX are two distinct devices.

Relative frequency offsets exist between their local oscillator (LOs) due to natural effects such as impurities, electrical noise, and temperature differences.
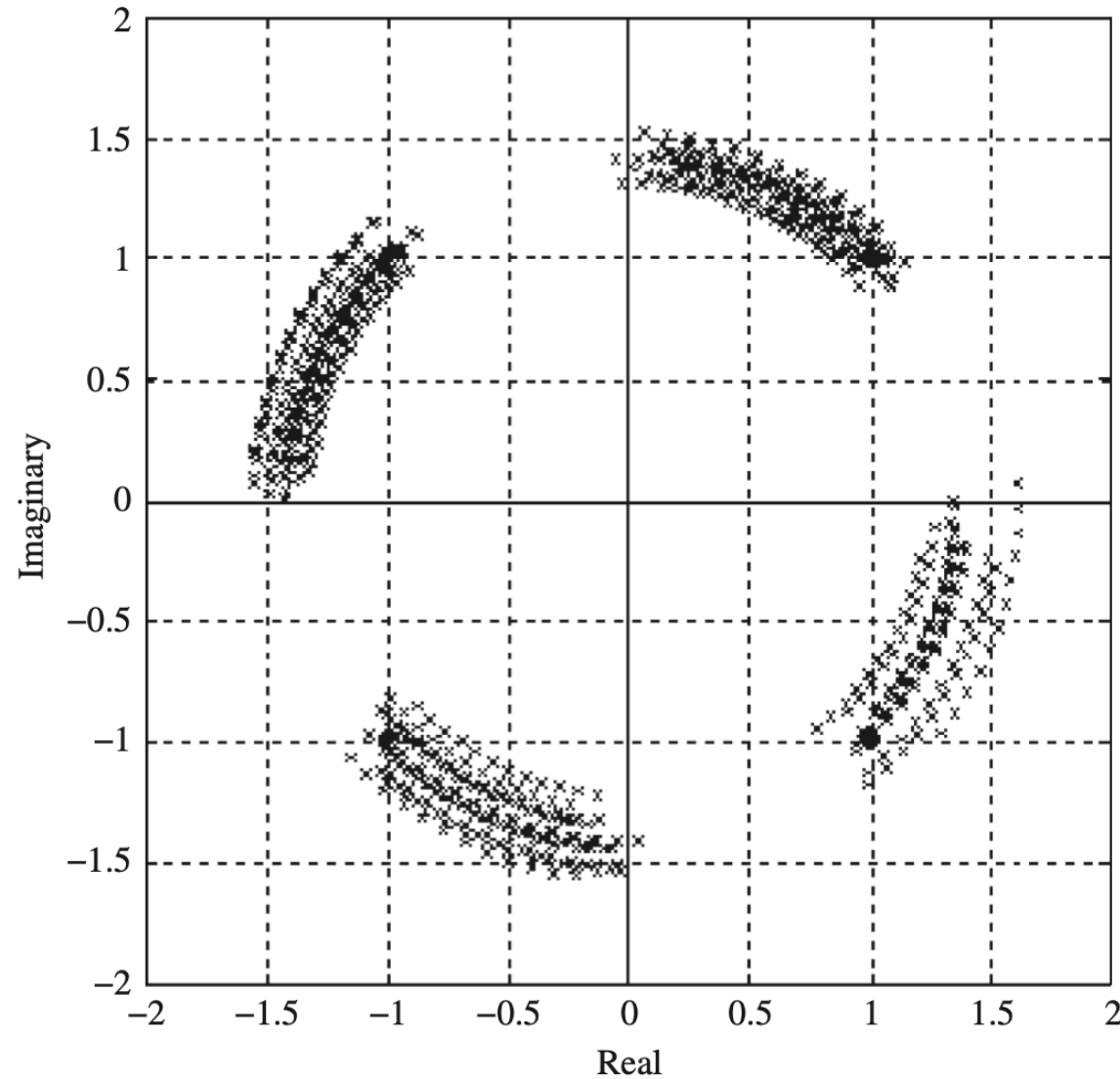
$$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$$

RX signal
(n-th sample)

TX signal
(n-th sample)

Δf: Carrier frequency offset

f_s: sample rate

Received Symbol with CFO

$$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$$

RX signal
(n-th sample)

TX signal
(n-th sample)

Δf : Carrier frequency offset

f_s : sample rate

4

Freq. Offset of Commercial Oscillators: PPM

PPM: parts per million

Define the maximum carrier offset

$$\Delta f_{max} = \frac{f_c \times \text{PPM}}{10^6}$$

We only need to calibrate this much

[Example]

Wi-Fi 802.11: ± 20 ppm
$\Rightarrow$ Carrier difference: $[-40, 40]$ ppm

At 2.4 GHz
$\Delta f_{max} = \frac{2.4 \times 10^9 \times 20}{10^6} = 48$ KHz

CFO Estimation Using Two Identical Halves

Transmit repetitive signals
(e.g., 10 Short Training Symbols, 2 Long Training Symbols)

For two <u>consecutively</u> received copies
(L: the length of each copy, $L = 16$ for STS, $L = 64$ for LTS)

$$p(k) = \sum_{m=0}^{L-1} r^*(k+m)\, r(k+m+L)$$

$r(n) = s(n)\, e^{\frac{j2\pi\,\Delta f\, n}{f_s}}$

$$= \sum_{m=0}^{L-1} s^*(k+m)\, e^{\frac{-j2\pi\,\Delta f\,(k+m)}{f_s}}\; s(k+m+L)\, e^{\frac{j2\pi\,\Delta f\,(k+m+L)}{f_s}}$$

$$= e^{\frac{j2\pi\,\Delta f\, L}{f_s}} \sum_{m=0}^{L-1} s^*(k+m)\, s(k+m+L)$$

$s(k) = s(k+L)$

$$= e^{\frac{j2\pi\,\Delta f\, L}{f_s}} \sum_{m=0}^{L-1} |s(k+m)|^2$$

Frequency Offset $\Rightarrow$ Phase diff between 1st & 2nd RX copy

$$\phi = \frac{2\pi L\,\Delta f}{f_s}$$

6

The Largest Correctable Frequency Offset

$$\Delta \hat{f} = \frac{\hat{\phi} \, f_s}{2\pi L}$$

$$\hat{\phi} = \tan^{-1}\left(\frac{\mathcal{Im}(P(k))}{\mathcal{Re}(P(k))}\right)$$

ϕ can be at most π: $\Delta f_{max} = \frac{f_s}{2L}$

Recall from the earlier calculation

Wi-Fi 802.11: ± 20 ppm
$\Rightarrow$ Carrier difference: $[-40, 40]$ ppm

Wi-Fi 802.11 with 20 MHz bandwidth:

$$\Delta f_{max,STS} = \frac{20\times10^6}{2\times16} = 625 \text{ KHz}$$

$$\Delta f_{max,LTS} = \frac{20\times10^6}{2\times64} = 156.25 \text{ KHz}$$

Coarse Freq Correction

Fine Freq Correction

At 2.4 GHz

$$\Delta f_{max} = \frac{2.4 \times 10^9 \times 20}{10^6} = 48 \text{ KHz}$$
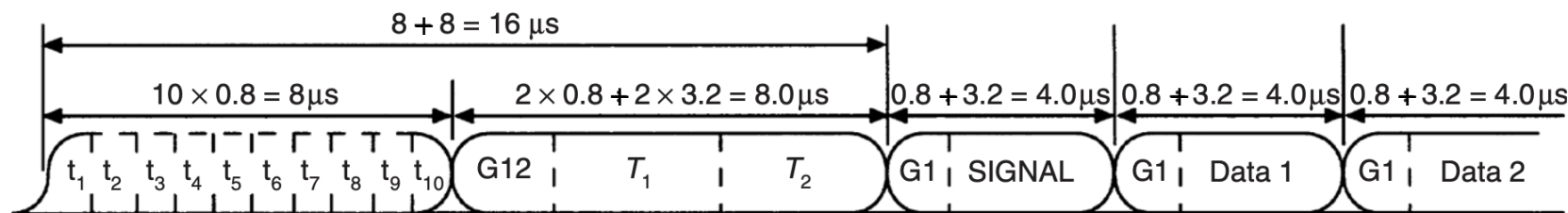
Apply The Estimated Carrier Frequency Offset

$$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$$

RX signal   TX signal

$$\Longrightarrow$$

$$r_{CFO}(n) = r(n)\, e^{\frac{-j2\pi\, \Delta\hat{f}\, n}{f_s}}$$

CFO Corrected

- What is the order of (a) channel estimation, (b) CFO estimation, and (c) CFO correction?

- Which parts of the signal require CFO correction? Starting from (a) STS, (b) LTS, or (c) only the OFDM symbols?

- When is "$n = 0$" when correcting the CFO? Does it matter?

Antenna Array

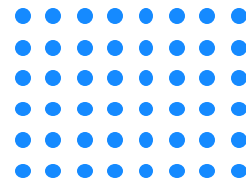## Set of antennas commonly organized in a structure
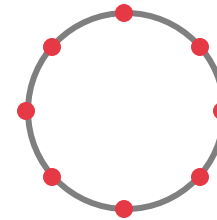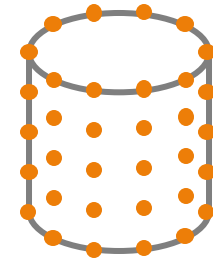
Linear Array   Planar Array   Circular Array   Cylindrical array

└ Uniform linear array   └ Uniform planar array

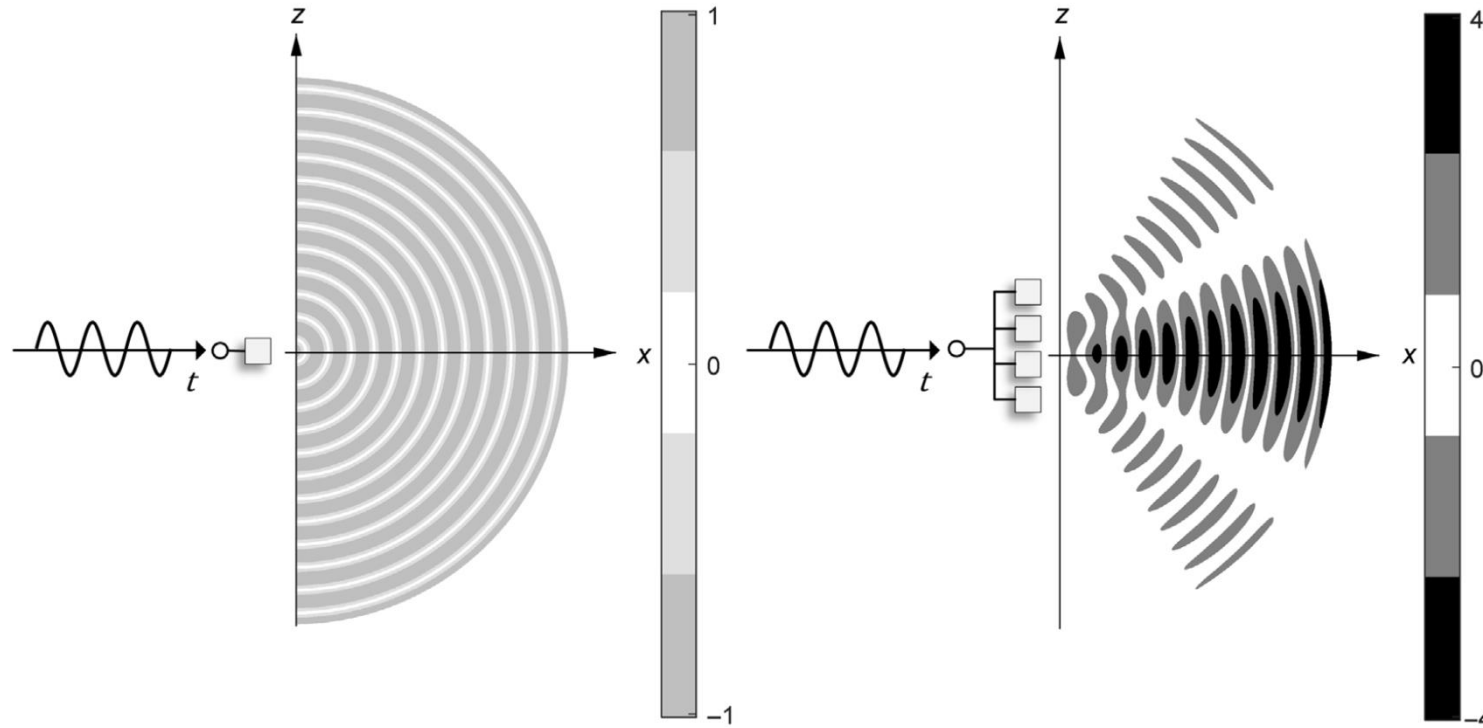## For Both transmission and reception

Gain (signal amplification)
from transmitting/receiving different versions of the same signal from all antennas

Gain direction control
Steer to the intended user, reduce interference to/from other users

N antennas → N times gain

X-axis direction, with the same transmit power

1 Antenna

4 Antennas

$A\sin(\theta) \implies$ **Power = A^2**

$4 \times \left(\dfrac{1}{\sqrt{4}} A\sin(\theta) \right) = 2A\sin(\theta) \implies$ **Power = $4A^2$**

Normalize to maintain the same transmit power

Change the Direction of the Maximum Gain

Adjust the amplitude and delay of the signal
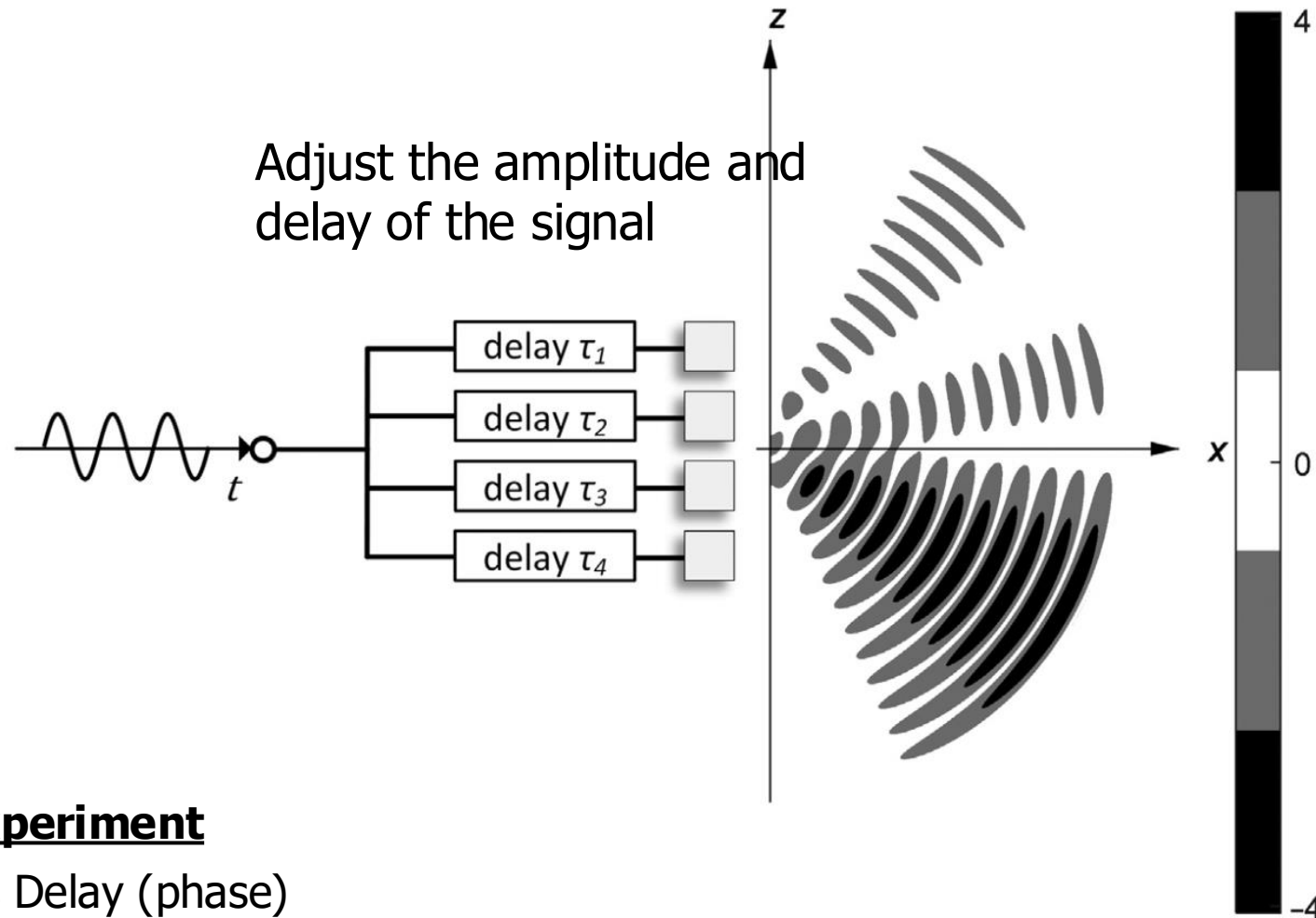
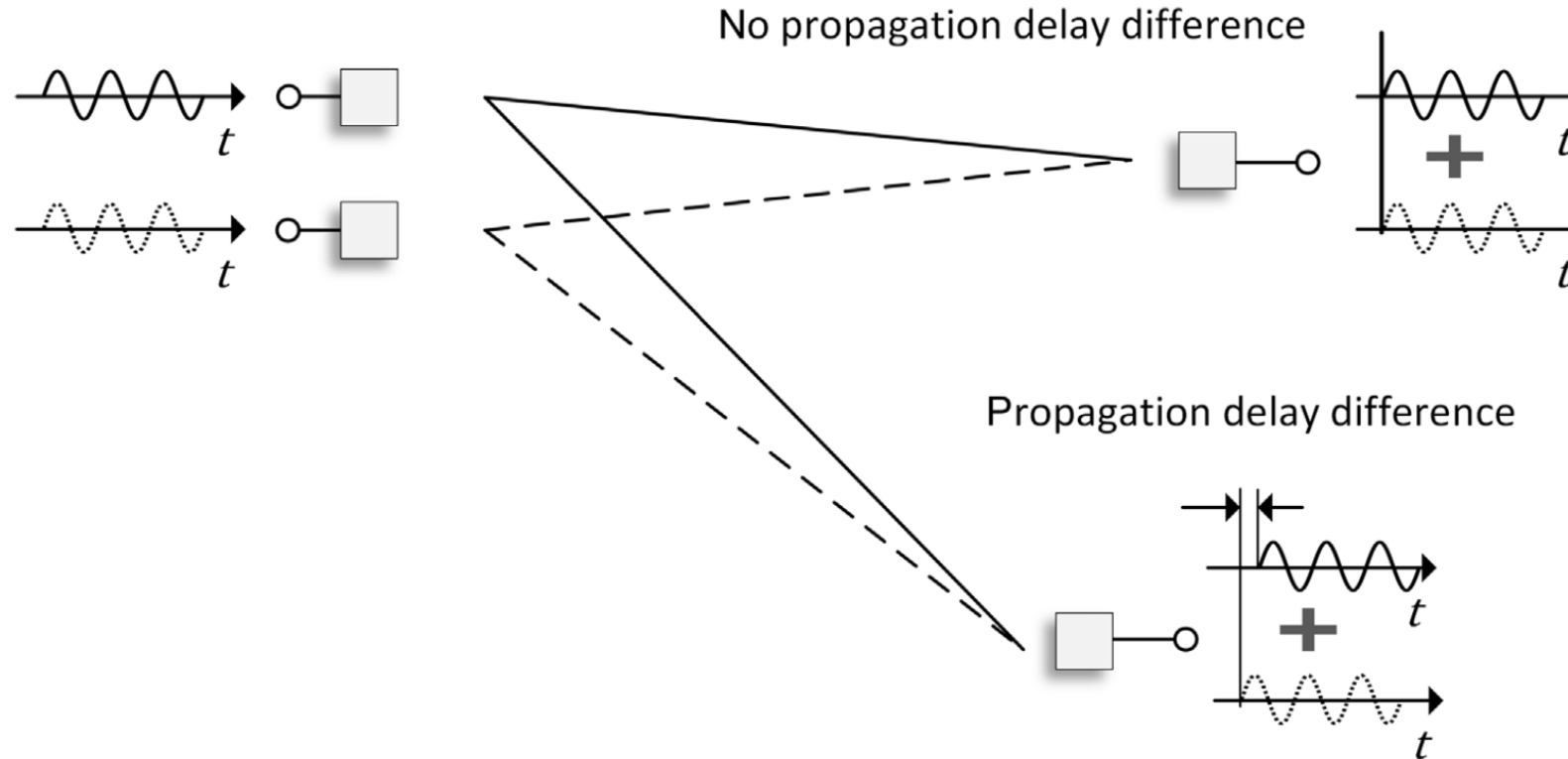**Thought experiment**

Amplitude vs Delay (phase)

Which is more crucial in changing the maximum direction?

Let's Start with Two

No propagation delay difference

Propagation delay difference

The received signal's amplitude depends on the difference in propagation delays

Propagation delay depends on the receiver's direction

Question: How can we find the "gain" at different angles?
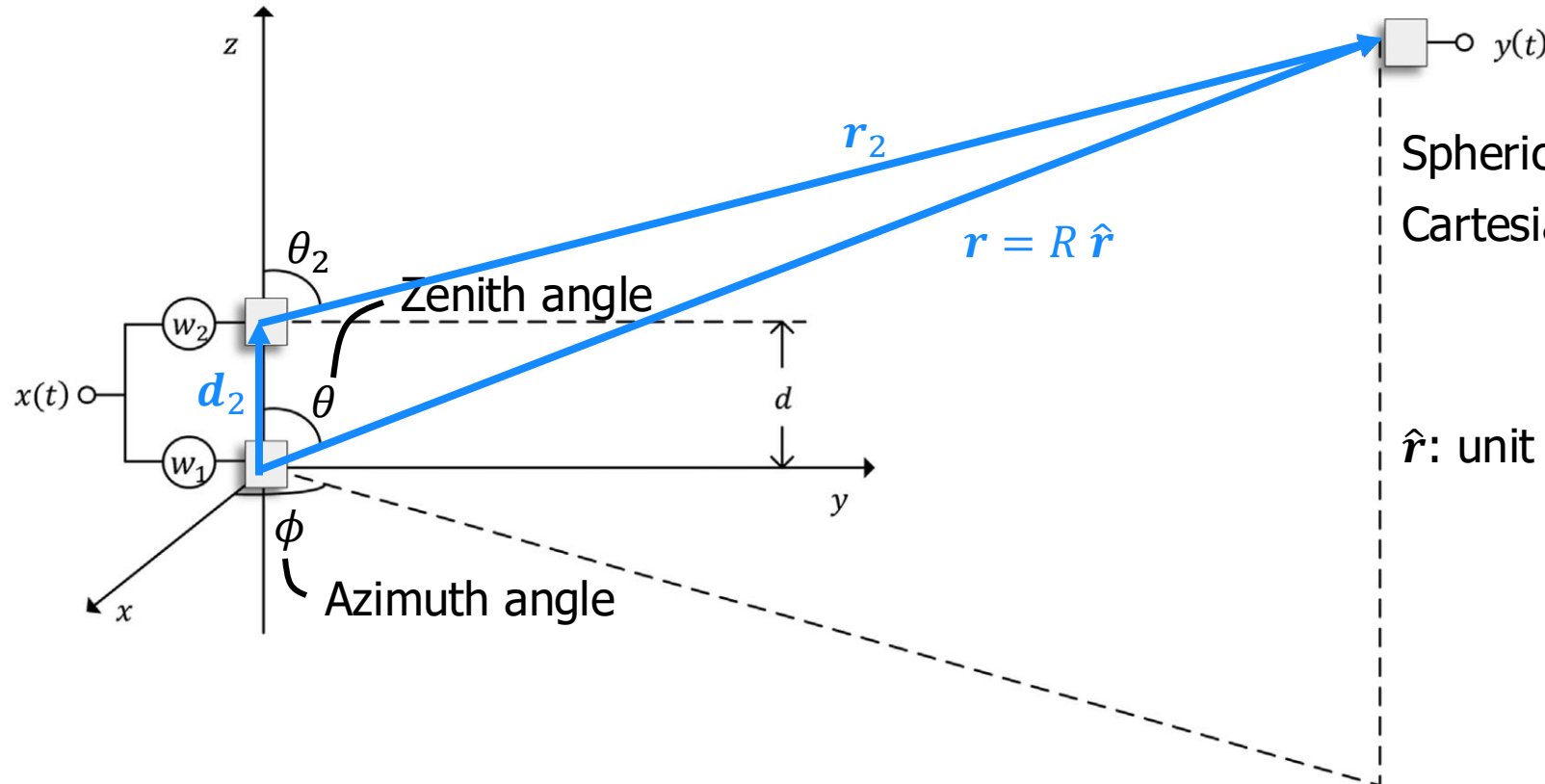
Array with two elements

Assume free space (only line-of-sight path)

R is large
$\Rightarrow r$ and r_2 approximately parallel
$\Rightarrow r_2 = R_2\, \hat{r}$
$\Rightarrow R_2 = R - d\cos\theta = R - \mathbf{d}_2^{\mathrm{T}}\, \hat{r}$
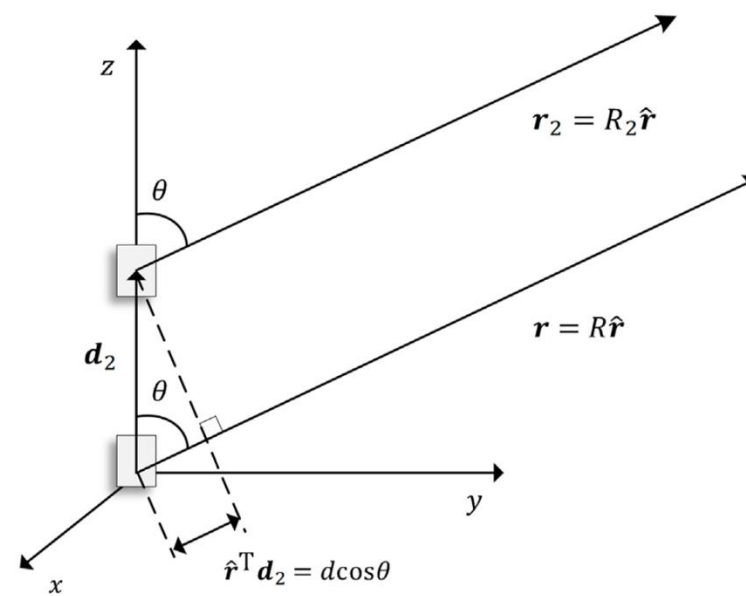
Spherical coordinate: (R, θ, ϕ)
Cartesian coordinate:

$$r = R\hat{r} = R \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$\hat{r}$: unit vector of r

Zenith angle

Azimuth angle

Array with two elements

For simplicity, consider transmitting a sinusoid at radio frequency

$x(t) = X$ and $y(t) = Y$

$$Y = \alpha e^{-j2\pi f_c \tau_1} \left(w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}} \right) X$$

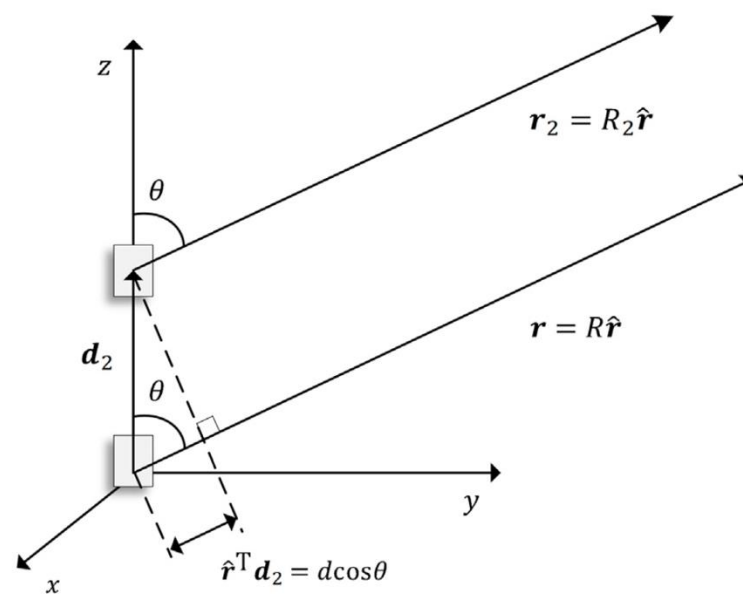Array Factor (AF)

$$AF(\theta, \phi) = w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}}$$

$$\boxed{|Y|^2 = \alpha^2 |AF(\theta, \phi)|^2 |X|^2}$$

The gain pattern for an array of isotropic elements relative to a single isotropic antenna

Also referred to as the (free-space) array gain

Assume all elements isotropic

$$y(t) = h_1 x_1(t - \tau_1) + h_2 x_2(t - \tau_2)$$

where $h_n = \alpha e^{-j2\pi f_c \tau_n}$

$$\tau_1 = \frac{R}{c}$$

$$\tau_2 = \frac{R_2}{c} = \tau_1 - \frac{d\cos\theta}{c}$$

$$y(t) = \alpha e^{-j2\pi f_c \tau_1} x_1(t - \tau_1) + \alpha e^{-j2\pi f_c \tau_2} x_2(t - \tau_2)$$

$$y(t) = \alpha e^{-j2\pi f_c \tau_1} w_1 x(t - \tau_1) + \alpha e^{-j2\pi f_c \tau_2} w_2 x(t - \tau_2)$$

Figure diagram: $r_2 = R_2 \hat{r}$, $r = R\hat{r}$, d_2, θ, $\hat{r}^T d_2 = d\cos\theta$, axes x, y, z

14

Array with two elements

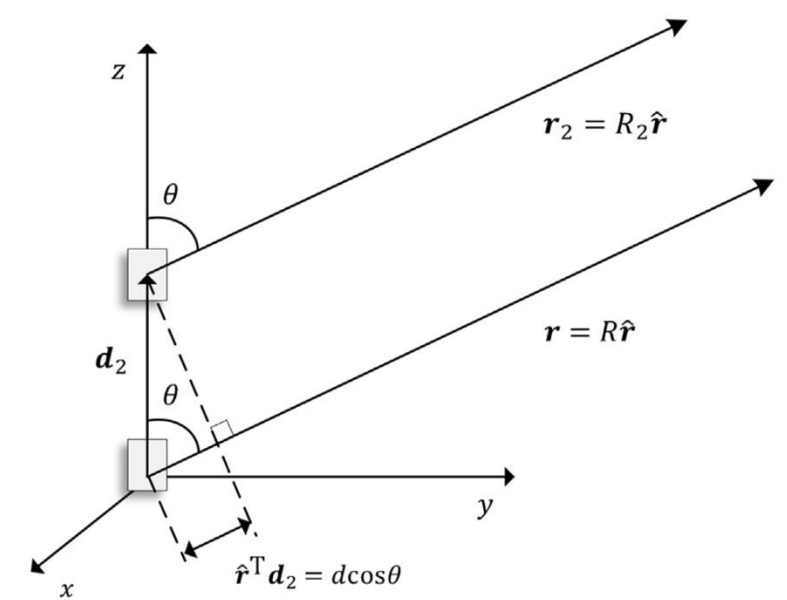Example: Antenna space = 1 wavelength
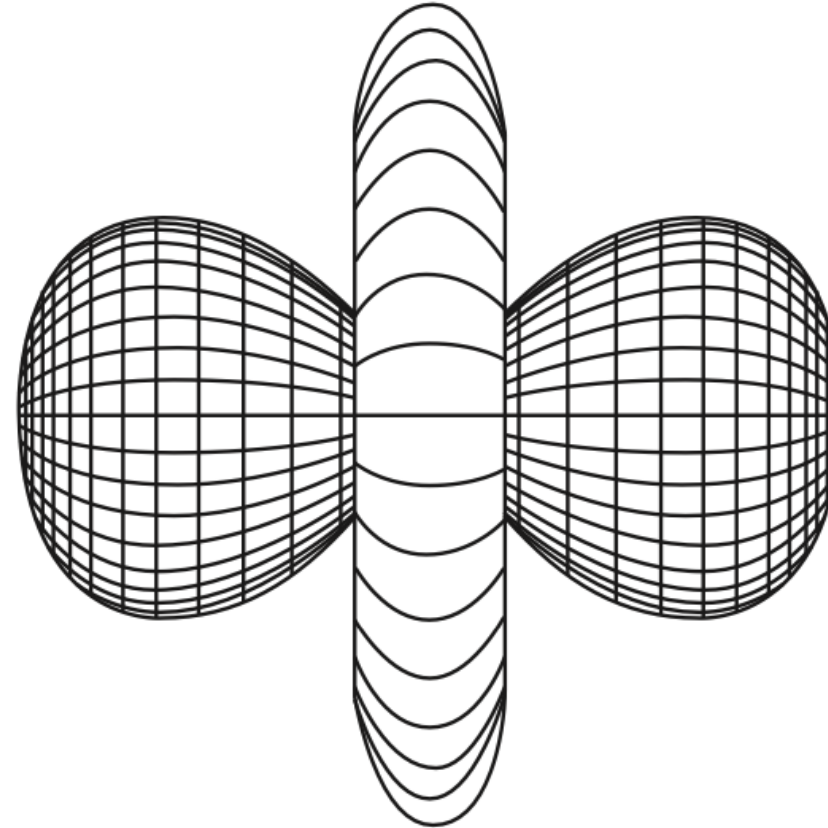
Extension to General Form

From Array with Two Elements

$$Y = \alpha e^{-j2\pi f_c \tau_1} \left(w_1 + w_2 e^{j2\pi f_c \frac{d \cos \theta}{c}} \right) X$$

$$= \alpha e^{-j2\pi f_c \tau_1} \left(w_1 e^{j2\pi f_c \frac{d_1^T \hat{r}(\theta,\phi)}{c}} + w_2 e^{-j2\pi f_c \frac{d_2^T \hat{r}(\theta,\phi)}{c}} \right) X$$

$$= \alpha e^{-j2\pi f_c \tau_1} [w_1 \quad w_2] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} X \qquad \text{where } a_n = e^{j2\pi f_c \frac{d_n^T \hat{r}(\theta,\phi)}{c}}$$

$$\boxed{y(t) = \boldsymbol{w}^T \boldsymbol{a}(\theta, \phi)\, \alpha e^{-j2\pi f_c \tau} x(t)}$$

Can this form be used for rectangular arrays?

Can this form be used for arrays with arbitrary placement?

Beamforming Weight Vector

Array Response Vector

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad \boldsymbol{a}(\theta, \phi) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}, \quad a_n = e^{j2\pi f_c \frac{d_n^T \hat{r}(\theta,\phi)}{c}}, \quad \hat{\boldsymbol{r}}(\theta, \phi) = \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix}$$
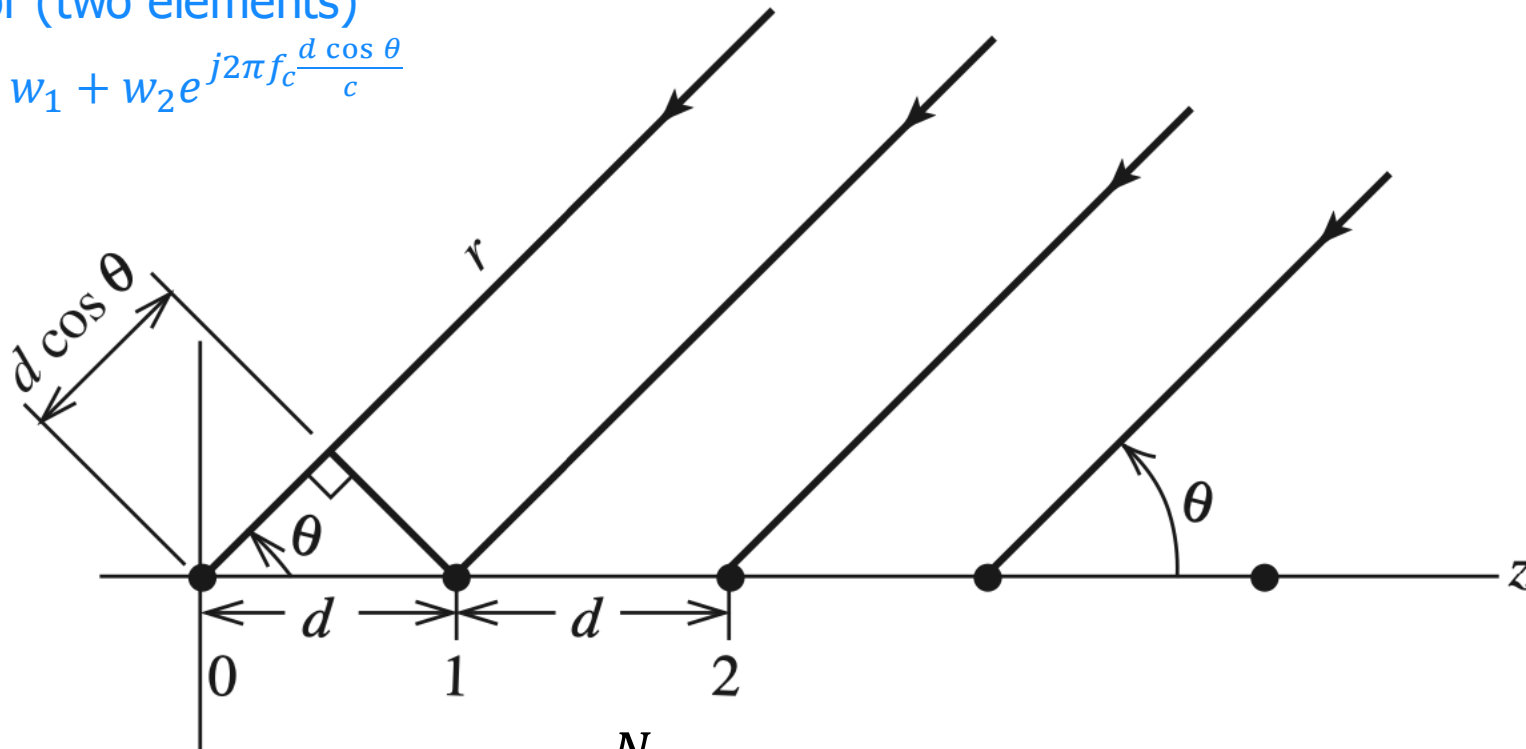
Uniform Linear Array with N Elements

Array Factor (two elements)

$$AF(\theta, \phi) = w_1 + w_2 e^{j2\pi f_c \frac{d \cos \theta}{c}}$$

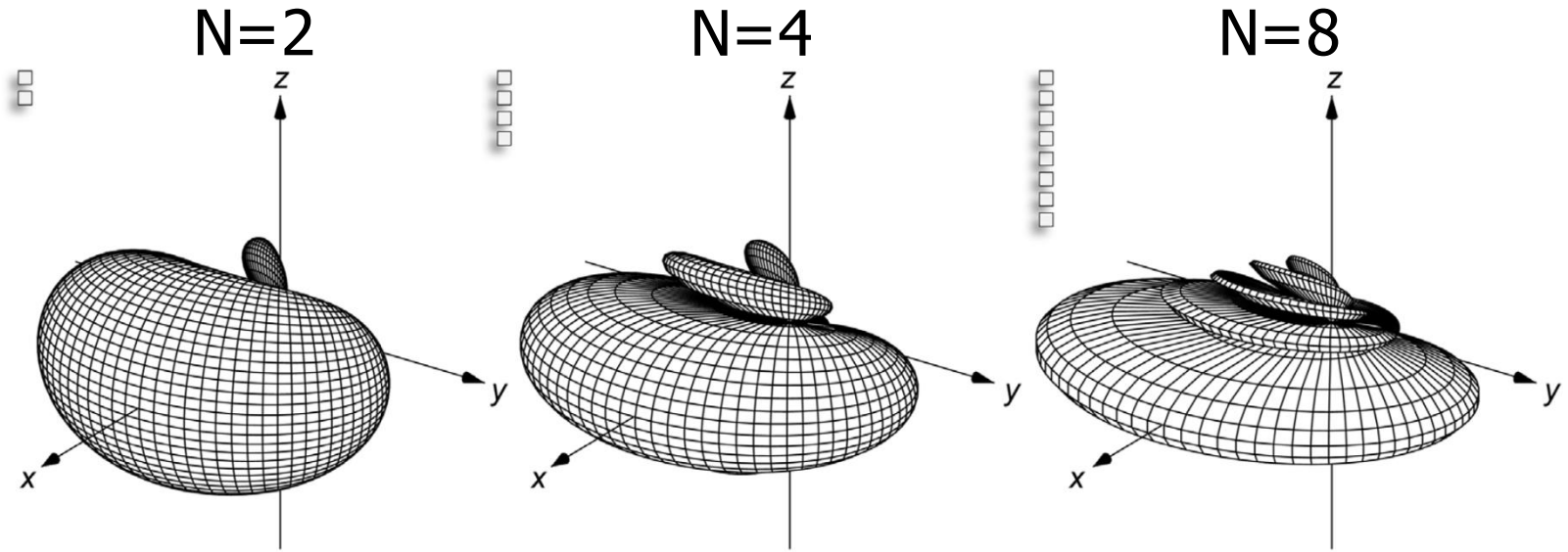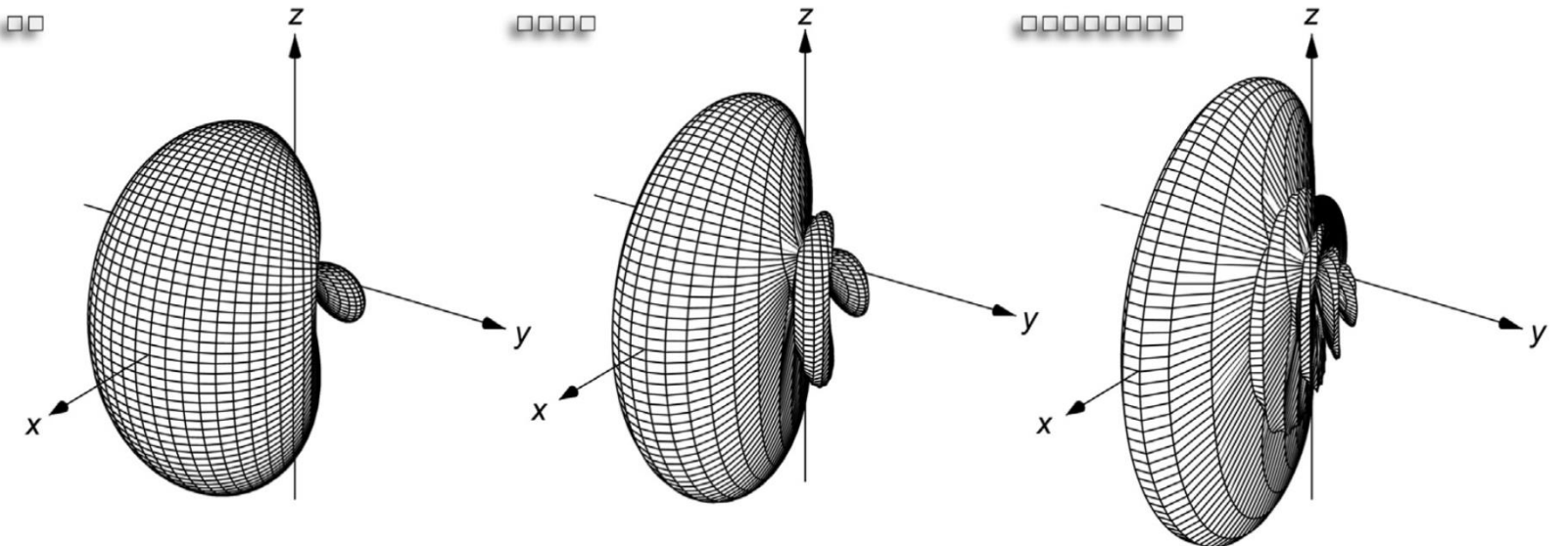$$AF(\theta, \phi) = \sum_{n=1}^{N} w_n \, e^{j(n-1)d \cos \theta \frac{2\pi f_c}{c}}$$

Basic Gain Pattern

$$w = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Vertical Arrays

Horizontal Arrays

N=2   N=4   N=8

Multi-Antenna Systems

Gains in Multiple Antenna Systems

- Beamforming gain

- Diversity gain

- Spatial multiplexing gain

- Can be applied at TX & RX

Beamforming Gain: N antennas → N times gain
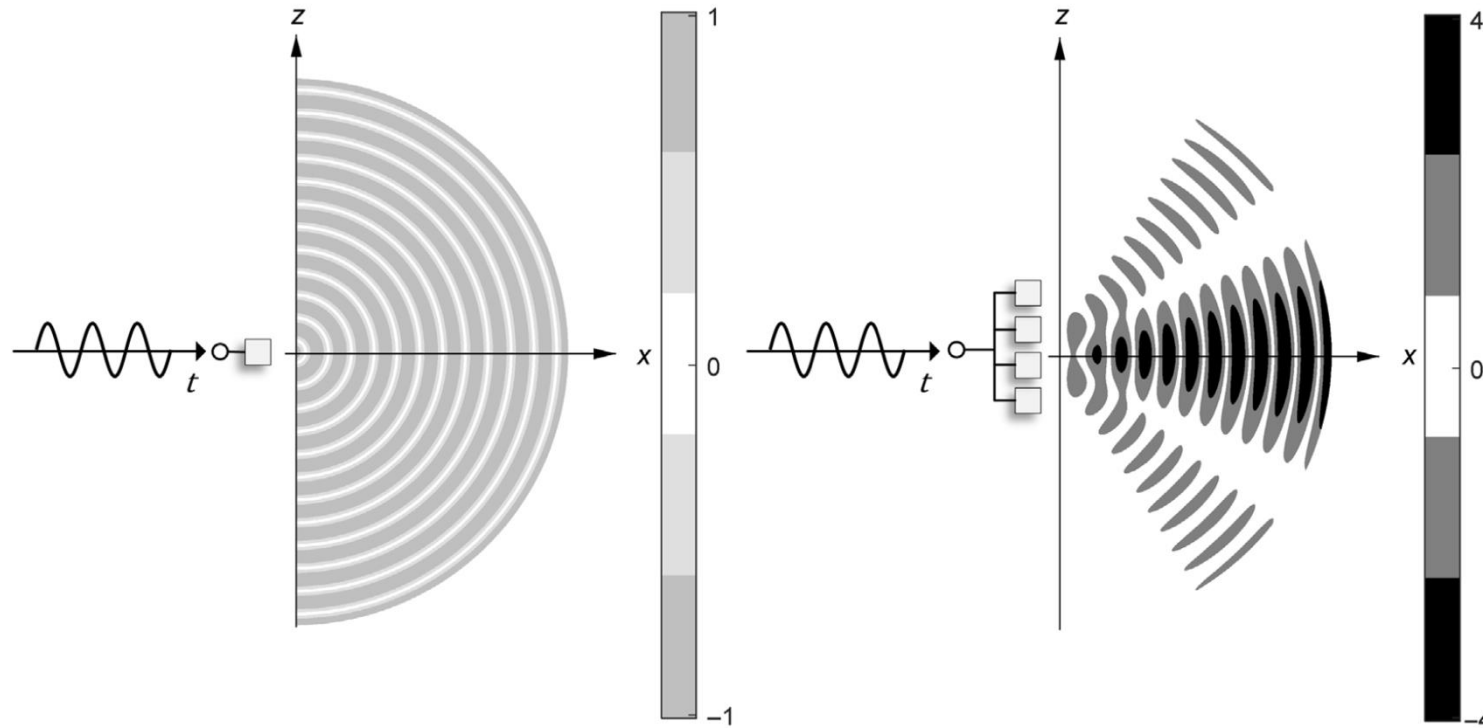
X-axis direction, with the same transmit power

1 Antenna

$A \sin(\theta) \implies$ **Power = A^2**

4 Antennas

$4 \times \left(\dfrac{1}{\sqrt{4}} A \sin(\theta) \right) = 2A \sin(\theta) \implies$ **Power = $4A^2$**

Normalize to maintain the same transmit power

Diversity Gain: Less Likely that All Branches Suffer

- Fading: Even for nearby locations, signal strength varies.
  - The extreme case: Rayleigh fading

- Diversity: Mitigating the effects of fading by sending or receiving through statistically independent channels.
 - **Spatial** diversity: several antenna elements separated in space.
  - **Temporal** diversity: transmission of the signal at different times.
  - **Frequency** diversity: transmission of the signal on different frequencies.
  - **Angular** diversity: multiple antennas with different antenna patterns.
  - **Polarization** diversity: antennas with different polarizations (e.g., vertical, horizontal).

- Independent, or low correlation, is preferred
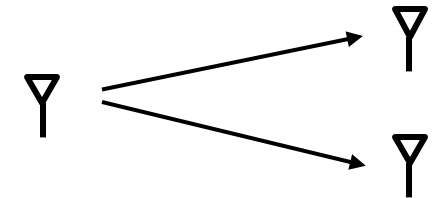
Question: Is there diversity gain by using multiple antennas in the free-space scenario?

Exploiting Diversity

- Selection: the "best" branch is selected
  - Pro: Simple
  - Con:
    At TX – no beamforming gain
    At RX – waste signal energy by discarding $(N-1)$ copies of the received signal

- Combining: all copies of the signal are combined
  - To maximize SNR
    At RX: Maximum Ratio Combining
    At TX: Maximum Ratio Transmission (or Conjugate Beamforming)

Maximum Ratio Combining to Maximize SNR

$\underline{N}$ RX antennas, independent AWGN

$$\boldsymbol{y} = \boldsymbol{h}x + \boldsymbol{n} \qquad n_i \sim CN\left(0, \frac{\sigma^2}{2}\right), \text{ i.i.d.}$$

$N \times 1$

Apply a linear combiner $\boldsymbol{w}$

$$z = \boldsymbol{w}^T \boldsymbol{y} = \boldsymbol{w}^T \boldsymbol{h} x + \boldsymbol{w}^T \boldsymbol{n}$$

$$\longrightarrow SNR = \frac{|\boldsymbol{w}^T \boldsymbol{h}|^2}{\sigma^2 \|\boldsymbol{w}\|^2} = \frac{\left|\sum_{n=1}^{N} w_n h_n\right|^2}{\sigma^2 \sum_{n=1}^{N} |w_n|^2}$$

Maximize SNR (Cauchy-Schwarz inequality)

$$\left|\sum_{n=1}^{N} w_n h_n\right|^2 \leq \sum_{n=1}^{N} |w_n|^2 \sum_{n=1}^{N} |h_n|^2$$

Equality holds if and only if $w_n = \alpha h_n^*$

Maximum Ratio Combiner

$$\boldsymbol{w} = \boldsymbol{h}^H$$

New SNR: sum of branch SNRs

$$SNR = \frac{\left|\sum_{n=1}^{N} h_n^* h_n\right|^2}{\sigma^2 \sum_{n=1}^{N} |h_n^*|^2} = \frac{\left|\sum_{n=1}^{N} |h_n|^2\right|^2}{\sigma^2 \sum_{n=1}^{N} |h_n|^2}$$

$$= \frac{\sum_{n=1}^{N} |h_n|^2}{\sigma^2} = \sum_{n=1}^{N} \frac{|h_n|^2}{\sigma^2}$$

Can we use the same derivation for analyzing TX?

24

Diversity Gain vs Beamforming Gain

- **Diversity Gain**
  It is improbable that several antenna elements are in a fading dip simultaneously. The probability for very low signal levels is thus decreased by the use of multiple antenna elements.

- **Beamforming Gain**
  The combiner performs an averaging over the noise at different antennas. Thus, even if the signal levels at all antenna elements are identical, the combiner output SNR is larger than the SNR at a single-antenna element.
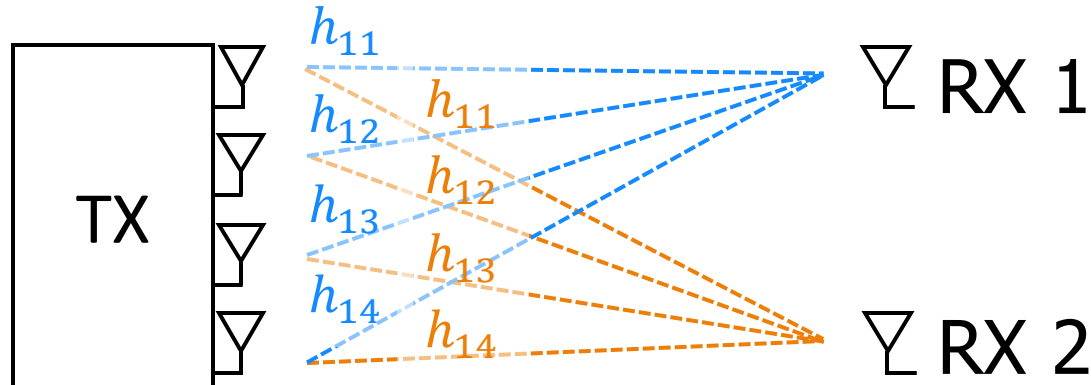
Spatial Multiplexing Gain

More antennas → More simultaneous data streams

Data stream 1
0110100111...

Data stream 2
1011111010...

$$y_1 = \overbrace{[-\,\boldsymbol{w}_1\,-]\begin{bmatrix}|\\ \boldsymbol{h}_1 \\ |\end{bmatrix} s_1}^{\textbf{Signal}} + \overbrace{[-\,\boldsymbol{w}_2\,-]\begin{bmatrix}|\\ \boldsymbol{h}_1 \\ |\end{bmatrix} s_2}^{\textbf{Interference}} + \overbrace{N_1}^{\textbf{Noise}}$$

$$y_2 = \underbrace{[-\,\boldsymbol{w}_1\,-]\begin{bmatrix}|\\ \boldsymbol{h}_2 \\ |\end{bmatrix} s_1}_{\textbf{Interference}} + \underbrace{[-\,\boldsymbol{w}_2\,-]\begin{bmatrix}|\\ \boldsymbol{h}_2 \\ |\end{bmatrix} s_2}_{\textbf{Signal}} + \underbrace{N_2}_{\textbf{Noise}}$$

No interference if

$$[-\,\boldsymbol{w}_2\,-]\begin{bmatrix}|\\ \boldsymbol{h}_1 \\ |\end{bmatrix} = 0 \quad \textbf{\&} \quad [-\,\boldsymbol{w}_1\,-]\begin{bmatrix}|\\ \boldsymbol{h}_2 \\ |\end{bmatrix} = 0$$

In addition, want strong signal

Common Precoding Strategies

Maximum Ratio Transmission (Conjugate Beamforming)

- Max signal at the intended user
- Inter-user interference exists

$$w_i = h_i^H$$

$$y_1 = [-\,w_1\,-]\overbrace{\begin{bmatrix}|\\h_1\\|\end{bmatrix}}^{\text{Signal}} s_1 + [-\,w_2\,-]\overbrace{\begin{bmatrix}|\\h_1\\|\end{bmatrix}}^{\text{Interference}} s_2 + \overbrace{N_1}^{\text{Noise}}$$

$$y_2 = [-\,w_1\,-]\underbrace{\begin{bmatrix}|\\h_2\\|\end{bmatrix}}_{\text{Interference}} s_1 + [-\,w_2\,-]\underbrace{\begin{bmatrix}|\\h_2\\|\end{bmatrix}}_{\text{Signal}} s_2 + \underbrace{N_2}_{\text{Noise}}$$

Zero-forcing Precoding

- Null the inter-user interference
- Lose some signal gain

$$W = pinv(H)$$

$$\overset{pinv(H)}{\begin{bmatrix}\bigcirc\ w_1\ \bigcirc\\\bigcirc\ w_2\ \bigcirc\end{bmatrix}}\overset{H}{\begin{bmatrix}h_1 & h_2\end{bmatrix}} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Reference

- Andreas F. Molisch, *Wireless communications*. John Wiley & Sons, 2012. [NTU Library Link]
    - Chap. 13: Diversity
    - Chap. 20: Multiantenna Systems

- Joachim Speidel. *Introduction to digital communications*. Springer Nature, 2021. [NTU Library Link]
    - Chap. 14: Principles of Linear MIMO Receivers
    - Chap. 18: MIMO systems with Precoding

- Henrik Asplund, Jonas Karlsson, Fredric Kronestedt, Erik Larsson, David Astely, Peter von Butovitsch, Thomas Chapman et al. *Advanced Antenna Systems for 5G Network Deployments: Bridging the Gap Between Theory and Practice*. Academic Press, 2020. [NTU Library Link]
    - Chap. 6: Multi-Antenna Technologies

Paper Debate

Debate Format

20 minutes	Defense Team
10 minutes	Offense Team
5 minutes	Preparation time
10 minutes	Follow up arguments
5 minutes	Questions and comments from class

Timing will be strictly enforced!

Paper 3: MULoc: Towards Millimeter-Accurate Localization for Unlimited UWB Tags via Anchor Overhearing

https://forms.gle/92T9hycv7eyDFKeW9

Only the audience vote!

Presenters: Upload your slides
(With 分工表)