# 114-1 電工實驗（通信專題）

# OFDM & Frame Synchronization

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# Paper 1 Debate Review

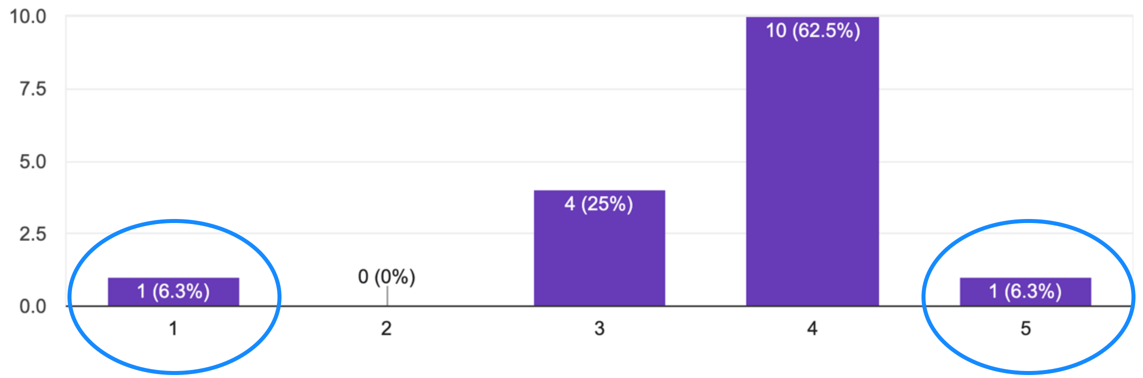
# Feedback from the audience

- Since everyone has read the paper, the defense side can spend less time on introducing paper in detail. Emphasizing strengths would make statements stronger. The offense side performs well. They spot multiple problems, and the most impressive one is that they spot the feeding channel is missing. Also, BER near 90% is pretty funny. It’s a shame that I didn’t spot that stupid mistake. (Perhaps the author scaled it, though.)
- 正反方都盡力了 好殘酷
- 感覺不一定要逐字護航，而是想辦法轉移焦點，例如強調實驗的創新貢獻，畢竟在一個新的方法裡面完美的做好每個細節跟scalability是有點難的。
- 我覺得論文作者對於實驗細節的確沒有寫的很清楚，另外正方沒有回答到傳輸流量比RF低這個問題。但是論文的想法是好的、想法架構也很完整，所以我還是認為它有相應的學術價值。
- 雖然反方提出的觀點都很棒，但因為這篇paper的重點還是在確實實作出了一個能結合sensing 跟 communication的系統，在實驗的環境中表現上的確也明顯優於OWC，但也省略了不少實作的細節，不過整體來說我認為這篇paper還是提供了一個先驅，把概念性的想法實作出來，因此我還是認為可以weakly accept.

# How the paper is rated among the class

Before the debate, how do you rate the paper?

16 responses

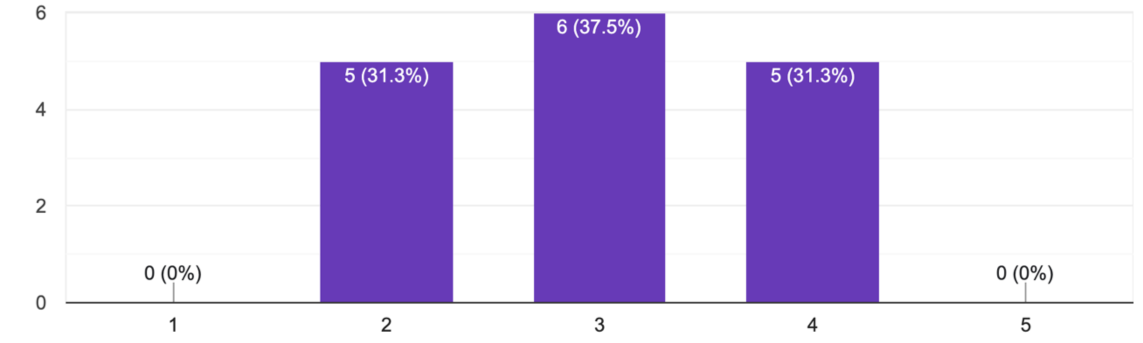



Before the debate

Average = 3.625

After the debate, how do you rate the paper?

16 responses



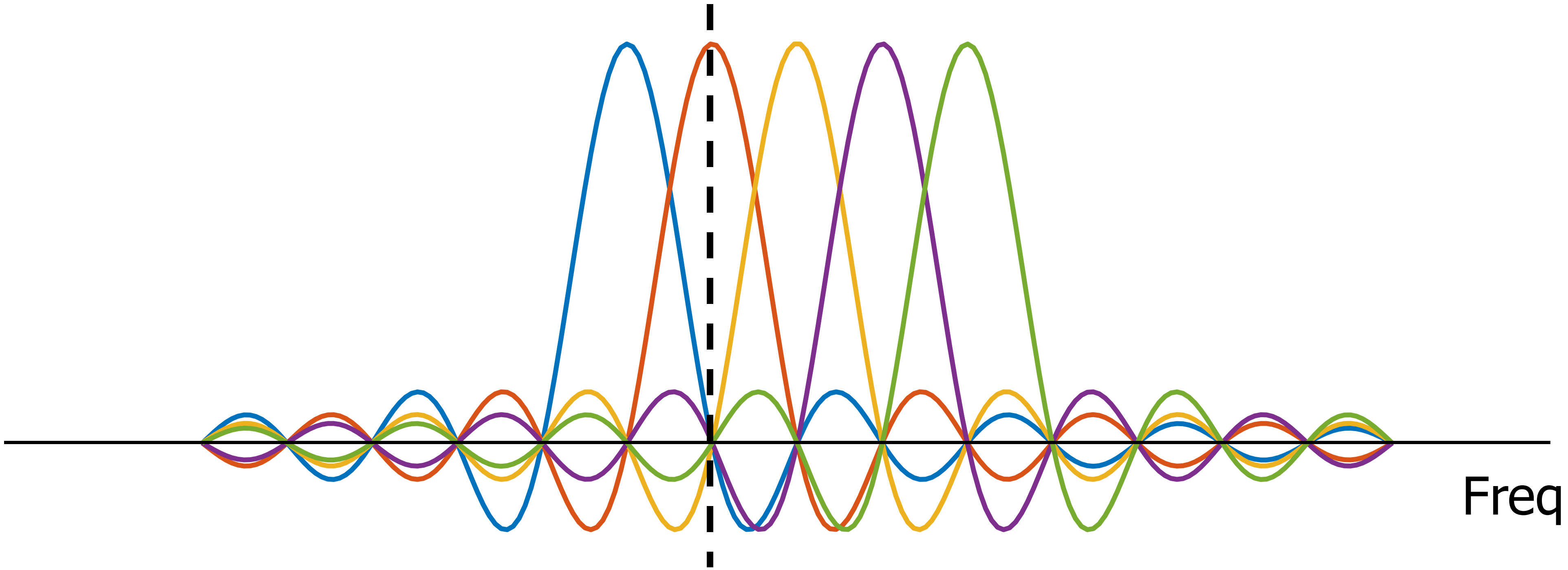

After the debate

Average = 3

# OFDM

Orthogonal Frequency-Division Multiplexing

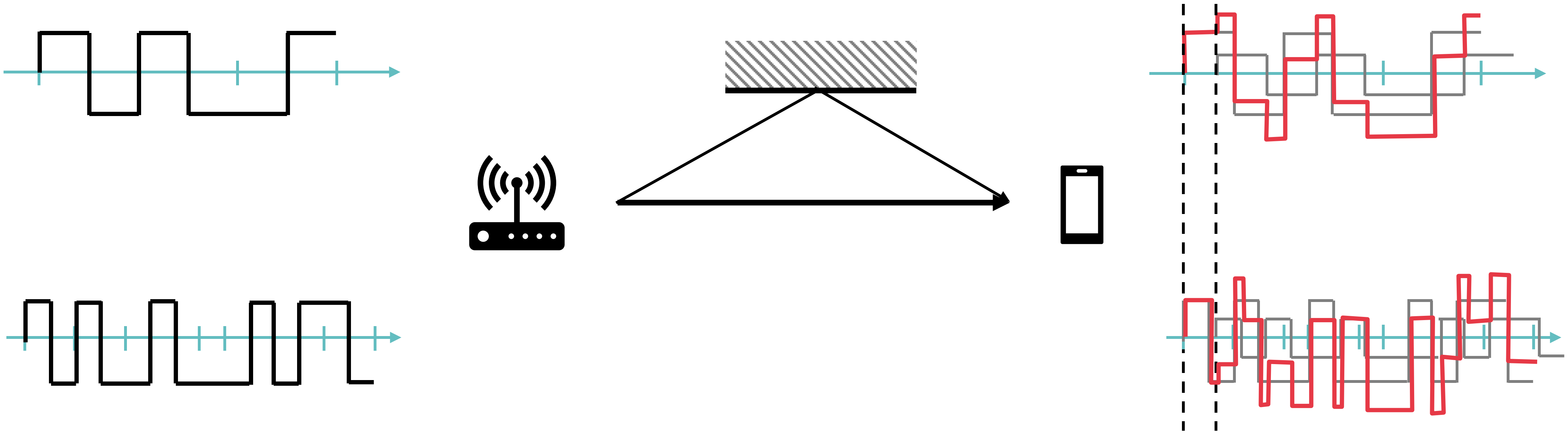
# OFDM: Orthogonal Frequency-Division Multiplexing




Spectra of different modulated carriers overlap
But each carrier is in the spectral nulls of all other carriers

Wi-Fi (IEEE 802.11a, g, n, ac, ax), LTE

# Motivation of OFDM

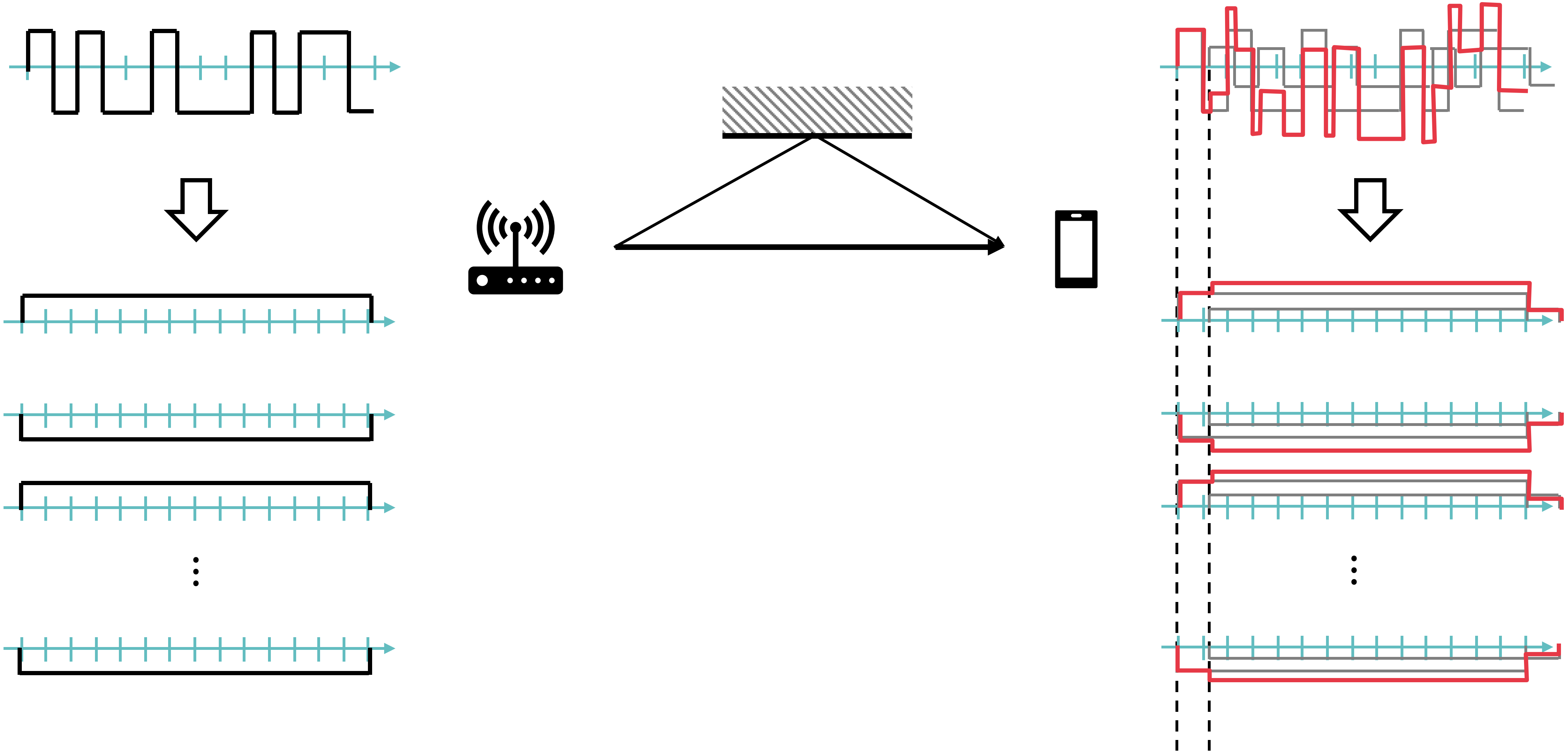


Higher data rate

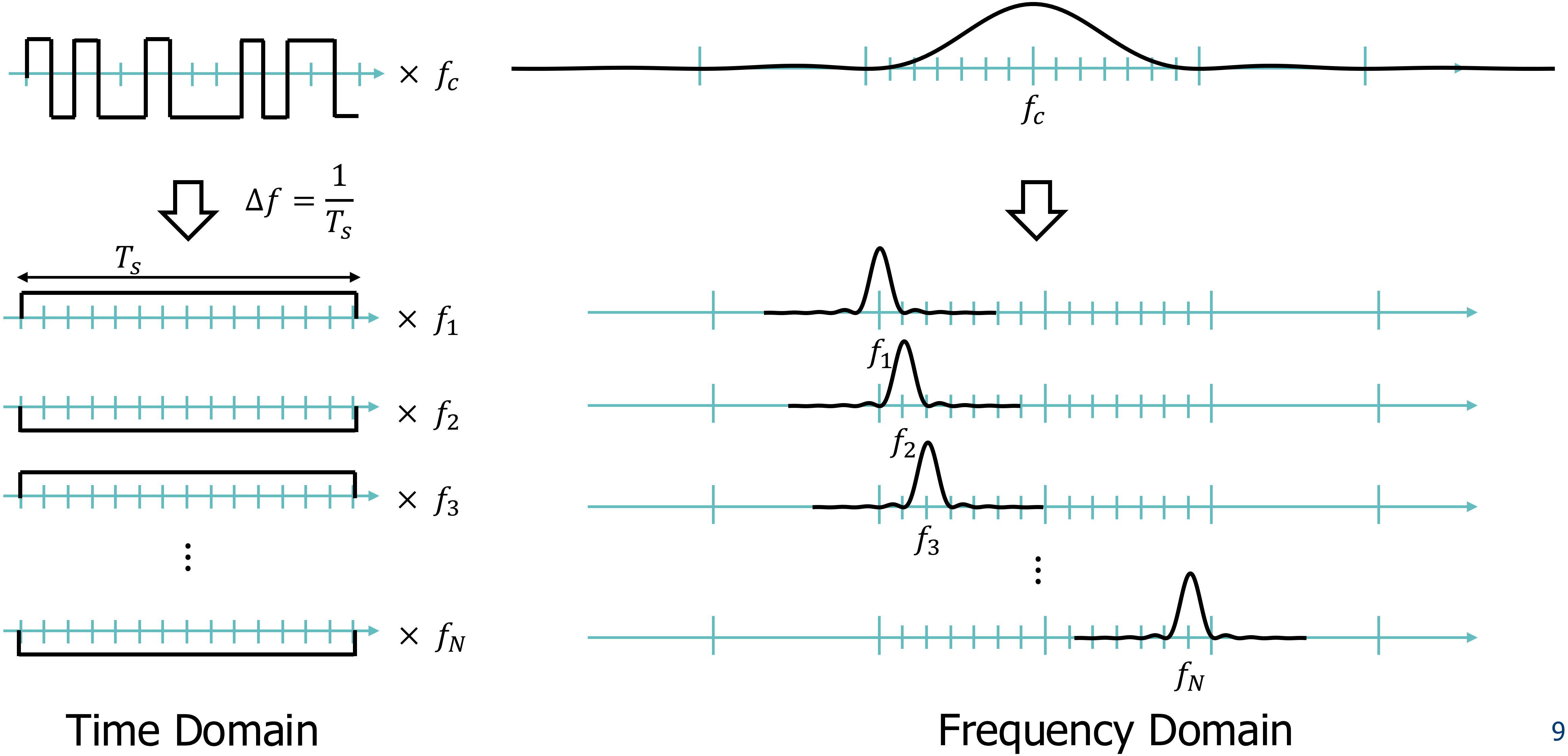
⇒ Shorter symbol time

⇒ Inter-symbol interference more significant

# Motivation of OFDM



# OFDM: Orthogonal Frequency-Division Multiplexing



# OFDM – Analog View

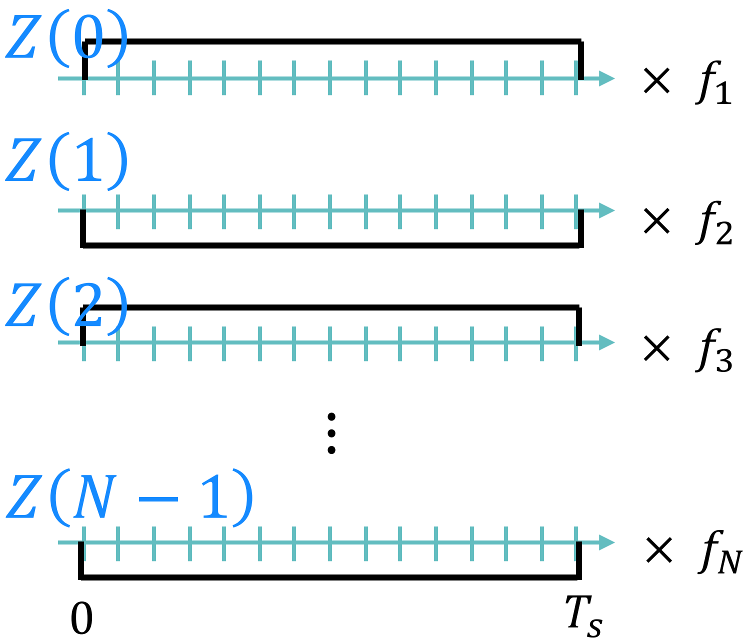
$$s_k(t) = Re\left\{Z(k)e^{j[2\pi(f_c+k\Delta f)t]}\right\},\ 0 \le t < T_s,\ \Delta f = 1/T_s$$

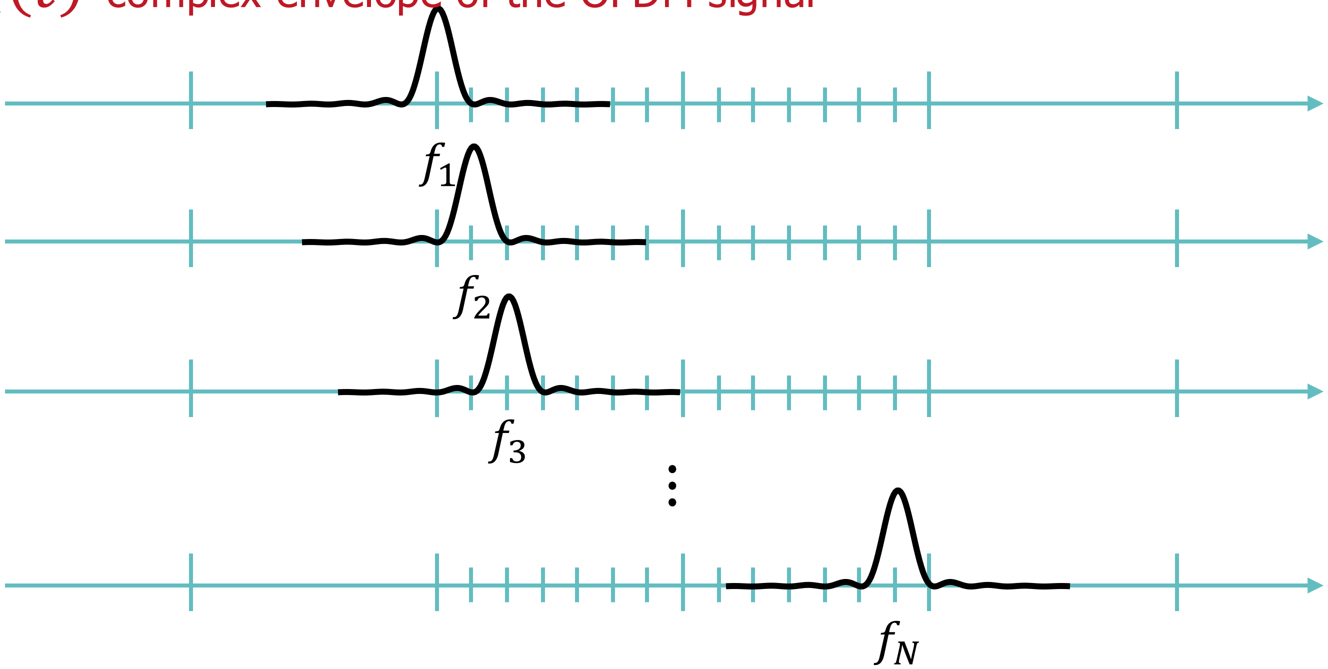
Modulated Subcarrier

$$s(t) = \frac{1}{N}\sum_{k=0}^{N-1} s_k(t) = Re\left\{\underbrace{\left(\frac{1}{N}\sum_{k=0}^{N-1} Z(k)e^{j2\pi k\Delta f t}\right)}_{s_L(t)} e^{j2\pi f_c t}\right\}, 0 \le t < T_s$$

OFDM signal

$s_L(t)$ complex envelope of the OFDM signal




Complex envelope of the $k$-th subcarrier, constant during a symbol interval

# OFDM – Digital View

$$s_k(t) = Re\{Z(k)e^{j[2\pi(f_c+k\Delta f)t]}\},\ 0 \le t < T_s,\ \Delta f = 1/T_s$$

Modulated Subcarrier

OFDM signal

$$s(t) = \frac{1}{N}\sum_{k=0}^{N-1} s_k(t) = Re\left\{\boxed{\left\{\frac{1}{N}\sum_{k=0}^{N-1} Z(k)e^{j2\pi k\Delta f t}\right\}} e^{j2\pi f_c t}\right\}, 0 \le t < T_s$$

$s_L(t)$ complex envelope of the OFDM signal

Sampling time $\frac{T_s}{N}$

(sample at $t = \frac{nT_s}{N}, n = 0, 1, \cdots, N-1$)

$$s_L(n) = \frac{1}{N}\sum_{k=0}^{N-1} Z(k)e^{j2\pi kn/N}, n = 0, 1, \cdots, N-1$$

$$= IDFT\{Z(k)\}$$

→ Generate time sequence $s_L(n)$ from frequency samples

$$\boxed{\Delta f = \frac{1}{T_s}, \qquad \Delta t = \frac{T_s}{N}}$$

$$Z(k) = \sum_{n=0}^{N-1} s_L(n)e^{-j2\pi kn/N}, k = 0, 1, \cdots, N-1$$

$$= DFT\{s_L(n)\}$$

# In practice, the zero subcarrier is DC

$$k = \{0, 1, \cdots, N-1\}$$

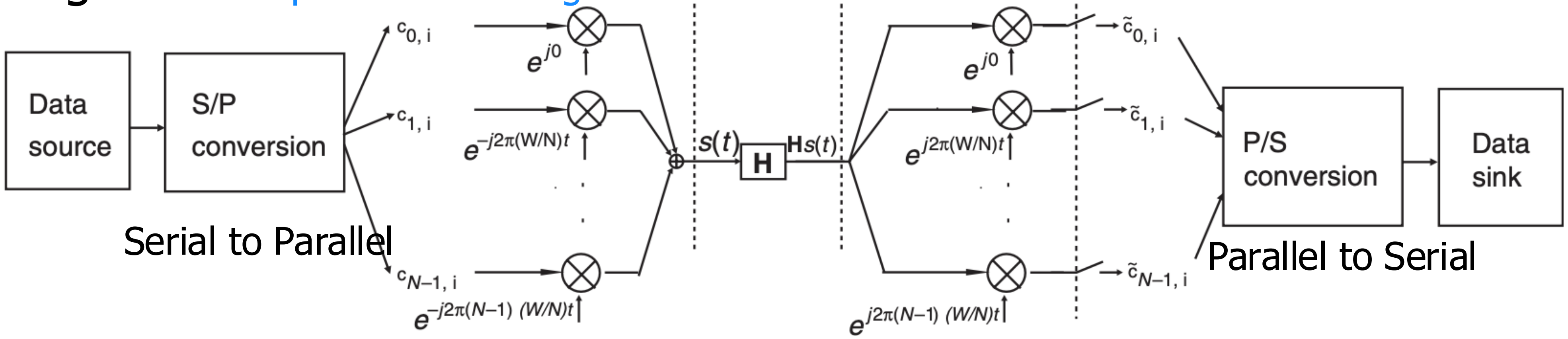
$$\Downarrow$$

$$k = \left\{-\frac{N}{2}, -\frac{N}{2}+1, \cdots, 0, \cdots, \frac{N}{2}-2, \frac{N}{2}-1\right\}$$

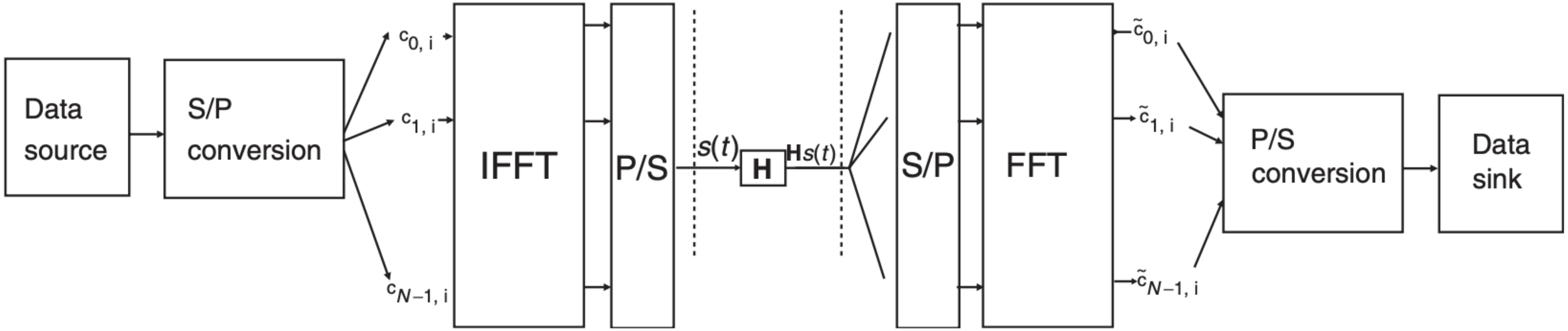
- The “0” subcarrier is associated with the center frequency
- The “0” subcarrier is omitted and filled with zero values (leakage of the LO signal)

# OFDM Representation

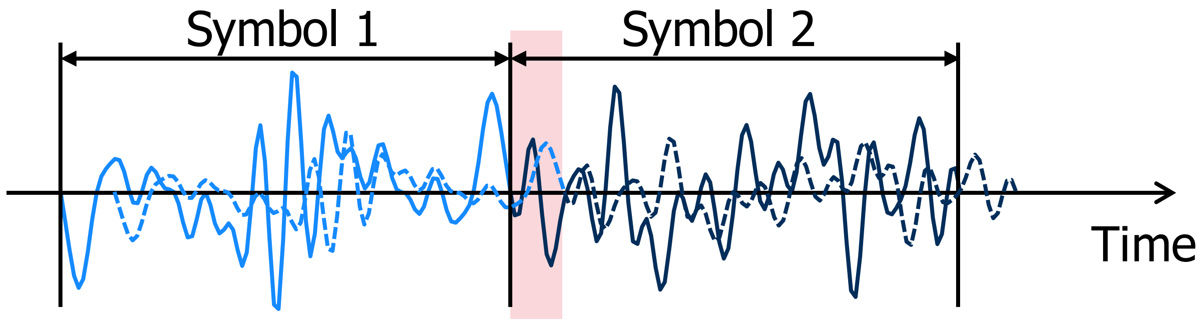
Analog For Concept Understanding




Digital Actual implementation

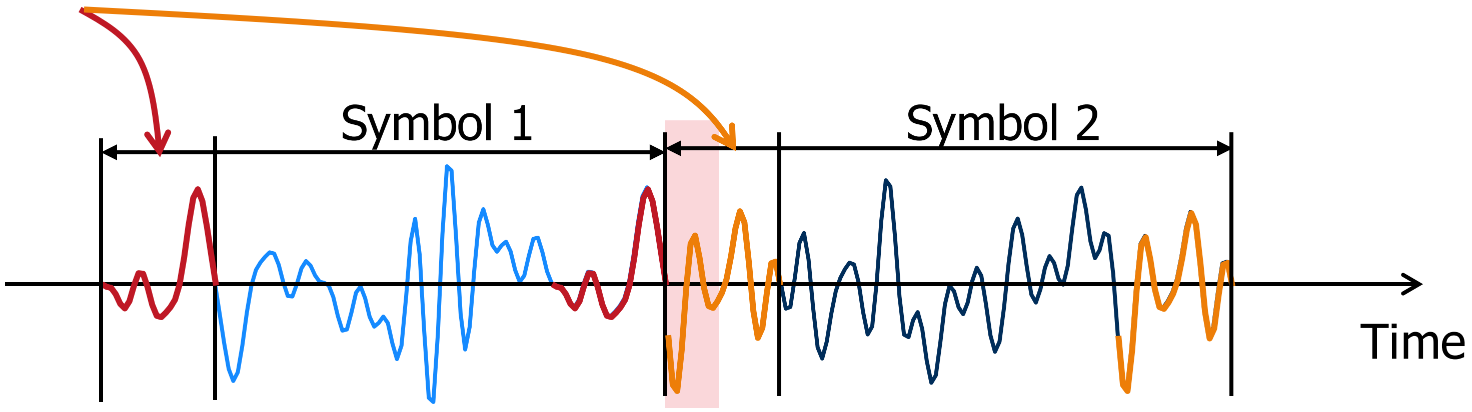


# Cyclic Prefix to Combat Delay Dispersion

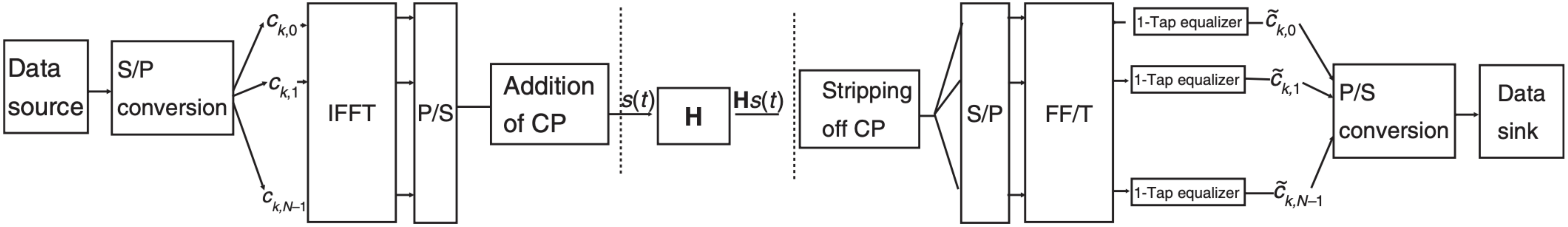


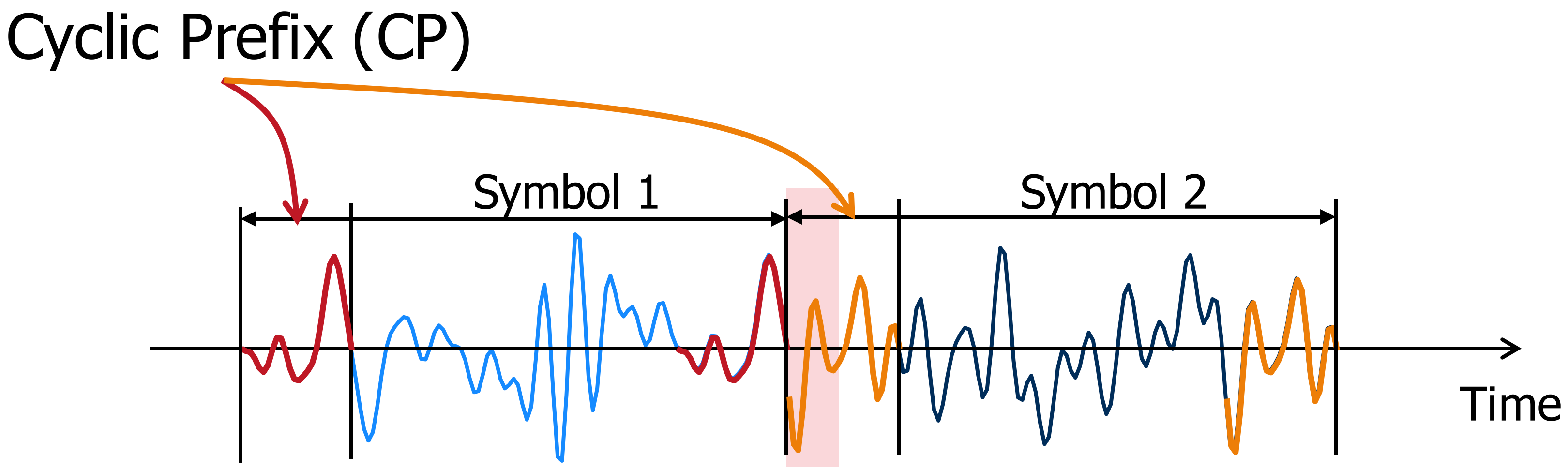

Inter-Symbol Interference

Cyclic Prefix (CP)



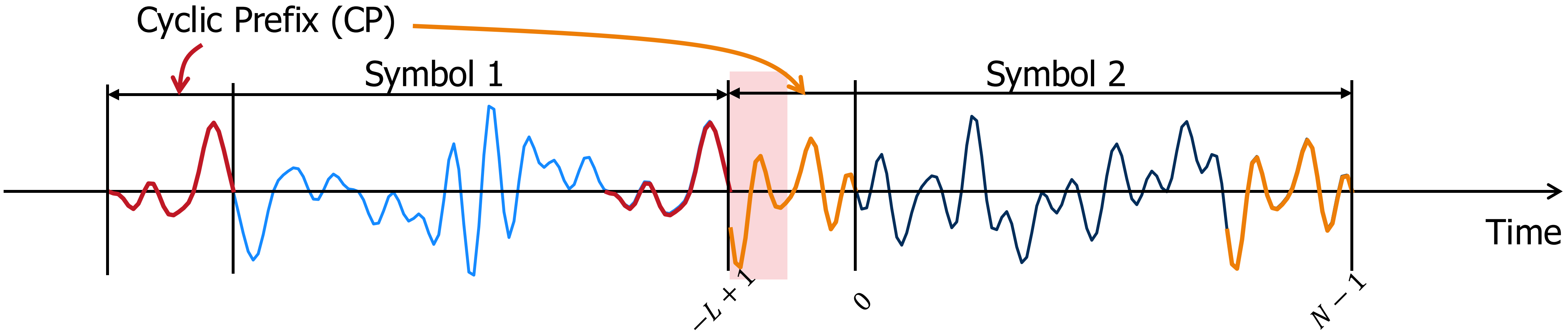
# OFDM Implementation with CP



# OFDM Transmission Model with Cyclic Prefix

$$y(n) = h(n) * \tilde{s}_L(n) = \sum_{k=0}^{N+L-1} h(k)\, \tilde{s}_L(n-k) = \sum_{k=0}^{N+L-1} h(k)\, s_L(n-k)_N = \sum_{k=0}^{L-1} h(k)\, s_L(n-k)_N$$




length $L-1$

length $N$

$s_L(n),\ 0 \le n \le N-1$

Channel: $h(n)$ of length $L$

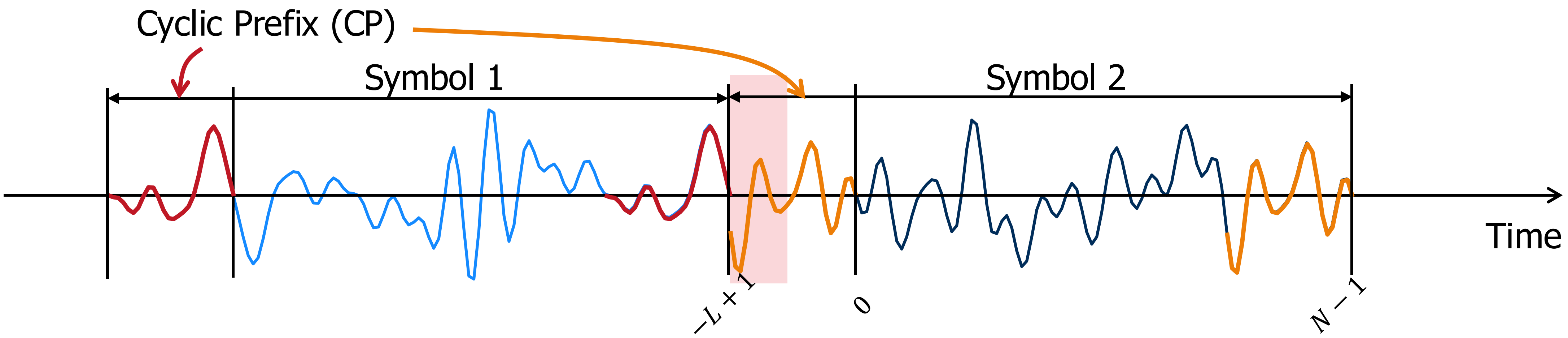
→ expand to length $N+L-1$ with zero padding

length $N+L-1$

$\tilde{s}_L(n) = s_L(n)_N,\ -L+1 \le n \le N-1$

# OFDM Transmission Model with Cyclic Prefix

$$y(n) = h(n) * \tilde{s}_L(n) = \sum_{k=0}^{N+L-1} h(k)\, \tilde{s}_L(n-k) = \sum_{k=0}^{N+L-1} h(k)\, s_L(n-k)_N = \sum_{k=0}^{L-1} h(k)\, s_L(n-k)_N$$




Channel: $h(n)$ of length $L$

→ expand to length $N+L-1$ with zero padding

| | $s_L(0)$ | $s_L(1)$ | $s_L(2)\cdots$ | $s_L(N-2)$ | $s_L(N-1)$ |
|---|---|---|---|---|---|
| $n=0$ | $h(0)$ | | | $h(2)$ | $h(1)$ |
| $n=1$ | $h(1)$ | $h(0)$ | | | $h(2)$ |
| $n=2$ | $h(2)$ | $h(1)$ | $h(0)$ | | |

$\Longrightarrow$ Circular Convolution ($\otimes$)

# OFDM Transmission Model with Cyclic Prefix

$$y(n) = h(n) \otimes s_L(n) = \sum_{k=0}^{N-1} h(k) \boxed{s_L(n-k)_N}$$

Circular Convolution

$s_L(m)_N = s_L[(m) \bmod N]$

circular shift of $s_L(n)$

Using the notations earlier

$$Y(k) = H(k)Z(k), \qquad k = 0, 1, \cdots, N-1$$

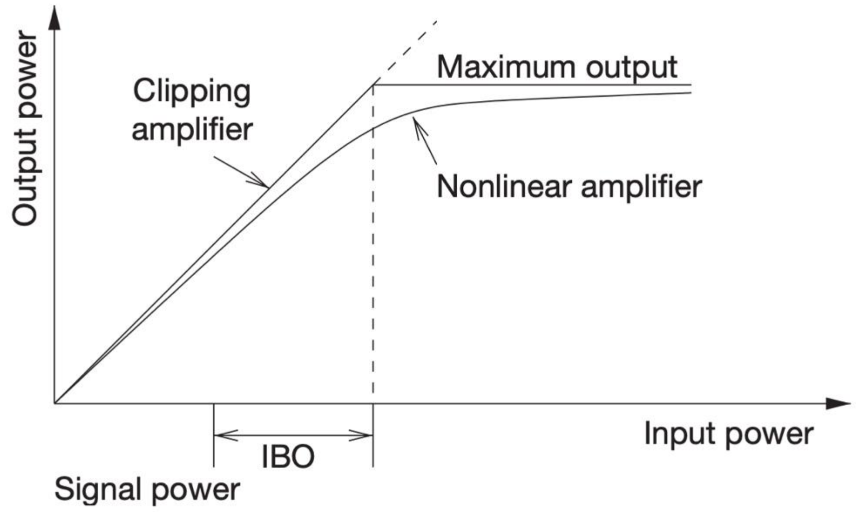
For each subcarrier $k$,

the TX & RX symbols are characterized by a constant $H(k)$

# Issue: High Peak-to-Average Power Ratio (PAPR)

OFDM signal: superposition of sinusoidal signals on $N$ subcarriers

- On average: power $\propto N$
- However, if signals of all subcarriers add up constructively, the amplitude of the signal $\propto N$, and power $\propto N^2$
- Worst case PAPR $\propto N$




High PAPR

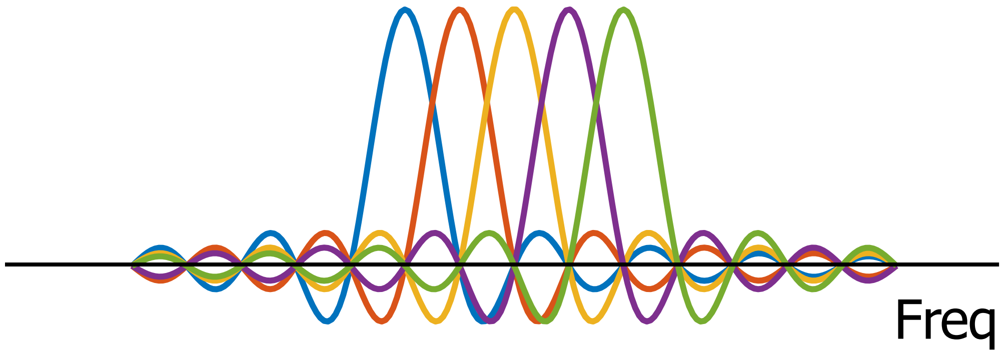
→ clipping of the signal amplitude in the D/A converter

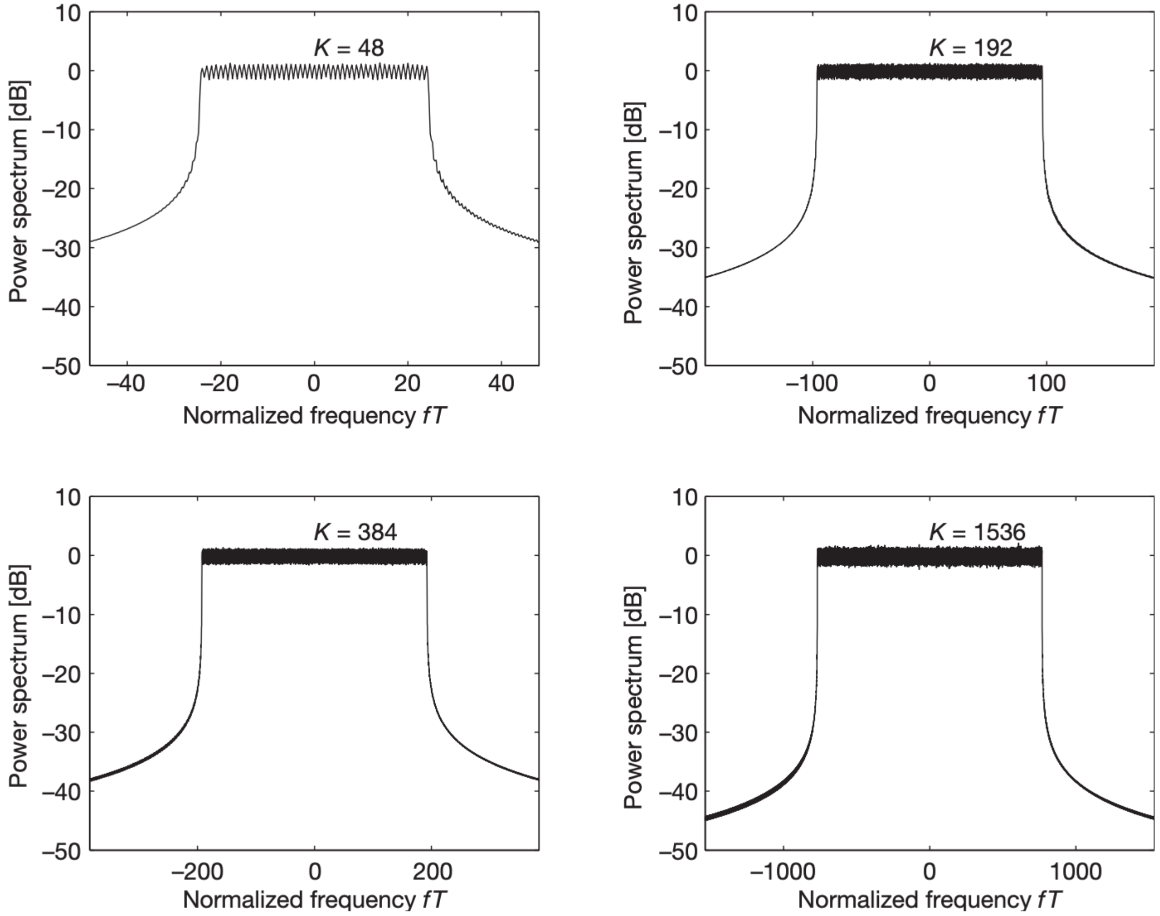
→ saturate the transmitter power amplifier

PAPR Reduction Techniques

# Issue: Spectral shaping for OFDM systems

- Severe out-of-band radiation due to poor decay of sinc function
- More subcarriers reduces out-of-band radiation
  - Spectrum of individual subcarrier (main lobe & side lobes) becomes narrower with increasing number of subcarriers



# Examples of OFDM Systems: WiFi, WiMax, LTE

**Table 5-1:** Examples of OFDM systems with CP

| System | DFT size (N) | Used sub-channel | Sub carrier spacing kHz | Sample rate MHz | Bandwidth MHz | OFDM symbol time usec |
|---|---|---|---|---|---|---|
| IEEE 802.11a | 64 | 52 (48 data + 4 pilot) | 312.5 | 20 | 16.56 | 4.0 (3.2 + 0.8) |
| WiMax | 2048 | 1681 | 7.81 | 16 | 20 | 144 (128 + 16) |
| LTE | 2048 | 1200 | 15 | 30.72 | 20 | 83.37 (66.67 + 16.7) |

# Reference - OFDM

- Andreas F. Molisch. *Wireless Communications*. John Wiley & Sons, 2011. [NTU Library Link]
  - Chap. 19: Orthogonal Frequency Division Multiplexing (OFDM)

- Sung-Moon Michael Yang. *Modern Digital Radio Communication Signals and Systems*. Springer, 2020. [NTU Library Link]
  - Chap. 5: OFDM Signals and Systems

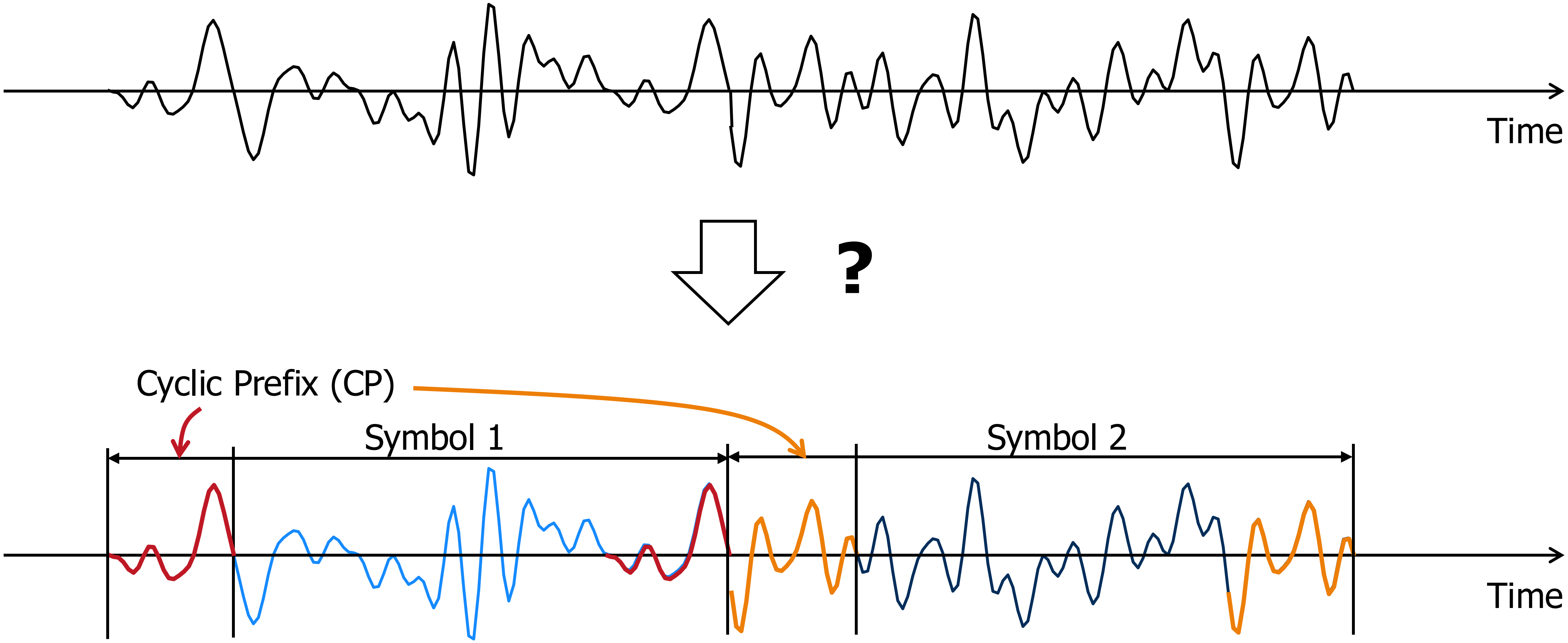
- Tri T. Ha. *Theory and Design of Digital Communication Systems*. Cambridge University Press, 2010.
  - 6.18 Orthogonal frequency division multiplexing (OFDM)
  - 7.23 OFDM demodulation

- Henrik Schulze and Christian Lüders. *Theory and applications of OFDM and CDMA: Wideband wireless communications*. John Wiley & Sons, 2005.
  - Chap. 4: OFDM

# Frame Synchronization

Wi-Fi 802.11a/g as an example

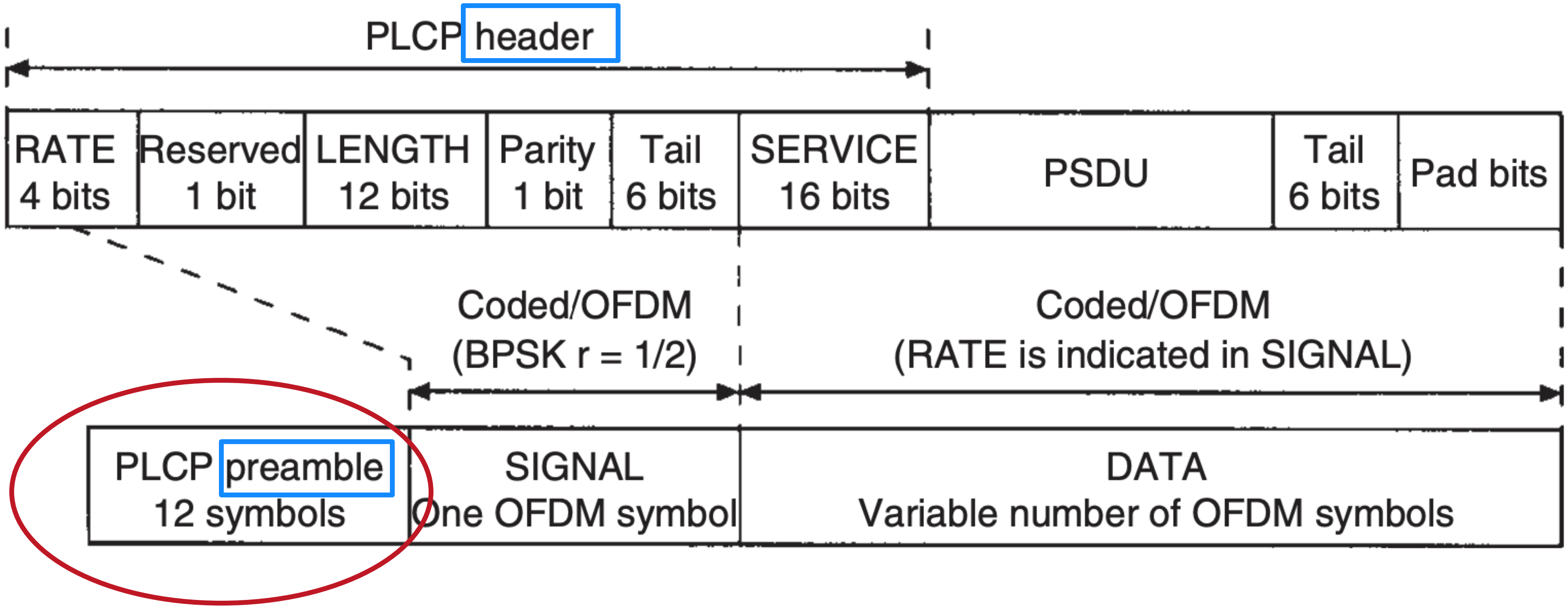
# How to find symbol boundaries?




*Wrong symbol boundaries will lead to wrong decoded data!*

# 802.11 a/g PHY Frame Format

Start the frame with predefined known signals

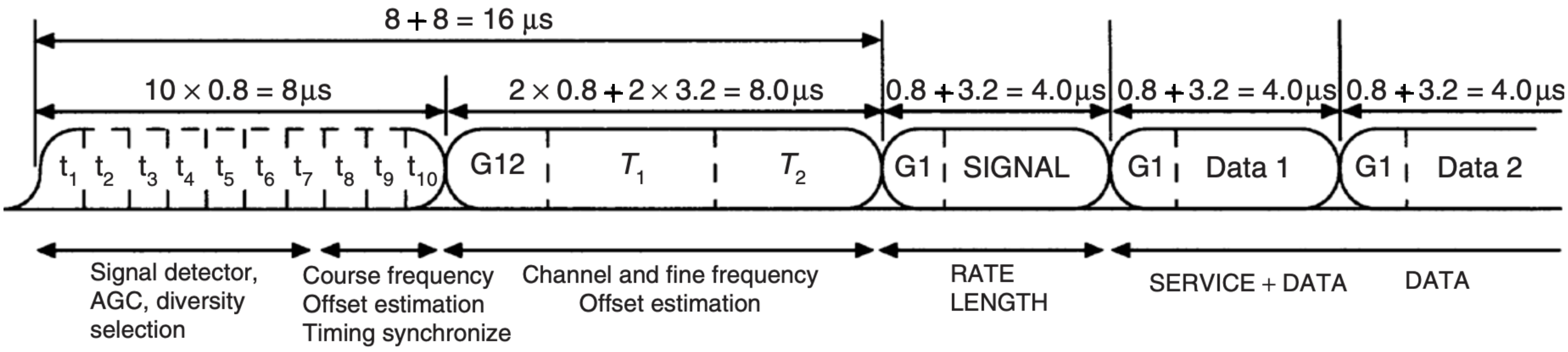



Focus on this part

PLCP: Physical Layer Convergence Procedure

PSDU: Physical Layer Service Data Unit

# 802.11 a/g PLCP Preamble




Short Training Sequence

10x Short Training Symbol

Long Training Sequence

2.5x Long Training Symbol

# Short Training Symbol & Long Training Symbol

**Short OFDM Training Symbol**

4x Expansion in Frequency

4x Contraction in Time

$$S_{-26,26} = \overset{\text{Power Normalization}}{\boxed{\sqrt{(13/6)}}}\{0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 0, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0\}$$

IFFT ⇨ 4 Short Training Symbols

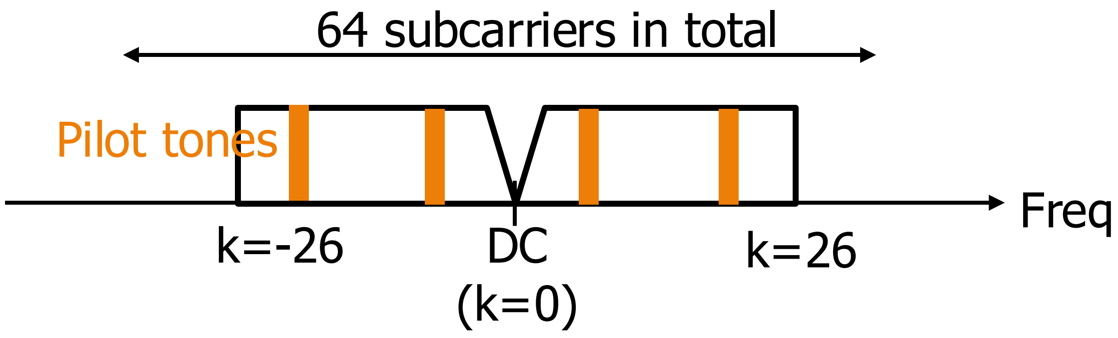
**Long OFDM Training Symbol**

$$L_{-26,26} = \{1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, \overset{\substack{\text{DC}\\ \downarrow}}{0}, 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1\}$$

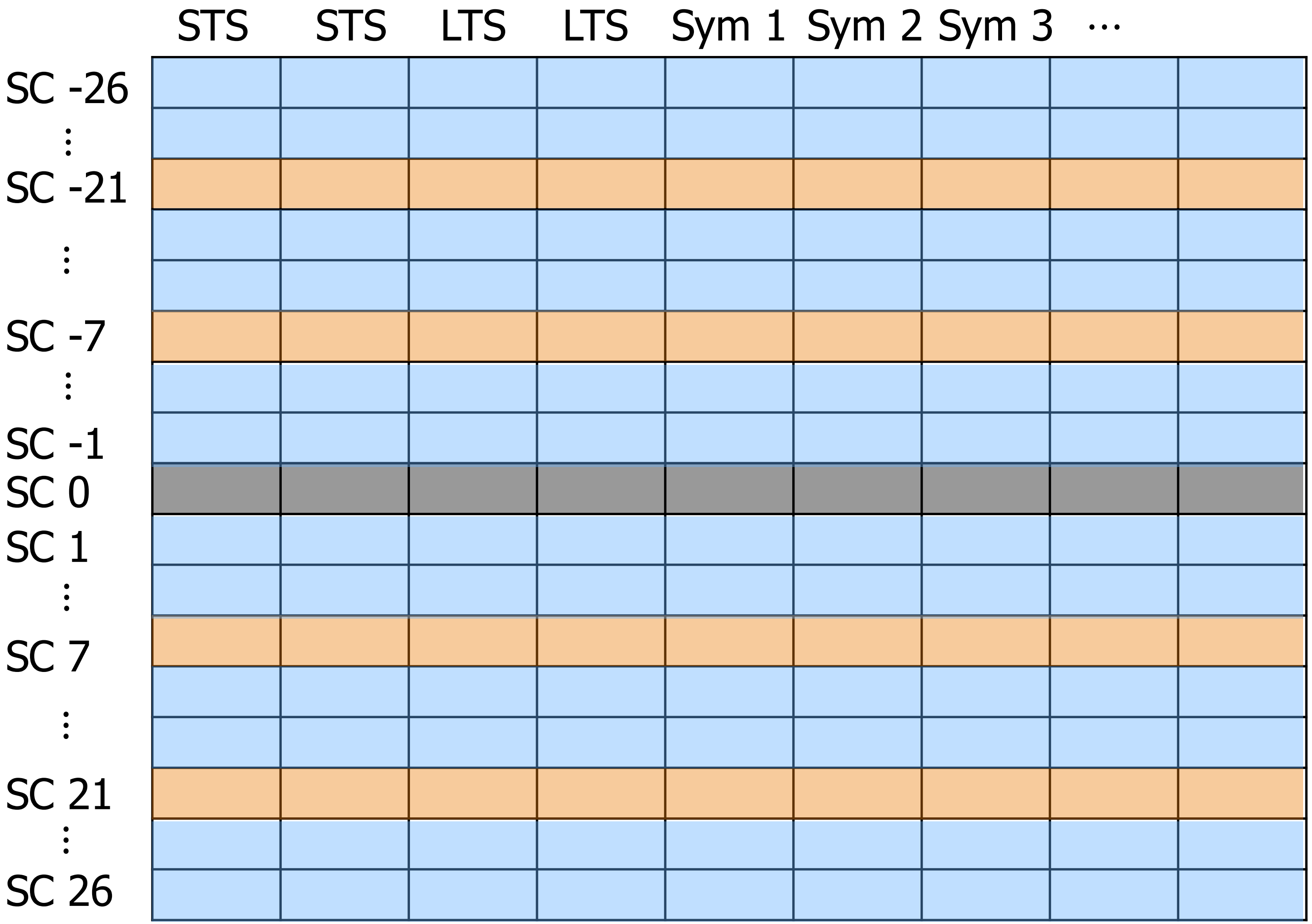
IFFT ⇨ 1 Long Training Symbol

$N_{FFT} = 64$

- $N_{Data} = 48$
- $N_{Pilot} = 4;\ \{-21, -7, 7, 21\}$
- 12 null-carriers



# Data & Pilot Tones




Data Carrier

- QAM

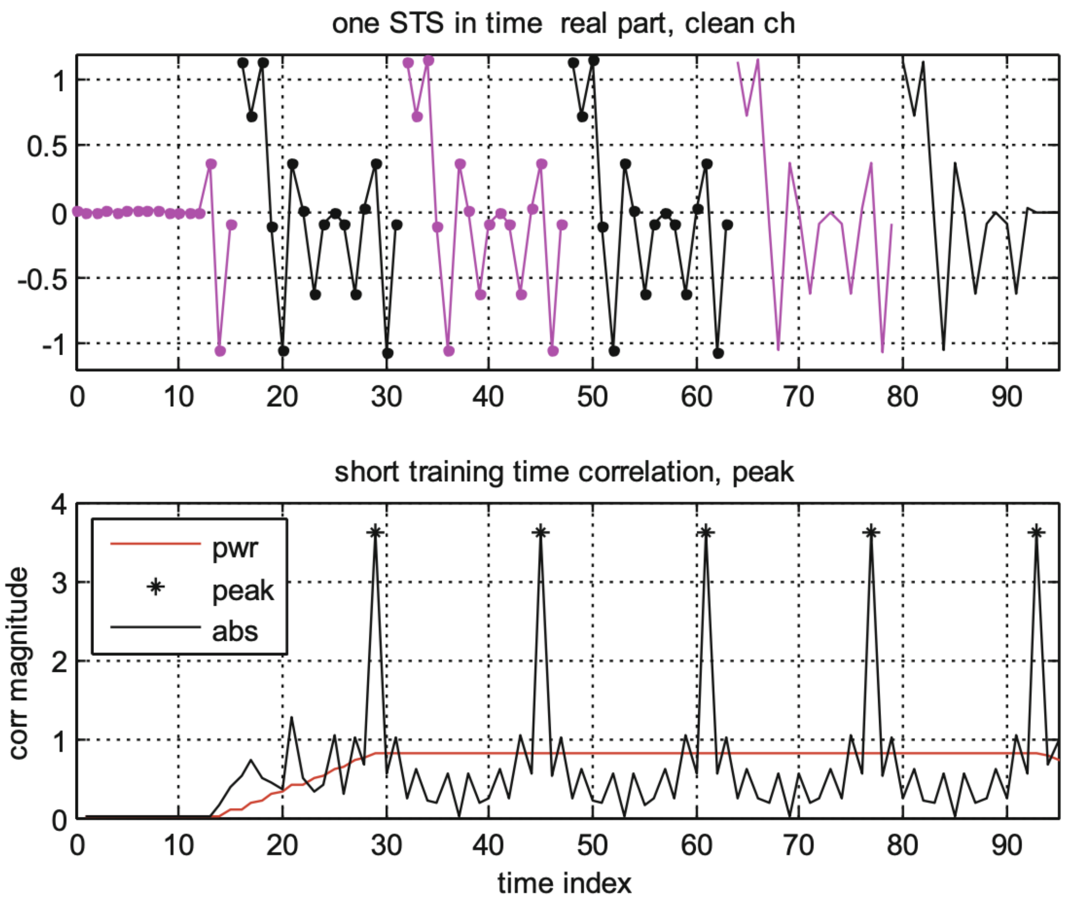
Pilot Carrier

- BPSK modulated

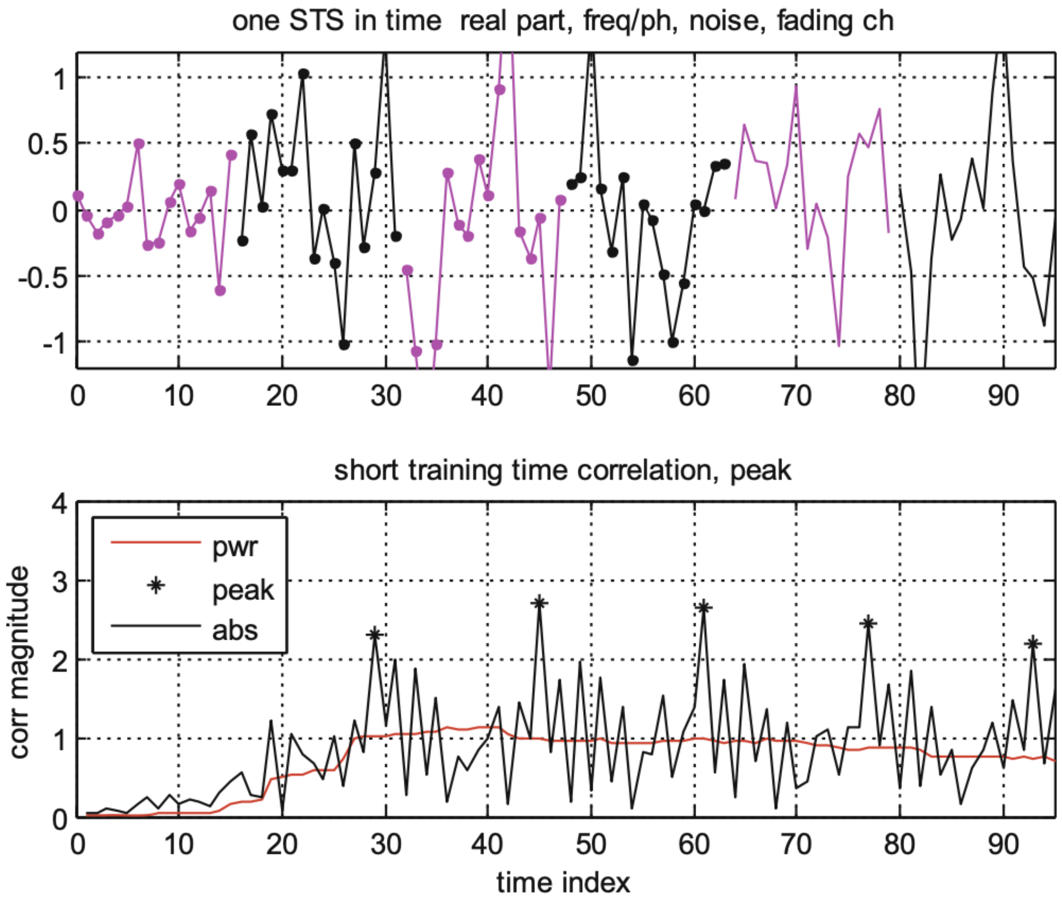
Null Carrier

# Matched Filter to Detect STS & LTS

Clean




Frequency offset + phase offset
+ noise + frequency selective fading



# Reference - Preamble


- Ali Grami. *Introduction to Digital Communications*. Academic Press, 2020. [NTU Library Link]
    - Chap. 8: Synchronization
- Sung-Moon Michael Yang. *Modern Digital Radio Communication Signals and Systems*. Springer, 2020. [NTU Library Link]
    - Chap. 7: Synchronization of Frame, Symbol Timing and Carrier
- Ahmad R.S. Bahai, Burton R. Saltzberg, and Mustafa Ergen. *Multi-carrier digital communications theory and applications of OFDM*. Springer New York, 2004. [NTU Library Link]
    - Chap. 5: Synchronization