114-1 電工實驗（通信專題）

Carrier Frequency Offset, Linear Array

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Review

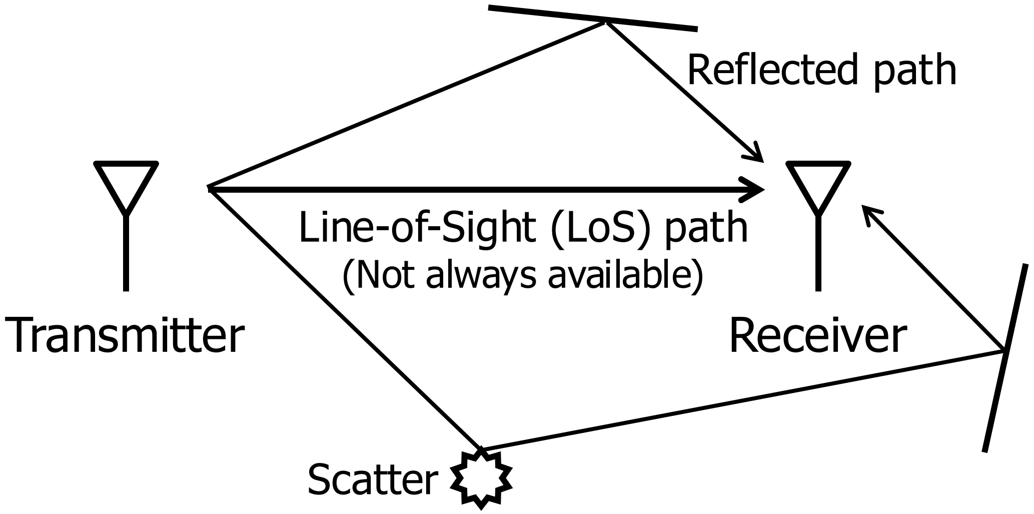
Channel Estimation

Software-Defined Radio & USRP

Channel Impulse Response & Channel Transfer Function

Channel Impulse Response

$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k}\, \delta(t - \tau_k)$$



Channel transfer function

$$H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{\underbrace{-j2\pi f \tau_k}}$$

Frequency-dependent

phase lag caused by delay τ_k

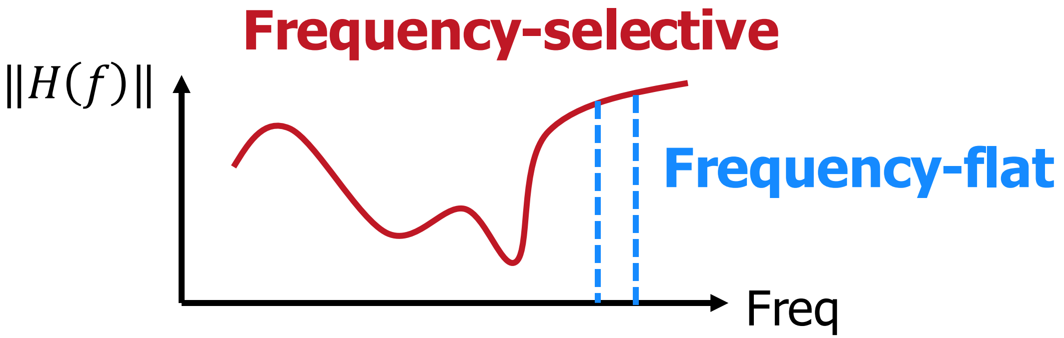
i.i.d., uniformly distributed in $[0, 2\pi]$

(f_c is large, small changes in delay τ_k cause large changes in the phase $2\pi f_c \tau_k$)

Frequency-Flat vs Frequency-Selective

Channel Impulse Response

$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k} \, \delta(t - \tau_k)$$



Narrowband frequency-flat approximation

The channel transfer function is approximately constant over a small band around f_0

Channel transfer function

$$H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{\underbrace{-j2\pi f \tau_k}}$$

↓ Frequency-dependent

phase lag caused by delay τ_k

i.i.d., uniformly distributed in $[0, 2\pi]$

(f_c is large, small changes in delay τ_k cause large changes in the phase $2\pi f_c \tau_k$)

$$h \approx H(f_0) = \sum_{k=1}^{M} A_k e^{j\gamma_k}$$

$\gamma_k = \theta_k - 2\pi f_0 \tau_k \mod 2\pi$

γ_k: i.i.d., uniform over $[0, 2\pi]$

Narrowband Rayleigh & Rician fading models

$$h \approx H(f_0) = \sum_{k=1}^{M} A_k e^{j\gamma_k}$$

$\gamma_k = \theta_k - 2\pi f_0 \tau_k \mod 2\pi$

γ_k: i.i.d., uniform over $[0, 2\pi]$

M is large & no dominant component

Narrowband Rayleigh fading

$$H(f_0) \sim CN\left(0, \sum_{K=1}^{M} A_k^2\right)$$

1 dominant component + many smaller multipath components

$$h = A_1 e^{j\gamma_1} + h_{\text{diffuse}}$$

Complex Gaussian

Narrowband Rician fading

$$H(f_0) \sim CN\left(A_1 e^{j\gamma_1}, \sum_{k=2}^{M} A_K^2\right)$$

OFDM Channel Estimation - Pilot Symbols

For OFDM:

$$y(n) = h(n) \otimes x(n) \quad \Leftrightarrow \quad Y(k) = H(k)X(k), \qquad k = 0, 1, \cdots, N-1$$

To estimate $H(k)$, transmit known pilot symbols

Received symbol

$$H_{LS}(k) = \frac{Y(k)}{X(k)}$$

A complex number representing the phase and amplitude of the channel response

Known pilot symbol

Least Squares Channel Estimate

802.11 a/g: Channel Estimation Using LTS

Long OFDM Training Symbol

$$L_{-26,26} = \{1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, 1,$$
$$-1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1\}$$

Received symbol

$$H_{LS}(k) = \frac{Y(k)}{X(k)}$$

Long Training Symbol, either 1 or -1

- $\frac{Y(k)}{1} = Y(k) \cdot 1$
- $\frac{Y(k)}{(-1)} = Y(k) \cdot (-1)$

$$H_{LS}(k) = Y(k) \cdot X(k)$$

Avoid the divide-by-0 problem

Equalization

Use channel estimates to normalize phase/amplitude for OFDM data symbols

$$Y(k) = H(k)\,X(k)$$

$Y(k)$	$H(k)$	$X(k)$	
Known (observation)	Unknown	Known (pilot symbols)	**Channel Estimation**
Known (observation)	Known (estimate)	Unknown	**Equalization**

Received symbol

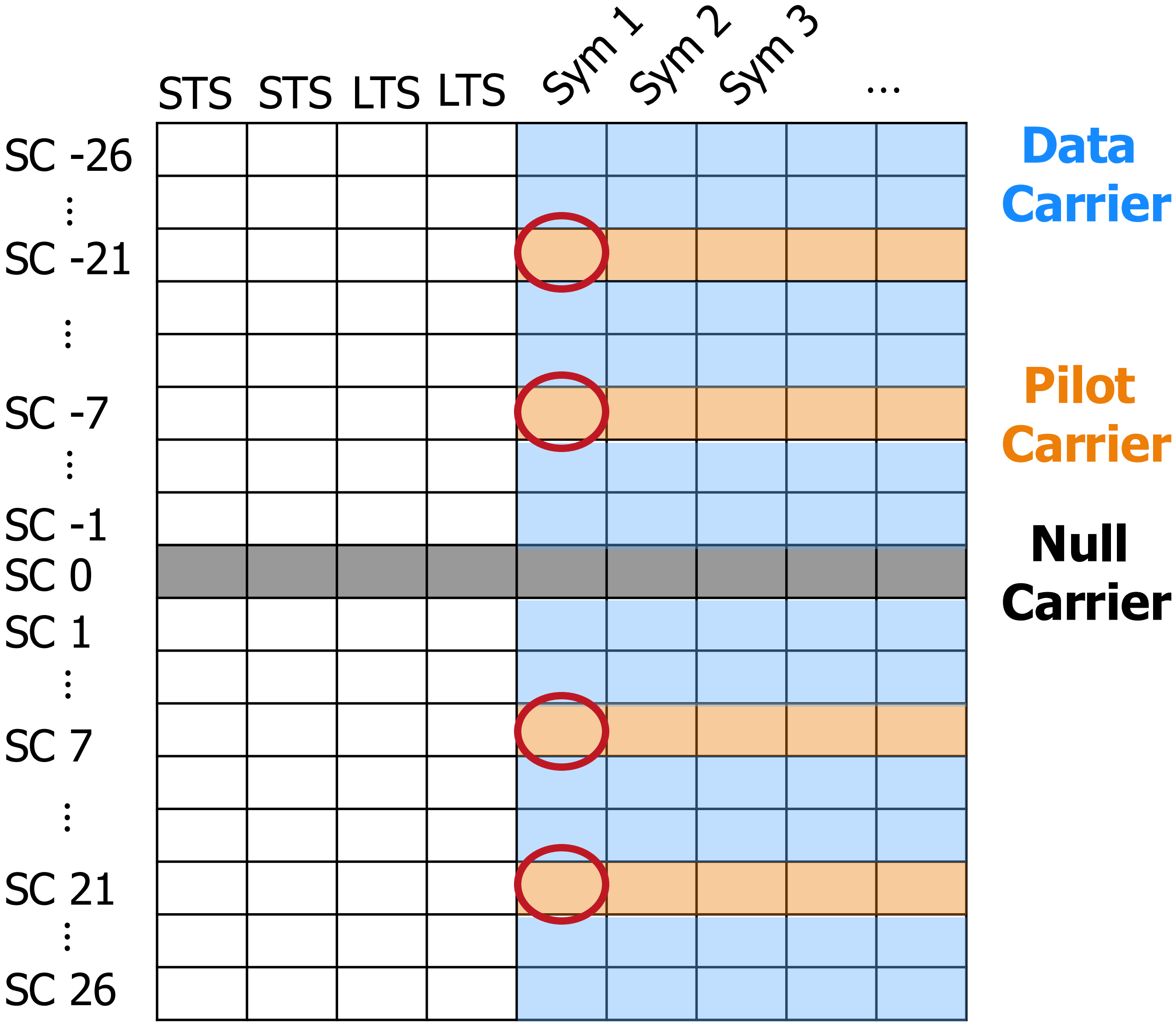
$$\hat{X}(k) = \frac{Y(k)}{H_{LS}(k)}$$

Equalized symbol

Used for demodulation

Estimated channel

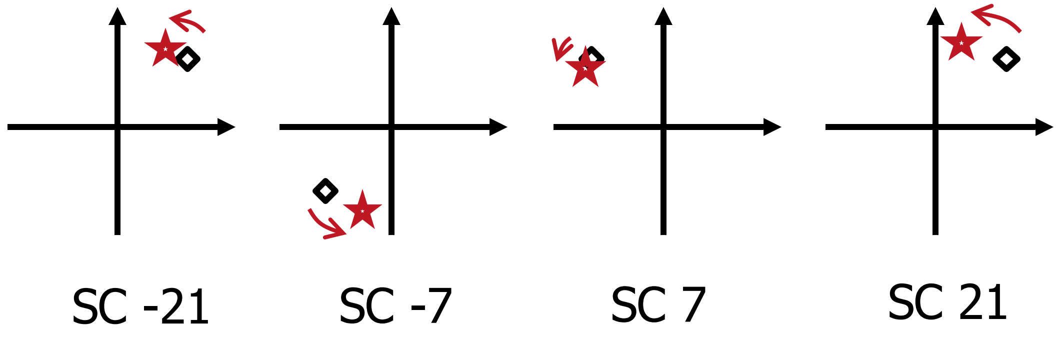
Tracking with Pilot Tones



Transmit known symbols in pilot tones

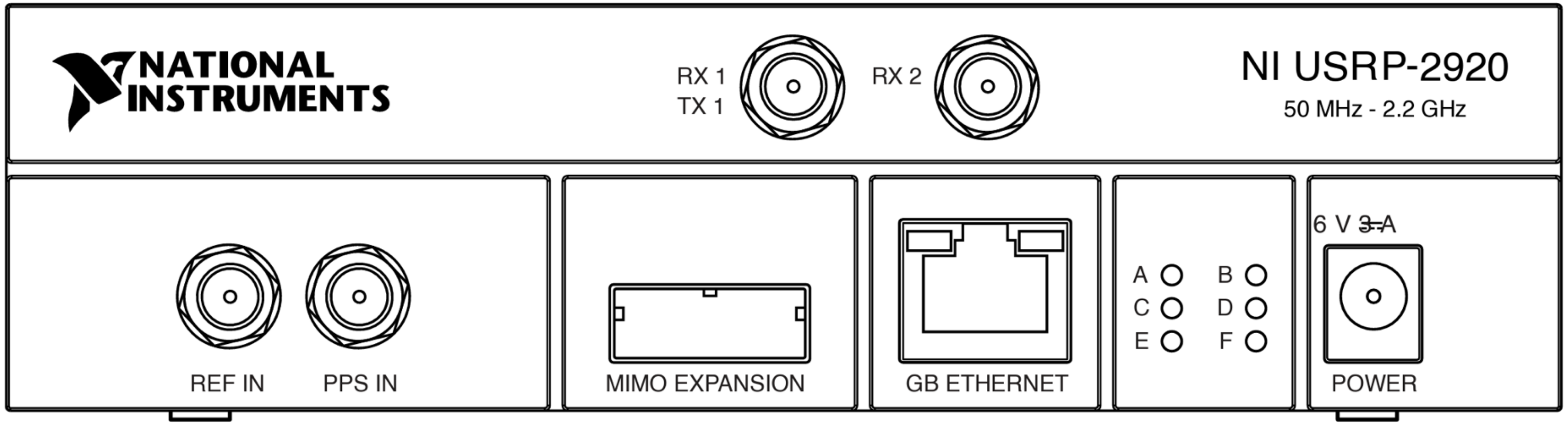
Correct for drift in the channel over time

For example, after equalization, obtain additional correction term using the four pilot symbols

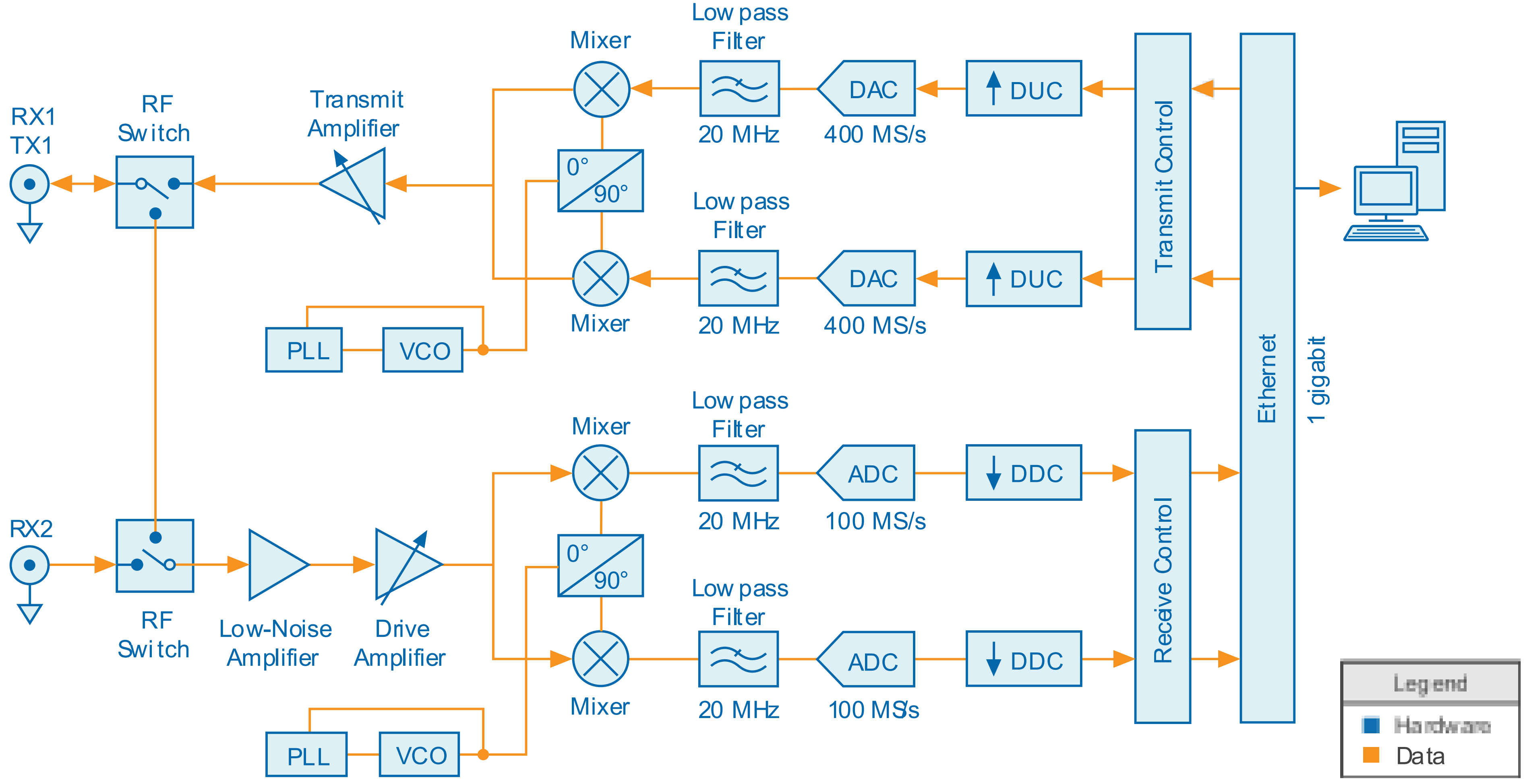


Universal Software Radio Peripheral

USRP-2920

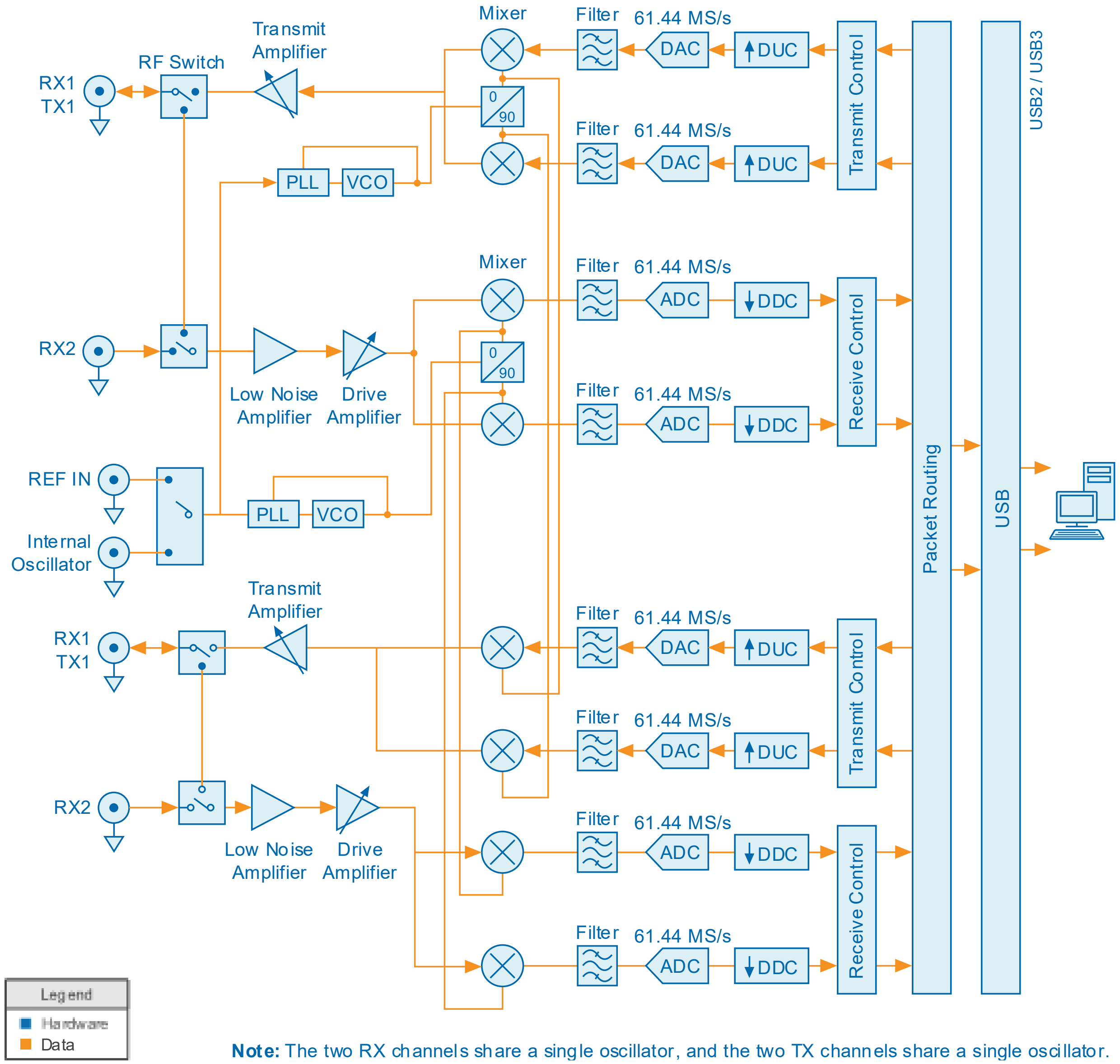


Block Diagram of USRP-2920



Can we use both antennas to receive simultaneously? No!

Block Diagram of USRP-2901



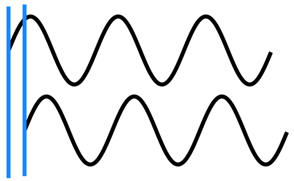
Note: The two RX channels share a single oscillator, and the two TX channels share a single oscillator.

Can we use all 4 antennas simultaneously?

How many TX, RX?

Do the two TX channels share the same oscillator? Why is it important?

If two TX use different oscillators that exhibit a phase difference ϕ (ϕ can vary each time the device is turned on)



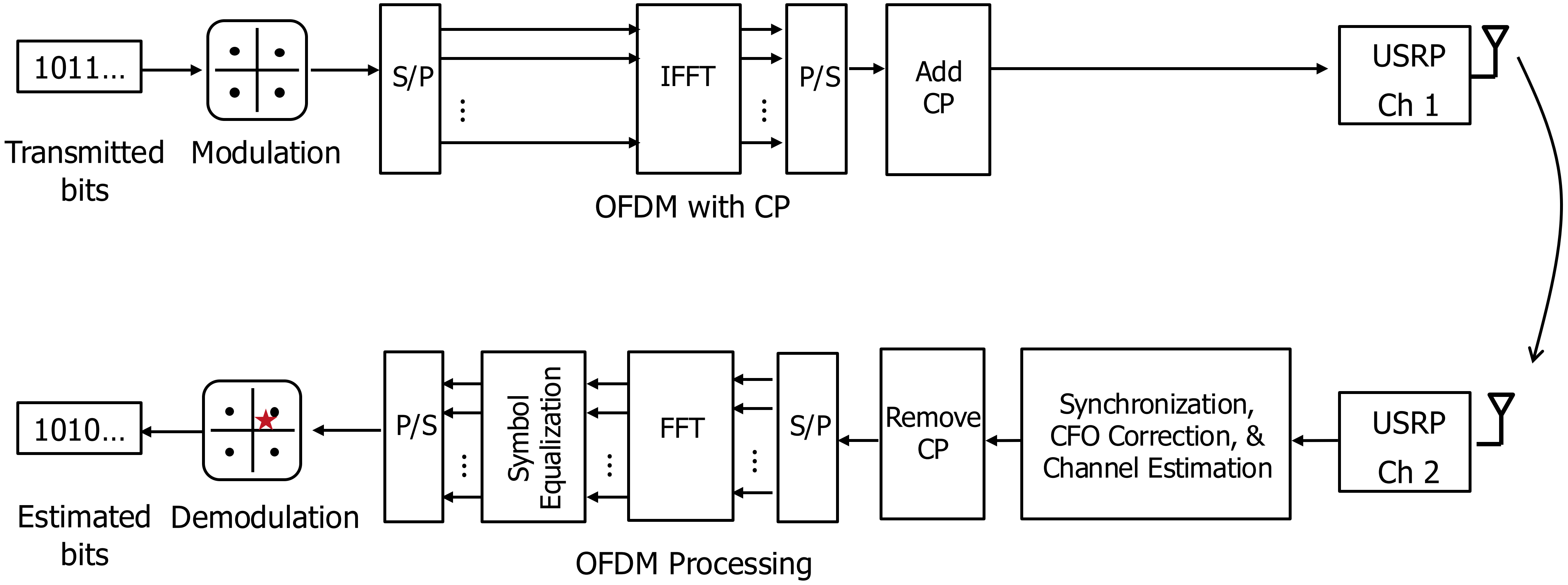
Up-conversion:

$$y_1(t) = Re\{x(t)e^{j2\pi f_c t}\}$$

$$y_2(t) = Re\{x(t)e^{j2\pi f_c t+\phi}\}$$

$$= Re\{x(t)\underbrace{\boxed{e^{\phi}}}_{\text{Phase}} \cdot e^{j2\pi f_c t}\}$$

Lab 4 – Part 1



Focus: synchronization, channel estimation, equalization

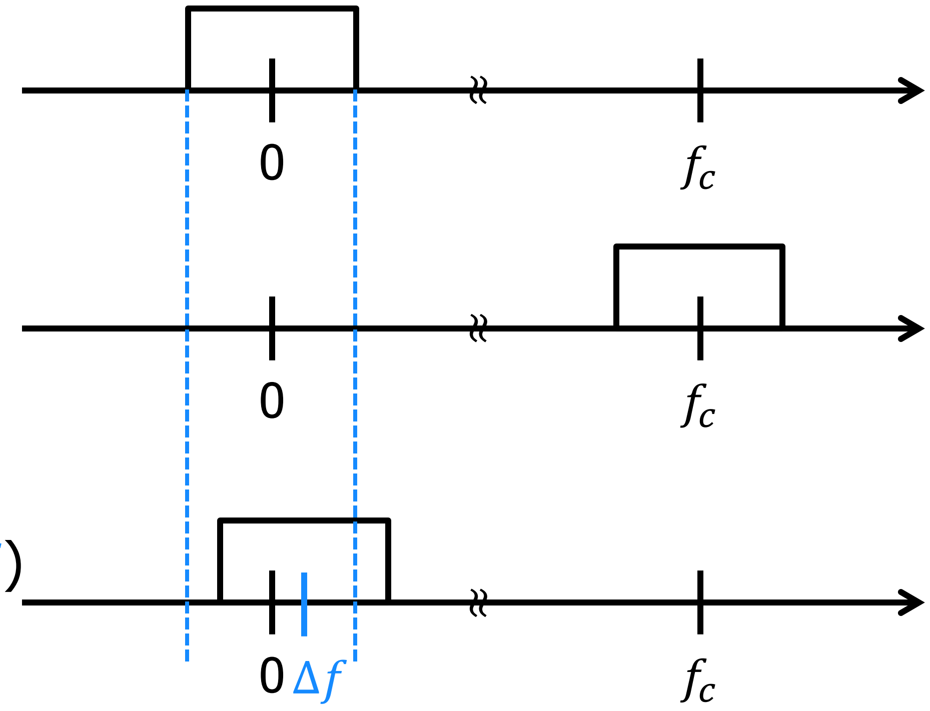
Carrier Frequency Offset

Carrier Frequency Offset (CFO)

Baseband

Upconverted (f_c)
(at TX)

Downconverted ($f_c - \Delta f$)
(at RX)



TX & RX are two distinct devices.

Relative frequency offsets exist between their local oscillator (LOs) due to natural effects such as impurities, electrical noise, and temperature differences.

$$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$$

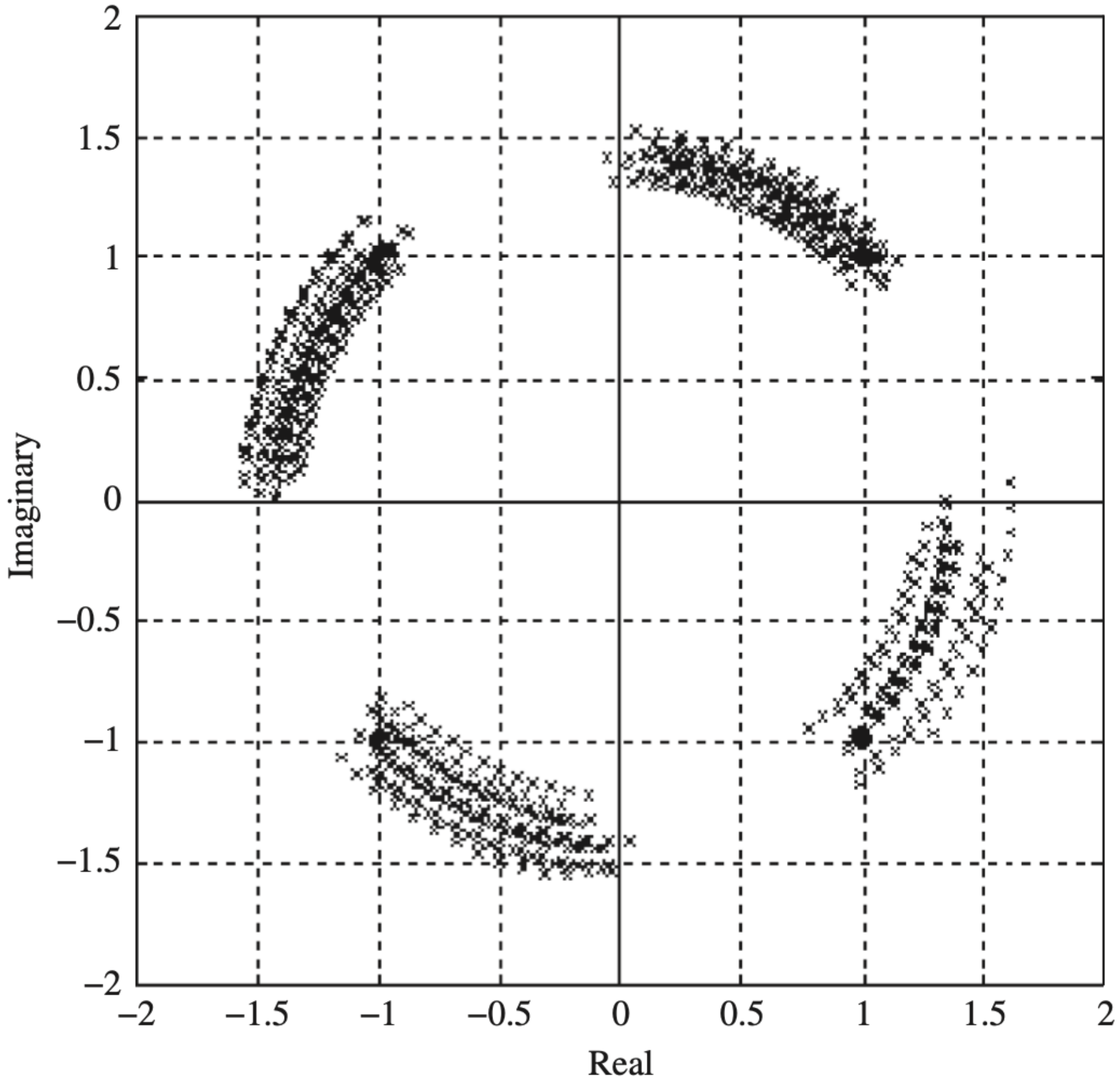
$r(n)$: RX signal (n-th sample)

$s(n)$: TX signal (n-th sample)

Δf: Carrier frequency offset

f_s: sample rate

Received Symbol with CFO



$$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$$

RX signal (n-th sample)

TX signal (n-th sample)

Δf: Carrier frequency offset

f_s: sample rate

Freq. Offset of Commercial Oscillators: PPM

PPM: parts per million

Define the maximum carrier offset

$$\Delta f_{max} = \frac{f_c \times \text{PPM}}{10^6}$$

We only need to calibrate this much

[Example]

Wi-Fi 802.11: ± 20 ppm

$\Longrightarrow$ Carrier difference: $[-40, 40]$ ppm

At 2.4 GHz

$\Delta f_{max} = \frac{2.4 \times 10^9 \times 20}{10^6} = 48$ KHz

CFO Estimation Using Two Identical Halves

Transmit repetitive signals
(e.g., 10 Short Training Symbols, 2 Long Training Symbols)

For two <u>consecutively</u> received copies
(L: the length of each copy, $L = 16$ for STS, $L = 64$ for LTS)

$$p(k) = \sum_{m=0}^{L-1} r^*(k+m)\, r(k+m+L)$$

$r(n) = s(n)\, e^{\frac{j2\pi\,\Delta f\, n}{f_s}}$

$$= \sum_{m=0}^{L-1} s^*(k+m)\, e^{\frac{-j2\pi\,\Delta f\,(k+m)}{f_s}}\, s(k+m+L)\, e^{\frac{j2\pi\,\Delta f\,(k+m+L)}{f_s}}$$

$$= e^{\frac{j2\pi\,\Delta f\, L}{f_s}} \sum_{m=0}^{L-1} s^*(k+m)\, s(k+m+L)$$

$\mathrm{s}(k) = \mathrm{s}(k+L)$

$$= e^{\frac{j2\pi\,\Delta f\, L}{f_s}} \sum_{m=0}^{L-1} |s(k+m)|^2$$

Frequency Offset $\Longrightarrow$ Phase diff between 1st & 2nd RX copy

$$\phi = \frac{2\pi L\,\Delta f}{f_s}$$

CFO Estimation Using Two Identical Halves

$$\boxed{\Delta\hat{f} = \frac{\hat{\phi}\, f_s}{2\pi L}}$$

$$\hat{\phi} = \tan^{-1}\left(\frac{\mathcal{Im}\big(P(k)\big)}{\mathcal{Re}\big(P(k)\big)}\right)$$

$$p(k) = \sum_{m=0}^{L-1} r^*(k+m)\, r(k+m+L)$$

$r(n) = s(n)\, e^{\frac{j2\pi\, \Delta f\, n}{f_s}}$

$$= \sum_{m=0}^{L-1} s^*(k+m)\, e^{\frac{-j2\pi\, \Delta f\, (k+m)}{f_s}}\, s(k+m+L)\, e^{\frac{j2\pi\, \Delta f\, (k+m+L)}{f_s}}$$

$$= e^{\frac{j2\pi\, \Delta f\, L}{f_s}} \sum_{m=0}^{L-1} s^*(k+m)\, s(k+m+L)$$

$\mathrm{s}(k) = \mathrm{s}(k+L)$

$$= e^{\frac{j2\pi\, \Delta f\, L}{f_s}} \sum_{m=0}^{L-1} |s(k+m)|^2$$

Frequency Offset $\Longrightarrow$ Phase diff between 1st & 2nd RX copy

$$\phi = \frac{2\pi L\, \Delta f}{f_s}$$

The Largest Correctable Frequency Offset

$$\boxed{\Delta\hat{f} = \frac{\hat{\phi}\, f_s}{2\pi L}}$$

$$\hat{\phi} = \tan^{-1}\left(\frac{\mathcal{Im}\big(P(k)\big)}{\mathcal{Re}\big(P(k)\big)}\right)$$

ϕ can be at most π: $\Delta f_{max} = \dfrac{f_s}{2L}$

Wi-Fi 802.11 with 20 MHz bandwidth:

$\Delta f_{max,STS} = \dfrac{20\times10^6}{2\times16} = 625$ KHz — Coarse Freq Correction

$\downarrow$

$\Delta f_{max,LTS} = \dfrac{20\times10^6}{2\times64} = 156.25$ KHz — Fine Freq Correction

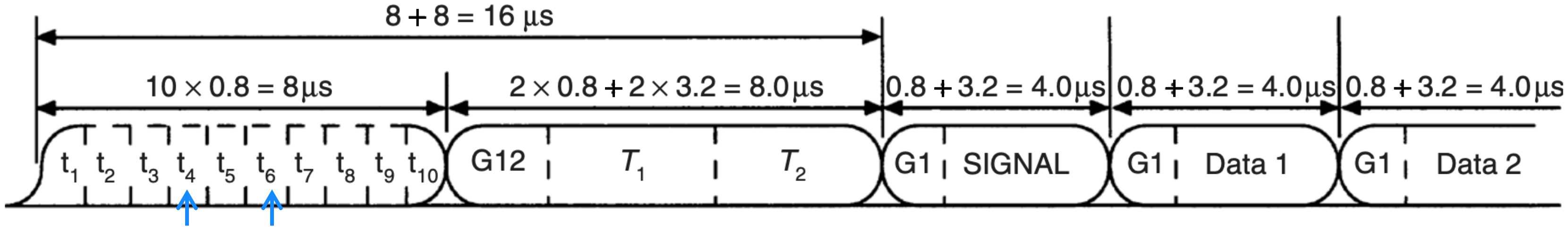
Recall from the earlier calculation

Wi-Fi 802.11: ± 20 ppm
$\Longrightarrow$ Carrier difference: $[-40, 40]$ ppm

At 2.4 GHz

$\Delta f_{max} = \dfrac{2.4\times10^9\times20}{10^6} = 48$ KHz

Exercise: Non-Consecutive Copies for CFO Estimation



- Can we use non-consecutive received copies for CFO Estimation?
- Let's say we use the 4th & the 6th short training symbol for CFO estimation, how should the equation be modified? (L: the length of each copy)

$$p(k) = \sum_{m=0}^{\overset{?}{L-1}} r^*(k+m)\, r(k+m+\overset{2L}{L}) \; ?$$

$r(n) = s(n)\, e^{\frac{j2\pi\,\Delta f\, n}{f_s}}$

$$= \sum_{m=0}^{L-1} s^*(k+m)\, e^{\frac{-j2\pi\,\Delta f\,(k+m)}{f_s}}\, s(k+m+\cancelto{2L}{L})\, e^{\frac{j2\pi\,\Delta f\,(k+m+\cancelto{2L}{L})}{f_s}}$$

$$= e^{\frac{j2\pi\,\Delta f\,\cancelto{2L}{L}}{f_s}} \sum_{m=0}^{L-1} s^*(k+m)\, s(k+m+\cancelto{2L}{L})$$

$s(k) = s(k+L)$

$$= e^{\frac{j2\pi\,\Delta f\,\cancelto{2L}{L}}{f_s}} \sum_{m=0}^{L-1} |s(k+m)|^2$$

Phase offset

$$\phi = \frac{2\pi \cancelto{2L}{L}\, \Delta f}{f_s}$$

Does this imply coarser or finer frequency offset correction?

Apply The Estimated Carrier Frequency Offset

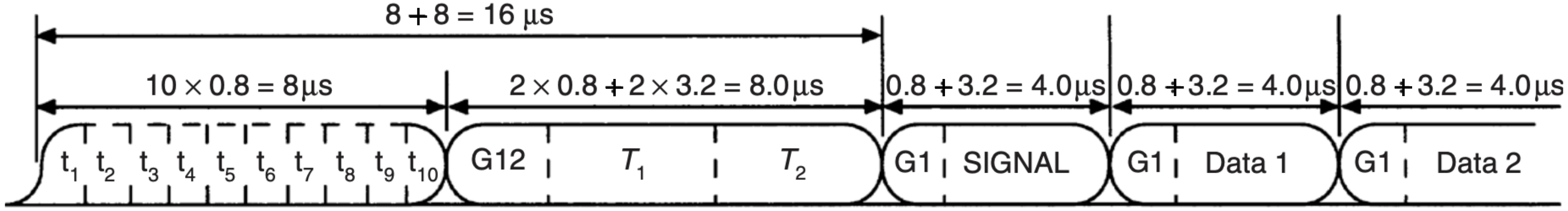
$$r(t) = s(t)\, e^{\frac{j2\pi\,\Delta f\, t}{f_s}}$$

RX signal TX signal

$$\Longrightarrow$$

$$r_{CFO}(t) = r(t)\, e^{\frac{-j2\pi\,\Delta\hat{f}\, t}{f_s}}$$

CFO Corrected



- What is the order of (a) channel estimation, (b) CFO estimation, and (c) CFO correction?
- Which parts of the signal require CFO correction? Starting from (a) STS, (b) LTS, or (c) only the OFDM symbols?
- When is "$t = 0$" when correcting the CFO? Does it matter?

Reference – OFDM CFO

- Travis Collins, Robin Getz, Alexander Wyglinski, Di Pu. *Software-Defined Radio for Engineers*. Artech, 2018. [NTU Library Link]
  - Chap. 7: Carrier Synchronization
  - Chap. 10: Orthogonal Frequency Division Multiplexing
- Tri T. Ha. *Theory and Design of Digital Communication Systems*. Cambridge University Press, 2010.
  - 7.23 OFDM demodulation - Carrier phase synchronization
- Ahmad R.S. Bahai, Burton R. Saltzberg, and Mustafa Ergen. *Multi-carrier digital communications theory and applications of OFDM*. Springer New York, 2004. [NTU Library Link]
  - Chap. 5.4: Frequency Offset Estimation
- E. Sourour, H. El-Ghoroury and D. McNeill, "Frequency offset estimation and correction in the IEEE 802.11a WLAN," IEEE VTC2004-Fall. Los Angeles, CA. [Link]

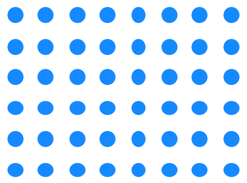
Linear Array

Antenna Array

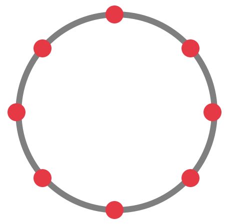
Set of antennas commonly organized in a structure

Linear Array
└ Uniform linear array

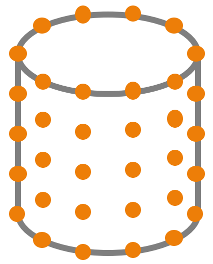
Planar Array
└ Uniform planar array



Circular Array



Cylindrical array



For Both transmission and reception

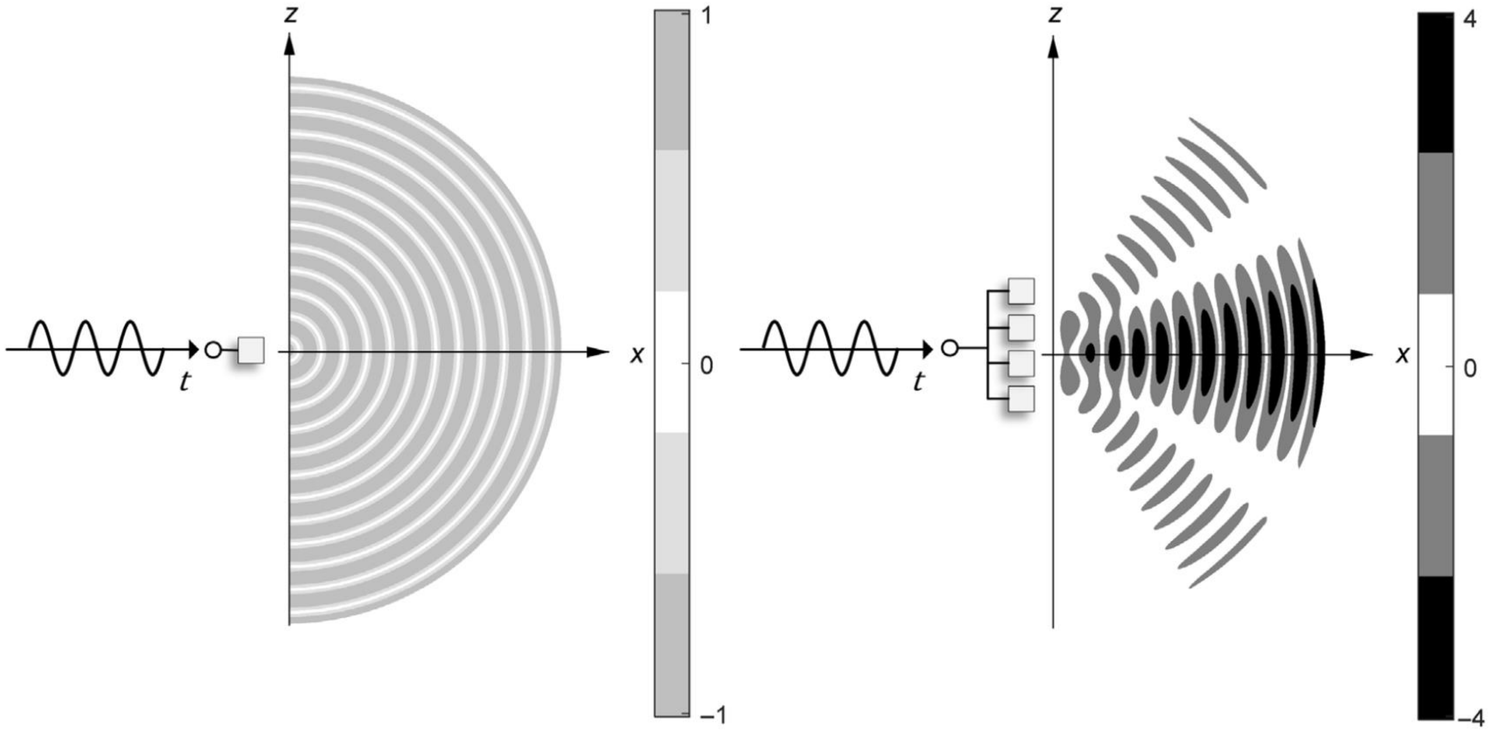
Gain (signal amplification)

from transmitting/receiving different versions of the same signal from all antennas

Gain direction control

Steer to the intended user, reduce interference to/from other users

N antennas → N times gain



X-axis direction, with the same transmit power

$\boxed{\text{1 Antenna}}$

$A\sin(\theta) \quad \Longrightarrow \quad$ Power $= A^2$

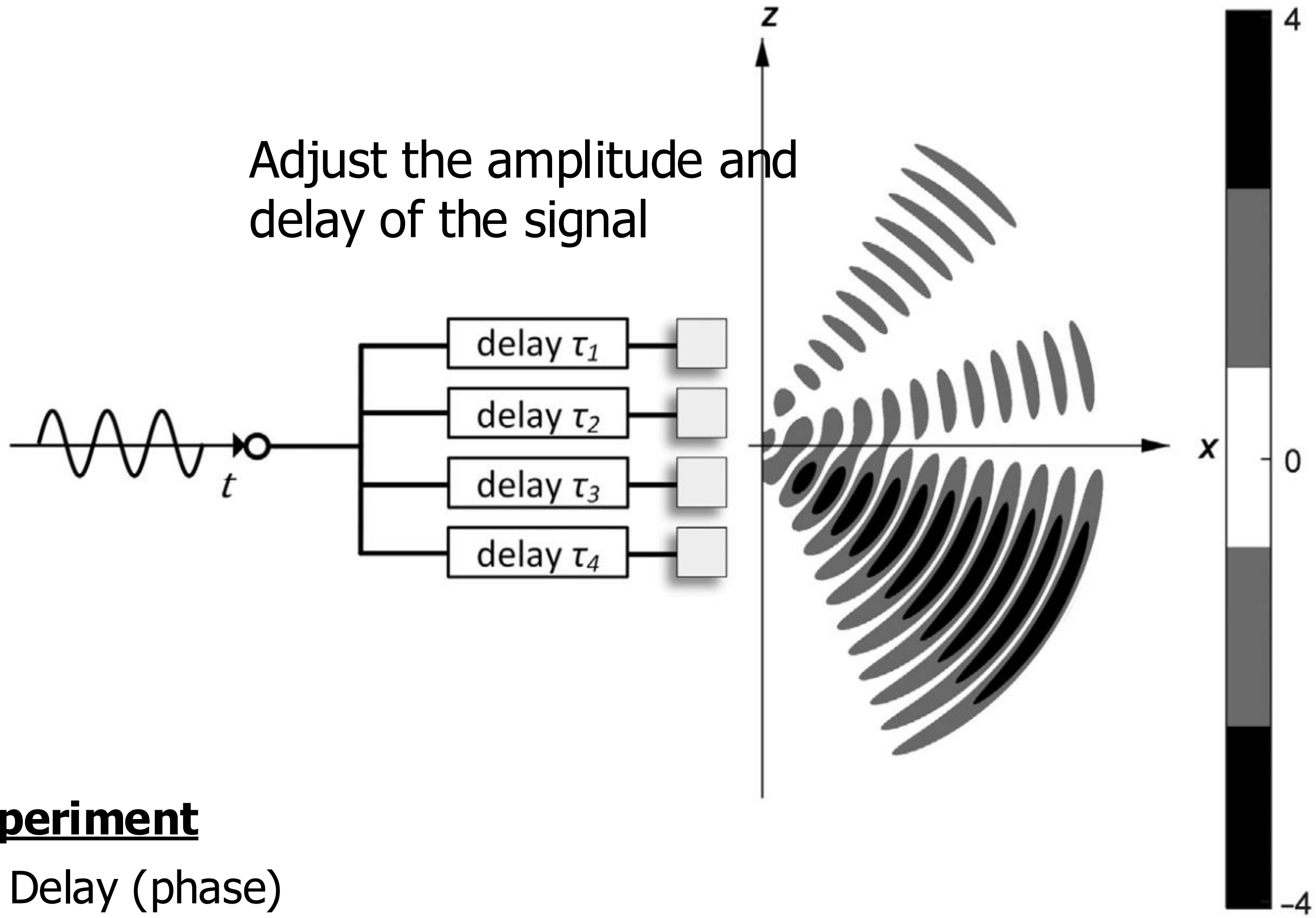
$\boxed{\text{4 Antennas}}$

$4 \times \left(\boxed{\frac{1}{\sqrt{4}}} A\sin(\theta) \right) = 2A\sin(\theta) \quad \Longrightarrow \quad$ Power $= 4A^2$

Normalize to maintain the same transmit power

Change the Direction of the Maximum Gain

Adjust the amplitude and delay of the signal

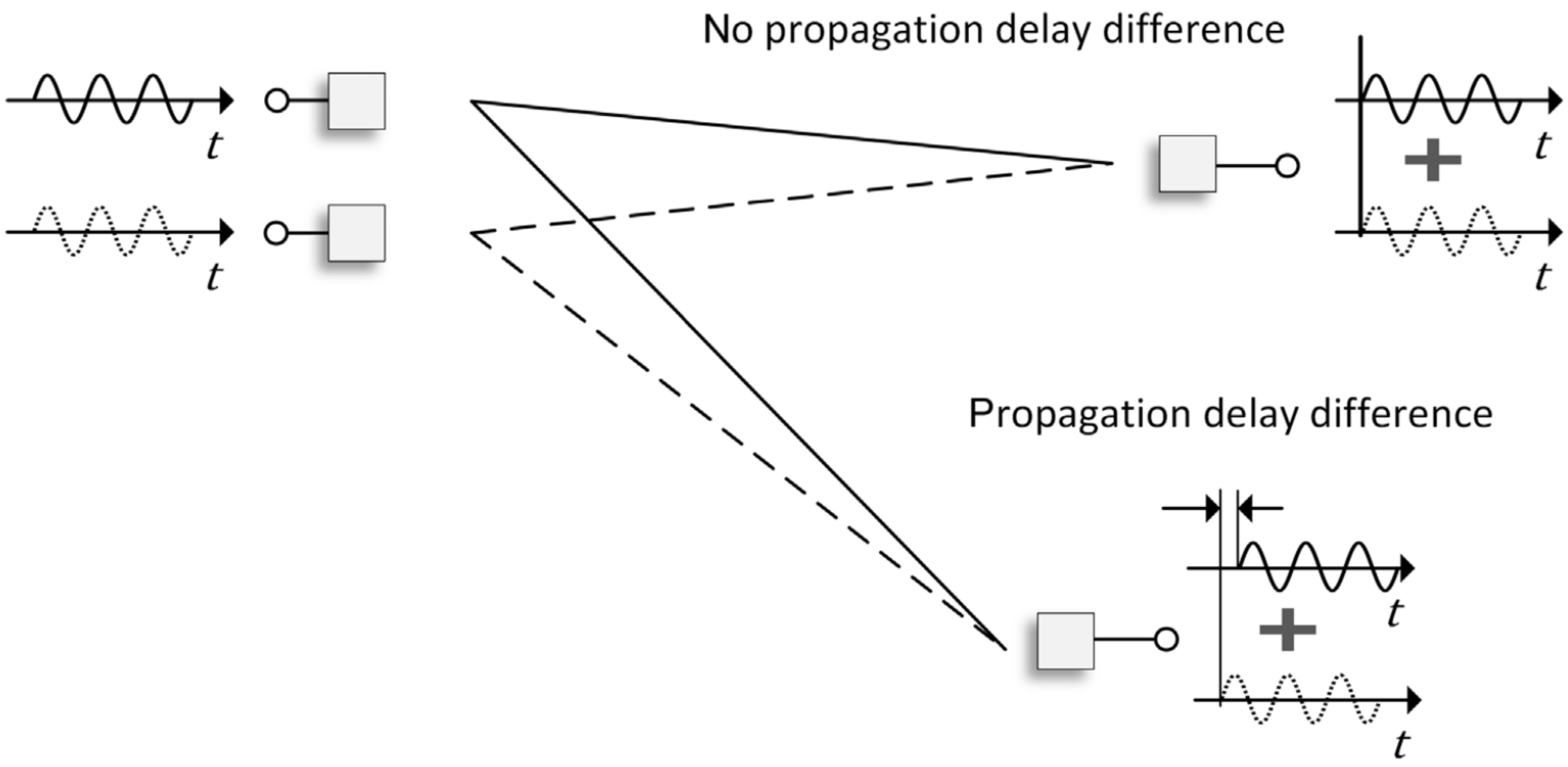


Thought experiment

Amplitude vs Delay (phase)

Which is more crucial in changing the maximum direction?

Let's Start with Two



The received signal's amplitude depends on the difference in propagation delays

Propagation delay depends on the receiver's direction

Array with two elements

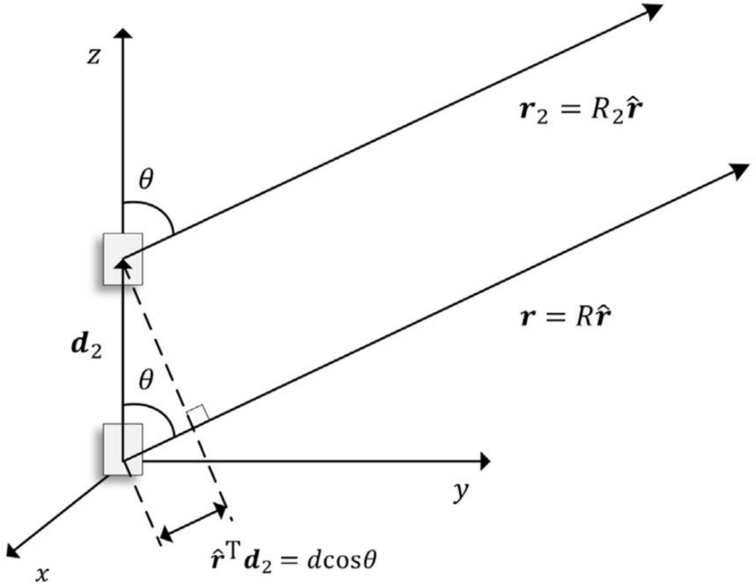
Assume free space (only line-of-sight path)

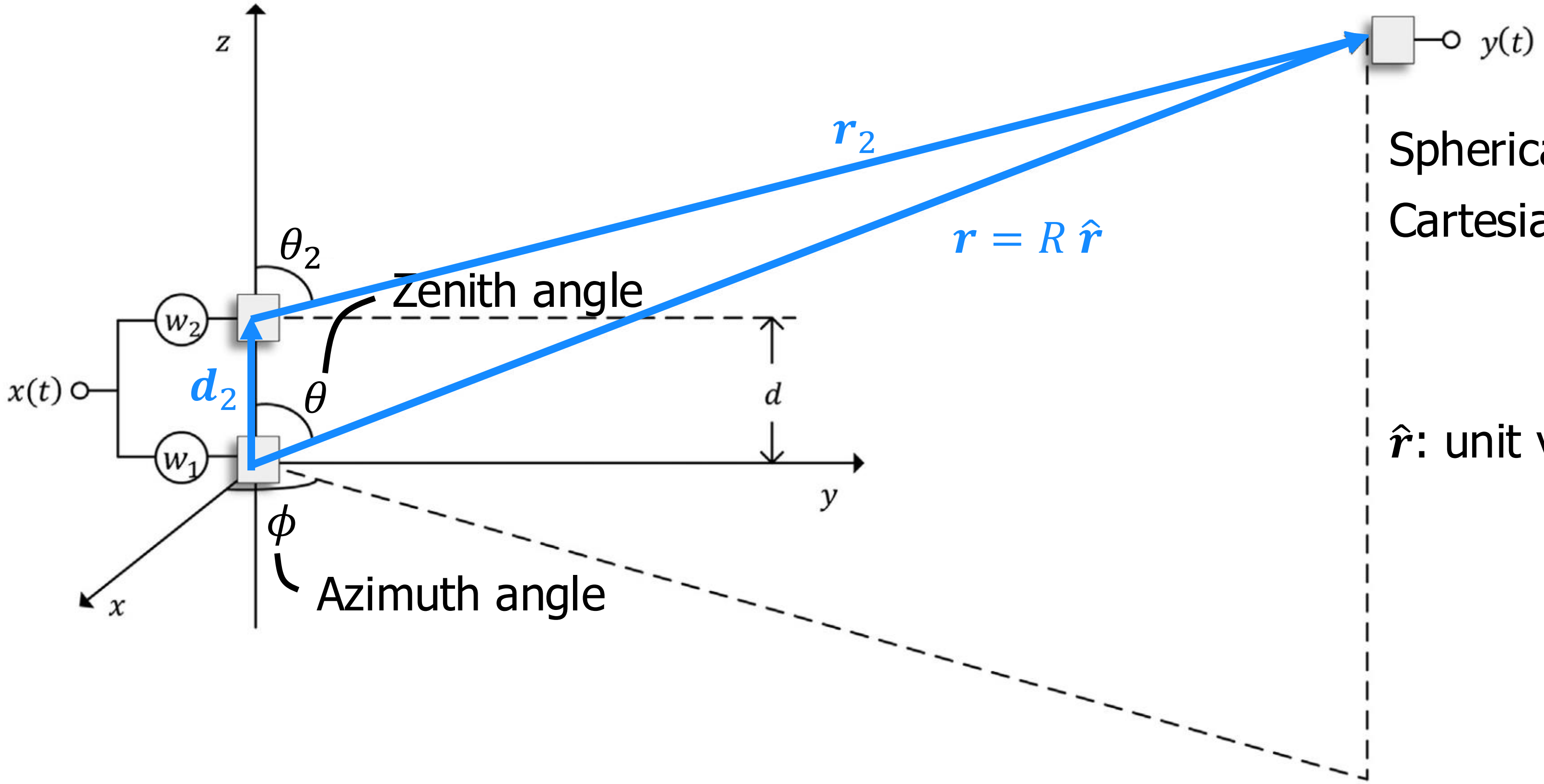
R is large

$\Longrightarrow$ $\boldsymbol{r}$ and $\boldsymbol{r}_2$ approximately parallel

$\Longrightarrow$ $\boldsymbol{r}_2 = R_2\,\hat{\boldsymbol{r}}$

$\Longrightarrow$ $R_2 = R - d\cos\theta = R - \mathbf{d}_2^{\mathrm{T}}\,\hat{\boldsymbol{r}}$





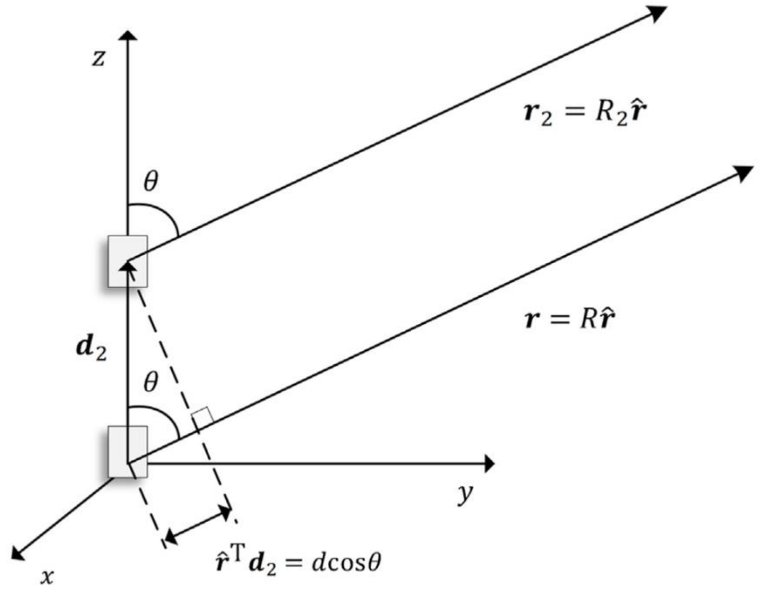
Spherical coordinate: (R, θ, ϕ)

Cartesian coordinate:

$$\boldsymbol{r} = R\hat{\boldsymbol{r}} = R\begin{bmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{bmatrix}$$

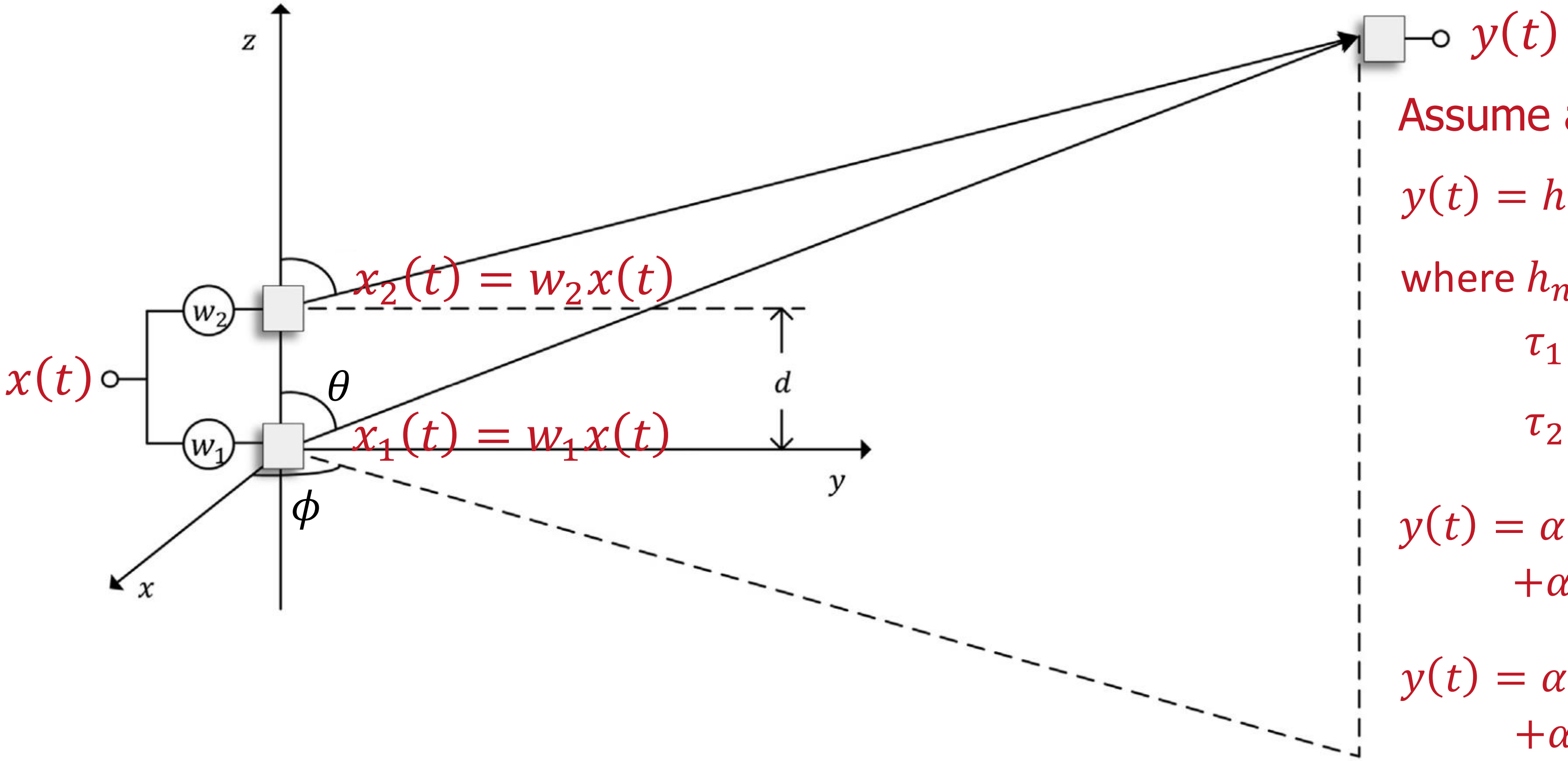
$\hat{\boldsymbol{r}}$: unit vector of $\boldsymbol{r}$

Array with two elements



Assume free space (only line-of-sight path)

R is large
$\Longrightarrow$ $\boldsymbol{r}$ and $\boldsymbol{r}_2$ approximately parallel
$\Longrightarrow \boldsymbol{r}_2 = R_2\,\hat{\boldsymbol{r}}$
$\Longrightarrow R_2 = R - d\cos\theta = R - \mathbf{d}_2^{\mathrm{T}}\,\hat{\boldsymbol{r}}$



Assume all elements isotropic

$$y(t) = h_1 x_1(t-\tau_1) + h_2 x_2(t-\tau_2)$$

where $h_n = \alpha e^{-j2\pi f_c \tau_n}$

$$\tau_1 = \frac{R}{c}$$

$$\tau_2 = \frac{R_2}{c} = \tau_1 - \frac{d\cos\theta}{c}$$

$$y(t) = \alpha e^{-j2\pi f_c\tau_1} x_1(t-\tau_1) + \alpha e^{-j2\pi f_c\tau_2} x_2(t-\tau_2)$$

$$y(t) = \alpha e^{-j2\pi f_c\tau_1} w_1 x(t-\tau_1) + \alpha e^{-j2\pi f_c\tau_2} w_2 x(t-\tau_2)$$

Array with two elements

For simplicity, consider transmitting a sinusoid at radio frequency

$x(t) = X$ and $y(t) = Y$

$$Y = \alpha e^{-j2\pi f_c \tau_1} \underbrace{\left(w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}}\right)} X$$

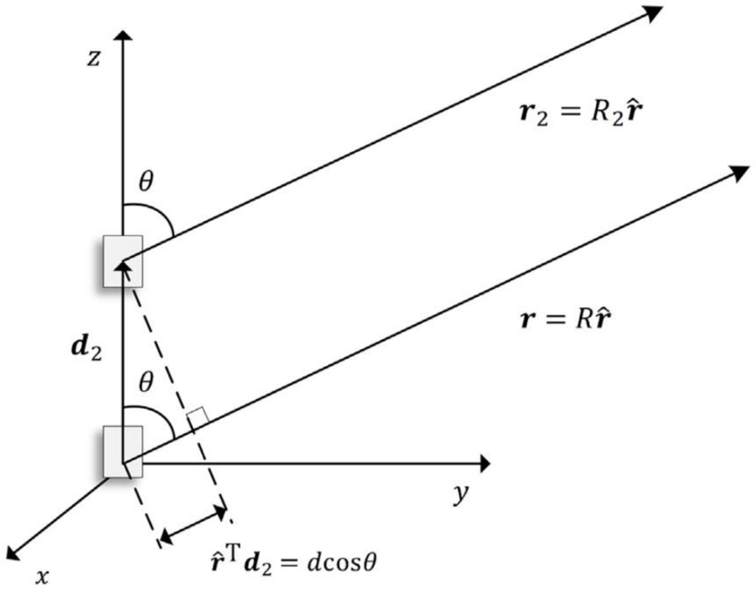
Array Factor (AF)

$$AF(\theta,\phi) = w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}}$$

$$\boxed{|Y|^2 = \alpha^2 \underbrace{|AF(\theta,\phi)|^2} |X|^2}$$

The gain pattern for an array of isotropic elements relative to a single isotropic antenna

Also referred to as the (free-space) array gain



$y(t)$

Assume all elements isotropic

$$y(t) = h_1 x_1(t-\tau_1) + h_2 x_2(t-\tau_2)$$

where $h_n = \alpha e^{-j2\pi f_c \tau_n}$

$$\tau_1 = \frac{R}{c}$$

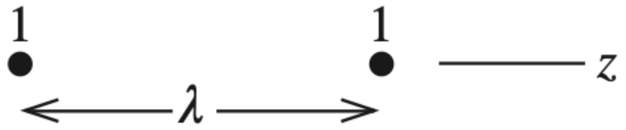
$$\tau_2 = \frac{R_2}{c} = \tau_1 - \frac{d\cos\theta}{c}$$

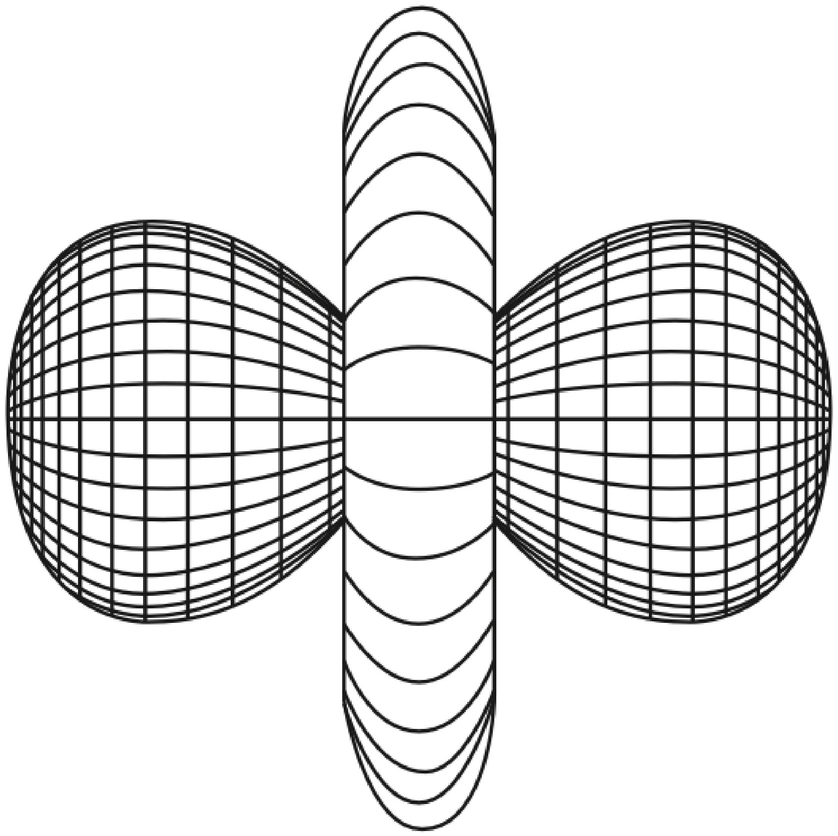
$$y(t) = \alpha e^{-j2\pi f_c \tau_1} x_1(t-\tau_1) + \alpha e^{-j2\pi f_c \tau_2} x_2(t-\tau_2)$$

$$y(t) = \alpha e^{-j2\pi f_c \tau_1} w_1 x(t-\tau_1) + \alpha e^{-j2\pi f_c \tau_2} w_2 x(t-\tau_2)$$

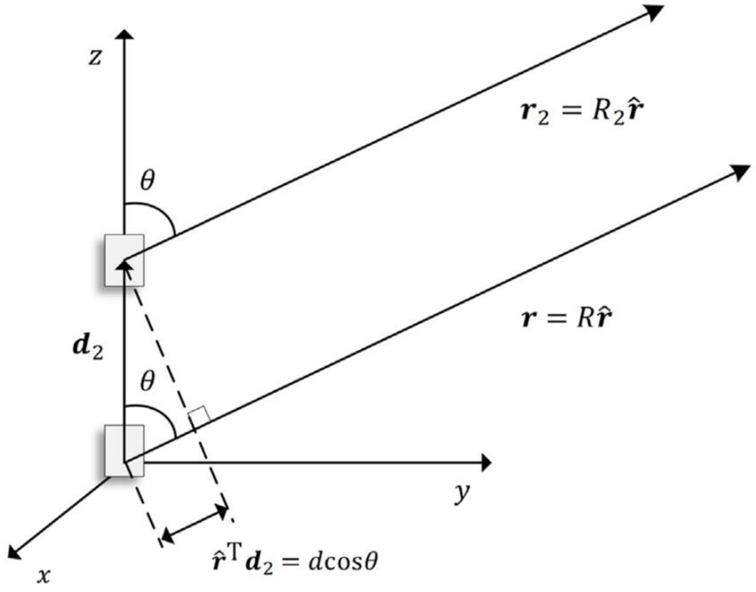
Array with two elements

Example: Antenna space = 1 wavelength





Extension to General Form



From Array with Two Elements

$$Y = \alpha e^{-j2\pi f_c \tau_1}\left(w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}}\right)X$$

$$= \alpha e^{-j2\pi f_c \tau_1}\left(w_1 e^{j2\pi f_c \frac{\boldsymbol{d}_1^T \hat{\boldsymbol{r}}(\theta,\phi)}{c}} + w_2 e^{-j2\pi f_c \frac{\boldsymbol{d}_2^T \hat{\boldsymbol{r}}(\theta,\phi)}{c}}\right)X$$

$$= \alpha e^{-j2\pi f_c \tau_1}[w_1 \quad w_2]\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} X \qquad \text{where } a_n = e^{j2\pi f_c \frac{\boldsymbol{d}_n^T \hat{\boldsymbol{r}}(\theta,\phi)}{c}}$$

$$\boxed{y(t) = \boldsymbol{w}^T \boldsymbol{a}(\theta,\phi)\, \alpha e^{-j2\pi f_c \tau} x(t)}$$

Beamforming Weight Vector

Array Response Vector

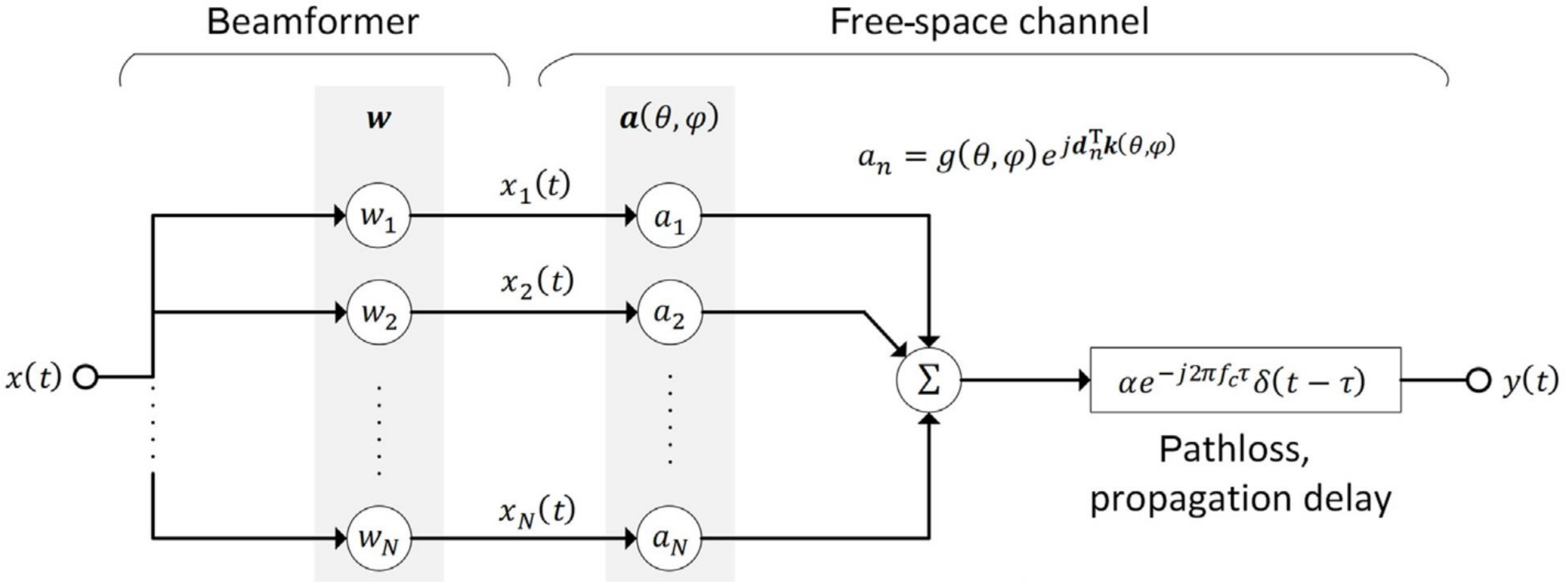
$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad \boldsymbol{a}(\theta,\phi) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}, \quad a_n = e^{j2\pi f_c \frac{\boldsymbol{d}_n^T \hat{\boldsymbol{r}}(\theta,\phi)}{c}}, \quad \hat{\boldsymbol{r}}(\theta,\phi) = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

Can this form be used for rectangular arrays?

Can this form be used for arrays with arbitrary placement?

Extension to General Form

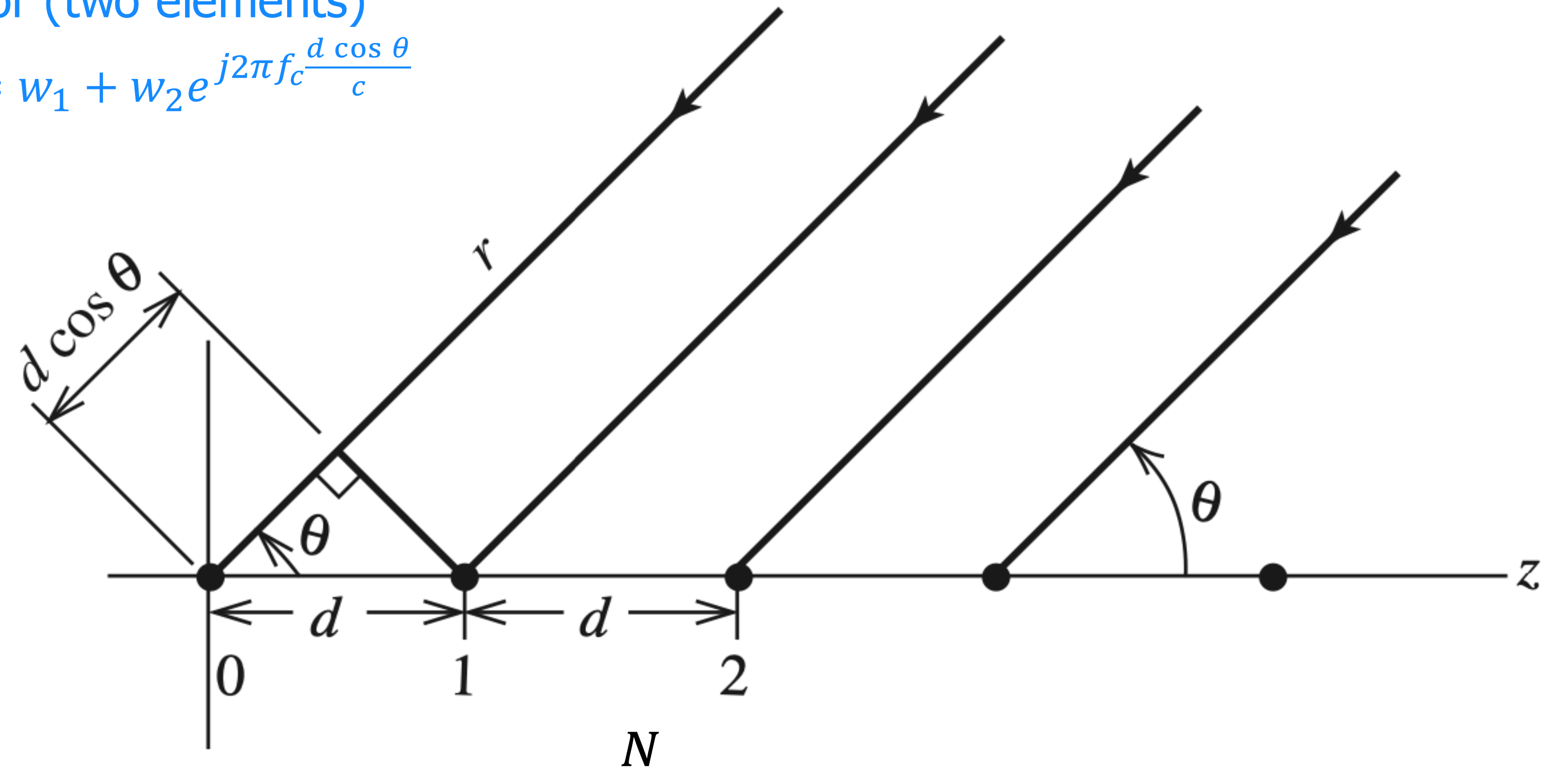
$$\boxed{y(t) = \boldsymbol{w}^T \boldsymbol{a}(\theta, \phi)\, \alpha e^{-j2\pi f_c \tau} x(t)}$$



Uniform Linear Array with N Elements

Array Factor (two elements)

$AF(\theta,\phi) = w_1 + w_2 e^{j2\pi f_c \frac{d\cos\theta}{c}}$



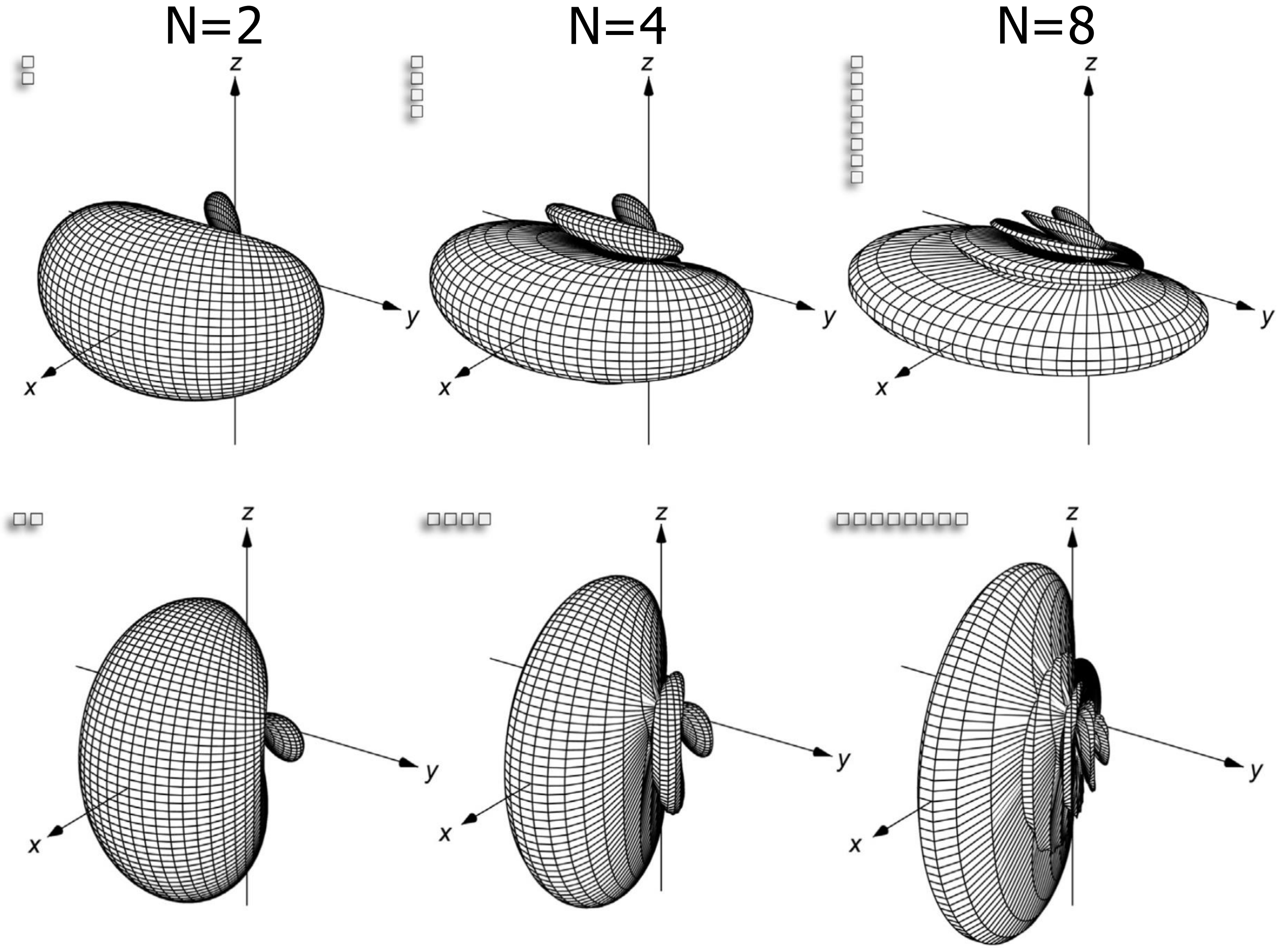
$$AF(\theta,\phi) = \sum_{n=1}^{N} w_n \, e^{j(n-1)d\cos\theta \frac{2\pi f_c}{c}}$$

Basic Gain Pattern

$$w = \frac{1}{\sqrt{N}}\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}$$

Vertical Arrays

Horizontal Arrays



Reference

- Constantine A Balanis. *Antenna Theory: Analysis and Design*. John Wiley & Sons, 2016. [NTU Library Link]
  - Chap. 6: Arrays: Linear, Planar, and Circular
- Warren L. Stutzman and Gary A. Thiele. *Antenna theory and design*. John Wiley & Sons, 2012
  - Chap. 8: Array Antennas
- Henrik Asplund, Jonas Karlsson, Fredric Kronestedt, Erik Larsson, David Astely, Peter von Butovitsch, Thomas Chapman et al. *Advanced Antenna Systems for 5G Network Deployments: Bridging the Gap Between Theory and Practice*. Academic Press, 2020. [NTU Library Link]
  - Chap. 4: Antenna arrays and classical beamforming