114-1 電工實驗（通信專題）

Channel Estimation

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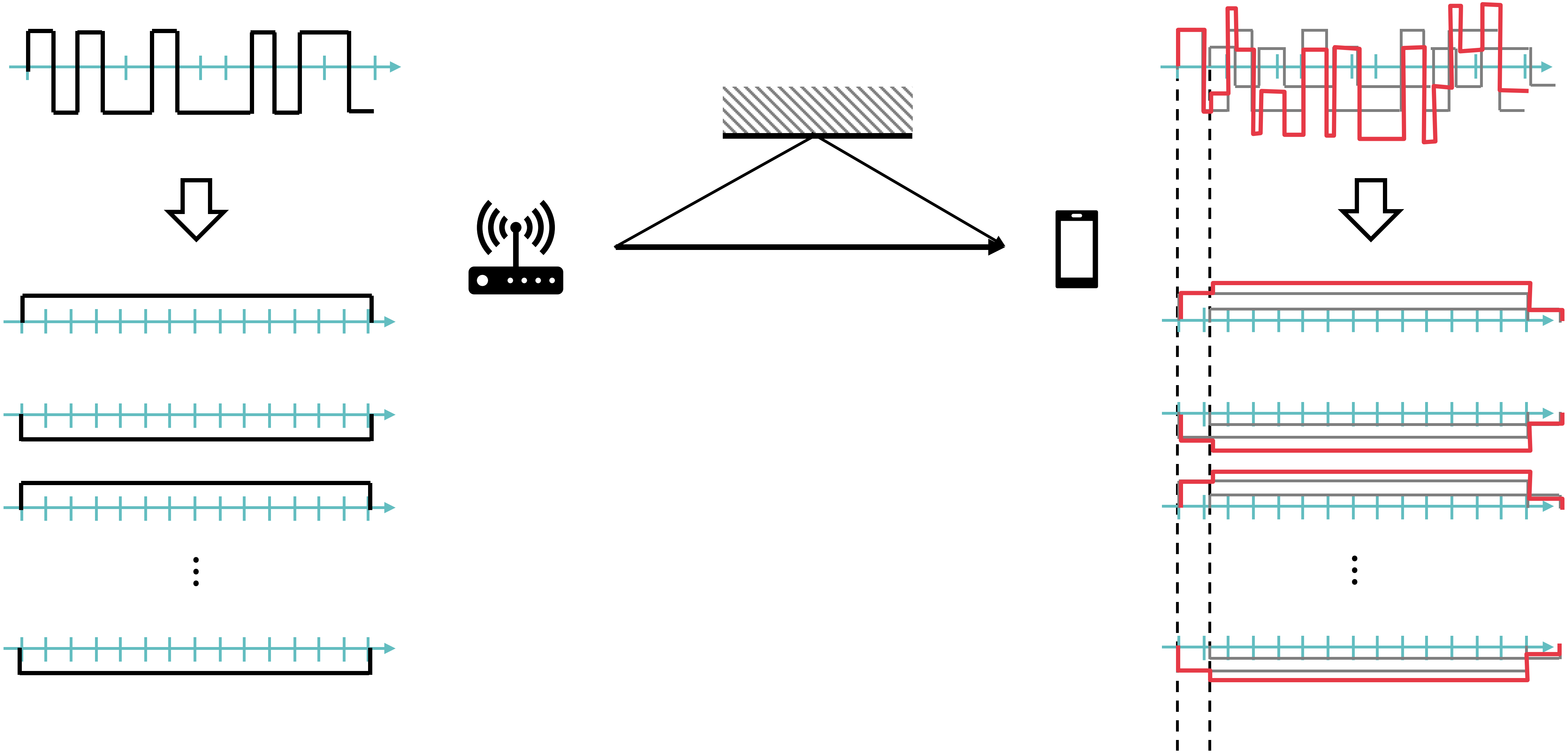
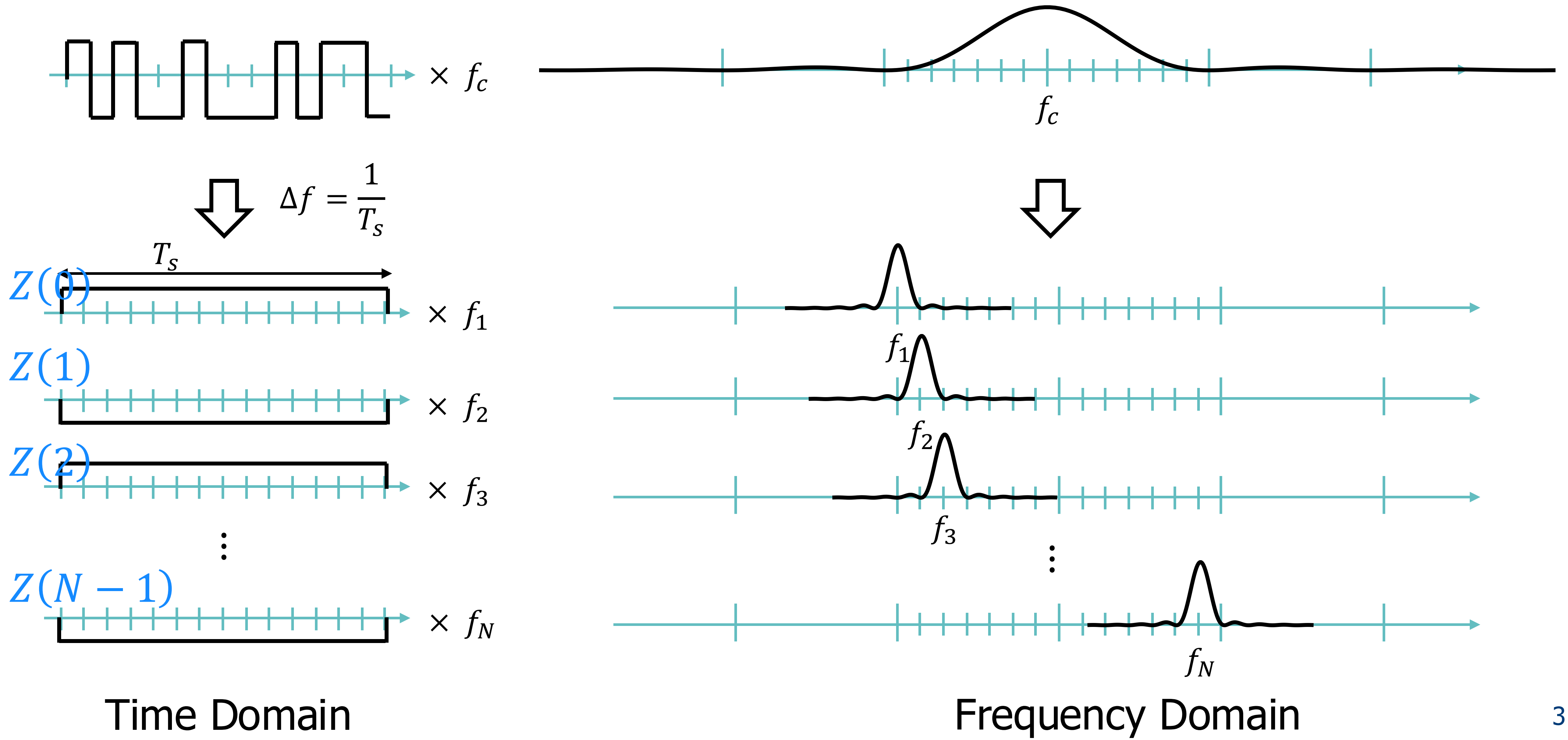
Motivation of OFDM



OFDM: Orthogonal Frequency-Division Multiplexing



OFDM – Analog & Digital View

$$s_k(t) = Re\{Z(k)e^{j[2\pi(f_c+k\Delta f)t]}\},\ 0 \le t < T_s,\ \Delta f = 1/T_s$$

Modulated Subcarrier

$$s(t) = \frac{1}{N}\sum_{k=0}^{N-1} s_k(t) = Re\left\{\boxed{\left(\frac{1}{N}\sum_{k=0}^{N-1} Z(k)e^{j2\pi k\Delta f t}\right)} e^{j2\pi f_c t}\right\}, 0 \le t < T_s$$

OFDM signal

$s_L(t)$ complex envelope of the OFDM signal

Analog

Digital

Sampling time $\frac{T_s}{N}$

(sample at $t = \frac{nT_s}{N}, n = 0, 1, \cdots, N-1$)

$$s_L(n) = \frac{1}{N}\sum_{k=0}^{N-1} Z(k)e^{j2\pi kn/N}, n = 0, 1, \cdots, N-1$$

$$= IDFT\{Z(k)\}$$

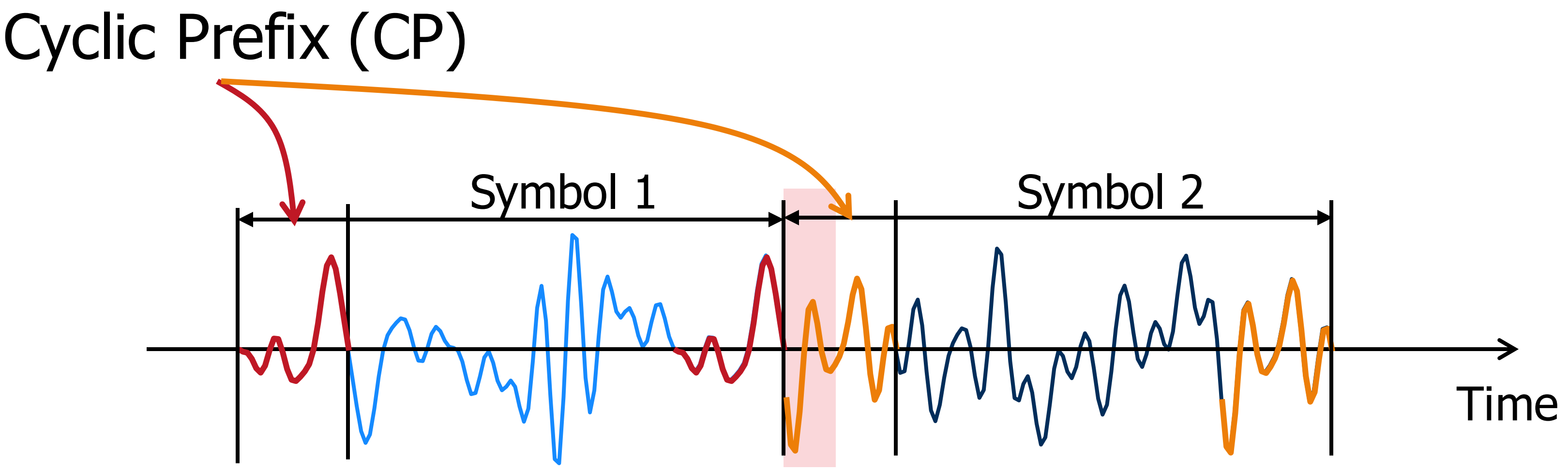
→ Generate time sequence $s_L(n)$ from frequency samples

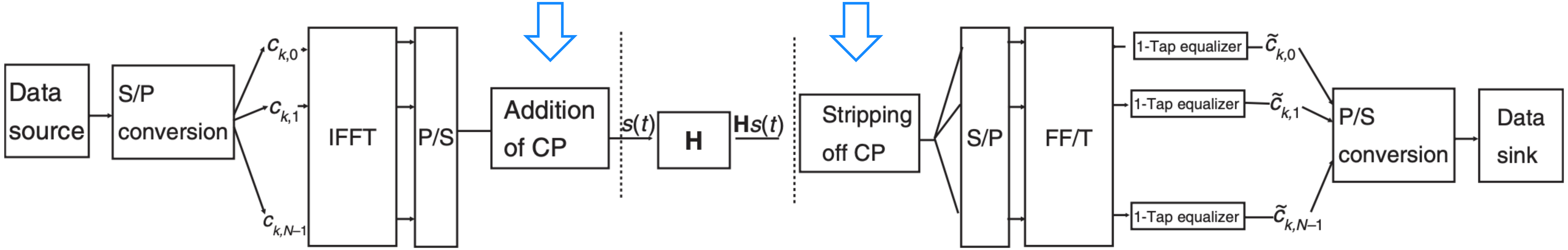
$$Z(k) = \sum_{n=0}^{N-1} s_L(n)e^{-j2\pi kn/N}, k = 0, 1, \cdots, N-1$$

$$= DFT\{s_L(n)\}$$

$$\boxed{\Delta f = \frac{1}{T_s}, \qquad \Delta t = \frac{T_s}{N}}$$

OFDM Implementation with CP to Combat Delay Dispersion





OFDM Transmission Model with Cyclic Prefix

$$y(n) = h(n) \otimes s_L(n) = \sum_{k=0}^{N-1} h(k) \boxed{s_L(n-k)_N}$$

Circular Convolution

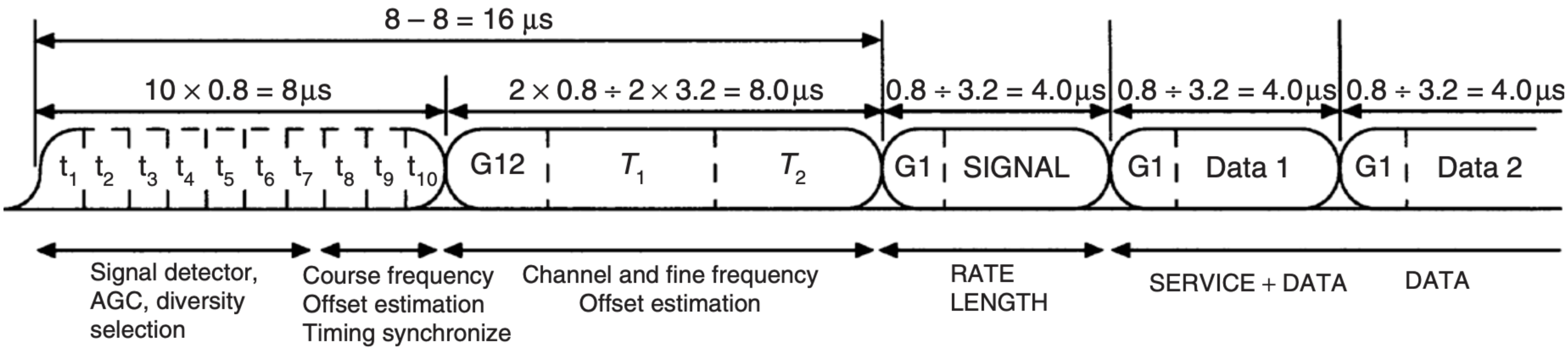
$s_L(m)_N = s_L[(m) \bmod N]$
circular shift of $s_L(n)$

Using the notations earlier

$$Y(k) = H(k)Z(k), \qquad k = 0, 1, \cdots, N-1$$

For each subcarrier k,
the TX & RX symbols are characterized by a constant $H(k)$

802.11 a/g PLCP Preamble



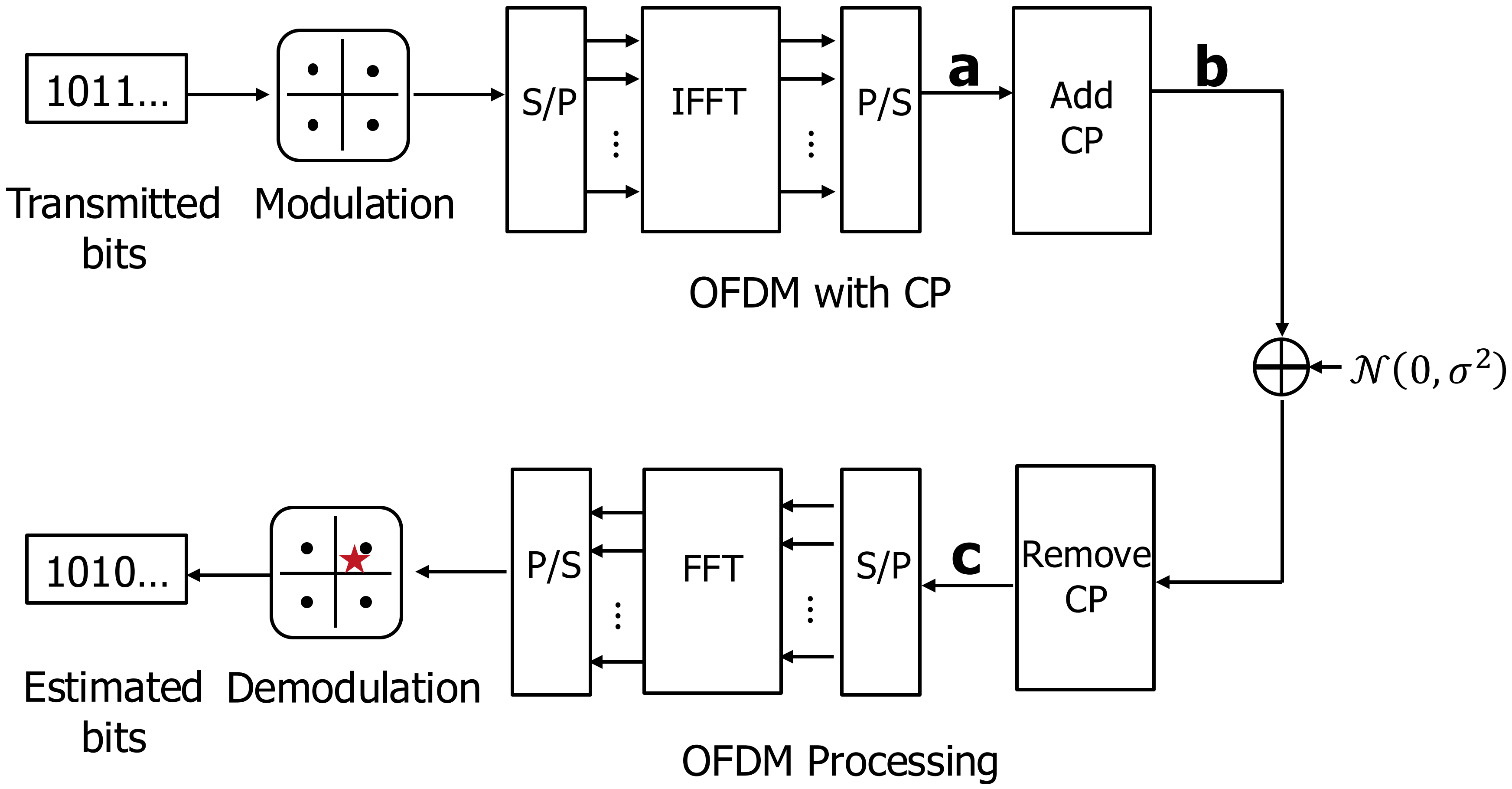
Short Training Sequence

10x Short Training Symbol

Long Training Sequence

2.5x Long Training Symbol

Lab 3 - Part 1: OFDM signal generation and reception



- Generate & receive 1 OFDM symbol (noiseless)
- Simulate OFDM transmissions for different SNRs

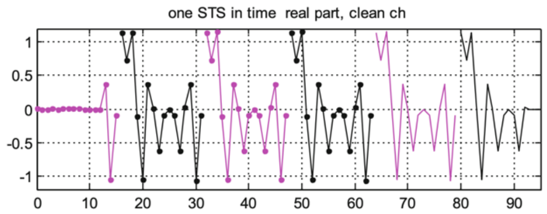
Lab 3 - Part 2: STS & LTS for sync & equalization

Q: Why a normalization factor of $\sqrt{13/6}$?

Short OFDM Training Symbol

Power Normalization

$$S_{-26,26} = \boxed{\sqrt{(13/6)}}\{0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 0, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0\}$$



IFFT ⇨ 4 Short Training Symbols

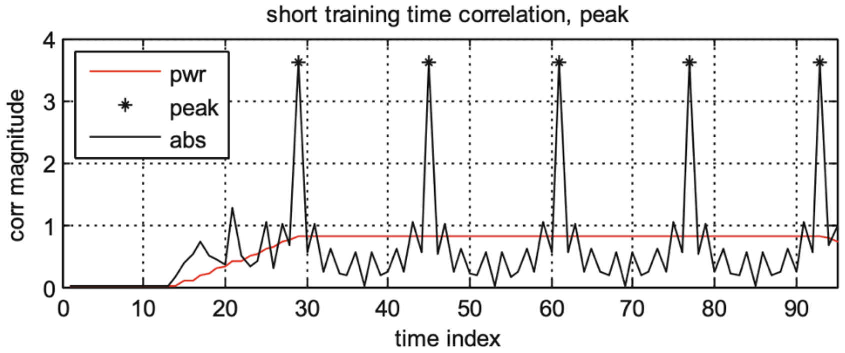
Long OFDM Training Symbol

DC

$$L_{-26,26} = \{1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1\}$$

IFFT ⇨ 1 Long Training Symbol

- Frame synchronization (noiseless and noisy)
- Channel estimation and equalization for a multipath channel



Channel Estimation

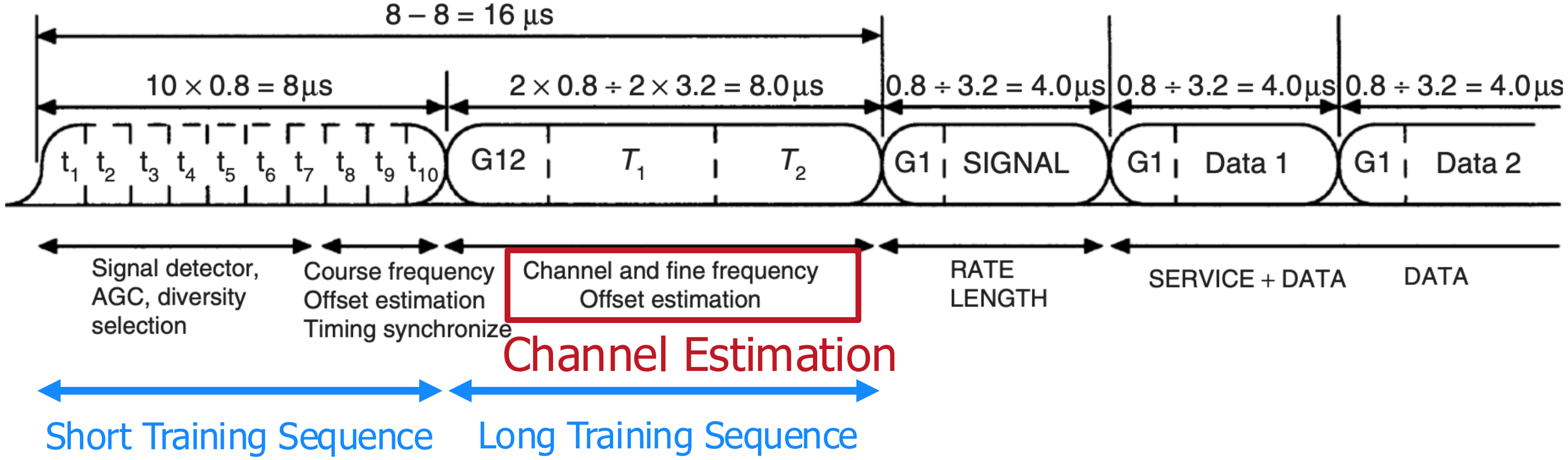
Goal: Channel Estimation of 802.11 a/g

Step 0

Channel model Rayleigh fading & Rician fading

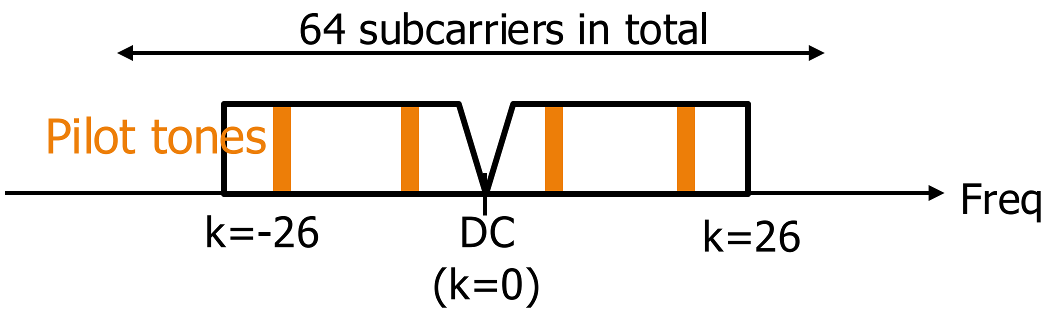
Step 1

Initial estimation using long training symbols



Step 2

Tracking with pilot tones



Channel Model

$u(t)$: transmitted complex baseband signal

$y(t)$: received complex baseband signal

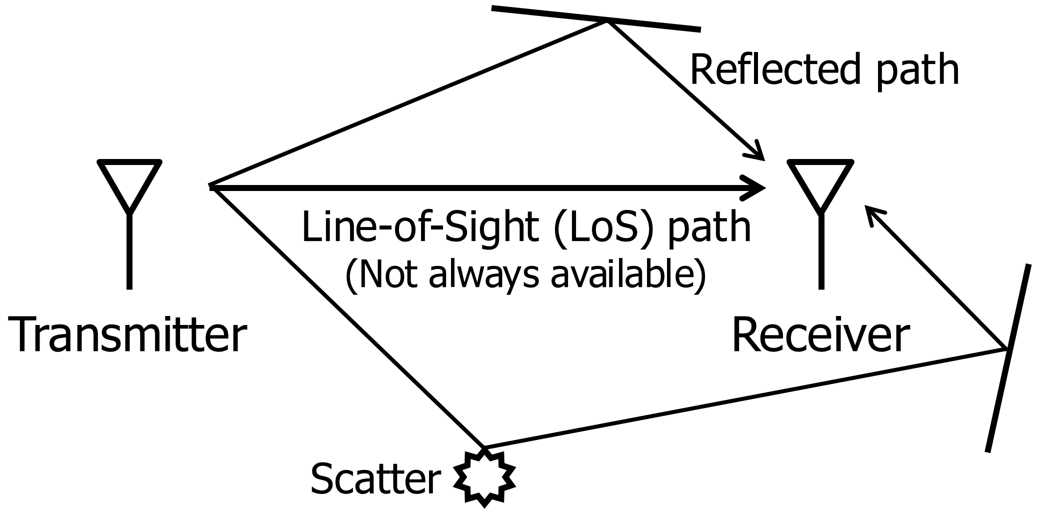
A_k: Amplitude

θ_k: Phase (due to the scattering)

$$y(t) = \sum_{k=1}^{M} \boxed{A_k e^{j\theta_k}} u(t - \tau_k)$$

k-th multipath component

relative delay



$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k}\, \delta(t - \tau_k)$$

Impulse response of complex baseband channel

Channel Impulse Response & Channel Transfer Function

Channel Impulse Response

$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k}\, \delta(t - \tau_k)$$

$\{\theta_k\}$: i.i.d., uniformly over $[0, 2\pi]$

Channel transfer function

$$H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{-\underbrace{j2\pi f\tau_k}}$$

Frequency-dependent

phase lag caused by delay τ_k

i.i.d., uniformly distributed in $[0, 2\pi]$

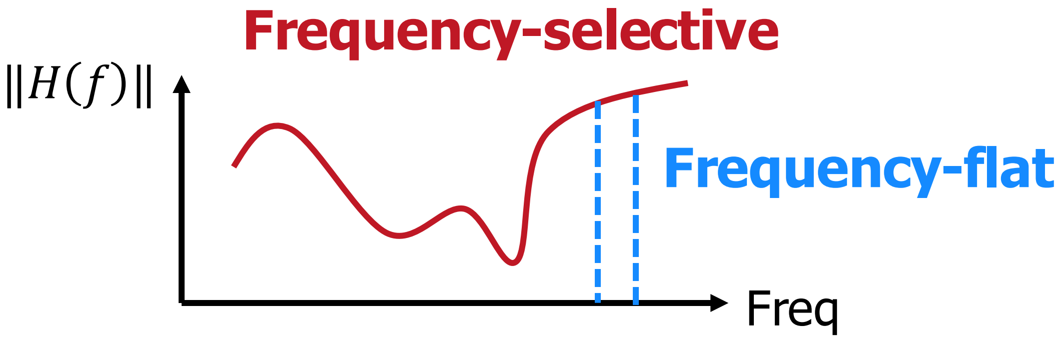
(f_c is large, small changes in delay τ_k cause large changes in the phase $2\pi f_c \tau_k$)

Frequency-Flat vs Frequency-Selective

Channel Impulse Response

$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k} \delta(t - \tau_k)$$

$\{\theta_k\}$: i.i.d., uniformly over $[0, 2\pi]$



Channel transfer function

$$H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{\underbrace{-j2\pi f \tau_k}_{}}$$

$H(f)$ → Frequency-dependent

phase lag caused by delay τ_k

i.i.d., uniformly distributed in $[0, 2\pi]$

(f_c is large, small changes in delay τ_k cause large changes in the phase $2\pi f_c \tau_k$)

Narrowband frequency-flat approximation

The channel transfer function is approximately constant over a small band around f_0

$$h \approx H(f_0) = \sum_{k=1}^{M} A_k e^{j\gamma_k}$$

$\gamma_k = \theta_k - 2\pi f_0 \tau_k \mod 2\pi$

γ_k: i.i.d., uniform over $[0, 2\pi]$

Narrowband Rayleigh & Rician fading models

$$h \approx H(f_0) = \sum_{k=1}^{M} A_k e^{j\gamma_k}$$

$\gamma_k = \theta_k - 2\pi f_0 \tau_k \mod 2\pi$

γ_k: i.i.d., uniform over $[0, 2\pi]$

M is large & no dominant component → **Narrowband Rayleigh fading**

$$H(f_0) \sim CN\left(0, \sum_{K=1}^{M} A_k^2\right)$$

1 dominant component + many smaller multipath components → **Narrowband Rician fading**

$$H(f_0) \sim CN\left(A_1 e^{j\gamma_1}, \sum_{k=2}^{M} A_K^2\right)$$

$$h = A_1 e^{j\gamma_1} + h_{\text{diffuse}}$$

Complex Gaussian

Thought Experiment: What if only one path exists?

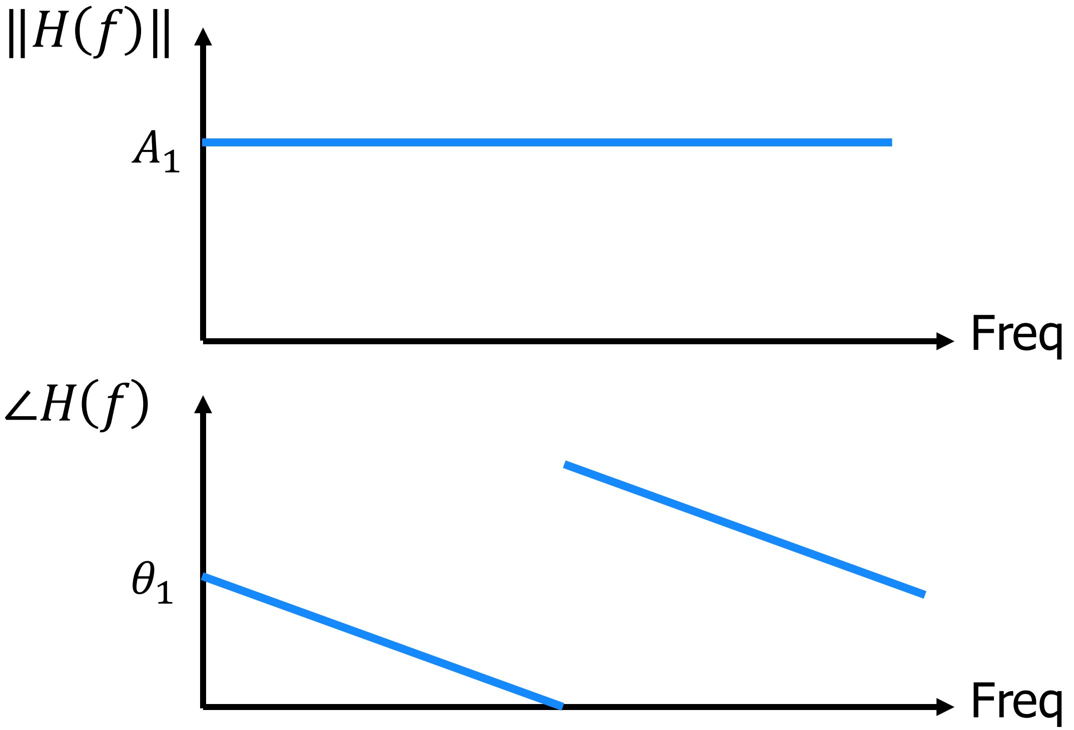
Channel transfer function

$$H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{-j2\pi f \tau_k}$$

$\{\theta_k\}$: i.i.d., uniformly over $[0, 2\pi]$

$\Downarrow$

$$H(f) = A_1 e^{j\theta_1} e^{-j2\pi f \tau_1}$$



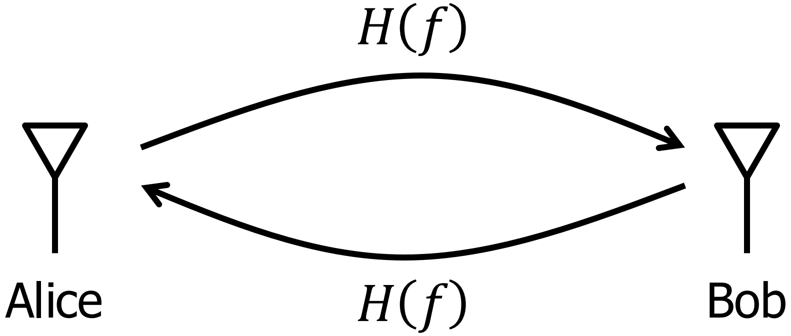
Is it frequency-flat?

What if τ_1 becomes larger?

A Few Things About Channel

- Often referred to as CSI (Channel State Information)
  - Statistical CSI
  - Instantaneous CSI

- Channel reciprocity
  - Physical channel is reciprocal (However, hardware is not)
  - Explicit vs implicit channel estimation



- Known CSI at TX vs Known CSI at RX

OFDM Channel Estimation - Pilot Symbols

Last class:

$$y(n) = h(n) \otimes x(n) \quad \Leftrightarrow \quad Y(k) = H(k)X(k), \qquad k = 0, 1, \cdots, N-1$$

To estimate $H(k)$, transmit known pilot symbols

Received symbol

$$H_{LS}(k) = \frac{Y(k)}{X(k)}$$

A complex number representing the phase and amplitude of the channel response

Known pilot symbol

Least Squares Channel Estimate

802.11 a/g: Channel Estimation Using LTS

Long OFDM Training Symbol

$$L_{-26,26} = \{1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, 1, \\ -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1\}$$

Received symbol

$$H_{LS}(k) = \frac{Y(k)}{X(k)}$$

Long Training Symbol, either 1 or -1

- $\frac{Y(k)}{1} = Y(k) \cdot 1$
- $\frac{Y(k)}{(-1)} = Y(k) \cdot (-1)$

$$H_{LS}(k) = Y(k) \cdot X(k)$$

Avoid the divide-by-0 problem

Using both LTSs

There are two LTSs!

Why two?
→ For carrier frequency offset estimation

How to use both for channel estimation?
→ Average

Equalization

Use channel estimates to normalize phase/amplitude for OFDM data symbols

$$Y(k) = H(k)\, X(k)$$

$Y(k)$	$H(k)$	$X(k)$	
Known (observation)	Unknown	Known (pilot symbols)	**Channel Estimation**
Known (observation)	Known (estimate)	Unknown	**Equalization**

Received symbol

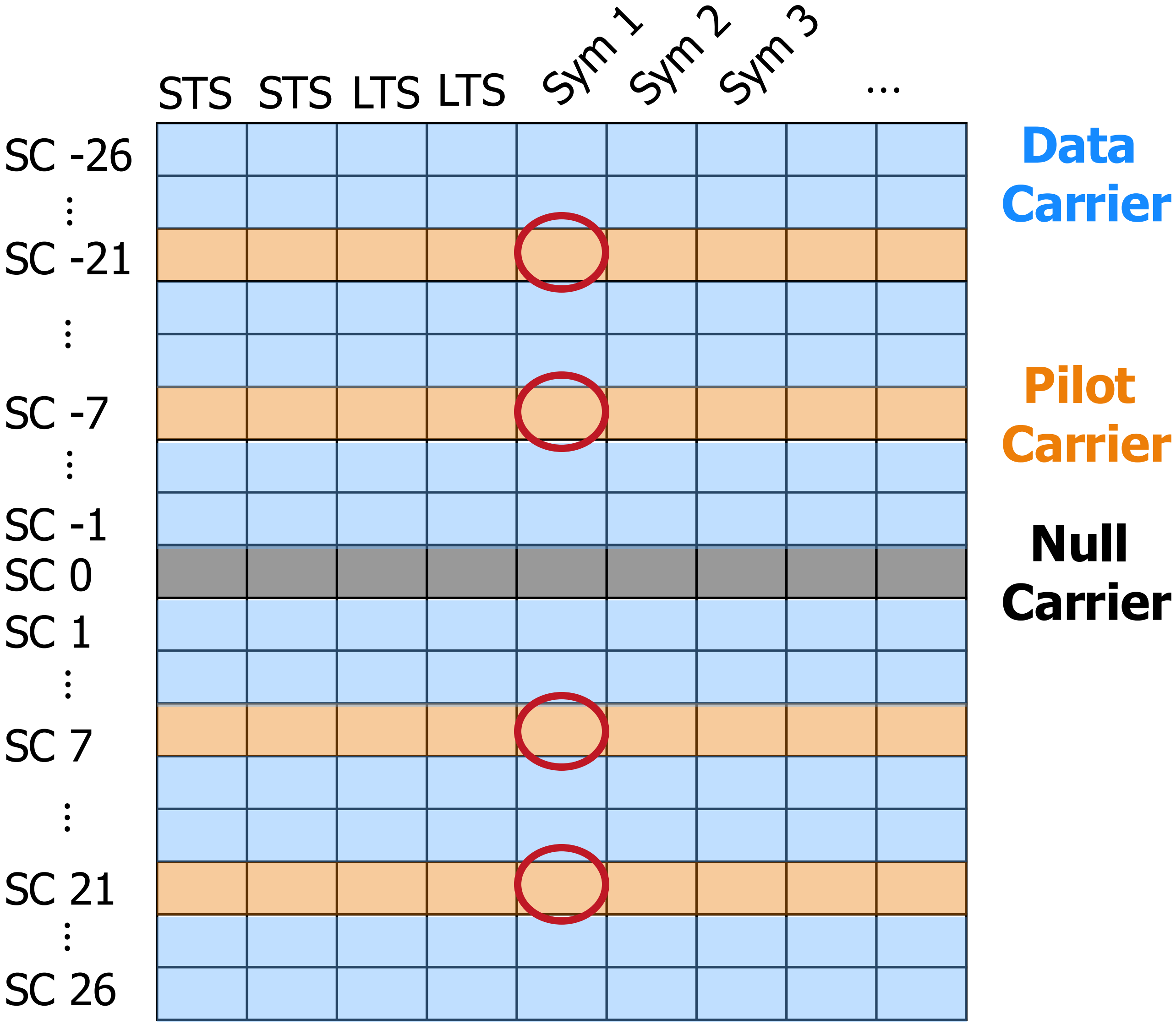
$$\hat{X}(k) = \frac{Y(k)}{H_{LS}(k)}$$

Equalized symbol

Used for demodulation

Estimated channel

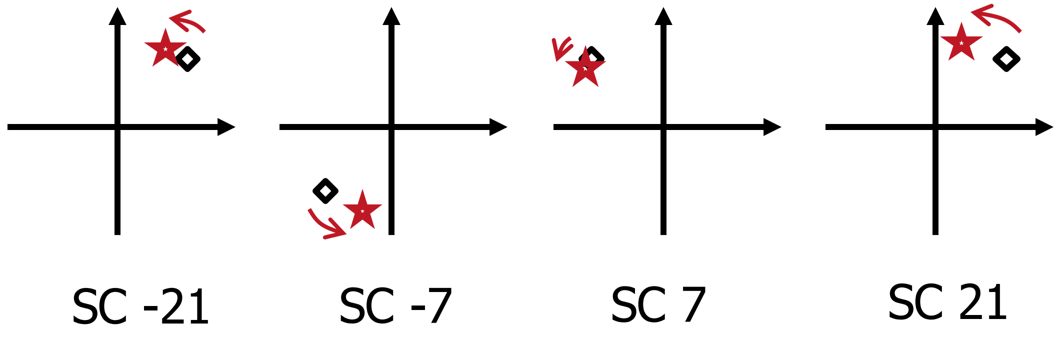
Tracking with Pilot Tones



Transmit known symbols in pilot tones

Correct for drift in the channel over time

For example, after equalization, obtain additional correction term using the four pilot symbols



Summary

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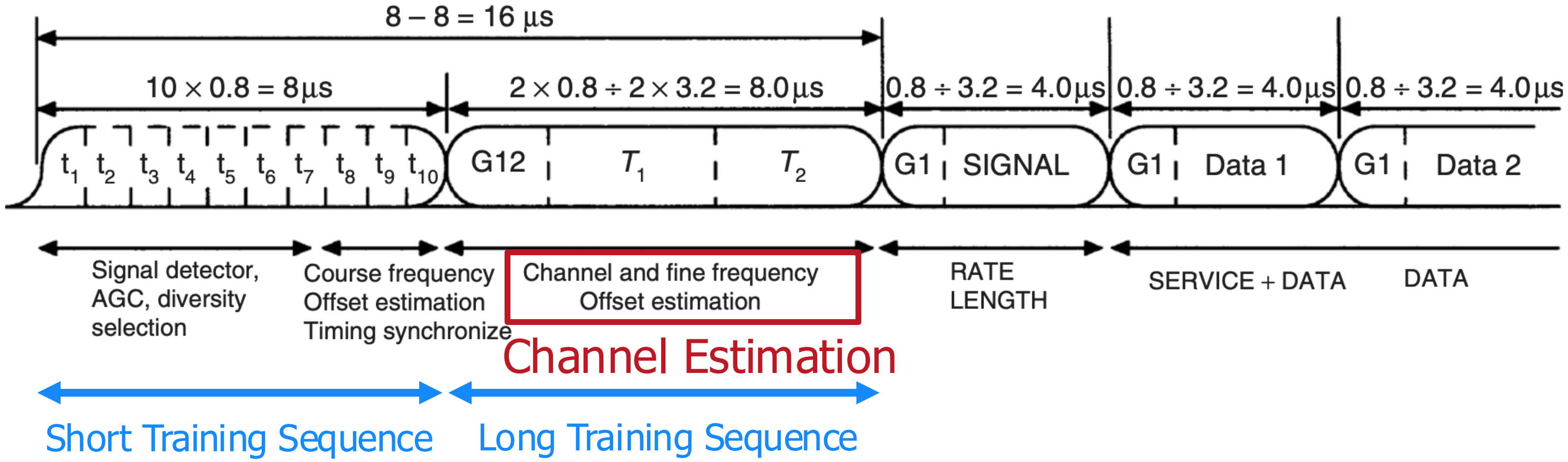
Channel model Rayleigh fading & Rician fading

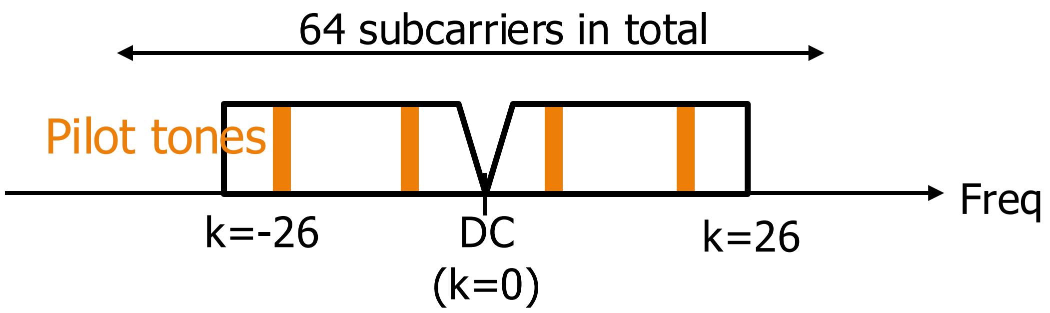
Step 1

Initial estimation using long training symbols



Step 2

Tracking with pilot tones



Reference - Channel

- Digital communications- Principles and systems by Ifiok Otung
  - Chap. 2: Linear channels and systems
- Fundamentals of Digital Communication by Upamanyu Madhow
  - Chap. 8.1: Channel Modeling

Reference – OFDM Channel Estimation

- Wireless Communications by Andreas F. Molisch
  - Chap. 19.5: Channel Estimation
- Modern Digital Radio Communication Signals and Systems by Sung-Moon Michael Yang
  - Chap. 5.1.5: Receiver Processing When the Channel Dispersion < CP
- Multi-carrier digital communications theory and applications of OFDM, Ahmad R.S. Bahai, Burton R. Saltzberg, Mustafa Ergen
  - Chap. 6: Channel Estimation and Equalization

Next Week: First USRP Experiment

- First, a class on SDR, and then move to MD335 for the lab
- TA 奕昕 will lead the lab

Paper Debate

Debate Format

20 minutes	Defense Team
10 minutes	Offense Team
5 minutes	Preparation time
10 minutes	Follow up arguments
5 minutes	Questions and comments from class

Timing will be strictly enforced!

Paper 2: Enriching Multi-User OFDMA in Wi-Fi Networks with Frequency-Selective Channel Awareness

https://forms.gle/HKvxKpuCHu4oKxrg7

Only the audience vote!

Presenters: Upload your slides
(With 分工表)