Homework 1 (Due: 25th Sept.)

(1) What is the advantage and the disadvantage of the time-frequency analysis when compared with the Fourier transform? (10 scores)

Advantage: time-frequency analysis allows us to observe how the frequency varies with time, while the Fourier transform only shows the spectrum and does not contain information about time.

Disadvantage: ① time-frequency analysis cannot compute convolution like Fourier transform
② The computation load is heavier since it has two dimensions.

(2) Suppose that $x(t) = \exp[j(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots\cdots)]$.

In what condition the instantaneous frequency of $x(t)$ is invariant with time? (10 scores)

$\phi(t) = a_0 + a_1 t + \cdots$, instantaneous freq. $= \frac{\phi'(t)}{2\pi} = 2a_2 t + a_1 + \cdots$

$\Rightarrow$ instantaneous freq. is invariant with time iff $a_2 = a_3 = \cdots = a_n = 0$ and a_1, a_2 are constant.

(3) Write at least three conditions where the chirp signal may be generated. (10 scores)

① Radar system ② Animal Voice ③ Doppler Effect ④ seismic waves

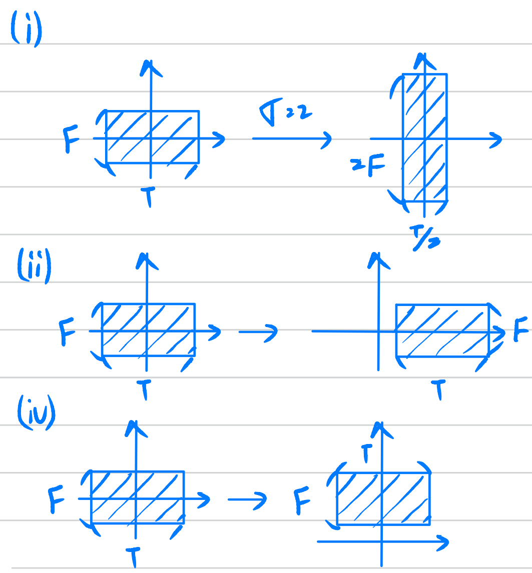
(4) Which of the following operation may varying the number of sampling points required for a function? Why? (i) Scaling; (ii) time-shifting; (iii) multiplied by t; (iv) multiplying by $\exp(jt)$ (10 scores)

(i) no, as $g(t) \rightarrow g(\sigma t)$, $G(f) \rightarrow \frac{1}{|\sigma|} G(\frac{f}{\sigma})$, this does not change the area of time-frequency distribution.

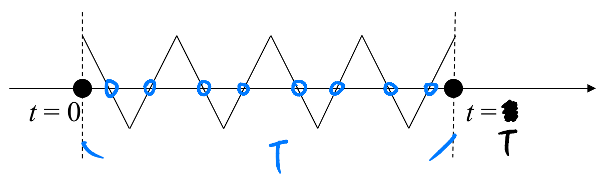
(ii) no, time shifting does not change the area

(iii) no, multiply by t corresponds to a differentiation in frequency domain: $F\{t \cdot x(t)\} = j\frac{d}{d\omega} X(\omega)$, this doesn't change the time interval or bandwidth, it only changes the shape ∴ The area (sampling point) remains the same.

(iv) no, multiply by $e^{j\omega_0 t}$ corresponds to a frequency shift in frequency domain $F\{x(t)e^{j\omega_0 t}\} = X(\omega - \omega_0)$, this doesn't change the time interval or bandwidth ∴ The area (sampling point) remains the same.



(5) How do we compute the frequency by zero crossings? (5 scores)



Count the number of zero crossing in a time interval T,

$f \approx \frac{\text{number of zero crossing}}{2T}$ (approximate, since in number of zero crossing is not necessary 2 in a period).

(6) Why the discrete wavelet transform is useful for (a) image compression and (b) directional edge detection? (10 scores)

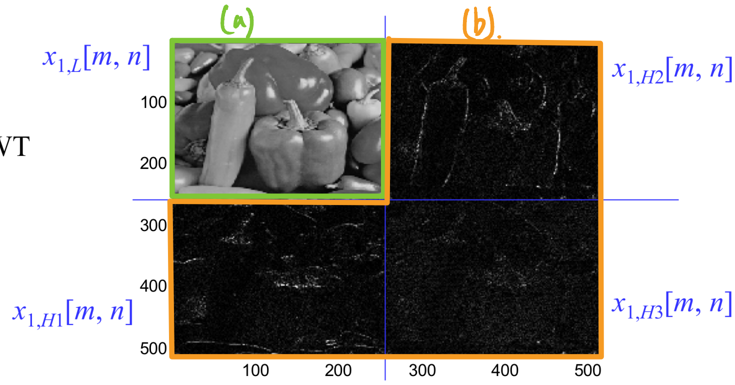
It uses two stages of filtering to create 4 outputs, which represent high/low freq. along x-axis and high/low freq. along y-axis respectively.

(a). Since most part of the image has a gradual change, it belongs to low freq. part along both x and y-axis. Thus, the output (low, low) retains most information in the image, but only use 1/4 amount of data, so it came be used as image compression. See the image in (a) below

(b). Similarly, vertical, horizontal, and corner edge correspond to (high, low), (low, high), and (high, high) frequency respectively. Thus, it can be used as edge detection. See the image in (b) below.

2-D DWT

的結果



Moreover, we can conduct the wavelet transform several times to further increase the effectiveness.

(7) (a) How does the window width B affect the resolution of the rec-STFT? (b) What is the advantage of the STFT with an asymmetric window? (c) Why better time-frequency analysis result can be obtained if one uses the Gaussian window instead of the rectangular window? (15 scores)

(a). $B\uparrow$, t-domain resolution$\downarrow$, f-domain resolution$\uparrow$

$B\downarrow$, t-domain resolution$\uparrow$, f-domain resolution$\downarrow$

(b). Asymmetric window is useful for real-time or causal system, where future data isn't available. This can be used in seismic wave analysis or collision detection

(c). Because rectangular window has a tradeoff between the resolution in time and frequency axis. Moreover, convolution of many rectangular function increase the mainlobes and decrease the sidelobes. Gaussian function is convolution of infinite rectangular function. In addition, the Fourier transform of Gaussian function is still Gaussian, ensuring good resolution in both domains, as proof in uncertainty principle.

(8) Determine the Gabor transform (standard definition) of the Gaussian function $\exp(-\pi t^2)$? (Hint: Using the scaling and shifting properties of the FT). (10 scores)

$$\int_{-\infty}^{\infty} e^{-(a\tau^2+b\tau)}d\tau = \sqrt{\frac{\pi}{a}}e^{b^2/4a}$$

$$a=2\pi,\ b=(j2\pi f-2\pi t)$$

$$X(t,f)=\int_{-\infty}^{\infty} e^{-\pi(\tau-t)^2} e^{-\pi\tau^2} e^{-j2\pi f\tau}d\tau = \int_{-\infty}^{\infty} e^{-2\pi\tau^2+2\pi\tau t-\pi t^2} e^{-j2\pi f\tau}d\tau = e^{-\pi t^2}\left[\int_{-\infty}^{\infty} e^{-(2\pi\tau^2+(j2\pi f-2\pi t)\tau)}d\tau\right]$$

$$= e^{-\pi t^2}\sqrt{\frac{\pi}{2\pi}}\, e^{(j2\pi f-2\pi t)^2/8\pi} = e^{-\pi t^2}\frac{1}{\sqrt{2}}\, e^{\frac{-4\pi^2 f^2-j8\pi^2 f t+4\pi^2 t^2}{8\pi}} = \frac{1}{\sqrt{2}}e^{-\pi t^2-\frac{1}{2}\pi f^2+\frac{\pi t^2}{2}-j\pi f t} = \frac{1}{\sqrt{2}}e^{-\frac{\pi}{2}(t^2+f^2)}e^{-j\pi f t}.$$

(9) Write a Matlab or Python program that can generate a *.wav file whose instantaneous frequency is $\pm(at^2+bt+c)$ Hz, the length of the file is T second, and the sampling frequency is Fs Hz.

gwave (a, b, c, T, Fs)

The code should be handed out by $\boxed{\text{NTUCool}}$ together with homework.

(20 scores)