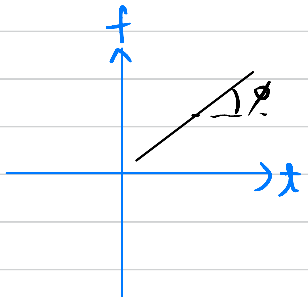
(1) Suppose that $x(t)$ is a chirp function. Which of the following functions are also chirps? Why? (i) $FT(x(t))$; (ii) the FrFT of $x(t)$; (iii) the generalized modulation of $x(t)$; (iv) $x(t) * x(t)$ where * means the convolution.
(Note that the time-frequency distribution of $x(t)$ is a line). (10 scores)

Chirp function = instantaneous freq. changes with time



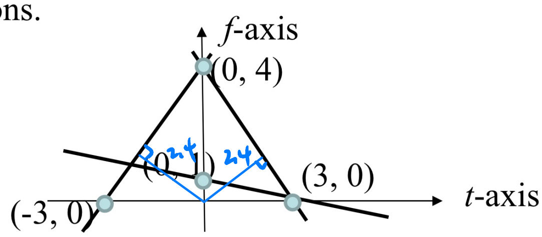
(i) Rotate 0.5π → chirp

(ii) Rotate ϕ → chirp, except when the rotation makes the plot completely horizontal

(iii) $e^{j\phi(t)}x(t) = y(t)$, $\phi(t) = \sum_{k=0} a_k t^k$ → if $k \le 2$, it is only a linear transform → chirp.
But if $k > 2$, nonlinear transform, a line turns into a curve → not chirp

(iv) Convolution with a chirp function → t-axis shearing → still a line → chirp #

(2) Suppose that the time-frequency distribution of a signal is as the following triangular regions.



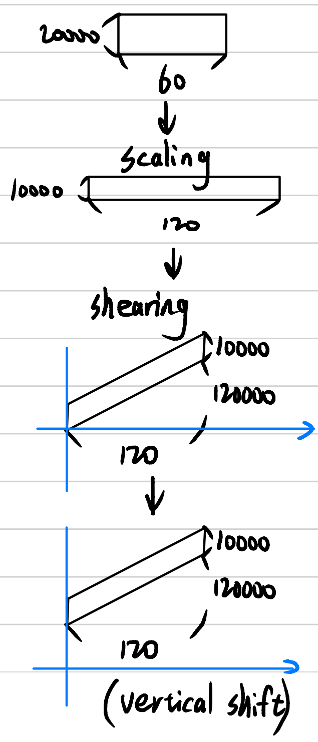
Suppose that the signal is interfered by the white noise How do we use the FrFT filters to reduce the effect of white noise? $H(u)$ and α of each FrFT filter should be shown. (15 scores)

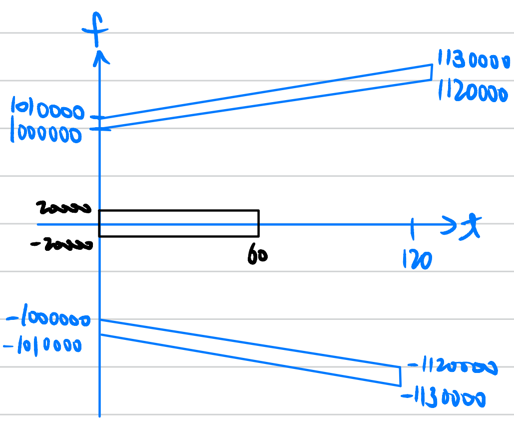
① $X_1(t) = OF^{-\alpha_1}(OF^{\alpha_1}(x(t))H_1(u))$, $H_1(u) = \begin{cases} 1 & \text{if } u \le 2.4 \\ 0 & \text{if } u > 2.4 \end{cases}$, $\alpha_1 = \tan^{-1}(\frac{3}{4})$

$X_2(t) = OF^{-\alpha_2}(OF^{\alpha_2}(X_1(t))H_2(u))$, $H_2(u) = \begin{cases} 1 & \text{if } u \le 2.4 \\ 0 & \text{if } u > 2.4 \end{cases}$, $\alpha_2 = \pi + \tan^{-1}(-\frac{3}{4})$ ⇒ 3 FrFT is needed #

$X_3(t) = OF^{-\alpha_3}(OF^{\alpha_3}(X_2(t))H_3(u))$, $H_3(u) = \begin{cases} 1 & \text{if } u \ge 3/\sqrt{10} \\ 0 & \text{if } u < 3/\sqrt{10} \end{cases}$, $\alpha_3 = \tan^{-1}(3)$

(3) Suppose that there are 6 signals. Their time length are all 10 seconds and their frequencies are all from -20000Hz ~ 20000Hz. Also suppose that the channel is almost full except for $f \in [1000000 + 1000t, 1010000 + 1000t]$ and $f \in [-1000000 - 1000t, -1010000 - 1000t]$, $0 < t < 120$. How do we transmit the 6 signals by the channel? (15 scores)





$a = 1000$

Transmit 6 signals $x(t)$ in series, then :

① Scaling: $X_1(t) \to \frac{1}{\sqrt{2}} x(\frac{1}{2}t)$
$S_x(t,f) \to S_x(\frac{1}{2}t, 2f)$

② Shearing: $X_2(t) \to e^{j\pi 1000 t^2} X_1(t)$
$S_x(t,f) \to S_x(t - 1000f, f)$

③ Vertical shift $X_3(t) = e^{j2\pi 1000000 t} X_2(t)$
$S_x(t,f) \to S_x(t, f - 1000000)$

(4) (a) What are the <u>two main differences</u> between the IMF and a sinusoid function?

(b) Which function is an IMF? Why? (i) $\sin(\pi t^3)$; (ii) $(3 - \cos(12\pi t))\cos(\pi t)$

(10 scores)

(a) ①: The amplitude and frequency can vary with time for IMF, but not for sinusoid function

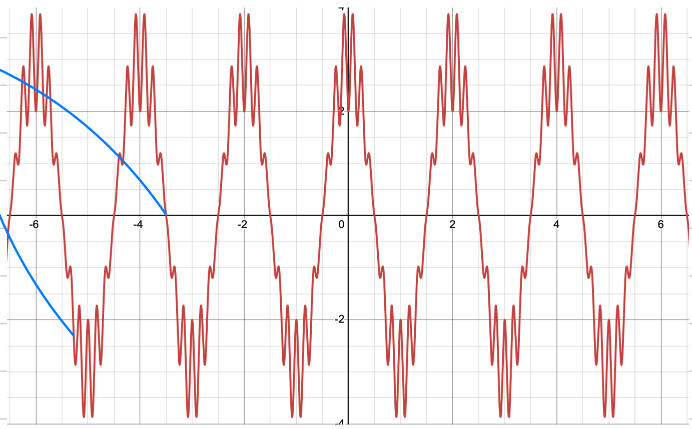
②: The waveform is limited to sinusoid for sinusoid function, but IMF can have many different shape.

(b). For IMF, $|\#\text{extremes} - \#\text{zero-crossing}| \le 1$, and mean of envelope defined by local max and min is near to zero at any point. From the plot below, (i) satisfy the condition and (ii) does not, so (i) is an IMF. //#

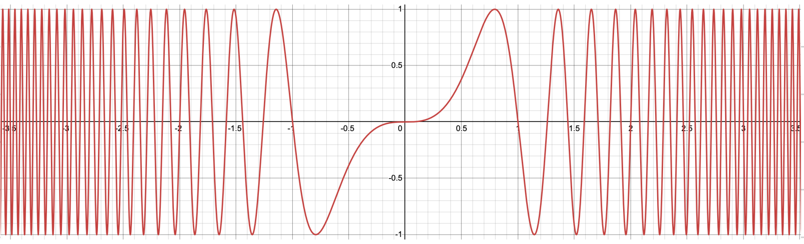
(ii) $(3-\cos(12\pi t))\cos\pi t$

few zero-cross

many local max/min



(i) $\sin(\pi t^3)$



(5) (a) What is the most important advantage of the Haar transform nowadays?

(b) Write the 6th row of the 16-point Haar transform. (10 scores)

(a). Edge detection: Find the places that signal changes significantly

Example: $H_8 = \begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&1&1&1&-1&-1&-1&-1 \\ 1&1&-1&-1&0&0&0&0 \\ 0&0&0&0&1&1&-1&-1 \\ 1&-1&0&0&0&0&0&0 \\ 0&0&1&-1&0&0&0&0 \\ 0&0&0&0&1&-1&0&0 \\ 0&0&0&0&0&0&1&-1 \end{bmatrix}$, signal $= \begin{bmatrix} 1.2 \\ 1.2 \\ \boxed{\begin{matrix} 1.8 \\ 0.8 \end{matrix}} \\ \boxed{\begin{matrix} 2 \\ 2 \end{matrix}} \\ 1.9 \\ 2.1 \end{bmatrix}$ big change, small change, $H_8 \begin{bmatrix} 1.2 \\ 1.2 \\ 1.8 \\ 0.8 \\ 2 \\ 2 \\ 1.9 \\ 2.1 \end{bmatrix} = \begin{bmatrix} 13 \\ -3 \\ -0.2 \\ 0 \\ 0 \\ \boxed{1} \\ \boxed{0} \\ -0.2 \end{bmatrix}$ big, small

(b) $H_{16} = \begin{bmatrix} H_8 \otimes [1\ 1] \\ I_8 \otimes [1\ -1] \end{bmatrix}$, 6th row $= [0\,0\,1\,-1\,0\,0\,0\,0] \otimes [1\ 1] = [0\,0\,0\,0\,1\,1\,-1\,-1\,0\,0\,0\,0\,0\,0\,0\,0]$ //#

(6) (a) What is the role of the vanish moment in the wavelet transform?

(b) Suppose that $x(t) = a + bt + t^2$ for $-2 < t < 2$, $x(t) = 0$ otherwise.

If $x(t)$ has the <u>vanish moment of 2,</u> determine a and b. (10 scores)

(a) Vanish moment determines the order of signal that will be filtered out after the inner product.

If $\psi(t)$ has vanish moment p, $\psi(t)$ is orthogonal to $\sum_{k=0}^{p-1} C_k t^k$. Thus, higher the vanish moment, more low freq. component is filtered.

(b). $m_0=\int_{-\infty}^{\infty}x(t)dt=\int_{-2}^{2}a+bt+t^2dt=\frac{1}{3}t^3+\frac{1}{2}bt^2+at\Big|_{-2}^{2}=\frac{16}{3}+4a=0 \rightarrow a=-\frac{4}{3}$

$m_1=\int_{-2}^{2}at+bt^2+\frac{1}{3}t^3dt=\frac{1}{2}at^2+\frac{1}{3}bt^3+\frac{1}{12}t^4\Big|_{-2}^{2}=\frac{16}{3}b=0 \rightarrow b=0$

$\therefore (a,b)=(-\frac{4}{3},0)$ #

(7) Write a Matlab or Python program of the Hilbert-Huang transform.

y = hht(x, t, thr)

x: input, y: output (each row of y is one of the IMFs of x), t: samples on the *t*-axis, thr : the threshold used in Step 7.

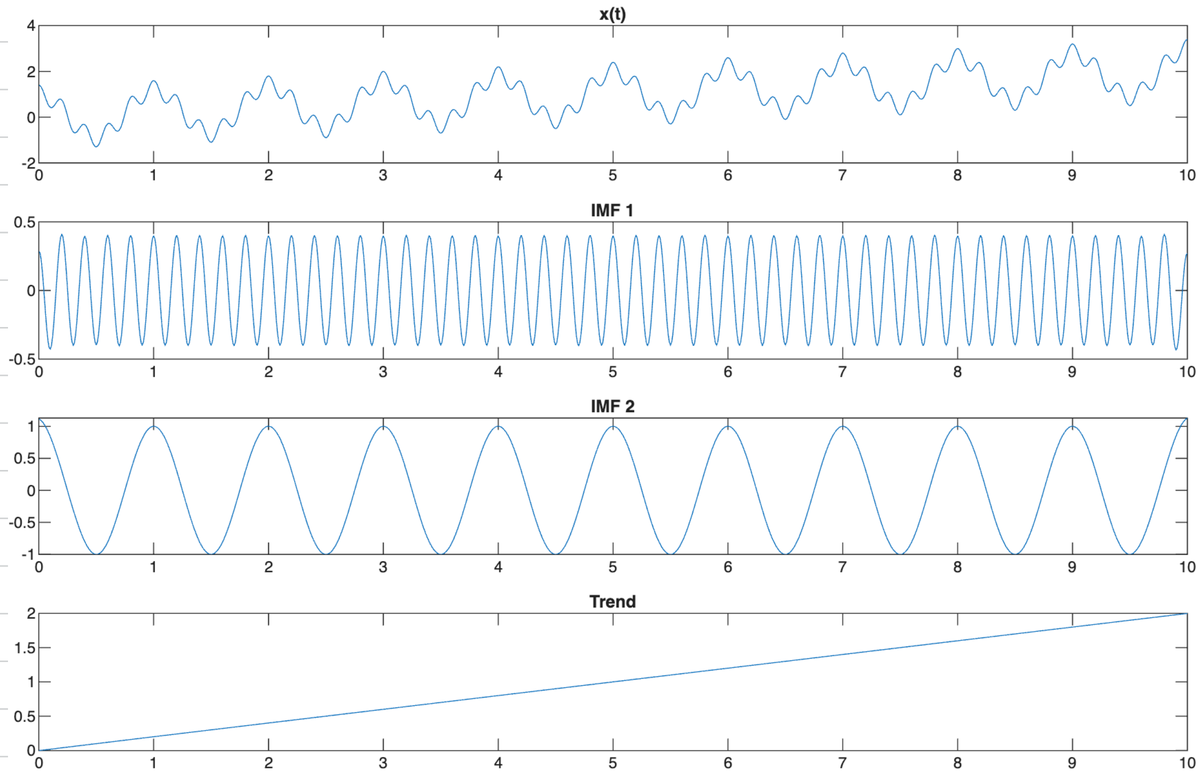
In Step 8, the number of non-boundary extremes can be no more than 3.

Step 9 is not required. The code should be handed out by NTUCool.

(30 scores)

Example:
```
t = [0: 0.01: 10];
x = 0.2*t + cos(2*pi*t) + 0.4*cos(10*pi*t);
thr = 0.2;
y = hht(x, t, thr);
```

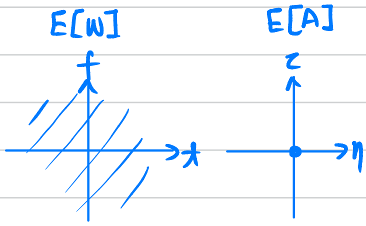
The local max / min only consider non-boundary points, $K=20$ #



Extra(学号尾数4) = If $x(t)$ is white noise which is also white? ① $x(\sigma t)$ ② $e^{j2\pi t^2}x(t)$ ③ $e^{j2\pi t^2} * x(t)$.

For white noise $R_x(\tau)=\sigma\delta(\tau)$, $S_x(f)=\sigma$ $\Rightarrow E[W_x(t,f)]=\sigma$, $E[A_x(\eta,\tau)]=\sigma\delta(\tau)\delta(\eta)$

$\therefore$ For scaling $(x(\sigma t))$, f-axis shearing $(e^{j2\pi t^2}x(t))$, t-axis shearing $(e^{j2\pi t^2} * x(t))$



$E[W_x(t,f)]=S_x(f)=\sigma$ remains the same $\Rightarrow$ ①, ②, ③ are still white. #