Parameter Estimation and Adaptive Design of an Autonomous Racing Car

A thesis submitted in partial fulfilment of the degree of
Master of Engineering in Mechatronics

by

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This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of the author’s knowledge, it contains no material previously published or written by another person, except where due reference is made in the text.

<Boyu Zhao>
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Abstract

This thesis proposes a parameter estimation method for a scale-down autonomous racing car. The dynamic single-track model of Euler integration method is used for parameter estimation based on the measurements generated by the multi-body model in state-space form. By implementing optimization methods, the vehicle mass, moment of inertia for entire mass about z axis, distance from centre of gravity to rear/front axle, centre of gravity height of total mass and cornering stiffness coefficients of front/rear tires have been successfully estimated. The accuracies of the estimated parameters were also demonstrated. Moreover, an idea of adaptive design for the estimated parameters have been proposed in this thesis. By adjusting the selected parameters, the vehicle can achieve the performance of a complex model under a simple model.

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Nomenclature

The following notations are defined and used in this thesis.

Table 1 Variables.

Variables	**Notations**	**Units**
Global Position of the vehicle	x, y	m
Velocity	v	$\frac{m}{s}$
Horizontal/Vertical velocity	v_x, v_y	$\frac{m}{s}$
Acceleration	a	$\frac{m}{s^2}$
Horizontal/Vertical Acceleration	a_x, a_y	$\frac{m}{s^2}$
Yaw angle	θ	rad
Yaw rate	ω	rad/s
Yaw acceleration	$\dot{\omega}$	rad/s^2
Steering angle	δ	rad
Steering angle velocity	v_δ	rad/s
Slip Angle	β	rad
Distance from center of gravity to front/rear axle	l_f, l_r	m
Length of the front/rear axle tracks	t_f, t_r	m
The length of wheelbase	l_{wb}	m
Mass	m	kg
Longitudinal forces of front/rear wheels	$F_{f,x}, F_{r,x}$	N
Lateral forces of front/rear wheels	$F_{f,y}, F_{r,y}$	N
Longitudinal forces of front/rear, left/right wheels	$F_{f,l,x}, F_{r,l,x}, F_{f,r,x}, F_{r,r,x}$	N
Lateral forces of front/rear, left/right wheels	$F_{f,l,y}, F_{r,l,y}, F_{f,r,y}, F_{r,r,y}$	N
Moment of inertia for entire mass about z axis	I_z	kgm^2
Cornering stiffness coefficient front/rear	$C_{s,f}, C_{s,r}$	$1/rad$
Friction coefficient	μ	--
Gravitational acceleration	g	m/s^2
Center of gravity height of total mass	h_{cg}	m

Chapter 1 Introduction and Background

Autonomous driving racing vehicles are currently widely researched topics. Due to high cost and immature autonomous driving technology, scaled down vehicles have come into the sight of researchers. It has the advantages of low cost, simple calibration, rapid deployment and so on. F1/10 autonomous racing cars are equipped with a camera and a LiDAR. The idea is to design a model that can make the self-driving car drive as fast as possible around a racing track. Both the scale down autonomous racing cars and the competition rules restrict the capabilities of the computational hardware, where the efficiency is a crucial requirement. Thus, it is essential to find a vehicle model that is simple but can approximate the states of the car in racing scenario as much as possible. And an effective with low-cost control algorithm is significantly important in the development of the autonomous racing car.

In [1], F1TENTH Foundation introduced the build and environment configuration for an F1/10 racing car. Studies [2] and [3] developed a complete development and design process of a scale down racing car, including vehicle models, tire models, drivetrain models and controller design. Studies [4] and [5] introduced model predictive control and model predictive contouring control in controller design. In [6] and [7], Pacejka magic formula and Fiala tire model are introduced, which have been widely used in tire modelling in industry. In [2], a simplified drivetrain model for 1:43 scale electric is utilized. In [8], models for vehicle dynamics are implemented ranging from point-mass model, kinematic single-track model, dynamics single-track model and multibody model. In [9], a dynamic dual track model was used to estimate the states of vehicle for use in active safety systems.

It is essential to understand the kinematics of the vehicle before designing a controller for it. Car is a complex dynamic system, which is a combination of many variables, like position, velocity, and yaw angle. In the field of vehicle active safety system, there have been many studies on the states of vehicle moving process [10] [11] [12], which main focused on the studies of slip angle yaw rate. In study [13], a parameter estimation method for Unmanned ground vehicle (UGVs) has been introduced. The direct measurement of the vehicle motion states such as lateral velocity, slip angle requires expensive additional sensors.

In this thesis, various models of vehicle dynamics are discussed in order to estimate the parameters of the autonomous racing car as accurately as possible using a simple model according to the states of it, which is finding a balance between the model complexity and computing speed. Based on the results of the parameter estimation, an adaptive design approach is also proposed to refine the parameter estimation and approximate the performance of a complex model under a simple model of vehicle dynamics.

Chapter 2 Problem Formulation

A car itself is a complex dynamic system. It is difficult to measure and calculate the parameters such as moment of inertia, the centre of gravity height, etc. for a scaled-down autonomous racing car [13]. In this thesis, a method for estimating the selected parameters of the autonomous racing car based on the measurements is proposed. Due to the limitations of real conditions, there is no real racing car to obtain measurement data, so in this paper, different driving scenarios are set for the complex vehicle dynamics model, and the results are used as measurement values. Based on these measurements, a simple model is used to fit the result to estimate the parameters of the car that can enable the simple model to meet the performance of the complex model.

In addition, this thesis proposes a design idea, that is, by adjusting the parameters of the autonomous racing car, the car can approximate the performance of the complex model under the simple model, which is called adaptive design.

Chapter 3 Model Selection

In this chapter, the relevant models involved in this project are introduced, including models of vehicle dynamics, tire models and drivetrain model. For a moving car, the dynamics model can calculate the state of the car at each moment, where the longitudinal force is provided by the drivetrain model, and the lateral force can be obtained by lateral dynamics analysis, which is based on the tire model. The figure below shows the relationship between different models.

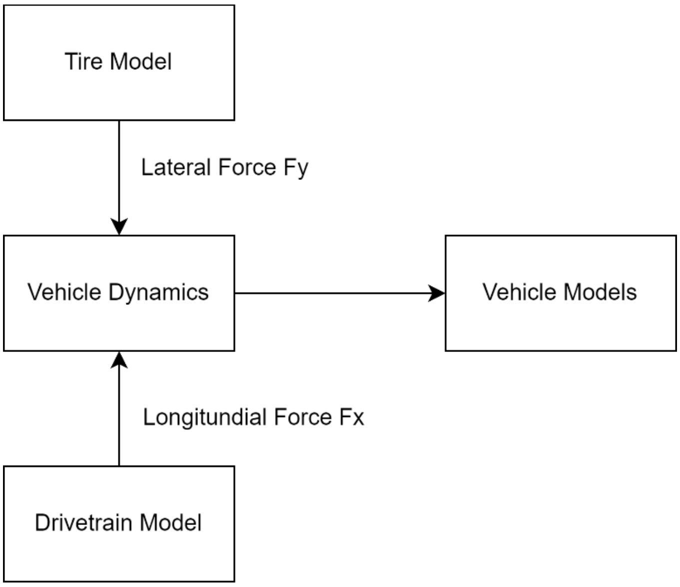

Figure 1 Vehicle models.

3.1 Vehicle Dynamics

In this section, the vehicle models for vehicle dynamics are introduced from simple to complex. Vehicle dynamics is the study of vehicle motion [14]. The models consider increasingly complex lateral vehicle dynamic and tire models ranging from kinematic single-track model, dynamic single-track model, dynamic dual-track model and multi-body model.

3.1.1 Kinematic Single-Track Model

Kinematic single-track model, known as kinematic bicycle model, is a classical model which simplified a four-wheel vehicle into two-wheel model by lumping the front and rear wheel pairs each into one wheel. In this way, a four-wheel and two-steering-angle issue becomes two-wheel and one-steering-angle question, as the image below shows.

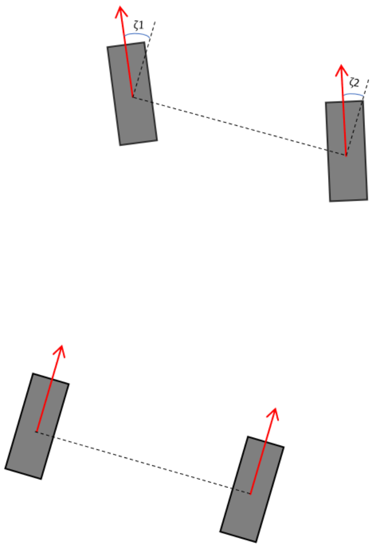

Figure 2 Four-wheel kinematic model.

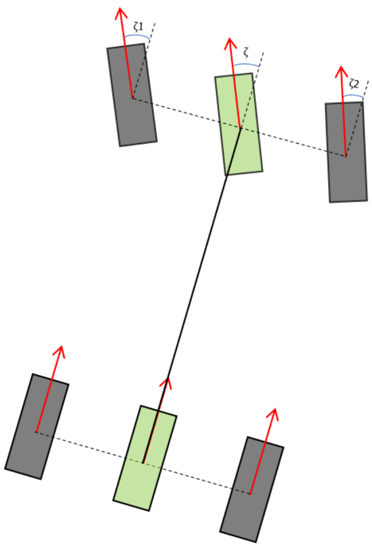

Figure 3 Kinematic single-track model.

The simplified model is based on the following assumptions:

- There is no rear wheel steering.
- The wheels of one axle are combined.
- The vehicle is driving at a low speed.
- The centre of gravity is at road level, which means there is no rolling, no pitching and no wheel load differences.
- There are no slip angles and lateral forces.
- There is an instantaneous centre of rotation when the vehicle is turning.
- There is no lateral acceleration, which means the vehicle doesn’t turn or turns slowly.

In [15] and [16], a kinematics single track model is introduced. If the state of the vehicle in this model is defined as $(s_x, s_y, \Psi, \mathrm{v})$, then it can be derived from the vehicle dynamics model as the equations below.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \Psi \\ v \end{pmatrix} = \begin{pmatrix} vcos(\Theta) \\ vsin(\Theta) \\ \frac{vtan(\delta)}{\mathrm{L_{wb}}} \\ a \end{pmatrix} \tag{1.}$$

Convert it in state-space form in this thesis, if the states are,

$$x_1 = s_x, x_2 = s_y, x_3 = \delta, x_4 = v, x_5 = \Psi, \tag{2.}$$

the input variables are,

$$u_1 = v_\delta, u_2 = a_{long}, \tag{3.}$$

and so the kinematic single-track model is

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} x_4 cos(\mathrm{x_5}) \\ x_4 sin(\mathrm{x_5}) \\ u_1 \\ u_2 \\ \frac{x_4 \tan(x_3)}{l_{wb}} \end{pmatrix} \tag{4.}$$

3.1.2 Dynamic Single-Track Model

The single-track model allows the approximate and physically plausible description of the lateral dynamics of vehicle, which is based on the following simplifications [17].

- The velocity of the vehicle’s centre of gravity is considered to be constant along the longitude of its trajectory.
- All lifting, rolling and pitching motion will be neglected.
- The vehicle’s mass is assumed to be concentrated at the centre of gravity.
- The front and the rear tires will be represented as one single tire on each axle. The imaginary tire contact points V and H, which the tire forces are to act upon, lie along the centre of the axle.
- The pneumatic trail and the aligning torque resulting from the slip angle of the tire will be neglected.
- The wheel-load distribution between front and rear axle is assumed to be constant.
- The longitudinal forces on the tires, resulting from the assumption of a constant longitudinal velocity, will be neglected [17].

After the above assumptions on the model, the only possible motion left is the yaw angle Ψ and the slip angle β. The yaw angle Ψ occurs in the form of the yaw rate $\dot{\Psi}$. The slip angle β represents the direction of the deviation of the centre of gravity from the vehicle's steering axis. The dynamic single-track model works when perform planning of evasive manoeuvres closer to physical limits [8].

In this model, the load transfer of the vehicle due to longitudinal acceleration is considered, such that the vertical forces the front and rear axis $F_{z,f}$ and $F_{z,r}$ become [8]

$$F_{z,f} = \frac{mgl_r - ma_{\text{long}}\,h_{cg}}{l_r + l_f}, \tag{5.}$$

$$F_{z,r} = \frac{mgl_f + ma_{\text{long}}\,h_{cg}}{l_r + l_f} \tag{6.}$$

The forces are derived from the slip angle β and yaw rate $\dot{\Psi}$.

The $\dot{\beta}$ and $\ddot{\Psi}$ can be calculated with the equations below.

$$\begin{aligned}
\dot{\beta} = {} & \frac{\mu}{v(l_r + l_f)}\Big(C_{S,f}(gl_r - a_{\text{long}}\,h_{cg})\delta - \Big(C_{S,r}(gl_f + a_{\text{long}}\,h_{cg}) + C_{S,f}(gl_r - a_{\text{long}}\,h_{cg})\Big)\beta \\
& + \big(C_{S,r}(gl_f + a_{\text{long}}\,h_{cg})l_r - C_{S,f}(gl_r - a_{\text{long}}\,h_{cg})l_f\big)\frac{\dot{\Psi}}{v}\Big) - \dot{\Psi}, \\
\ddot{\Psi} = {} & \frac{\mu m}{I_z(l_r + l_f)}\big(l_f C_{S,f}(gl_r - a_{\text{long}}\,h_{cg})\delta \\
& + \Big(l_r C_{S,r}(gl_f + a_{\text{long}}\,h_{cg}) - l_f C_{S,f}(gl_r - a_{\text{long}}\,h_{cg})\Big)\beta \\
& - \Big(l_f^2 C_{S,f}(gl_r - a_{\text{long}}\,h_{cg}) + l_r^2 C_{S,r}(gl_f + a_{\text{long}}\,h_{cg})\Big)\frac{\dot{\Psi}}{v}\Big).
\end{aligned} \tag{7.}$$

Thus, with tire cornering stiffness coefficient, in state-space model, the model can be written as below with states,

$$x_1 = s_x, x_2 = s_y, x_3 = \delta, x_4 = v, x_5 = \Psi, x_6 = \dot{\Psi}, x_7 = \beta, \tag{8.}$$

input variables

$$u_1 = v_\delta, u_2 = a_{\text{long}}, \tag{9.}$$

and state-space form

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} x_4 \cos(x_5 + x_7) \\ x_4 \sin(x_5 + x_7) \\ u_1 \\ u_2 \\ x_6 \\ \frac{\mu m}{I_z(l_r+l_f)}\Big(l_f C_{S,f}\big(g l_r - u_2 h_{cg}\big)x_3 + \Big(l_r C_{S,r}\big(g l_f + u_2 h_{cg}\big) - l_f C_{S,f}\big(g l_r - u_2 h_{cg}\big)\Big)x_7 \\ -\Big(l_f^2 C_{S,f}\big(g l_r - u_2 h_{cg}\big) + l_r^2 C_{S,r}\big(g l_f + u_2 h_{cg}\big)\Big)\frac{x_6}{x_4}\Big) \\ \frac{\mu}{x_4(l_r+l_f)}\Big(C_{S,f}\big(g l_r - u_2 h_{cg}\big)x_3 - \Big(C_{S,r}\big(g l_f + u_2 h_{cg}\big) + C_{S,f}\big(g l_r - u_2 h_{cg}\big)\Big)x_7 \\ +\big(C_{S,r}\big(g l_f + u_2 h_{cg}\big)l_r - C_{S,f}\big(g l_r - u_2 h_{cg}\big)l_f\big)\frac{x_6}{x_4}\Big) - x_6 \end{pmatrix} \tag{10.}$$

3.1.3 Dynamic Dual-Track Model

Compared to the previous models, the dynamic dual-track model represents all four wheels instead of merging each axle into a single wheel. It represents the vehicle's geometry more accurately and allows it to consider that the normal force acting on each wheel of the front or rear axle is different, which describes the behaviour of the vehicle in more detail and models the dynamic forces acting on the car more accurately when there are lateral forces. The model is typically used when modelling laterally asymmetric vehicles.

It is a complicated model of the vehicle. Each wheel is modelled individually with an individual wheel load and individual latitude/longitude forces. In this way, the axle loads are also different in front and rear.

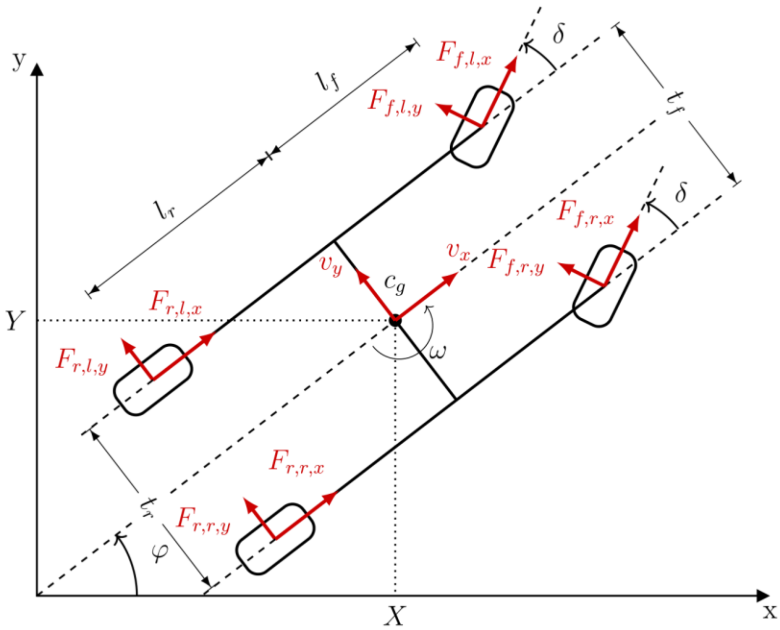

Figure 4 Dynamic dual-track model [3].

In [9], a dynamic dual-track model is implemented for a robust vehicle motion states estimation method. If written it in state space form, the model is described by the following differential equations

$$x_1 = s_x, x_2 = s_y, x_3 = \delta, x_4 = v, x_5 = \Psi, x_6 = \dot{\Psi}, \tag{11.}$$

and input variables

$$u_1 = v_\delta, u_2 = a_{\text{long}}, u_3 = M_z, \tag{12.}$$

and state space form

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} x_4 \cos(x_6) - x_4 \sin(x_6) \\ x_4 \sin(x_6) + x_4 \cos(x_6) \\ u_1 + x_4 x_6 \\ \mathrm{u}_2 - x_4 x_6 \\ x_6 \\ \dfrac{u_3}{I_z} \end{pmatrix}, \tag{13.}$$

where u_1,u_2 and u_3 is given by:

$$\begin{aligned} u_1 &= \frac{(F_{f,l,x} + F_{f,r,x})\cos(\delta) - (F_{f,l,y} + F_{f,r,y})\sin(\delta) + F_{r,l,x} + F_{r,r,x}}{m}, \\ u_2 &= \frac{(F_{f,l,x} + F_{f,r,x})\sin(\delta) + (F_{f,l,y} + F_{f,r,y})\cos(\delta) + F_{r,l,y} + F_{r,r,y}}{m}, \\ u_3 &= l_f\Big((F_{f,l,x} + F_{f,r,x})\sin(\delta) + (F_{f,l,y} + F_{f,r,y})\cos(\delta)\Big) - l_r(F_{r,l,y} + F_{r,r,y}) \\ &\quad - \frac{t_f}{2}\Big((F_{f,l,x} - F_{f,r,x})\cos(\delta) - (F_{f,l,y} - F_{f,r,y})\sin(\delta)\Big) - \frac{t_r}{2}(F_{r,l,x} - F_{r,r,x}) \end{aligned} \tag{14.}$$

3.1.4 Multi-Body Model

The previously introduced models consider many effects of the vehicle dynamics, however, there are still variables missing. They do not consider the vertical load of the four wheels. The multi-body model takes the vehicles' pitch, roll, yaw, the individual spin and slip, and non-linear tire model dynamics. The multi-body dynamics is described by 3 masses: The unsprung mass and the sprung masses of the front and rear axles. The forces between these masses are described by the dynamics of the suspension and the tire model [8]. The model contains more than 29 state variables. In [8], there is a detailed explanation on the multi-body dynamic model. In this thesis, since the multibody model has a very high accuracy for the simulation of the vehicle states when moving, it is used to generate measurements data for singe-track model to estimate the vehicle parameters.

The vehicle states for multi-body are shown below [8]:

Table 2 State variables.

	State variables	Description
$\boldsymbol{x_1}$	s_x	x-position in a global coordinate system
$\boldsymbol{x_2}$	s_y	y-position in a global coordinate system
$\boldsymbol{x_3}$	δ	steering angle of front wheels
$\boldsymbol{x_4}$	v_x	velocity in longitudinal direction in the vehicle-fixed coordinate system
$\boldsymbol{x_5}$	Ψ	yaw angle
$\boldsymbol{x_6}$	$\dot{\Psi}$	yaw rate
$\boldsymbol{x_7}$	Φ_S	roll angle
$\boldsymbol{x_8}$	$\dot{\Phi}_S$	roll rate
$\boldsymbol{x_9}$	θ_S	pitch angle
$\boldsymbol{x_{10}}$	$\dot{\theta}_S$	pitch rate
$\boldsymbol{x_{11}}$	v_y	velocity in lateral direction in the vehicle-fixed coordinate system
$\boldsymbol{x_{12}}$	S_z	z-position (height) from ground
$\boldsymbol{x_{13}}$	v_z	velocity in vertical direction perpendicular to road plane
$\boldsymbol{x_{14}}$	Φ_{UF}	roll angle front
$\boldsymbol{x_{15}}$	$\dot{\Phi}_{UF}$	roll rate front
$\boldsymbol{x_{16}}$	$v_{y,UF}$	velocity in y-direction front
$\boldsymbol{x_{17}}$	$s_{z,UF}$	z-position front
$\boldsymbol{x_{18}}$	$v_{z,UF}$	velocity in z-direction front
$\boldsymbol{x_{18}}$	Φ_{UR}	roll angle rear
$\boldsymbol{x_{20}}$	$\dot{\Phi}_{UR}$	roll rate rear
$\boldsymbol{x_{21}}$	$v_{y,UR}$	velocity in y-direction rear
$\boldsymbol{x_{22}}$	$s_{z,UR}$	z-position rear
$\boldsymbol{x_{23}}$	$v_{z,UR}$	velocity in z-direction rear
$\boldsymbol{x_{24}}$	ω_{LF}	left front wheel angular velocity
$\boldsymbol{x_{25}}$	ω_{RF}	right front wheel angular velocity
$\boldsymbol{x_{26}}$	ω_{LR}	left rear wheel angular velocity
$\boldsymbol{x_{27}}$	ω_{RR}	right rear wheel angular velocity
$\boldsymbol{x_{28}}$	$\delta_{y,f}$	front lateral displacement of sprung mass due to roll
$\boldsymbol{x_{29}}$	$\delta_{y,r}$	rear lateral displacement of sprung mass due to roll

The above state variables are considered in the multi-body model. The inputs are as follow:

$$u_1 = v_\delta, u_2 = a_{long} \tag{15.}$$

The details of multi-body state space form are in [18].

3.2 Tire Models

As mentioned in Section 3.1, a tire provides the lateral and longitudinal tire forces in vehicle dynamics. It creates the contact between the vehicle and road. In addition, a tire can apply more forces if there is a higher friction coefficient. Thus, the parameters depend both on the parameters of the tire and the road the vehicle is driving on.

3.2.1 Pacejka Magic Formula

Pacejka is an empirical formula to describe the longitudinal or lateral force and slip generated by a tire. It is given by the formula,

$$F = D \times \sin\big(C \times \arctan\big(B \times x - E \times (B \times x - \arctan(B \times x))\big)\big), \tag{16.}$$

with

$$T(X) = y(x) + S_v \tag{17.}$$

$$x = X + S_h \tag{18.}$$

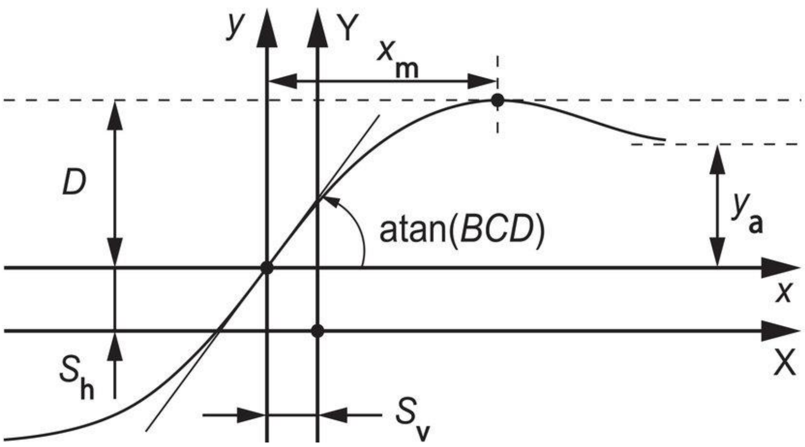

Figure 5 Curve produced by the general form of the Magic Formula [19].

where F corresponds to either the lateral force F_y, or the longitudinal force F_x. The x corresponds to the input variable, which is the longitudinal slip for F_x, or the lateral slip for F_y. The parameters B, C, D, E are called Pacejka parameters. The parameter B is known as the stiffness factor, C as the shape factor, D as the peak value and E as the curvature factor. The S_h and S_v are two shift parameters, corresponding to the horizontal and vertical shift, respectively [6].

In this thesis, the Pacejka magic formula is used to compute the longitudinal tire forces and lateral tire forces.

3.2.2 Fiala Tire Model

Fiala tire model was widely used for control of high side-slip maneuvers, which was originally developed to estimate lateral tire force generation [7] but has been extended to take consideration of both lateral and longitudinal forces based on the assumption that the later and longitudinal tire stiffness are equal [20]. It is written as:

$$F_y(\alpha) = \begin{cases} -C_\alpha \tan(\alpha) + \dfrac{C_\alpha^2\left(2 - \frac{\mu_s}{\mu_p}\right)}{3\mu_p F_z}|\tan(\alpha)|\tan(\alpha) - \dfrac{C_\alpha^3\left(1 - \frac{\mu_s}{m_p}\right)}{9\mu_p^2 F_z^2}\tan^3(\alpha) & \text{if } |\alpha| < \alpha_{sl}\,, \\ -\mu_s F_z \operatorname{sgn}(\alpha) & \text{if } |\alpha| \geq \alpha_{sl} \end{cases} \quad (19.)$$

where F_y is the lateral force the tire generates, C_α is the tire cornering stiffness, F_z is the normal load of the tire, μ_p is the peak friction coefficient between the tire and the road, μ_s is the sliding friction coefficient of the tire and α is the tire's slip angle. The α_{sl} is computed as:

$$\alpha_{sl} = \arctan\left(\frac{3\mu_p F_z}{C_\alpha}\right) \quad (20.)$$

3.3 Drivetrain Models

For a tire model, it is important to obtain the longitudinal force F_x which is acting on the wheels. In [2], a rear-wheel-drive model using a motor model for the DC electric motor as well as a friction model for the rolling resistance and the drag was introduced.

$$F_{r,x} = (C_{m1} - C_{m2}v_x)d - C_r - C_d v_x^2, \quad (21.)$$

where d is the PWM applied to the DC motor, $Cm1$ and $Cm2$ are empirical parameters representing the characteristics of the motor. Cr and Cd are coefficients representing the rolling and drag resistances of the car.

In [2], it introduced a speed controller that accepts engine revolutions per minute (ERPM), which is denoted as $\tilde{d}$. $\tilde{d} \in [0,1]$. When $\tilde{d} = 0$, it is the minimal ERPM and $\tilde{d} = 1$ corresponds to the maximal ERPM. As the equation below shows:

$$F_x = (C_{m1} - C_{m2}v_x)\tilde{d} - C_{m3} - C_{m4}v_x^2 \quad (22.)$$

In this thesis, the driving scenarios have been designed in advance. The input steering angle velocity and longitudinal acceleration of the vehicle have been set up directly, which means the drivetrain model can be skipped when doing simulation.

3.4 Summary

In this section, the models of vehicle dynamics, tire models and drivetrain models are introduced. For vehicle dynamics, complex models mean more variables, higher demands on computing units and longer decision times for the controllers. It is important to find a balance between model accuracy and decision time for an autonomous racing car. The task was to find a model that described the state of the car as realistically as possible in the shortest possible time. Therefore, in this thesis, the dynamic single-track model is chosen for parameter estimation. Instead, using a multibody model to generate the measurements.

Since the single-track model has only 7 state variables, while the multibody model has 29 state variables. There are misalignments among these state variables. Therefore, the state variables slip angle and velocity of the multibody model need to be dealt with as the equations below show.

$$slip\ angle = \arctan\left(\frac{v_x}{v_y}\right), \tag{23.}$$

$$velocity = \sqrt{v_x^2 + v_y^2} \tag{24.}$$

In this way, seven state variables corresponding to the single-track model can be selected in the multi-body model, and the parameter estimation can be performed based on the measurements generated by multi-body model.

Chapter 4 System Identification

In this chapter, the setting of the driving scenarios, the optimization method of parameter estimation, and the idea of adaptive design are introduced.

4.1 Driving Scenarios

The method of parameter estimation is designed for an autonomous racing car. Therefore, in the design of the driving scenarios, it should be as close as possible to the racing scenarios. In racing competitions, the most common scenario is to drive on a curve, including decelerating into the corner and accelerating out of the corner [21], which are also the basis of the driving scenario designs.

In this thesis, three driving scenarios at the same initial position with different steering angle velocities and longitudinal accelerations are designed respectively.

1. Turn left at a constant speed.
2. Decelerate into the corner.
3. Accelerate into the corner.

4.2 Parameter Estimation

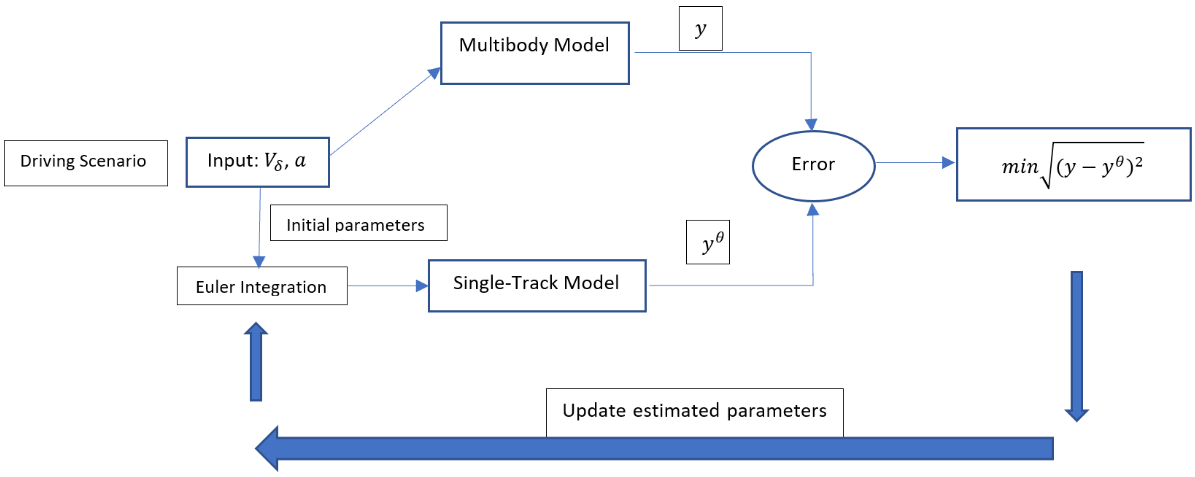

Figure 6 Parameter estimation workflow.

As the figure above shows, based on different driving scenarios, the workflow of the parameter estimation method is shown below.

1. The multibody model is used to generate measurements of the vehicle states.
2. The estimated initial parameters are used to calculate the estimated vehicle states in using single-track model with Euler integration.
3. Calculate the error between the measurements and the estimated states.
4. Update the estimated parameters until the 2-norm of the error is minimized.
5. Finally, output the final estimated parameters, which are the optimized estimated vehicle parameters.

As the equation below shows,

$$\min(error) = \sqrt{\left(state_{multibody} - state_{singletrack}\right)^2}. \tag{25.}$$

4.2.1 Euler Integration for Dynamic Single-Track Model

Based on the state space form in section 3.1.2, the dynamic single-track model can be written in Euler Integration form as follows. k represents the state of the vehicle.

$$\begin{aligned}
x_{k+1} &= x_k + \Delta T \times \cos\left(\beta_k + \varphi_k\right) \\
y_{k+1} &= y_k + \Delta T \times \sin\left(\beta_k + \varphi_k\right) \\
\delta_{k+1} &= \delta_k + \Delta T \times V_\delta \\
v_{k+1} &= v_k + \Delta T \times a \\
\varphi_{k+1} &= \varphi_k + \Delta T \times \dot{\varphi}_k \\
\dot{\varphi}_{k+1} &= \dot{\varphi}_k + \Delta T \times \Bigg(\frac{\mu m}{I_z\left(l_r + l_f\right)}\Big(l_f C_{S,f}\left(g l_r - a h_{cg}\right)\delta + \Big(l_r C_{S,r}\left(g l_f + a h_{cg}\right) - l_f C_{S,f}\big(g l_r - \\
&\quad a h_{cg}\big)\Big)\beta - \left(l_f^2 C_{S,f}\left(g l_r - a h_{cg}\right) + l_r^2 C_{S,r}\left(g l_f + a h_{cg}\right)\right)\frac{\dot{\Psi}}{v}\Big)\Bigg) \\
\beta_{k+1} &= \beta_k + \Delta T \times \Bigg(\frac{\mu}{v\left(l_r + l_f\right)}\Big(C_{S,f}\left(g l_r - a_{\text{long}}\, h_{cg}\right)\delta - \Big(C_{S,r}\left(g l_f + a h_{cg}\right) + C_{S,f}\big(g l_r - \\
&\quad a h_{cg}\big)\Big)\beta + \left(C_{S,r}\left(g l_f + a h_{cg}\right) l_r - C_{S,f}\left(g l_r - a h_{cg}\right) l_f\right)\frac{\dot{\Psi}}{v}\Big) - \dot{\Psi}\Bigg).
\end{aligned} \tag{26.}$$

From the above equations, it can be found that the key variables are the yaw rate $\dot{\varphi}$ and slip angle β of the vehicle. With these two variables, other state variables of the vehicle can be calculated.

Thus, the estimated parameters are as follows, which are obtained from the equations of yaw rate $\dot{\varphi}$ and slip angle β:

- Vehicle mass m,
- Moment of inertia for entire mass about z axis I_z,
- Distance from center of gravity to rear axle l_r,
- Distance from center of gravity to front axle l_f,

- Center of gravity height of total mass h_{cg},
- Cornering stiffness coefficient (front) C_{sf},
- Cornering stiffness coefficient (rear) C_{sr}.

4.2.2 Optimization Algorithm

There are three optimization methods used in the practice to estimate the parameters $m, I_z, l_r, l_f, h_{cg}, C_{sf}, C_{sr}$, which are Nelder-Mead, Constrained Optimization BY Linear Approximation (COBYLA) and Sequential Least Squares Programming (SLSQP). A detail introduction for these methods is in [22]. Nelder-Mead cannot set constraints on the variables, while the COBYLA and SLSQP methods can set the constraints on the variables. In addition, the Jacobian matrix could be used for SLSQP to improve computational efficiency and results.

4.3 Adaptive Design

Through the parameter estimation method proposed above, the vehicle parameters are estimated from the measurements. But there are two questions remain. The first one is how to use these optimized estimated parameters in multi-body model simulation. Because the number of parameters in multi-body model is significantly more than it in single-track model, which means the estimated parameters are not enough to do simulation in multi-body model. The other question is that how to use the estimated parameters in single-track model to gain the performance that complex models have.

Given these questions, the idea of adaptive design is proposed. It is supposed that the autonomous racing car could be customized through changing vehicle mass, tires, distance from centre of gravity to front/rear axle or the centre of gravity height of total mass.

In the adaptive design, part of the parameters is selected, considered as customizable parameters, and participate in the optimization operation, so that the autonomous racing car in simple model can be closer to the effect of the complex model. In this thesis, cornering stiffness coefficients of the rear and front tires and vehicle mass are selected as customizable parameters in adaptive design. That is, it is assumed that the tires of the car can be replaced, and the mass can be adjusted, so that the car can achieve the performance of the multi-body model with the single-track model. Scenario “turn left” is taken as an example of achieving the idea in this thesis.

The process of adaptive design is as below.

1. Obtain the estimated parameters of the autonomous racing car based on the parameter estimation model.
2. Select the mass m and cornering stiffness coefficients $C_{s,f}$ and $C_{c,r}$ as adjustable parameters.
3. Bring them into the single-track model and calculate the error with the measurements generated by the multi-body model.
4. Using an optimization algorithm, minimize the 2-norm of the error.
5. Finally, the adjusted values of the selected parameters can be obtained.

The equation below shows the idea of adaptive design.

$$\min(error) = \min_{st} \min_{mb} \sqrt{\left(states_{multibody} - state_{singletrack}\right)^2} \qquad (27.)$$

Chapter 5 Experimental Setup and Result Analysis

This chapter introduces the car settings, driving scenario settings, etc. of the parameter estimation experiments of the autonomous racing car and presents figures to show the parameter estimation results. Finally, the results were analysed. The vehicle data and code of multi-body model used in this chapter is from open-source project “CommonRoad”, Technical University of Munich [23].

5.1 Experimental Setup

The vehicle data in this experiment is from BMW 320i, collected by the open-source project [23], where there is a detailed vehicle related parameters such as vehicle, length of wheelbase and tire parameter cornering stiffness coefficients, etc.

In order to implement the models of vehicle dynamics, in the experiment, the following parameters are set.

- Friction coefficient $\mu = 1.0489$,
- Gravitational acceleration $g = 9.81\ m/s^2$

Secondly, the initial values and constraints of the estimated parameters are set as the Table 3 shows.

Table 3 Estimated parameters initial values and constraints.

Parameters	Notation	Initial values	Constraints
Vehicle mass (kg)	m	1000	$m > 0$
Moment of inertia for entire mass about z axis (kgm^2)	I_z	1500	$I_z > 0$
Distance from center of gravity to rear axle (m)	l_r	1.5	$l_r > 0$ $l_r + l_f < 3$
Distance from center of gravity to front axle (m)	l_f	1	$l_f > 0$ $l_r + l_f < 3$
Center of gravity height of total mass (m)	h_{cg}	0.5	$h_{cg} > 0$
Cornering stiffness coefficient (front) ($1/rad$)	C_{sf}	20	$C_{sf} > 0$
Cornering stiffness coefficient (rear) ($1/rad$)	C_{sr}	20	$C_{sr} > 0$

Thirdly, the following driving scenarios are set in the experiments as the Table 4 shows. The experiment duration is set to 1 second.

Table 4 Driving scenarios.

Scenario Name	Inputs	Initial state
Turn left	$v_\delta = 0.2\ rad/s$ $a_{long} = 0\ m/s^2$	$[s_x, s_y, \delta, v, \Psi, \dot{\Psi}, \beta] = [0,0,0,15,0,0,0]$
Brake into the corner	$v_\delta = 0.15\ rad/s$ $a_{long} = -0.5g\ m/s^2$	$[s_x, s_y, \delta, v, \Psi, \dot{\Psi}, \beta] = [0,0,0,15,0,0,0]$
Accelerate out of the corner	$v_\delta = 0.15\ rad/s$ $a_{long} = 0.5g\ m/s^2$	$[s_x, s_y, \delta, v, \Psi, \dot{\Psi}, \beta] = [0,0,0,15,0,0,0]$

Fourthly, for adaptive design, the scenario "turn left" is taken as an example in this experiment, assuming that the mass of the car and the tires can be customized, which means the parameters mass m and cornering stiffness coefficient $C_{s,f}$ and $C_{s,r}$ can be adjusted. The other estimated parameters are held constant.

5.2 Experiment Results

5.2.1 Scenario：Turn Left

The Table 5 below shows the estimated parameters values in "COBYLA" and "SLSQP" methods in scenario "turn left", respectively.

Table 5 Parameter estimation result in "turn left" scenario.

Optimization Method	$\boldsymbol{m}$	$\boldsymbol{I_z}$	$\boldsymbol{l_r}$	$\boldsymbol{l_f}$	$\boldsymbol{h_{cg}}$	$\boldsymbol{C_{s,f}}$	$\boldsymbol{C_{s,r}}$
"COBYLA"	999.805612	1500.77353	1.58033175	1.41966825	1.20290118	20.8181788	19.5725357
"SLSQP"	1000.03652	1499.97571	1.26647953	1.73352047	1.78063348	24.9541496	17.0535105

The Figure 7, Figure 8, and Figure 9 show the simulation results of vehicle states in scenario "turn left" comparison between single-track model using estimated parameters in two optimization methods and multi-body model.

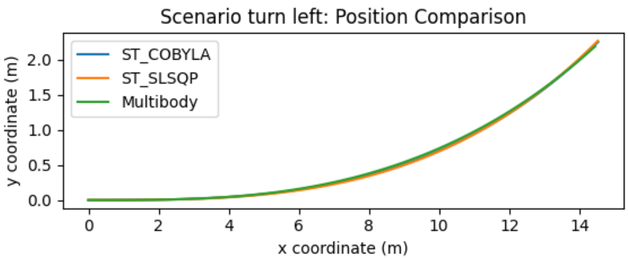

Figure 7 Scenario "turn left" position comparison.

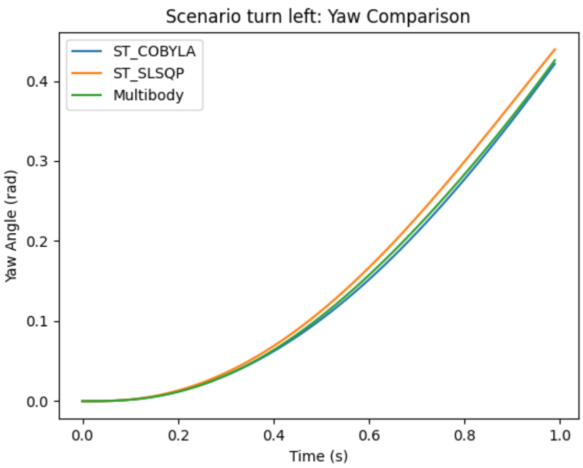

Figure 8 Scenario "turn left" yaw comparison.

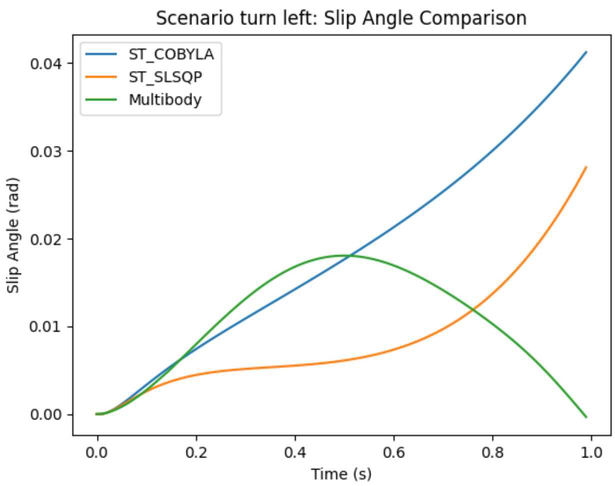

Figure 9 Scenario "turn left" slip angle comparison.

5.2.2 Scenario： Brake into the Corner

The Table 6 below shows the estimated parameters values in “COBYLA” and “SLSQP” methods in scenario “brake into the corner”, respectively.

Table 6 Parameter estimation result in "brake into the corner" scenario.

Optimization Method	$\boldsymbol{m}$	$\boldsymbol{I_z}$	$\boldsymbol{l_r}$	$\boldsymbol{l_f}$	$\boldsymbol{h_{cg}}$	$\boldsymbol{C_{s,f}}$	$\boldsymbol{C_{s,r}}$
“COBYLA”	1000.98505	1500.08749	1.68354331	1.31645669	0.708661358	28.6967037	12.6007674
“SLSQP”	1000.08266	1499.94503	1.61564469	1.47459617	0.385995017	34.0473623	12.7542268

The Figure 10, Figure 11, and Figure 12 show the simulation results of vehicle states in scenario “turn left” comparison between single-track model using estimated parameters in two optimization methods and multi-body model.

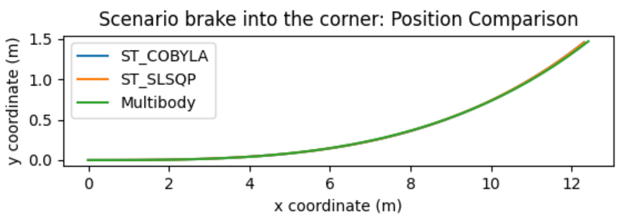

Figure 10 Scenario “brake into the corner” position comparison.

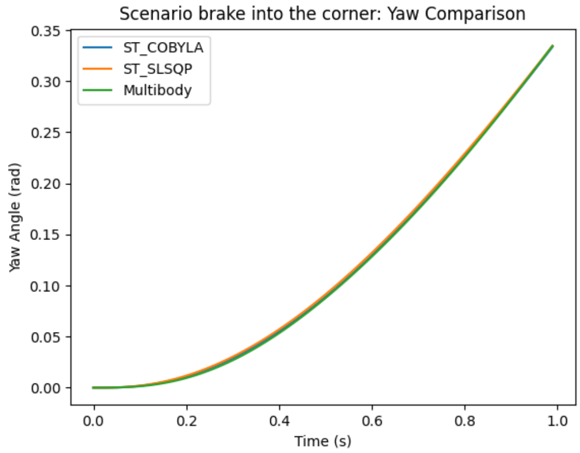

Figure 11 Scenario “brake into the corner” yaw comparison

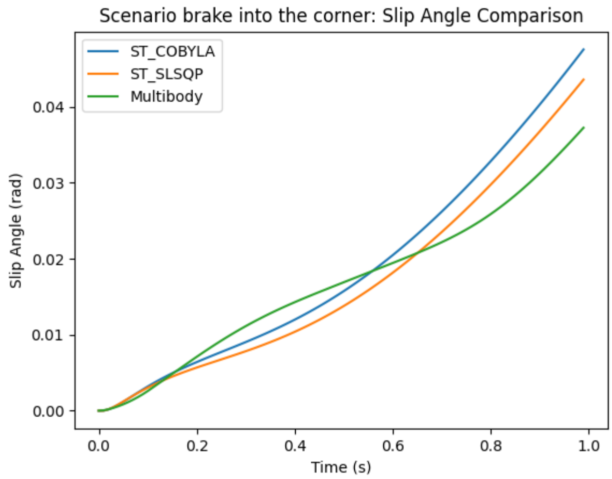

Figure 12 Scenario “brake into the corner" slip angle comparison.

5.2.3 Scenario： Accelerate out of the Corner

The Table 7 below shows the estimated parameters values in “COBYLA” and “SLSQP” methods in scenario “accelerate out of the corner”, respectively.

Table 7 Parameter estimation results in "accelerate out of the corner" scenario.

Optimization Method	m	I_z	l_r	l_f	h_{cg}	$C_{s,f}$	$C_{s,r}$
“COBYLA”	999.63 7290	1501.068 96	0.8328580 93	2.837828 98	1.044347 80	18.61229 25	19.88265 53
“SLSQP”	999.87 3279	1500.088 34	0.7609020 25	3.051899 23	1.660903 72	6.229356 06	31.32326 36

The Figure 13, Figure 14, and Figure 15 show the simulation results of vehicle states in scenario “turn left” comparison between single-track model using estimated parameters in two optimization methods and multi-body model.

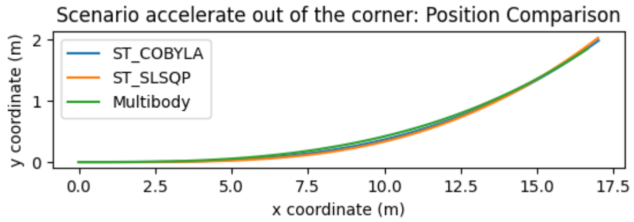

Figure 13 Scenario “accelerate out of the corner” position comparison.

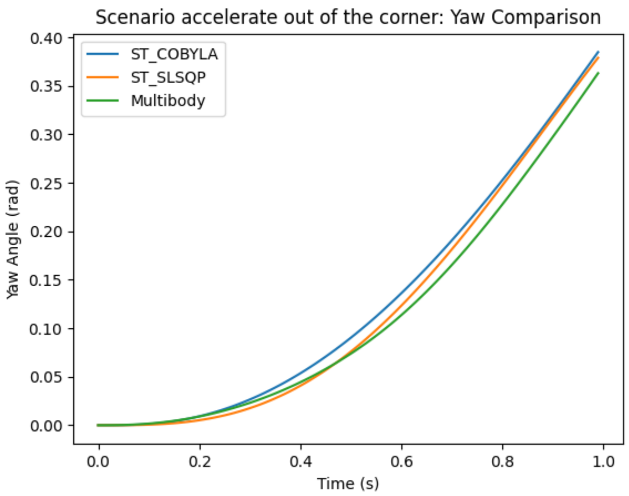

Figure 14 Scenario "accelerate out of the corner” yaw comparison.

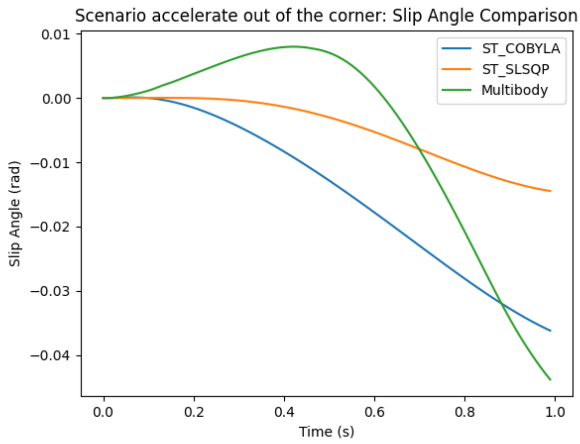

Figure 15 Scenario "accelerate out of the corner" slip angle comparison.

5.3 Results Analysis

According to the simulations of the estimated parameters of the car based on the single-track model in the figures above, the vehicle parameters can be obtained by these methods. Moreover, the single-track model under these estimated parameters can reflect the states of the car in the defined driving scenarios. The following conclusions can be drawn based on the simulations results.

Drawbacks of the single-track model. The estimated parameters can accurately simulate the car position and yaw angle. The simulation of the slip angle is not accurate enough for all the three driving scenarios. This is due to the inherent flaws of the single-track model for slip angle calculations. A more accurate model of the lateral dynamics in the single-track model will solve this problem, which shall bring in more state variables. In study [17], an improved single-track model with better performance on slip angle modeling has been developed.

The single-track model cannot present a good simulation on the vehicle velocity in real driving scenarios than multi-body model since it performs poorly on vehicle lateral dynamics. It cannot accurately reflect the speed loss of the vehicle in lateral motion.

Steering angle velocity. The steering angle velocity v_δ is used as the input of the simulations, and the single-track model will get the same results as the multi-body in the simulated car state steering angle in all driving scenarios.

Purely linear motion scenario. When the driving scenario is a purely linear motion, the single-track model cannot reflect the changes in the yaw angle and slip angle. If $v_\delta = 0$ and $a_{long} = -0.5g$ in the experiment, the simulation results of yaw angle and slip angle can be seen in the

Figure 16 and Figure 17 below. It can be found that the single-track cannot simulate the changes in yaw angle and slip angle. However, at the same time, it can be seen that the changes of yaw angle and slip angle are very small, so it is acceptable to ignore the changes of these two state variables in purely linear driving scenarios.

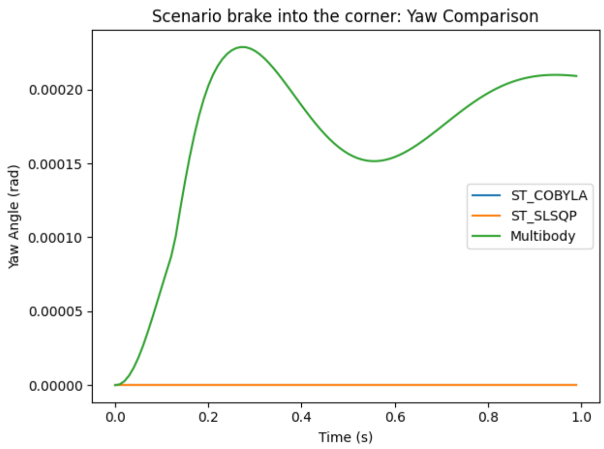

Figure 16 Purely linear motion yaw comparison.

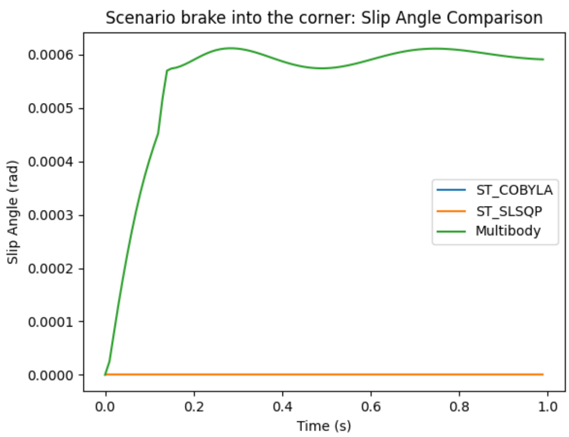

Figure 17 Purely linear motion slip angle comparison

In this thesis, a certain value is given to the input steering angle velocity v_δ when designing the driving scenarios, so as to obtain more accurate parameter estimation results. This is also consistent with the actual racing scenarios. In the real racing scenario [21], the racing lines are mostly curves, and there are few straight roads.

Comparison among optimization algorithms. For the car states position and yaw, the estimated parameters obtained by the COBYLA algorithm obtained better results in the “turn left” and “brake into the corner” scenarios. In the scenario “accelerate out of the corner”, the “SLSQP” algorithm got better results. In addition, the “Nelder-Mead” algorithm does not support adding constraints to variables. Although it is easy to deploy, in some driving scenarios, some parameters estimated by the algorithm are negative, such as h_{cg}, which is unreasonable. Therefore, in the final parameter estimation model, the algorithm “Nelder-Mead” was discarded.

Jacobian matrix. In the optimization algorithms, the use of the Jacobian matrix can obtain parameter estimation results more efficiently and accurately.

Constraints. The constraints of the estimated parameters can be adjusted. In the experiments, it is assumed that the front and rear tires of the car are different, which can be found in the parameter constraints in the Table 3. The tire cornering stiffness coefficients of front and rear tires are not specified to be equal in the constraints. Secondly, the parameters h_{cg} and l_r, l_f can also be given more specific constraints based on the requirements.

5.4 Experiment Results and Analysis of Adaptive Design

As mentioned in session 4.3, scenario "turn left" has been taken as the example of adaptive design with optimization algorithm "COBYLA". The Table 8 shows the results of adaptive design.

Table 8 Results of adaptive design.

Parameters	Values of parameter estimation in "SLSQP"	Values of adaptive design
$\boldsymbol{m}$	1000.03652	999.80418187
$\boldsymbol{C_{s,f}}$	24.9541496	24.94282201
$\boldsymbol{C_{s,r}}$	17.0535105	17.03475049

The Figure 18 below shows the position comparison after the adaptive design.

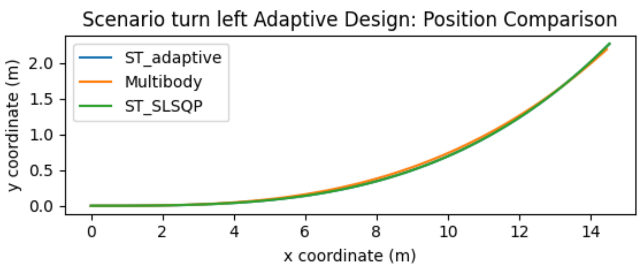

Figure 18 Adaptive design position comparison.

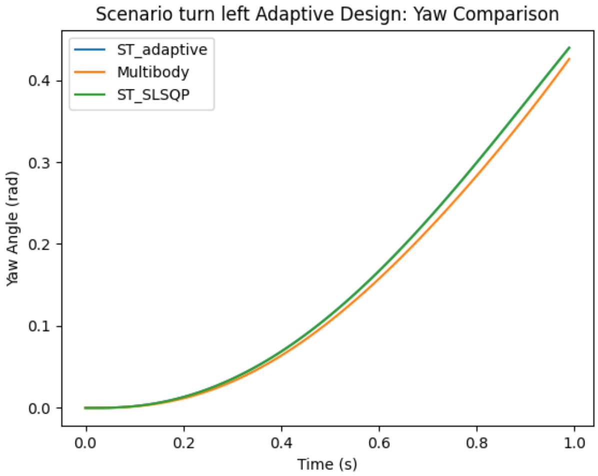

Figure 19 Adaptive design yaw comparison.

From the above results, the parameters of adaptive design did not bring greater changes to the model. But it provides an idea of customizing autonomous racing car, that is, by freely choosing the customizable parameter combinations of the car, the car can approach the performance of the complex model as much as possible under a simple model. In addition, it also provides a simulation method for the number and variety of estimated parameters obtained based on the single-track model is not enough to input back into the multi-body model for verification. The results of parameter estimation can also be improved based on this design.

Chapter 6 Conclusion and Future Work

6.1 Conclusion

In this thesis, a data estimation method is proposed to solve the problem that some parameters of the autonomous racing car are difficult to measure accurately. By implementing this method, the vehicle mass, moment of inertia for entire mass about z axis, distance from centre of gravity to rear/front axle, centre of gravity height of total mass and cornering stiffness coefficients of front/rear tires have been successfully estimated. The accuracies of the estimated parameters were also demonstrated. The feasibility and effectiveness of this method have been verified.

Secondly, an idea of adaptive design for the estimated parameters have been proposed and successfully implemented, which is based on the assumption that the vehicle can be customized. By adjusting the selected parameters, the vehicle can achieve the performance of a complex model under a simple model.

6.2 Future Work

First of all, as mentioned in session 5.3, the single-track model used in this thesis has defects in lateral dynamics, where an improved single-track model is required to support a better simulation in slip angle and velocity. However, more variables will bring higher requirements for computing power, which requires a balance between model complexity and cost.

Secondly, the current parameter estimation algorithm does not make adequate use of measurements. In the future, the Extended Kalman Filter for parameter estimation could be used in the model, which can lead to a more efficient use of the measurements. In study [13], the Extended Kalman Filter is used to estimate the tire parameters of Unmanned ground vehicle.

Thirdly, research on constraints of the autonomous racing car parameters is required. As the analysis on constraints in session 5.3, by designing more accurate constraints for autonomous racing car parameters, the results of parameter estimation can be further improved.

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