Supplementary Materials for Planning PEV Fast-Charging Stations Using Data-Driven Distributionally Robust Optimization Approach Based on ϕ-divergence

Abstract

The nomenclature appeared in the main document, including the abbreviations, the functions, the parameters and the decision variables, is listed before providing the appendices. In Appendix A, the proof of Theorem 2 is presented. In Appendix B, two examples for different choices of ϕ-divergence are given. In Appendix C, fast-charging station's location logic is provided. In Appendix D, the topology of transportation network and its parameters are shown. In Appendix E, plug-in electric vehicle flows over entire TN are presented. In Appendix F, the topology of distribution network and the coupling relationship between transportation network and distribution network are illustrated.

Nomenclature

Abbreviations:

CFRLM	Capacitated Flow Refueling Location Model.
CFRLM_SP	Capacitated Flow Refueling Location Model based on Sub-Path.
DN	Distribution Network.
DRO	Distributionally Robust Optimization.
O-D	Origin-Destination.
PEV	Plug-in Electric Vehicle.
RO	Robust Optimization.
SAA	Sample Average Approximate.
SP	Stochastic Programming.
TN	Transportation Network.

Functions:

$\mathbb{E}(\cdot)$	Expectation operator.
$\phi(\cdot)$	ϕ-divergence operator.

Parameters:

$\bar{I}_{mn}$	Upper limit of branch current of line (m,n).
$\bar{y}$	Sizing threshold of fast-charging stations.
$\boldsymbol{p}_i$	True probability distribution of charging demands at TN node i.
$\hat{\boldsymbol{p}}_i$	Empirical probability distribution of charging demands at TN node i.
$\hat{p}_{i,k}$	kth element of $\hat{\boldsymbol{p}}_i$.
$\overline{V}_m/\underline{V}_m$	Lower/Upper limit of nodal voltage at bus m.
ρ_i	ϕ-divergence threshold at TN node i.
$\tilde{y}^{pev}_{q,T}$	Uncertain charging demands on route $q \in \mathcal{Q}_i$ over time period T.
$c_{1,i}$	Cost for building a fast-charging station on route r.
$c_{2,i}$	Cost for building a fast-charging spot on route r.
$c_{3,i}$	Unit cost for distribution line on node i.
$c_{4,i}$	Unit cost for substraction capacity expansion on node i.
$c_{5,i}$	Unit penalty cost for unsatisfied PEV power on node i.
l_i	Required distribution line length to install a charging station on node i.
N_i	Sample size of charging demands at TN node i.
p^{spot}	Rated charging power of a charging spot.
$p^{sub}_{i,0}$	Initial substation capacity avaiable at node i.
$p_{i,k}$	kth element of $\boldsymbol{p}_i$.
s^b_m	Apparent base load at bus m.
T/T_k	Time period.
y^{pev}_{q,k,T_k}	Observed charging demands on route $q \in \mathcal{Q}_i$ over time period T_k for type k.
z_{mn}	Impedance of branch (m,n), z^H_{mn} is its conjugate.

Sets:

$\mathscr{E}$	Set of the directed links in TN.
$\mathscr{G}_{DN}$	Topology of DN.
$\mathscr{G}_{TN}$	Topology of TN.
$\mathscr{K}_i$	Set of PEV types at TN node i.
$\mathscr{L}$	Set of lines of the DN, in which (m, n) is in the order of bus m to bus n, and bus n lies between bus m and bus 0.
$\mathscr{O}_{q,k}$	Set of candidate location nodes of fast-charging stations for PEVs of type k on route q.
$\mathscr{P}_i$	Ambiguity set for charging demands at TN node i.
$\mathscr{Q}_i$	Set of routes that constain TN node i.
$\mathscr{V}$	Set of the vertexes in TN.
$\mathscr{W}$	Set of O-D pairs in TN.

Decision Variables:

η_i	Decision variable for TN node i.
$\gamma_{i,q,k}$	Binary variable, equals to 1, if PEVs of type k choose to get charged at TN node i on route q, and 0 otherwise.
λ_i	Positive variable for TN node i.
ξ_i	Decision variable for TN node i.
l_{mn}	Square of the magnitude of line (m, n)'s current.
$p_{i,un}^{pev}$	Unsatisfied active PEV power on node i.
p_i^{pev}	Active PEV power at node i.
$p_{i,0}^{sub}$	Initial substation capacity avaiable at node i.
s_m^{pev}	Apparent PEV power at bus m.
s_m	Total apparent power injection at bus m.
S_{mn}	Apparent power flow in line (m, n).
v_m	Square of the nodal voltage at bus m.
x_i	Binary variable, equals to 1 if there is a fast-charging station located on node i, and 0 otherwise.
y_i^{cs}	Integer variable, number of charging spots on node i.

Appendix A
Proof of Theorem 2

Proof: Let λ_i and η_i be the Lagrange multipliers corresponding to constraint (5) and (6), respectively. The Lagrange function $L(\boldsymbol{p}_i, \lambda_i, \eta_i)$ and the dual objective formulation $g(\lambda_i, \eta_i)$ for the optimization problem (4)-(7) are

$$\begin{aligned} L(\boldsymbol{p}_i, \lambda_i, \eta_i) \quad = \quad & \sum_{k \in \mathscr{K}_i} \sum_{q \in \mathscr{Q}_i} p_{i,k} y_{q,k,T_k}^{pev} \gamma_{i,q,k} + \lambda_i \left(\rho_i - \sum_{k \in \mathscr{K}_i} \hat{p}_{i,k} \phi \left(\frac{p_{i,k}}{\hat{p}_{i,k}} \right) \right) \\ & + \eta_i \left(1 - \sum_{k \in \mathscr{K}_i} p_{i,k} \right). \end{aligned}$$

and

$$g(\lambda_i, \eta_i) = \max_{\boldsymbol{p}_i} L(\boldsymbol{p}_i, \lambda_i, \eta_i).$$

The dual objective function satisfies

$$\begin{aligned}
g(\lambda_i, \eta_i) &= \max_{\boldsymbol{p}_i} L(\boldsymbol{p}_i, \lambda_i, \eta_i). \\
&= \lambda_i\rho_i + \eta_i + \max_{\boldsymbol{p}_i}\left(\sum_{k\in\mathcal{K}_i}\sum_{q\in\mathcal{Q}_i} p_{i,k} y^{pev}_{q,k,T_k}\gamma_{i,q,k}\right. \\
&\quad \left. -\lambda_i \sum_{k\in\mathcal{K}_i} \hat{p}_{i,k}\phi\left(\frac{p_{i,k}}{\hat{p}_{i,k}}\right) - \eta_i \sum_{k\in\mathcal{K}_i} p_{i,k}\right) \\
&= \lambda_i\rho_i + \eta_i \\
&\quad + \max_{\boldsymbol{p}_i} \sum_{k\in\mathcal{K}_i}\left(p_{i,k}\sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k} - \lambda_i\hat{p}_{i,k}\phi\left(\frac{p_{i,k}}{\hat{p}_{i,k}}\right) - \eta_i p_{i,k}\right) \\
&= \lambda_i\rho_i + \eta_i \\
&\quad + \max_{\boldsymbol{p}_i} \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\left(t_{i,k}\sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k} - \lambda_i\phi\left(t_{i,k}\right) - \eta_i t_{i,k}\right) \\
&= \lambda_i\rho_i + \eta_i \\
&\quad + \max_{\boldsymbol{p}_i} \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\left(t_{i,k}\left(\sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k} - \eta_i\right) - \lambda_i\phi\left(t_{i,k}\right)\right) \\
&= \lambda_i\rho_i + \eta_i + \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\left((\lambda_i\phi)^*\left(\sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k} - \eta_i\right)\right) \\
&= \lambda_i\rho_i + \eta_i + \lambda_i \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\,(\phi)^*\left(\frac{\sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k} - \eta_i}{\lambda_i}\right),
\end{aligned}$$

where $t_{i,k} = \frac{p_{i,k}}{\hat{p}_{i,k}}$ and relationship $(\lambda\phi)^*(u) = \lambda\phi^*(\frac{u}{\lambda})$. Since (4)-(7) is a convex program and there clearly exists at least one feasible solution, that is, $p_{i,k} = \hat{p}_{i,k}$. Strong duality holds based on Slater's condition. Therefore, service ability constraint (2) can be reformulated as (8), (9). The proof is completed. ■

Appendix B
Two examples for variation divergence and Hellinger divergence

Example 1. *Consider the variation divergence $\phi(t) = |t-1|$, whose conjugate $\phi^*(t)$ is $\phi^*(t) = \begin{cases} -1, & t \le -1, \\ t, & -1 \le t \le 1 \end{cases}$ and adjoint $\tilde{\phi}(t) = \phi(t)$. By employing the variation distance, (10)-(12) are equivalent to*

$$\begin{aligned}
&\begin{cases}
\lambda_i\rho_i + \eta_i + \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\xi_{i,k} \le y^{cs}_i, \\
\lambda_i\left(\tilde{\phi}\right)^*\left(-\frac{\xi_{i,k}}{\lambda_i}\right) \le \eta_i - \sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k}, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \textit{ for } k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases} \\
\Leftrightarrow &\begin{cases}
\lambda_i\rho_i + \eta_i + \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\xi_{i,k} \le y^{cs}_i, \\
\lambda_i \max\left\{-\frac{\xi_{i,k}}{\lambda_i}, -1\right\} \le \eta_i - \sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k}, \\
-\frac{\xi_{i,k}}{\lambda_i} \le 1, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \textit{ for } k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases} \\
\Leftrightarrow &\begin{cases}
\lambda_i\rho_i + \eta_i + \sum_{k\in\mathcal{K}_i}\hat{p}_{i,k}\xi_{i,k} \le y^{cs}_i, \\
\lambda_i + \eta_i \ge \sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k}, \\
\lambda_i - \xi_{i,k} \ge \sum_{q\in\mathcal{Q}_i} y^{pev}_{q,k,T_k}\gamma_{i,q,k}, \\
\lambda_i + \xi_{i,k} \ge 0, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \textit{ for } k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases}
\end{aligned}$$

The above formulation is a mixed integer linear model.

Example 2. *Consider the Hellinger divergence $\phi(t) = (\sqrt{t}-1)^2$, whose conjugate $\phi^*(t)$ is $\phi^*(t) = \frac{t}{1-t}, t \le 1$ and adjoint*

$\tilde{\phi}(t) = \phi(t)$. By employing the Hellinger distance, (10)-(12) are equivalent to

$$
\begin{cases}
\lambda_i \rho_i + \eta_i + \sum_{k \in \mathcal{K}_i} \hat{p}_{i,k} \xi_{i,k} \le y_i^{cs}, \\
\lambda_i \left(\tilde{\phi}\right)^* \left(-\frac{\xi_{i,k}}{\lambda_i}\right) \le \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k}, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \; for \; k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases}
$$

$$
\Leftrightarrow \begin{cases}
\lambda_i \rho_i + \eta_i + \sum_{k \in \mathcal{K}_i} \hat{p}_{i,k} \xi_{i,k} \le y_i^{cs}, \\
\lambda_i \left(-1 + \frac{\lambda_i}{\lambda_i + \xi_{i,k}}\right) \le \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k}, \\
\lambda_i + \xi_{i,k} \ge 0, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \; for \; k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases}
$$

$$
\Leftrightarrow \begin{cases}
\lambda_i \rho_i + \eta_i + \sum_{k \in \mathcal{K}_i} \hat{p}_{i,k} \xi_{i,k} \le y_i^{cs}, \\
\lambda_i^2 \le (\lambda_i + \xi_{i,k}) \left(\lambda_i + \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k}\right), \\
\lambda_i + \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k} \ge 0, \\
\lambda_i + \xi_{i,k} \ge 0, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \; for \; k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases}
$$

$$
\Leftrightarrow \begin{cases}
\lambda_i (\rho_i - 1) + \eta_i + \sum_{k \in \mathcal{K}_i} \hat{p}_{i,k} \zeta_{i,k} \le y_i^{cs}, \\
\sqrt{\lambda_i^2 + \frac{1}{4}\left(\zeta_{i,k} - \pi_{i,k}\right)^2} \le \frac{1}{2}\left(\zeta_{i,k} + \pi_{i,k}\right), \\
\lambda_i + \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k} \ge 0, \\
\lambda_i + \xi_{i,k} \ge 0, \\
\zeta_{i,k} = \lambda_i + \xi_{i,k}, \\
\pi_{i,k} = \lambda_i + \eta_i - \sum_{q \in \mathcal{Q}_i} y_{q,k,T_k}^{pev} \gamma_{i,q,k}, \\
\lambda_i \ge 0, \eta_i, \xi_{i,k} \in \mathbb{R}, \gamma_{i,q,k} \in \{0,1\} \; for \; k \in \mathcal{K}_i, q \in \mathcal{Q}_i
\end{cases}
$$

The above formulation is a mixed integer conic quadratic model.

Appendix C
Fast-charging stations' location logic

According to the operation rules of PEVs, they prefer to choose the routes that they do not require to get charged, that is, the distances of chosen routes are less than PEVs' driving ranges; if such choice is unavailable, the routes that require to get charged only once should be chosen instead. Thus, the fast-charging stations' location logic is explained by Fig. 1.

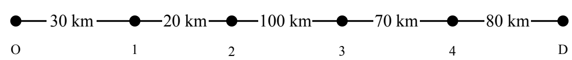

Fig. 1. Fast-charging station location logic between origin node O and destination node D with different driving ranges.

Consider the PEVs enter the route at node O and need to depart at node D. The route q between node O and node D consists of a set of nodes $\{O, 1, 2, 3, 4, D\}$. Thus, the candidate nodes for locating fast-charging station are nodes 1, 2, 3, 4. The distance between nodes are marked in Fig. 1. According to the operation rules, suppose there are 3 types of PEVs that plan to travel from node O to node D with different driving ranges $D_1 = 150$ km, $D_2 = 275$ km and $D_3 = 310$ km. The driving range constraints for PEVs on route q is that PEVs travel through route q with its distance longer than their driving ranges should cover at least one charging station so that the PEVs can travel through path q without depleting batteries. Denote $\mathcal{Q}_{q,k}$ be the candidate location set of fast-charging stations for PEVs of type k on route q. Then, for PEVs with driving range $D_1 = 150$ km (type 1), the fast-charging station should be located in node 3, that is $\mathcal{Q}_{q,1} = \{3\}$; for PEVs with driving range $D_2 = 270$ km (type 2), the fast-charging station can be located in node 1, 2, 3, 4, that is, $\mathcal{Q}_{q,2} = \{1, 2, 3, 4\}$; for PEVs with driving range $D_2 = 310$ km (type 3) which is longer than the distance between O-D pairs, according to the charging rule, PEVs would not get charged, that is, $\mathcal{Q}_{q,3} = \emptyset$.

Appendix D
Topology of Transportation Network and its parameters

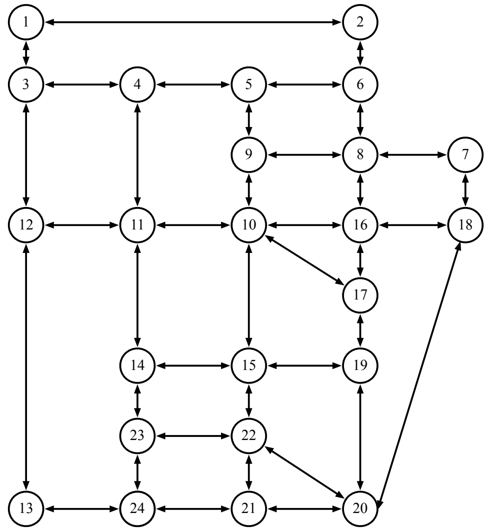

Fig. 2. The Sioux Falls Network, which contains 24 nodes and 78 links. The number in each circle in the node ID.

TABLE I
Link Distances for Sioux Falls Network.

Link	Length (km)	Link	Length (km)
(1, 2)/(2, 1)	60	(1, 3)/(3, 1)	40
(3, 4)/(4, 3)	40	(4, 5)/(5, 4)	20
(4, 11)/(11, 4)	60	(5, 6)/(6, 5)	40
(6, 8)/(8, 6)	20	(7, 8)/(8, 7)	30
(8, 9)/(9, 8)	100	(8, 16)/(16, 8)	50
(10, 11)/(11, 10)	50	(10, 15)/(10, 15)	60
(10, 17)/(17, 10)	60	(11, 12)/(12, 11)	60
(12, 13)/(13, 12)	30	(13, 24)/(24, 13)	40
(14, 23)/(23, 14)	40	(15, 19)/(19, 15)	30
(16, 17)/(17, 16)	20	(16, 18)/(18, 16)	30
(18, 20)/(20, 18)	40	(19, 20)/(19, 20)	40
(20, 22)/(22, 20)	50	(21, 22)/(22, 21)	30
(22, 23)/(23, 22)	40	(23, 24)/(24, 23)	20
(2, 6)/(6, 2)	60	(4, 5)/(5, 4)	20
(5, 9)/(9, 5)	50	(7, 18)/(18, 7)	20
(9, 10)/(10, 9)	30	(10, 16)/(16, 10)	40
(11, 14)/(14, 11)	40	(14, 15)/(15, 14)	50
(15, 22)/(22, 15)	30	(17, 19)/(19, 17)	20
(20, 21)/(21, 20)	60	(21, 24)/(24, 21)	30
(23, 24)/(24, 23)	20		

Appendix E
PEV flows over entire TN

TABLE II
Type 1 PEV flows between O-D pairs over 34 minutes.

O-D pairs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	25	25	125	50	75	125	200	125	325	125	50	125	75	0	125	100	25	75	75	25	100	75	25
2	25	0	25	50	25	100	50	100	50	150	50	25	75	25	25	100	50	0	25	25	0	25	0	0
3	25	25	0	50	25	75	25	50	25	75	75	50	25	25	25	50	25	0	0	0	0	25	25	0
4	125	50	50	0	125	100	100	175	175	300	375	150	150	125	125	200	125	25	50	75	50	100	125	50
5	50	25	25	125	0	50	50	125	200	250	125	50	50	25	50	125	50	0	25	25	25	50	25	0
6	75	100	75	100	50	0	100	200	100	200	100	50	50	25	50	225	125	25	50	75	25	50	25	25
7	125	50	25	100	50	100	0	250	150	475	125	175	100	50	125	100	250	50	100	125	50	125	50	25
8	200	100	50	175	125	200	250	0	200	400	200	150	150	100	150	550	350	75	175	225	100	125	75	50
9	125	50	25	175	200	100	150	200	0	700	350	150	150	150	250	350	225	50	100	150	75	175	125	50
10	325	150	75	300	250	200	475	400	700	0	975	500	475	525	1000	1100	975	175	450	625	300	650	450	200
11	125	50	75	350	125	100	125	200	350	1000	0	350	250	400	350	350	250	50	100	150	100	275	325	150
12	50	25	50	150	50	50	175	150	150	500	350	0	325	175	175	175	150	50	75	125	75	175	175	125
13	125	75	25	150	50	50	100	150	150	475	250	325	0	150	175	150	125	25	75	150	150	325	200	175
14	75	25	25	125	25	25	50	100	150	525	400	175	150	0	325	175	175	25	75	125	100	300	275	100
15	0	25	25	125	50	50	125	150	225	1000	350	175	175	325	0	300	375	50	200	275	200	650	250	100
16	125	100	50	200	125	225	350	550	350	1100	350	175	150	175	300	0	700	125	325	400	150	300	125	75
17	100	50	25	125	50	125	250	350	225	975	250	150	125	175	375	700	0	150	425	425	150	425	150	75
18	25	0	0	25	0	25	50	75	50	175	25	50	25	25	50	125	150	0	75	100	25	75	25	0
19	75	25	0	50	25	50	100	175	100	175	100	75	75	75	200	325	425	75	0	300	100	300	75	25
20	75	25	0	75	25	75	125	225	150	625	150	100	150	125	275	400	425	100	300	0	300	600	175	100
21	25	0	0	50	25	25	50	100	75	300	100	75	150	100	200	150	150	25	100	300	0	450	175	125
22	100	25	25	100	50	50	125	125	175	650	275	175	325	300	650	300	425	75	300	600	450	0	525	275
23	75	0	25	125	25	25	50	75	125	450	325	175	200	275	250	125	150	25	75	175	175	525	0	175
24	25	0	0	50	0	25	25	50	50	200	150	125	200	100	100	75	75	0	25	100	125	275	175	0

TABLE III
Type 2 PEV flows between O-D pairs over 36 minutes.

O-D pairs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	50	50	250	100	150	250	400	250	650	250	100	250	150	0	250	200	50	150	150	50	200	150	50
2	50	0	50	100	50	200	100	200	100	300	100	50	150	50	50	200	100	0	50	50	0	50	0	0
3	50	50	0	100	50	150	50	100	50	150	150	100	50	50	50	100	50	0	0	0	0	50	50	0
4	250	100	100	0	250	200	200	350	350	600	750	300	300	250	250	400	250	50	100	150	100	200	250	100
5	100	50	50	250	0	100	100	250	400	500	250	100	100	50	100	250	100	0	50	50	50	100	50	0
6	150	200	150	200	100	0	200	400	200	400	200	100	100	50	100	450	250	50	100	150	50	100	50	50
7	250	100	50	200	100	200	0	500	300	950	250	50	200	100	250	200	500	100	200	250	100	250	100	50
8	400	200	100	350	250	400	500	0	400	800	400	300	300	200	300	1100	700	150	350	450	200	250	150	100
9	250	100	50	350	400	200	300	400	0	1400	700	300	300	300	500	700	450	100	200	300	150	350	250	100
10	650	300	150	600	500	400	950	800	1400	0	1950	1000	950	1050	2000	2200	1950	350	900	1250	600	1300	900	400
11	250	100	150	700	250	200	250	400	700	2000	0	700	500	800	700	700	500	100	200	300	200	550	650	300
12	100	50	100	300	100	100	50	300	300	1000	700	0	650	350	350	350	300	100	150	250	150	350	350	250
13	250	150	50	300	100	100	200	300	300	950	500	650	0	300	350	300	250	50	150	300	300	650	400	350
14	150	50	50	250	50	50	100	200	300	1050	800	350	300	0	650	350	350	50	150	250	200	600	550	200
15	250	50	50	250	100	100	250	300	450	2000	700	350	350	650	0	600	750	100	400	550	400	1300	500	200
16	250	200	100	400	250	450	700	1100	700	2200	700	350	300	350	600	0	1400	250	650	800	300	600	250	150
17	200	100	50	250	100	250	500	700	450	1950	500	300	250	350	750	1400	0	300	850	850	300	850	300	150
18	50	0	0	50	0	50	100	150	100	350	50	100	50	50	100	250	300	0	150	200	50	150	50	0
19	150	50	0	100	50	100	200	350	200	350	200	150	150	150	400	650	850	150	0	600	200	600	150	50
20	150	50	0	150	50	150	250	450	300	1250	300	200	300	250	550	800	850	200	600	0	600	1200	350	200
21	50	0	0	100	50	50	100	200	150	600	200	150	300	200	400	300	300	50	200	600	0	900	350	250
22	200	50	50	200	100	100	250	250	350	1300	550	350	650	600	1300	600	850	150	600	1200	900	0	1050	550
23	150	0	50	250	50	50	100	150	250	900	650	350	400	550	500	250	300	50	150	350	350	1050	0	350
24	50	0	0	100	0	50	50	100	100	400	300	250	400	200	200	150	150	0	50	200	250	550	350	0

TABLE IV
TYPE 3 PEV FLOWS BETWEEN O-D PAIRS OVER 42 MINUTES.

O-D pairs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	15	15	75	30	45	75	120	75	195	75	30	75	45	0	75	60	15	45	45	15	60	45	15
2	15	0	15	30	15	60	30	60	30	90	30	15	45	15	15	60	30	0	15	15	0	15	0	0
3	15	15	0	30	15	45	15	30	15	45	45	30	15	15	15	30	15	0	0	0	0	15	15	0
4	75	30	30	0	75	60	60	105	105	180	225	90	90	75	75	120	75	15	30	45	30	60	75	30
5	30	15	15	75	0	30	30	75	120	150	75	30	30	15	30	75	30	0	15	15	15	30	15	0
6	45	60	45	60	30	0	60	120	60	120	60	30	30	15	30	135	75	15	30	45	15	30	15	15
7	75	30	15	60	30	60	0	150	90	285	75	105	60	30	75	60	150	30	60	75	30	75	30	15
8	120	60	30	105	75	120	150	0	120	240	120	90	90	60	90	330	210	45	105	135	60	75	45	30
9	75	30	15	105	120	60	90	120	0	420	210	90	90	90	150	210	135	30	60	90	45	105	75	30
10	195	90	45	180	150	120	285	240	420	0	585	300	285	315	600	660	585	105	270	375	180	390	270	120
11	75	30	45	210	75	60	75	120	210	600	0	210	150	240	210	210	150	30	60	90	60	165	195	90
12	30	15	30	90	30	30	105	90	90	300	210	0	195	105	105	105	90	30	45	75	45	105	105	75
13	75	45	15	90	30	30	60	90	90	285	150	195	0	90	105	90	75	15	45	90	90	195	120	105
14	45	15	15	75	15	15	30	60	90	315	240	105	90	0	195	105	105	15	45	75	60	180	165	60
15	0	15	15	75	30	30	75	90	135	600	210	105	105	195	0	180	225	30	120	165	120	390	150	60
16	75	60	30	120	75	135	210	330	210	660	210	105	90	105	180	0	420	75	195	240	90	180	75	45
17	60	30	15	75	30	75	150	210	135	585	150	90	75	105	225	420	0	90	255	255	90	255	90	45
18	15	0	0	15	0	15	30	45	30	105	15	30	15	15	30	75	90	0	45	60	15	45	15	0
19	45	15	0	30	15	30	60	105	60	105	60	45	45	45	120	195	255	45	0	180	60	180	45	15
20	45	15	0	45	15	45	75	135	90	375	90	60	90	75	165	240	255	60	180	0	180	360	105	60
21	15	0	0	30	15	15	30	60	45	180	60	45	90	60	120	90	90	15	60	180	0	270	105	75
22	60	15	15	60	30	30	75	75	105	390	165	105	195	180	390	180	255	45	180	360	270	0	315	165
23	45	0	15	75	15	15	30	45	75	270	195	105	120	165	150	75	90	15	45	105	105	315	0	105
24	15	0	0	30	0	15	15	30	30	120	90	75	120	60	60	45	45	0	15	60	75	165	105	0

TABLE V
TYPE 4 PEV FLOWS BETWEEN O-D PAIRS OVER 46 MINUTES.

O-D pairs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	10	10	50	20	30	50	80	50	130	50	20	50	30	0	50	40	10	30	30	10	40	30	10
2	10	0	10	20	10	40	20	40	20	60	20	10	30	10	10	40	20	0	10	10	0	10	0	0
3	10	10	0	20	10	30	10	20	10	30	30	20	10	10	10	20	10	0	0	0	0	10	10	0
4	50	20	20	0	50	40	40	70	70	120	150	60	60	50	50	80	50	10	20	30	20	40	50	20
5	20	10	10	50	0	20	20	50	80	100	50	20	20	10	20	50	20	0	10	10	10	20	10	0
6	30	40	30	40	20	0	40	80	40	80	40	20	20	10	20	90	50	10	20	30	10	20	10	10
7	50	20	10	40	20	40	0	100	60	190	50	70	40	20	50	40	100	20	40	50	20	50	20	10
8	80	40	20	70	50	80	100	0	80	160	80	60	60	40	60	220	140	30	70	90	40	50	30	20
9	50	20	10	70	80	40	60	80	0	280	140	60	60	60	100	140	90	20	40	60	30	70	50	20
10	130	60	30	120	100	80	190	160	280	0	390	200	190	210	400	440	390	70	180	250	120	260	180	80
11	50	20	30	140	50	40	50	80	140	400	0	140	100	160	140	140	100	20	40	60	40	110	130	60
12	20	10	20	60	20	20	70	60	60	200	140	0	130	70	70	70	60	20	30	50	30	70	70	50
13	50	30	10	60	20	20	40	60	60	190	100	130	0	60	70	60	50	10	30	60	60	130	80	70
14	30	10	10	50	10	10	20	40	60	210	160	70	60	0	130	70	70	10	30	50	40	120	110	40
15	0	10	10	50	20	20	50	60	90	400	140	70	70	130	0	120	150	20	80	110	80	260	100	40
16	50	40	20	80	50	90	140	220	140	440	140	70	60	70	120	0	280	50	130	160	60	120	50	30
17	40	20	10	50	20	50	100	140	90	390	100	60	50	70	150	280	0	60	170	170	60	170	60	30
18	10	0	0	10	0	10	20	30	20	70	10	20	10	10	20	50	60	0	30	40	10	30	10	0
19	30	10	0	20	10	20	40	70	40	70	40	30	30	30	80	130	170	30	0	120	40	120	30	10
20	30	10	0	30	10	30	50	90	60	250	60	40	60	50	110	160	170	40	120	0	120	240	70	40
21	10	0	0	20	10	10	20	40	30	120	40	30	60	40	80	60	60	10	40	120	0	180	70	50
22	40	10	10	40	20	20	50	50	70	260	110	70	130	120	260	120	170	30	120	240	180	0	210	110
23	30	0	10	50	10	10	20	30	50	180	130	70	80	110	100	50	60	10	30	70	70	210	0	70
24	10	0	0	20	0	10	10	20	20	80	60	50	80	40	40	30	30	0	10	40	50	110	70	0

APPENDIX F
TOPOLOGY OF DISTRIBUTION NETWORK AND THE NODE COUPLING RELATIONSHIP

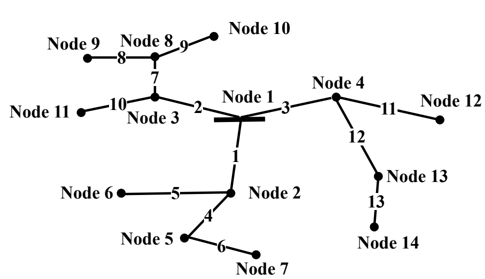

Fig. 3. The 110kV DN, in which node 1 is connected to a 220kV/110kV transformer with 150MVA capacity.

TABLE VI
NODE COUPLING RELATIONSHIP BETWEEN THE TN AND DN.

DN node ID	01	02	03	04	05	06	07
TN node ID	–	04	11	16	20	10	12
DN node ID	08	09	10	11	12	13	14
TN node ID	14	08	03	24	06	17	22