S228/406

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

BSc (Hons) in Computer Science

Stage 4

SEMESTER 2 EXAMINATIONS 2010

*** *SOLUTIONS* ***

ARTIFICIAL INTELLIGENCE 2

Dr. John Kelleher
Dr. D. Lillis
Dr. I. Arena

Duration: 2 Hours

Answer Question 1 (40 marks) **and**

any 2 Other Questions (30 marks each).

*** *SOLUTIONS* ***

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1. (a) Given the joint distribution for X and Y listed in Table 1 calculate:

   (i) $P(Y = y_2)$

   (5 marks)

   0.14 + 0.32 = 0.46

   (ii) $P(Y = y_2|X = x_1)$

   (5 marks)

   From the product rule: $P(a|b) = \frac{P(a \wedge b)}{P(b)} \rightarrow$
   $P(Y = y_2|X = x_1) = \frac{P(Y=y_2 \wedge X=x_1)}{P(X=x_1)} \rightarrow$
   $P(Y = y_2|X = x_1) = \frac{0.14}{0.26}$

   (b) After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. What are the chances that you actually have the disease?

   (10 marks)

   We are given the following information: $P(test|disease) = 0.99$
   $P(\neg test|\neg disease) = 0.99$
   $P(disease) = 0.0001$
   and the observation $test$.
   $P(a|b) = \frac{P(b|a)P(a)}{P(a)}$
   $P(disease|test) = \frac{P(test|disease)P(disease)}{P(test)} \rightarrow$
   $P(disease|test) = \frac{P(test|disease)P(disease)}{P(test|disease)P(disease)+P(test|\not{d}isease)P(\neg disease)} \rightarrow$
   $\frac{0.99 \times 0.0001}{(0.99 \times 0.0001)+(0.1 \times 0.9999)}$
   $P(disease|test) = .009804$

   (c) Let us say we have three classification algorithms. How can we order these three from best to worst?

   (20 marks)

Table 1: Joint Distribution for X and Y

	$X = x_1$	$X = x_2$
$Y = y_1$	0.02	0.30
$Y = y_2$	0.14	0.32
$Y = y_3$	0.10	0.12

This is a discursive question so giving a precise answer is not appropriate. However, key points that the student should touch on include:

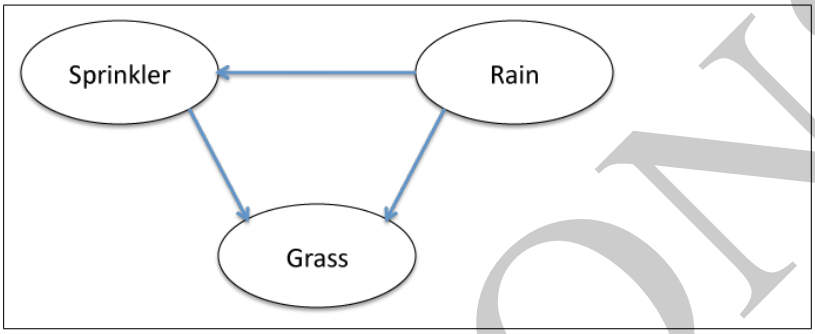
- Predictive accuracy
- Speed and scalability
  - Time to construct the model
  - Time to use the model
- Robustness (handling noise and missing values)
- Scalability
- Interpretability (understanding and insight provided by the model)

it should be noted also, that these evaluation criteria are application dependent.

2. (a) Consider the following scenario: *When it rains the grass usually gets wet but not always. When the sprinkler is on the grass sometimes gets wet but not always. Also, when it rains the sprinkler is often turned of, although sometimes its left on by mistake..*

(i) Using the following Boolean variables $Rain$ (true or false), $Sprinker$ (true or false), and $Grass$ (wet or dry) draw a Bayesian network that models this domain.

(5 marks)

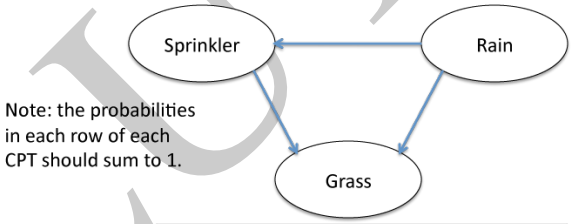


(ii) for each node in the network create a conditional probability table (CPT). Fill in the cells in the CPT tables with probabilities that you think are reasonable and that are not equal to 0 or 1.

(5 marks)

CPT SPRINKLER		
	S=T	S=F
R=T	P(S=T|R=T)	P(S=F|R=F)
R=F	P(S=F|R=T)	P(S=F|R=F)

CPT RAIN	
R=T	R=F
P(R=T)	P(R=F)



Note: the probabilities in each row of each CPT should sum to 1.

CPT GRASS			
		G=T	G=F
S=F	R=F	P(G=T|S=F,R=F)	P(G=F|S=F,R=F)
S=F	R=T	P(G=T|S=F,R=T)	P(G=F|S=F,R=T)
S=T	R=F	P(G=T|S=T,R=F)	P(G=F|S=T,R=F)
S=T	R=T	P(G=T|S=T,R=T)	P(G=F|S=T,R=F)

(iii) Using the Bayesian network and conditional probability tables you have created, compute the probability that it is raining give that the grass is wet?

(5 marks)

The answers student compute for this question will differ between students because they will each be using the probabilities they made up in step 2. However, irrespective of the specific probability values they use, they should compute $P(R = T|G = W)$ in the following manner:

$$P(R = T|G = W) = \frac{P(R{=}T,G{=}T)}{P(G{=}T)} = \frac{\sum_{S\in\{T,F\}} P(S,G{=}T,R{=}T)}{\sum_{S,R\in\{T,F\}} P(S,G{=}T,R)}$$

(b) You are on holidays on Fisher Island. The yearly weather on Fisher Island comes in five different varieties:

- there is a 10% chance that there will be rain everyday of the year.
- there is a 20% chance that there will be rain on 75% of the days of the year.
- there is a 40% chance that there will be rain on 50% of the days of the year.
- there is a 20% chance that there will be rain on 25% of the days of the year.
- there is a 10% chance that there will be no rain on any day of the year.

(i) given that it has rained on day 1 and 2 of the year compute the posterior probability of each of the 5 yearly weather patterns on day 2 of the year. Give your answer rounded to four places of precision.

(5 marks)

To begin we will define some notation. Let:

- h_1 denote the hypothesis that it will rain everyday, $P(h_1) = 0.1$.
- h_2 denote the hypothesis that it will rain on 75% of the days of the year, with prior $P(h_2) = 0.2$.
- h_3 denote the hypothesis that it will rain on 50% of the days of the year, with prior $P(h_3) = 0.4$.
- h_4 denote the hypothesis that it will rain on 25% of the days of the year, with prior $P(h_4) = 0.2$.
- h_5 denote the hypothesis that there will be no rain during the year, with prior $P(h_5) = 0.1$.

Also, if we use the notation $rain_x$ to represent the observation of rain on day x of the year, then the probability of rain on a day of the year given a particular hypothesis h is:

- $P(rain_x|h_1) = 1.0$.
- $P(rain_x|h_2) = 0.75$.
- $P(rain_x|h_3) = 0.5$.
- $P(rain_x|h_4) = 0.25$.
- $P(rain_x|h_5) = 0.0$.

Then:

- By Bayes' rule, we can compute the posterior probability of a hypothesis given the data so far using: $P(h_i|\mathbf{d}) = \alpha P(\mathbf{d}|h_i)P(h_i)$
- And, the likelihood of the data given a hypothesis is calculated using: $P(\mathbf{d}|h_i) = \prod_j P(d_j|h_i)$

So:

- $P(h_1|rain_1, rain_2) = \alpha(\prod_{j=1}^{2} P(rain_j|h_1))P(h_1) = \alpha 1.00^2 \times 0.1 = \alpha 0.1 = \frac{0.1}{0.325} \approx ..3077.$
- $P(h_2|rain_1, rain_2) = \alpha(\prod_{j=1}^{2} P(rain_j|h_2))P(h_1) = \alpha 0.75^2 \times 0.2 = \alpha 0.1125 = \frac{0.1125}{0.325} \approx .3461.$
- $P(h_3|rain_1, rain_2) = \alpha(\prod_{j=1}^{2} P(rain_j|h_3))P(h_1) = \alpha 0.50^2 \times 0.4 = \alpha 0.1 = \frac{0.1}{0.325} \approx ..3077.$
- $P(h_4|rain_1, rain_2) = \alpha(\prod_{j=1}^{2} P(rain_j|h_4))P(h_1) = \alpha 0.25^2 \times 0.2 = \alpha 0.0125 = \frac{0.0125}{0.325} \approx .0385.$
- $P(h_5|rain_1, rain_2) = \alpha(\prod_{j=1}^{2} P(rain_j|h_5))P(h_1) = \alpha 0.00^2 \times 0.1 = \alpha 0.0 = 0.0.$

(ii) given that after the first 10 days of the year the weather has been such that the posterior probabilities of each of the 5 varieties of the yearly weather on Fisher Island are: there is now a 90% chance that there will be rain everyday for the rest of the year; a 7% chance that there will be rain on 75% of the rest of the days of the year; a 2% chance that there will be rain on 50% of the rest of the days of the year; a 1% chance that there will be rain on 25% of the rest of the days of the year; and there is a 0% chance that there will be no rain for the rest of the year.

A. what is the Bayesian Prediction probability of rain on day 11.

(5 marks)

Bayesian predictions use a likelihood-weighted sum over the hypotheses: $P(X|\mathbf{d}) = \sum_i P(X|h_i)P(h_i|\mathbf{d})$ In this instance we get:
$P(rain_1 1|\mathbf{d}) = \sum_i P(rain_1 1|h_i)P(h_i|\mathbf{d})$
$= (1.0 * 0.9) + (.75 * .07) + (.5 * .02) + (.25 * .01) + 0$
$= 0.9 + .0525 + 0.01 + .0025 + 0$
$= 0.965$

B. what is the Maximum a Posterior (MAP) probability of rain on day 11.

(5 marks)

A MAP prediction just uses the prediction provided by the single most probable hypothesis. In this instance the single most probable hypothesis is the hypothesis that it will rain on every day of the year. This hypothesis would predict rain on day 11 with probability of 1.0 (i.e. certainty)

X	Y	Class
T	T	+
T	F	-
T	F	+
T	T	+
F	T	-

Table 2: X and Y Classification Data

3. (a) In the context of machine learning, distinguish between **supervised** and **unsupervised** learning.

(5 marks)

The distinction is that with **supervised learning** we know the actual label or category for each piece of data on which we train, whereas with **unsupervised learning** we do not know the classification of the data in the training sample. Unsupervised learning can thus often be viewed as a **clustering** task, while supervised learning can usually be seen as a **classification** task, or equivalently as a function-fitting task where one extrapolates the shape of a function based on some data points.

(b) In the context of machine learning, explain what is meant by **overfitting** the training data.

(5 marks)

Overfitting occurs when classifiers make decisions based on accidental properties of the training set that will lead to errors on the test set (or new data). As a result, whenever there is a large set of possible hypotheses, one has to be careful not to use the resulting freedom to find meaningless ”regularity” in the data.

(c) Discuss the advantages and disadvantages of k**-Nearest Neighbour** classification.

(10 marks)

Strengths

(i) No training involved lazy learning

(ii) New data can be added on the fly

(iii) Some explanation capabilities

(iv) Robust to noisy data by averaging k-nearest neighbors

Weaknesses

(i) Not the most powerful classification (generally its accuracy will be lower than an ANN or SVM model)

(ii) Slow classification

(iii) Curse of dimensionality (as you increase the number of features you need more and more examples to cover the problem space - kNN are particularly susceptible to this issue as they do not do any feature selection).

(d) Table 2 provides a classification for a data set of X Y pairs.

(i) Calculate the **entropy** for this classification.

(5 marks)

Entropy is $-\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.971$

(ii) Calculate the **information gain** for X and Y.

(5 marks)

Entropy for X = T $-\frac{3}{4}log_2\frac{3}{4} - \frac{1}{4}log_2\frac{1}{4} = 0.811$
Entropy for X = F $0 - \frac{1}{1}log_2\frac{1}{1} = 0$
Gain for X $0.971 - (\frac{4}{5} \times 0.811 + \frac{1}{5} \times 0) = 0.322$
Entropy for Y = T $-\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.918$
Entropy for Y = F $-\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2} = 1.0$
Gain for Y $0.971 - (\frac{3}{5} \times 0.918 + \frac{2}{5} \times 1) = 0.02$

4. Figure 1 shows a backprogation network that is currently processing the training vector $[1.0, 0.9, 0.9]$ which has an associated target vector $[0.1, 0.9, 1.0]$. Given that the output from unit B is 0.6 and from C is 0.8, and assuming that the activation function used at all nodes in the network is the logistic function (i.e., $f(x) = \frac{1}{1+\exp^{-x}}$):

   (a) Calculate the actual output vector (to 3 decimal places).

   (5 marks)

   Output of unit $i = f(\sum_{j=1}^{n} W_{j,i} \times activation_j)$
   First output unit input = -0.3 x 0.6 + 0.9 x 0.8 = 0.54 → f(0.54) = 0.632
   Second output unit input = -0.6 x 0.6 + -0.1 x 0.8 = -0.44 → f(-0.44) = 0.392
   Third output unit input = 0.4 x 0.6 + 1.2 x 0.8 = 1.2 → f(1.2)= 0.769

   (b) Calculate the error for each output unit.

   (5 marks)

   Error = target - output
   First output unit = (0.1 - 0.632) = - 0.532
   Second output unit = (0.9 - 0.392) = 0.508
   Third output unit = (0.1 - 0.769) = - 0.696

   (c) Calculate the error for each hidden unit B and C.

   (10 marks)

   Each hidden node j is responsible for some fraction of the error Err_i of each of the output units i to which it connects. Thus the Err_i values are divided according to the strengths of the connection between the hidden node and the output nodes and are propagated back to the hidden nodes. Where a hidden node feeds-forward into more than 1 output node the errors propagated back to it are summed: $Err_j = \sum_{i=1}^{n} W_{ji} \times Err_i$:
   $Err_B = (-0.3 \times -0.532) + (-0.6 \times 0.508) + (0.4 \times -0.696) = 0.1596 + -0.3048 + -0.2784 = -0.4236$
   $Err_C = (0.9 \times -0.532) + (-0.1 \times 0.508) + (1.2 \times -0.696) = -0.4788 + -0.0508 + -0.8352 = -1.3648$

   (d) The following sets express the mappings between predicates r, p, q, s, $class1$ and $class2$: $r \rightarrow \{a1, a2, a5, a6\}$, $p \rightarrow \{a2, a3, a5, a7\}$, $q \rightarrow \{a1, a2, a6\}$, $s \rightarrow \{(a2, f), (a1, 1), (a6, f)\}$, $class1 \rightarrow \{a2\}$, $class2 \rightarrow \{a2, a6\}$.

      (i) Given the above sets give a specialisation of the rule $class1(X) \leftarrow r(X) \wedge p(X)$ such that the rule is only satisfied by $class1$ members.

      (5 marks)

      $class1(X) \leftarrow r(X) \wedge p(X) \wedge q(X)$

      (ii) Given the above sets give a rule that will correctly classify only members of $class2$.

      (5 marks)

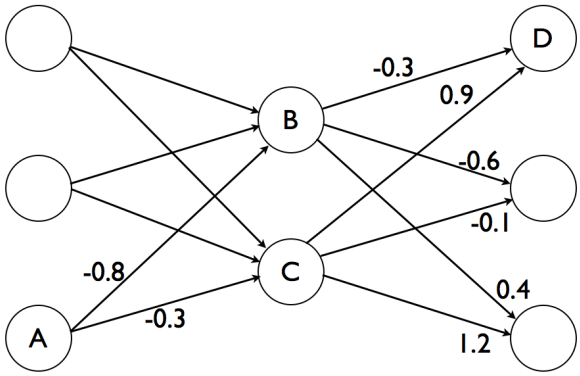


Figure 1: Example Neural Net

$class2(X) \leftarrow s(X, f)$