Digital Signatures

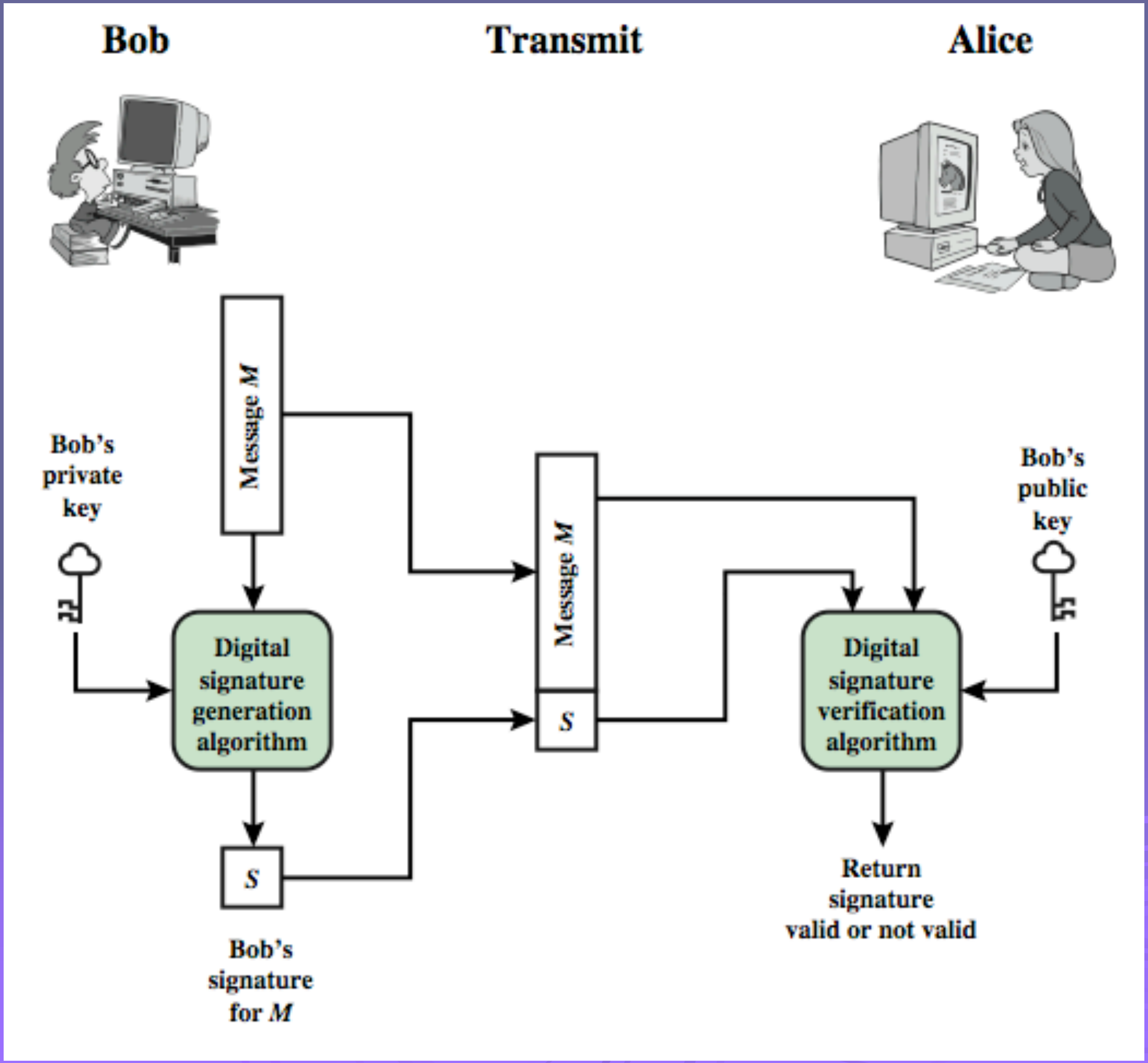
To guard against the baneful influence exerted by strangers is therefore an elementary dictate of savage prudence. Hence before strangers are allowed to enter a district, or at least before they are permitted to mingle freely with the inhabitants, certain ceremonies are often performed by the natives of the country for the purpose of disarming the strangers of their magical powers, or of disinfecting, so to speak, the tainted atmosphere by which they are supposed to be surrounded.

—*The Golden Bough*, Sir James George Frazer

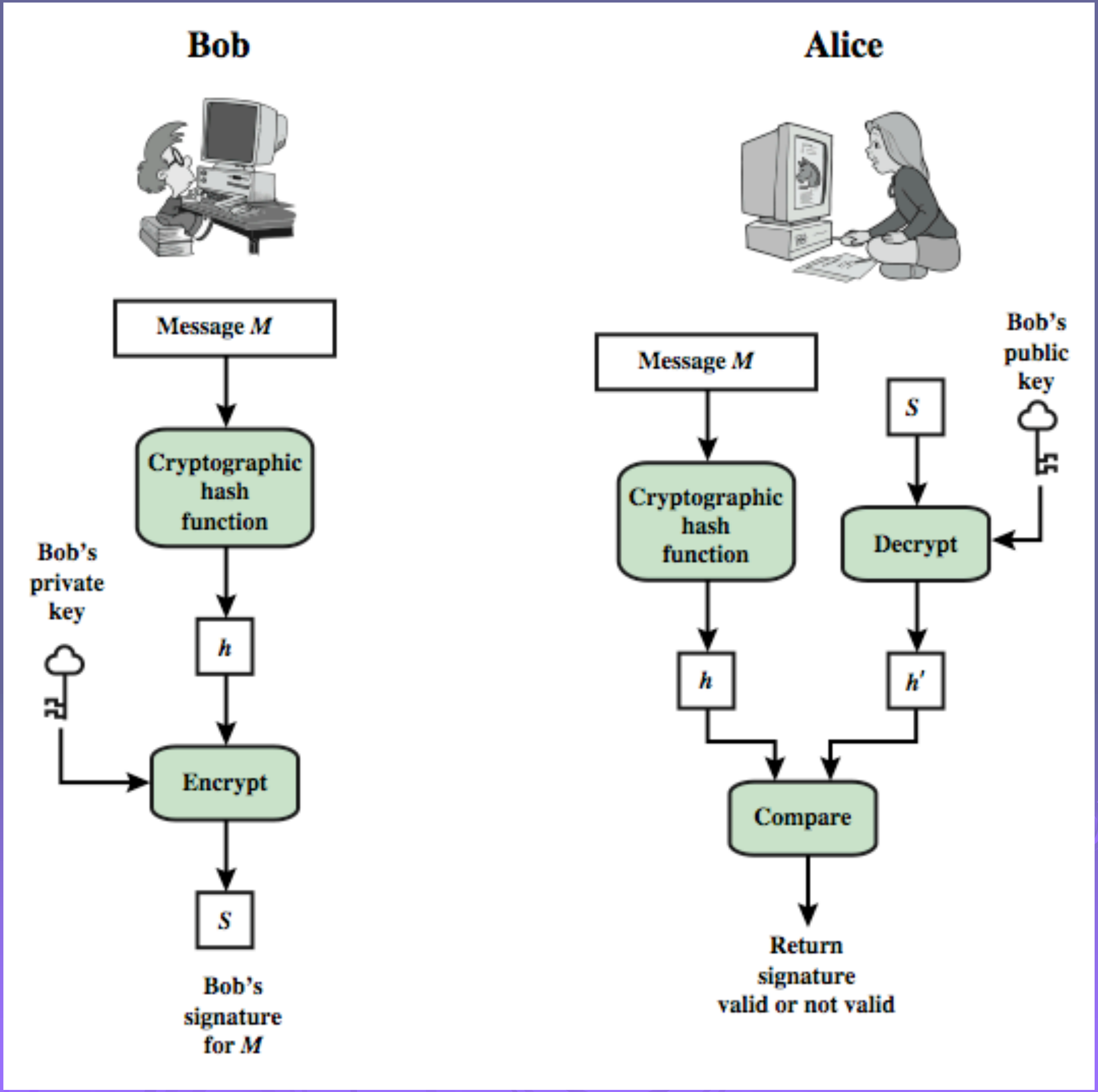
Digital Signatures

- have looked at message authentication
  - but does not address issues of lack of trust
- digital signatures provide the ability to:
  - verify author, date & time of signature
  - authenticate message contents
  - be verified by third parties to resolve disputes
- hence include authentication function with additional capabilities

Digital Signature Model


Bob
Transmit
Alice
Message M
Bob's private key
Digital signature generation algorithm
S
Bob's signature for M
Message M
S
Bob's public key
Digital signature verification algorithm
Return signature valid or not valid

Digital Signature Model



Attacks and Forgeries

- attacks
  - key-only attack
  - known message attack
  - generic chosen message attack
  - directed chosen message attack
  - adaptive chosen message attack
- break success levels
  - total break
  - selective forgery
  - existential forgery

Digital Signature Requirements

- must depend on the message signed
- must use information unique to sender
  - to prevent both forgery and denial
- must be relatively easy to produce
- must be relatively easy to recognize & verify
- be computationally infeasible to forge
  - with new message for existing digital signature
  - with fraudulent digital signature for given message
- be practical save digital signature in storage

Direct Digital Signatures

- involve only sender & receiver
- assumed receiver has sender's public-key
- digital signature made by sender signing entire message or hash with private-key
- can encrypt using receivers public-key
- important that sign first then encrypt message & signature
- security depends on sender's private-key

ElGamal Digital Signatures

- signature variant of ElGamal, related to D-H
  - so uses exponentiation in a finite (Galois)
  - with security based difficulty of computing discrete logarithms, as in D-H
- use private key for encryption (signing)
- uses public key for decryption (verification)
- each user (eg. A) generates their key
  - chooses a secret key (number): $1 < x_A < q-1$
  - compute their **public key**: $y_A = a^{x_A} \bmod q$

ElGamal Digital Signature

- Alice signs a message M to Bob by computing
  - the hash $m = H(M)$, $0 <= m <= (q-1)$
  - chose random integer K with $1 <= K <= (q-1)$ and $\gcd(K,q-1)=1$
  - compute temporary key: $S_1 = a^k \bmod q$
  - compute K^{-1} the inverse of $K \bmod (q-1)$
  - compute the value: $S_2 = K^{-1}(m-x_A S_1) \bmod (q-1)$
  - signature is: (S_1, S_2)
- any user B can verify the signature by computing
  - $V_1 = a^m \bmod q$
  - $V_2 = y_A^{S_1} S_1^{S_2} \bmod q$
  - signature is valid if $V_1 = V_2$

ElGamal Signature Example

- use field GF(19) $q=19$ and $a=10$
- Alice computes her key:
  - A chooses $x_A=16$ & computes $y_A=10^{16} \bmod 19 = 4$
- Alice signs message with hash $m=14$ as $(3,4)$:
  - choosing random $K=5$ which has $\gcd(18,5)=1$
  - computing $S_1 = 10^5 \bmod 19 = 3$
  - finding $K^{-1} \bmod (q-1) = 5^{-1} \bmod 18 = 11$
  - computing $S_2 = 11(14-16.3) \bmod 18 = 4$
- any user B can verify the signature by computing
  - $V_1 = 10^{14} \bmod 19 = 16$
  - $V_2 = 4^3.3^4 = 5184 = 16 \bmod 19$
  - since $16 = 16$ signature is valid

Schnorr Digital Signatures

- also uses exponentiation in a finite (Galois)
  - security based on discrete logarithms, as in D-H
- minimizes message dependent computation
  - multiplying a 2*n-bit* integer with an *n-bit* integer
- main work can be done in idle time
- have using a prime modulus p
  - $p-1$ has a prime factor q of appropriate size
  - typically p 1024-bit and q 160-bit numbers

Schnorr Key Setup

- choose suitable primes p , q
- choose a such that $a^q = 1 \bmod p$
- (a,p,q) are global parameters for all
- each user (eg. A) generates a key
  - chooses a secret key (number): $0 < s_A < q$
  - compute their **public key**: $v_A = a^{-s_A} \bmod q$

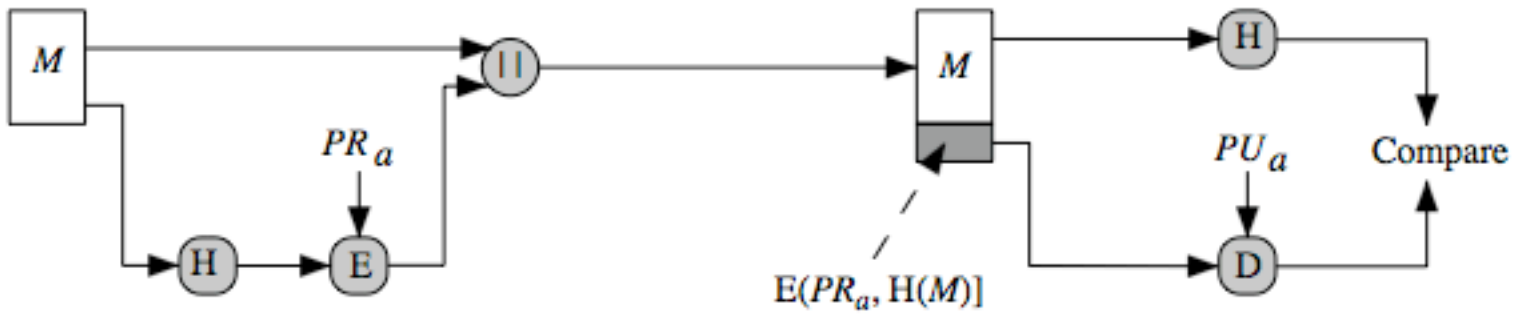
Schnorr Signature

- user signs message by
  - choosing random r with $0<r<q$ and computing $x = a^r \bmod p$
  - concatenate message with x and hash result to computing: $e = H(M || x)$
  - computing: $y = (r + se) \bmod q$
  - signature is pair (e, y)
- any other user can verify the signature as follows:
  - computing: $x' = a^y v^e \bmod p$
  - verifying that: $e = H(M || x')$

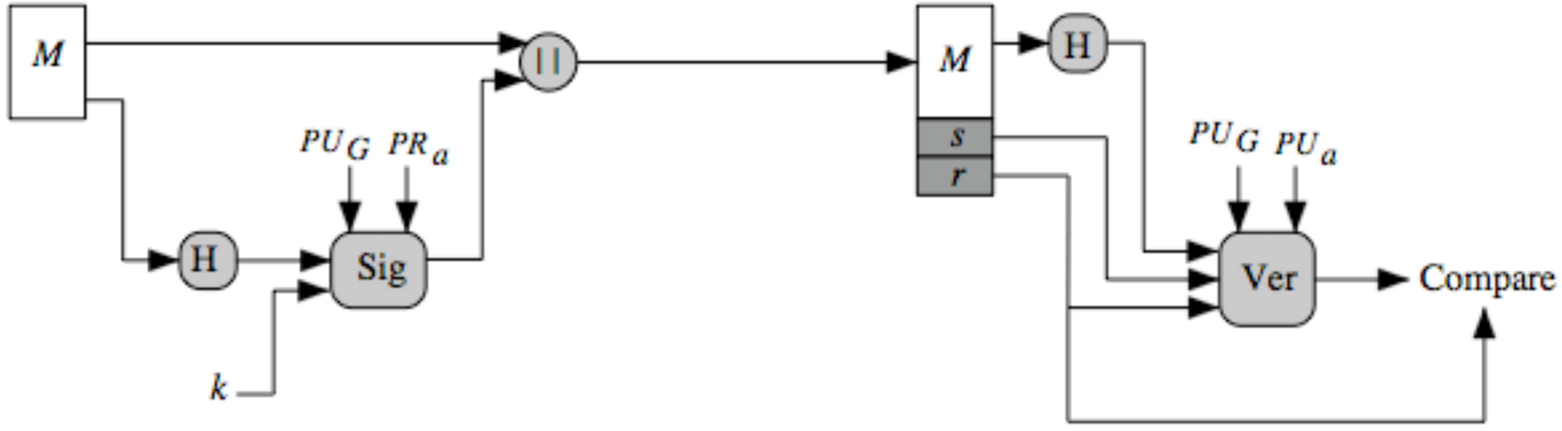
Digital Signature Standard (DSS)

- US Govt approved signature scheme
- designed by NIST & NSA in early 90's
- published as FIPS-186 in 1991
- revised in 1993, 1996 & then 2000
- uses the SHA hash algorithm
- DSS is the standard, DSA is the algorithm
- FIPS 186-2 (2000) includes alternative RSA & elliptic curve signature variants
- DSA is digital signature only unlike RSA
- is a public-key technique

DSS vs RSA Signatures



(a) RSA Approach



(b) DSS Approach

Digital Signature Algorithm (DSA)

- creates a 320 bit signature
- with 512-1024 bit security
- smaller and faster than RSA
- a digital signature scheme only
- security depends on difficulty of computing discrete logarithms
- variant of ElGamal & Schnorr schemes

DSA Key Generation

- have shared global public key values (p,q,g):
  - choose 160-bit prime number q
  - choose a large prime p with $2^{L-1} < p < 2^L$
    - where L= 512 to 1024 bits and is a multiple of 64
    - such that q is a 160 bit prime divisor of $(p-1)$
  - choose $g = h^{(p-1)/q}$
    - where $1<h<p-1$ and $h^{(p-1)/q} \bmod p > 1$
- users choose private & compute public key:
  - choose random private key: $x<q$
  - compute public key: $y = g^x \bmod p$

DSA Signature Creation

- to **sign** a message `M` the sender:
  - generates a random signature key `k`, $k<q$
  - nb. `k` must be random, be destroyed after use, and never be reused
- then computes signature pair:

$$r = (g^k \bmod p) \bmod q$$

$$s = [k^{-1}(H(M)+ xr)] \bmod q$$

- sends signature `(r,s)` with message `M`

DSA Signature Verification

- having received M & signature `(r,s)`
- to **verify** a signature, recipient computes:

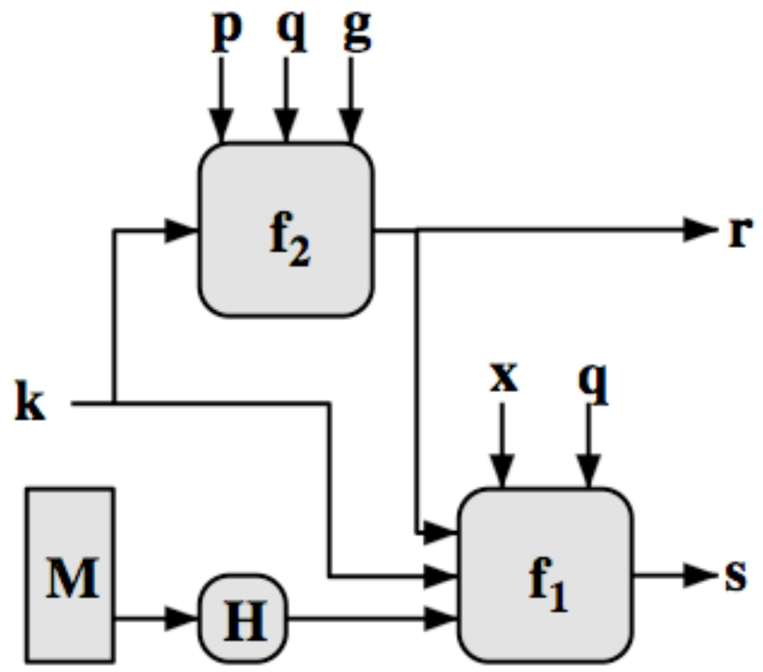
  $w = s^{-1} \bmod q$

  $u1 = [H(M)w] \bmod q$

  $u2 = (rw) \bmod q$

  $v = [(g^{u1} y^{u2}) \bmod p] \bmod q$
- if `v=r` then signature is verified

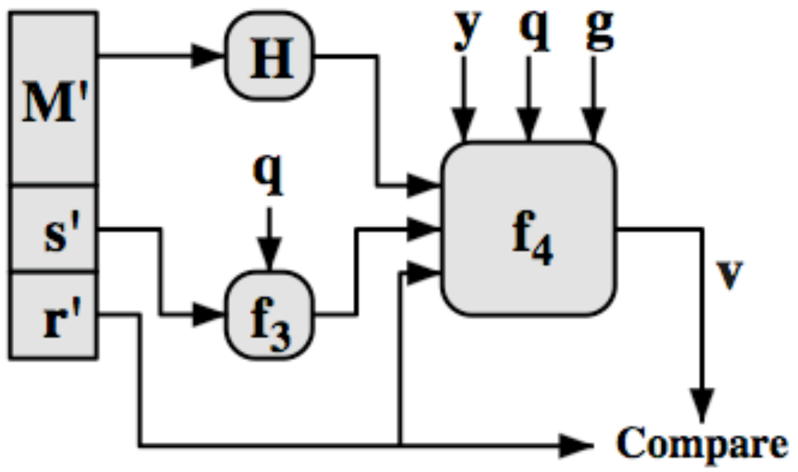
DSS Overview



$$s = f_1(H(M), k, x, r, q) = (k^{-1}(H(M) + xr)) \bmod q$$

$$r = f_2(k, p, q, g) = (g^k \bmod p) \bmod q$$

(a) Signing



$$w = f_3(s', q) = (s')^{-1} \bmod q$$

$$v = f_4(y, q, g, H(M'), w, r')$$

$$= ((g^{(H(M')w) \bmod q}\, y^{r'w \bmod q}) \bmod p) \bmod q$$

(b) Verifying

Summary

- have discussed:
  - digital signatures
  - ElGamal & Schnorr signature schemes
  - digital signature algorithm and standard