Mixed Integer Second Order Cone Programming Taking Appropriate Approximation for the UC in Hybrid AC-DC Grid

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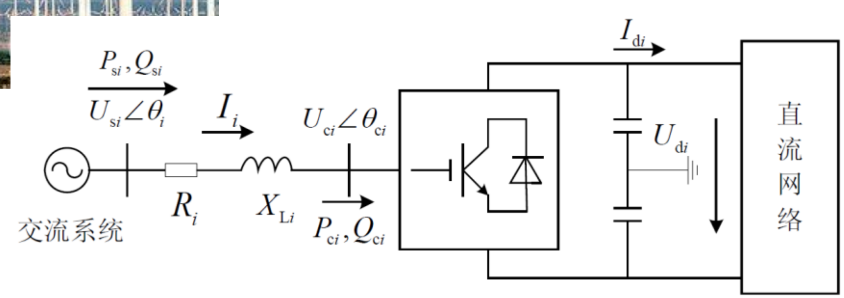
Contents

Background

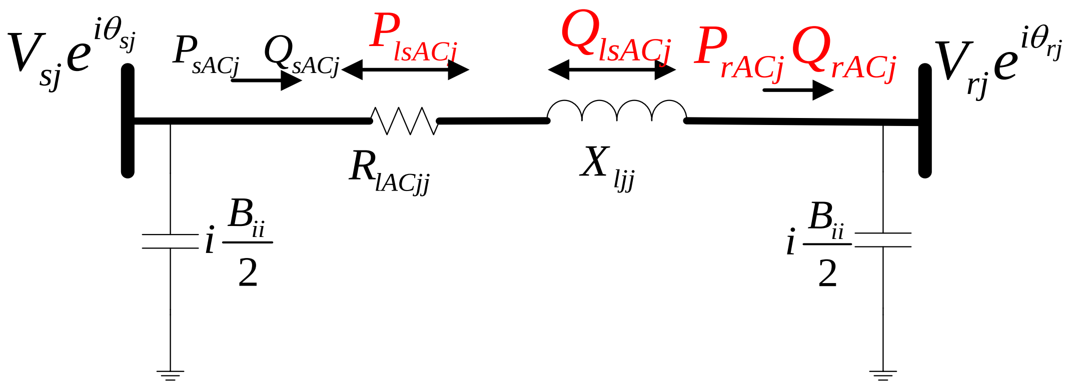
- The transmission and consumption of **large-scale wind power through VSC based HVDC links**
- **OPF and UC** for the hybrid AC-DC grid
- **Mixed integer second order cone programming** considering power flow constraints and converter model

AC/DC Power Transmission



Line Based Power Flow Model

AC lines model



Variables about power flow are introduced

Power angles are eliminated

Power loss can be expressed as quadratic form

Power balance

$$\begin{cases} P_{Gi} - P_{Di} = \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j})P_{rACj} + \sum_{j=1}^{n_{lAC}} M_{lAC}(i,j)P_{lsACj} \\ Q_{Gi} - Q_{Di} = \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j})Q_{rACj} + \sum_{j=1}^{n_{lAC}} M_{lAC}(i,j)Q_{lsACj} - B_{ii}V_i^2 \end{cases}$$

Loss relation

$$\left.\begin{cases} \boxed{\dfrac{P_{rACj}^2 + Q_{rACj}^2}{V_{rACj}^2} X_{ljj} = Q_{lsACj}} \\ \boxed{\dfrac{P_{rACj}^2 + Q_{rACj}^2}{V_{rACj}^2} R_{lACjj} = P_{lsACj}} \end{cases}\right\} \Rightarrow P_{lsACj}X_{ljj} = Q_{lsACj}R_{lACjj}$$

Can be replaced by a first order variable “W”

Line Based Power Flow Model

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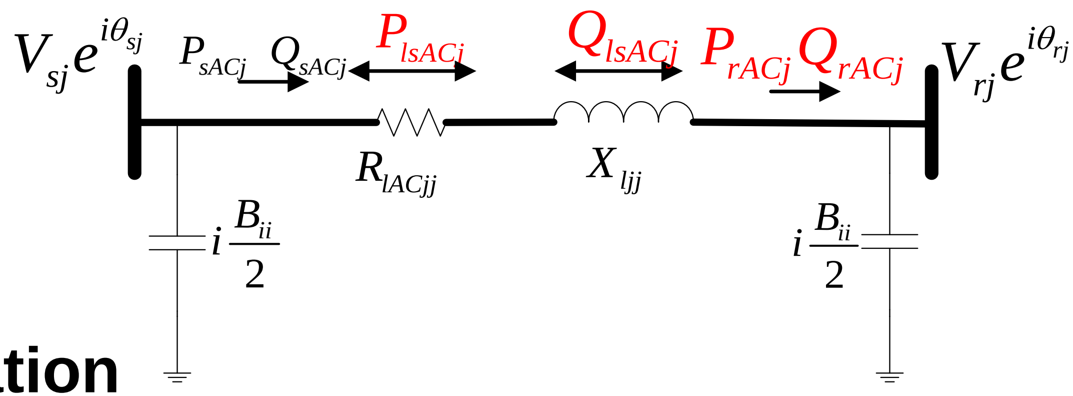
AC lines model



Voltage relation

$$V_{sj}e^{i\theta_{sj}} = V_{rj}e^{i\theta_{rj}} + \frac{P_{rACj} - iQ_{rACj}}{V_{rj}e^{-i\theta_{rj}}}(R_{lACjj} + iX_{ljj}) \Leftrightarrow V_{sj}V_{rj}e^{i\theta_{srj}} = V_{rj}^2 + (P_{rACj} - iQ_{rACj})(R_{lACjj} + iX_{ljj})$$

The real part>0 ⟺ **Actual power system**

Module $\boxed{V_{sj}^2 - V_{rj}^2 = 2R_{lACjj}P_{rACj} + 2X_{ljj}Q_{rACj} + 2R_{lACjj}P_{lsACj} + 2X_{ljj}Q_{lsACj}}$

Imaginary part $V_{sj}V_{rj}\sin\theta_{srj} = X_{ljj}P_{rACj} - R_{lACjj}Q_{rACj}$

Approximation $\boxed{\sin\theta_{srj} \approx \theta_{srj} \quad V_{sj}V_{rj} \approx 1}$

$\Rightarrow$ $\theta_{srj} = X_{ljj}P_{rACj} - R_{lACjj}Q_{rACj}$

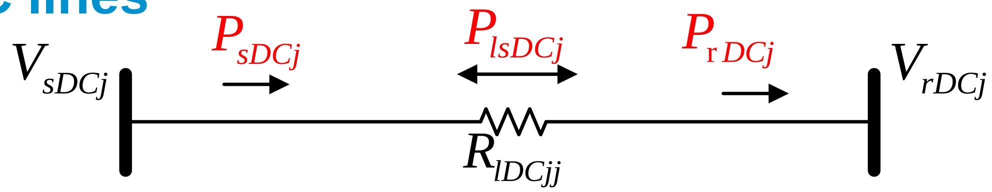
Fundamental Circuit Matrix

$\Rightarrow$ $\boxed{\sum_{k=1}^{n_C} C_{kj}X_{ljj}P_{rACj} - \sum_{k=1}^{n_C} C_{kj}R_{lACjj}Q_{rACj} = 0}$

Line Based Power Flow Model

DC lines model

Similar to AC lines



Power balance

$$P_{DCi} = -\sum_{j=1}^{n_{lDC}} M_{PDC}(\mathrm{i},\mathrm{j}) P_{\mathrm{r}DCj} - \sum_{j=1}^{n_{lDC}} M_{lDC}(i,j) P_{lossDCj}$$

Loss relation

$$\boxed{\frac{P_{rDCj}^2}{V_{rDCj}^2} R_{lDCjj} = P_{lsDCj}}$$

Voltage relation

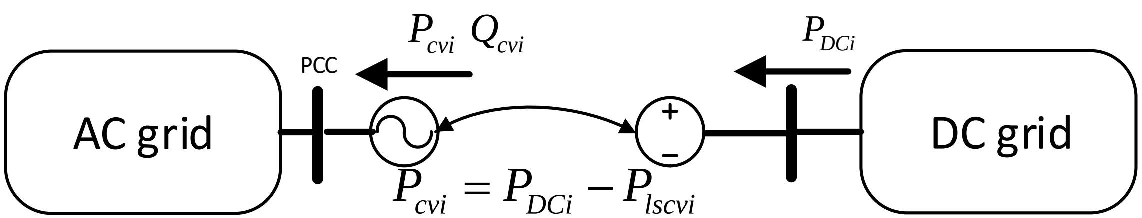
$$V_{sDCj}^2 - V_{rDCj}^2 = 2P_{rDCj} R_{DCjj} + P_{lsDCj} R_{DCjj}$$

Line Based Power Flow Model

Converter station model

the inner structure of converter station was neglected

only power flow of both sides are considered



Power transmission relation $$P_{DCi} = P_{cvi} + P_{lscvi}$$

Approximate loss relation $$\boxed{P_{lscvi} = \beta \left| P_{cvi} \right|} \qquad (\beta\text{=0.015})$$

Power balance at AC side

$$\begin{cases} P_{Gi} + P_{cvi} - P_{Di} = \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j}) P_{rACj} + \sum_{j=1}^{n_{lAC}} M_{lAC}(i,j) P_{lsACj} \\ Q_{Gi} + Q_{cvi} - Q_{Di} = \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j}) Q_{rACj} + \sum_{j=1}^{n_{lAC}} M_{lAC}(i,j) Q_{lsACj} - B_{ii} V_i^2 \end{cases}$$

MISOCP Formulation

SOC relaxation

0 or a small positive number

$$\frac{P_{rACj}^2 + Q_{rACj}^2}{V_{rACj}^2} R_{lACjj} = P_{lsACj} \Rightarrow R_{lACjj}(P_{rACj}^2 + Q_{rACj}^2) \le (1+\boxed{\varepsilon}) P_{lsACj} W_{rACj}$$

$$P_{lscvi} = \beta |P_{cvi}| \Leftrightarrow \beta^2 P_{cvi}^2 = P_{lscvi}^2 \Rightarrow \beta^2 P_{cvi}^2 \le (1+\varepsilon) P_{lscvi}^2$$

Non-convex

convex

Global optimal solution can be obtained

The correctness of SOC relaxation in distribution network has been validated.
Here the SOC relaxation is extended to transmission network.
Numerical results will be used to validated the correctness.

MISOCP Formulation

SOCP formulation for OPF in hybrid AC-DC grid

$$X^T = \Big[P_G^T \quad Q_G^T \quad W_{AC}^T \quad P_{rAC}^T \quad Q_{rAC}^T \quad P_{lsAC}^T \quad Q_{lsAC}^T$$
$$P_{cv}^T \quad Q_{cv}^T \quad P_{lscv}^T \quad W_{DC}^T \quad P_{rDC}^T \quad P_{lsDC}^T \Big]$$

$$Min(FX) = \sum_{i=1}^{n_G} b_i P_{Gi}$$

Subject to (all above equations)

$$P_{Gi} + P_{cvi} - \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j}) P_{rACj} - \sum_{j=1}^{n_{lAC}} M_l AC(i,j) P_{lsACj} = P_{Di}$$

$$Q_{Gi} + Q_{cvi} - \sum_{j=1}^{n_{lAC}} M_{PQAC}(\mathrm{i},\mathrm{j}) Q_{rACj} - \sum_{j=1}^{n_{lAC}} M_l AC(i,j) Q_{lsACj} + B_{ii} W_{DCi} = Q_{Di}$$

$$R_{lACjj} P_{rACj}^2 + R_{lACjj} Q_{rACj}^2 - (1+\varepsilon) P_{lsACj} W_{rACj} \le 0$$

$$X_{ljj} P_{rACj}^2 + X_{ljj} Q_{rACj}^2 - (1+\varepsilon) Q_{lsACj} W_{rACj} \le 0$$

$$X_{ljj} P_{lsACj} - R_{lACjj} Q_{lsACj} = 0$$

$$2R_{lACjj} P_{rACj} + 2X_{ljj} Q_{rACj} + R_{lACjj} P_{lsACj} + X_{ljj} Q_{lsACj} - \sum_{i=1}^{n_{bAC}} M_{W_{AC}}(j,i) W_{ACi} = 0$$

$$\sum_{k=1}^{n_C} C_{kj} X_{ljj} P_{rACj} - \sum_{k=1}^{n_C} C_{kj} R_{lACjj} Q_{rACj} = 0$$

$$\beta^2 P_{cvi}^2 - (1+\varepsilon) P_{lscvi}^2 \le 0$$

$$P_{cvi} + P_{lscvi} + \sum_{j=1}^{n_{lDC}} M_{PDC}(\mathrm{i},\mathrm{j}) P_{rDCj} + \sum_{j=1}^{n_{lDC}} M_{lDC}(i,j) P_{lsDCj} = 0$$

$$R_{lDCjj} P_{rDCj}^2 - (1+\varepsilon) P_{lsDCj} W_{rDCj} \le 0$$

$$2R_{DCjj} P_{rDCj} + R_{DCjj} P_{lsDCj} - \sum_{i=1}^{n_{bDC}} M_{W_{DC}}(j,i) W_{DCi} = 0$$

$$P_{Gi}^2 + Q_{Gi}^2 \le S_{\max Gi}^2 \qquad P_{cvi}^2 + Q_{cvi}^2 \le S_{\max cvi}^2$$

$P_{rAC}, Q_{rAC}, P_G, Q_G, W_{AC}, P_{cv}^T, Q_{cv}^T, W_{DC}^T, P_{rDC}^T$上下限

MISOCP Formulation

MISOCP formulation for UC in hybrid AC-DC grid

Added variables $\begin{bmatrix} PU_c^T & PD_c^T & U_c^T & C_{su}^T & C_{sd}^T \end{bmatrix}$

$$\text{minCost} = \sum_{t=1}^{T} \sum_{i=1}^{N_G} [b_i P_{Git} + C_{suit} + C_{sdit}]$$

Subject to

all OPF constraints

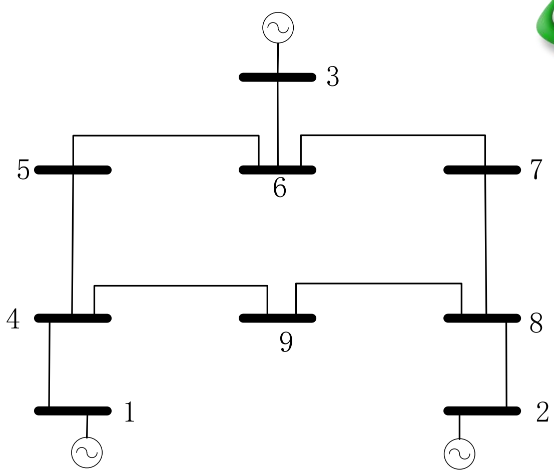
Constraints about UC:

1) SOC power flow constraints;
2) System spinning and operating reserve requirements;
3) Generating unit capacity;
4) Ramping up/down limits;
5) Maximum power output at up/down;
6) Minimum up/down time limits.

Case Study

OPF in IEEE 9-BUS system

Compared with results of MATPOWER



variables	MatPower	SOC	Difference (%)
Cost ($/h)	373.83	373.85	0.0054
P_G (MW)	324.8622	324.875	0.0039
Q_G (Mvar)	46.9145	46.7228	0.41
Q_{shant} (Mvar)	158.9155	159.179	0.17
P_{loss} (MW)	9.862	9.8752	0.13
Q_{loss} (Mvar)	90.83	90.9021	0.079

- All difference <1%
- Small error in cost and P_G
- ➢ the correctness get verified

output	bus	MatPower	SOC	Difference (%)
Pg(MW)	1	10	10	0
	2	44.86	44.88	0.045
	3	270	270	0
Qg(Mvar)	1	26.84	28.42	5.89
	2	7.66	7.00	8.62
	3	12.41	11.30	8.94

- Small error in P_G
- Relatively bigger error in Q_G
- ➢ Acceptable error
- ➢ Result from the approximation or the relaxation

Case Study

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OPF in IEEE 9-BUS system

$$V_{sj}e^{i\theta_{sj}} = V_{rj}e^{i\theta_{rj}} + \frac{P_{rACj} - iQ_{rACj}}{V_{rj}e^{-i\theta_{rj}}}(R_{lACjj} + iX_{ljj})$$

line	Left side	Right side	Difference (%)
1	0.0058	0.0048	16.22
2	-0.0367	-0.0316	13.89
3	-0.2224	-0.1900	14.56
4	0.1582	0.1316	16.85
5	0.1389	0.1167	15.99
6	0.0284	0.0239	15.89
7	-0.0280	-0.0233	17.07
8	0.1281	0.1101	14.05
9	-0.0363	-0.0314	13.32

- **Large error** in imaginary part, between 10% and 20%
  - **approximation** is one of the **error sources**.

$$\frac{P_{rACj}^2 + Q_{rACj}^2}{V_{rACj}^2} R_{lACjj} = P_{lsACj}$$

	line	Left side	Right side	Difference(%)
Active power loss (MW)	1	7.50E-06	7.50E-06	0.0203
	2	2.68E-01	2.68E-01	0.0010
	3	5.55E+00	5.55E+00	0.0010
	4	6.07E-04	6.07E-04	0.0007
	5	1.90E+00	1.90E+00	0.0010
	6	1.32E-01	1.32E-01	0.0010
	7	1.70E-06	1.70E-06	0.0006
	8	1.80E+00	1.80E+00	0.0010
	9	2.20E-01	2.20E-01	0.0010
Reactive power loss (MW)	1	4.32E-01	4.32E-01	0.0203
	2	1.45E+00	1.45E+00	0.0010
	3	2.42E+01	2.42E+01	0.0010
	4	3.56E+01	3.56E+01	0.0007
	5	1.61E+01	1.61E+01	0.0010
	6	1.12E+00	1.12E+00	0.0010
	7	1.07E+00	1.07E+00	0.0006
	8	9.08E+00	9.08E+00	0.0010
	9	1.87E+00	1.87E+00	0.0010

- **same results** on both sides
  - **relaxation** is **not error sources**

Case Study

OPF in Other IEEE Test system

Comparisons of results by MATPOWER, DC power flow model and proposed SOC power flow model in different systems

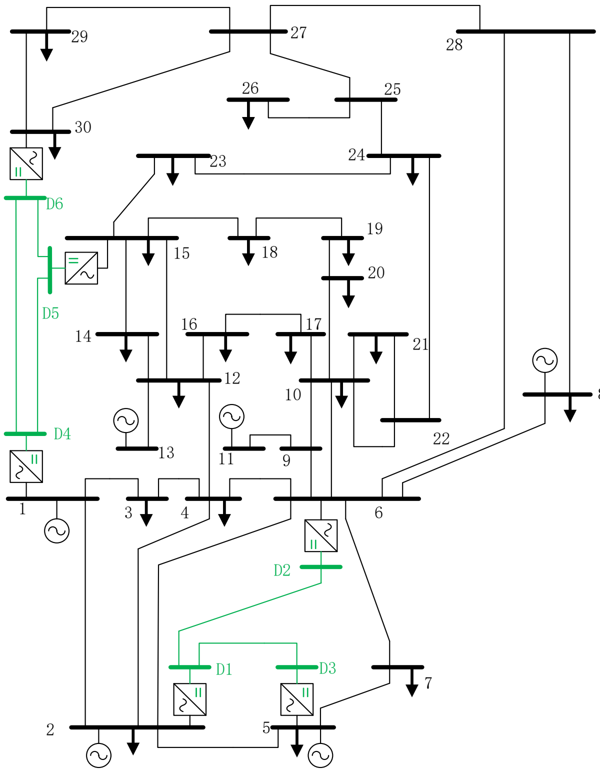
Test system		IEEE14	IEEE30	IEEE57
Cost ($/MW)	MatPower	5371.5	5927.6	25338.1
	DC	5180	5668	25016
	SOC	5370.3	5927	25309.3
Calculation time(s)	MatPower	0.09	0.13	0.16
	DC	0.01	0.01	0.01
	SOC	0.02	0.03	0.07

- **Much higher accuracy than DC power flow model**
- **Less calculation time than MATPOWER**

➢ **the SOC power flow constraints are much more accurate than DC power flow constraints and are calculated faster than MatPower**

Case Study

OPF in IEEE 30-BUS system + DC lines



system	Pure AC	Hybrid AC-DC
Cost($)	5927	5836.87
P_G(MW)	296.35	291.84
Q_G(MW)	121.41	64.04
Q_{shant}(MW)	58.948	61.04
P_{loss}(MW)	12.95	4.22
Q_{loss}(MW)	54.16	16.05
Q_{cv}(MW)	0	17.16
P_{lossDC}(MW)	0	0.4357

DC bus	Voltage(p.u.)	Pcv(MW)	Qcv(MW)
1	1.060	-74.0640	3.7782
2	1.058	17.2914	2.9877
3	1.055	54.2877	15.9021
4	1.060	-54.4211	-18.6728
5	1.057	37.3359	9.8445
6	1.058	15.3434	3.3247

DC lines:

Capacity: 100MW,
Resistance: 0.01p.u.
reference voltage: 320KV

DC bus voltage: 0.94~1.06p.u.

- **Cost, P_G and P_{loss} decrease**
- **P_{cv} and P_{lossDC} exist**

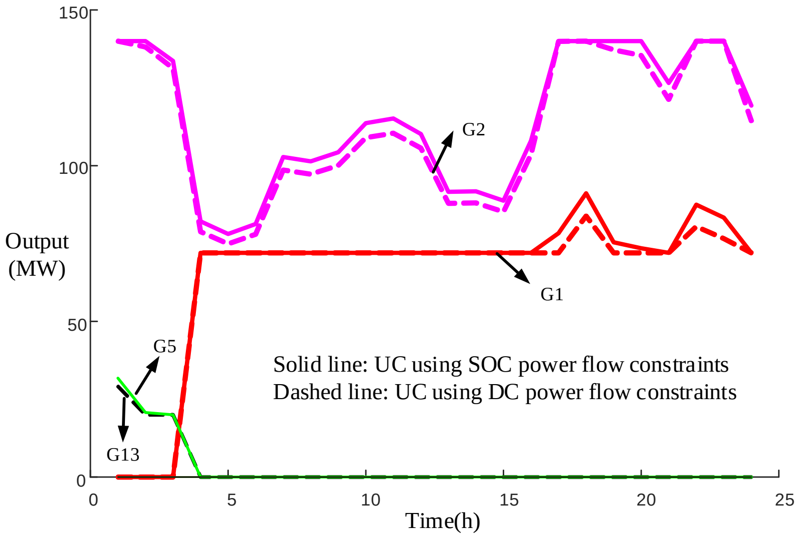
➢ **the applicability in the hybrid AC-DC grid get verified.**

Case Study

UC in Modified IEEE 30-BUS system

Modification:

	Pclimb	Tmin	Psu/Psd	Csu($)	U0
G1	1%/min	2h	80MW	3000	0
G2	1%/min	2h	30MW	3500	\
G5	1%/min	2h	25MW	3500	\
G8	1%/min	2h	25MW	3000	\
G11	1%/min	2h	25MW	3000	\
G13	1%/min	2h	25MW	3500	\



results:

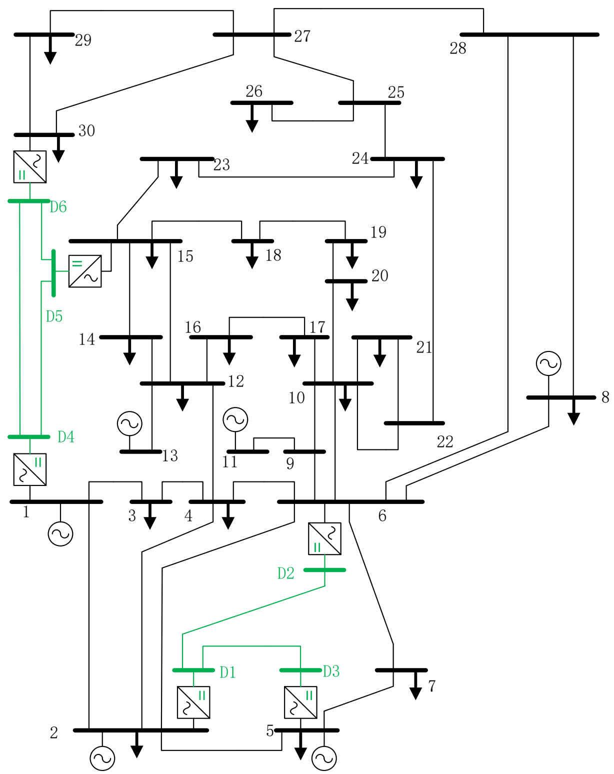
Model	Load (MWh)	Output (MWh)	Loss (MWh)	Coal cost($)	Start-up cost ($)	Total cost($)
SOC	4301	4411	110	89663	3000	92663
DC	4301	4301	0	87401	3000	90401

Number of time in one day		12	24	48	96
Calculation time(s)	SOC	13.88	22.67	78.33	202.98
	DC	0.022	0.14	0.359	1.349

- A little **higher cost**
- The **same output curve trend**
  - ➢ **Correctness** of SOC power flow constraints in UC get verified
- **Longer calculation time**

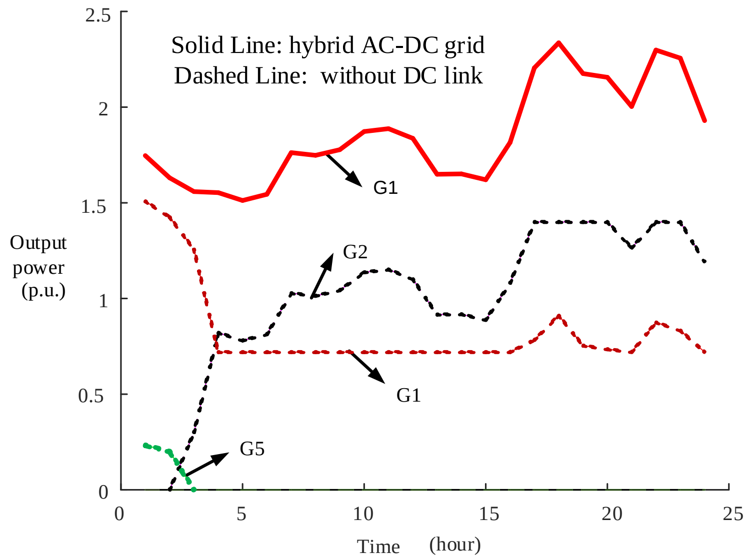
Case Study

UC in Modified IEEE 30-BUS system + DC lines



G1 on while G2 off at the beginning

System	Output (MWh)	Loss (MWh)	Coal cost($)	Start-up cost ($)	Total cost($)
AC	4416	115	89193	3500	92693
AC-DC	4453	152	89053	0	89053



- **output increase while cost decrease**
- **G1 can satisfy all the load demand**
- **Applicable for UC problem in the hybrid AC-DC grid**

Conclusions

1. A MISOCP formulation for the unit commitment in hybrid AC-DC power systems is proposed.
2. The AC and DC power flow constraints are relaxed using SOC approximation.
3. Power flow constraints in the hybrid AC-DC grid are considered in UC problem.

… Future work focuses on the robust unit commitment in hybrid AC-DC grid with wind power integrated.

Thank you for listening !

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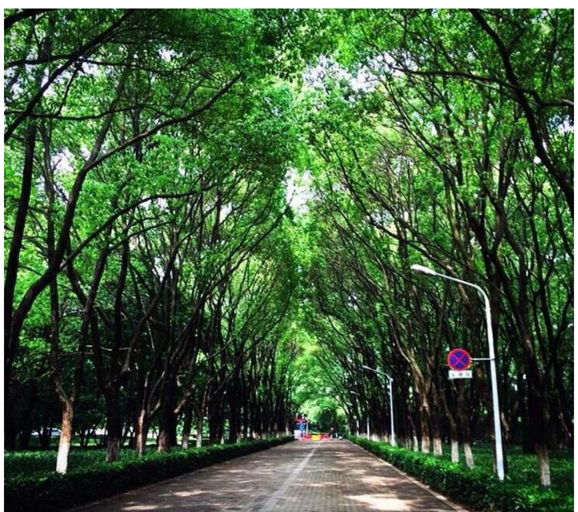

Thank you for listening ！

Q & A