Differential Algebraic Equation-constrained Frequency-secured Stochastic Unit Commitment

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Outline

- ❑Introduction and Problem Formulation
- ❑Solution Method
- ❑Case Study
- ❑Conclusions

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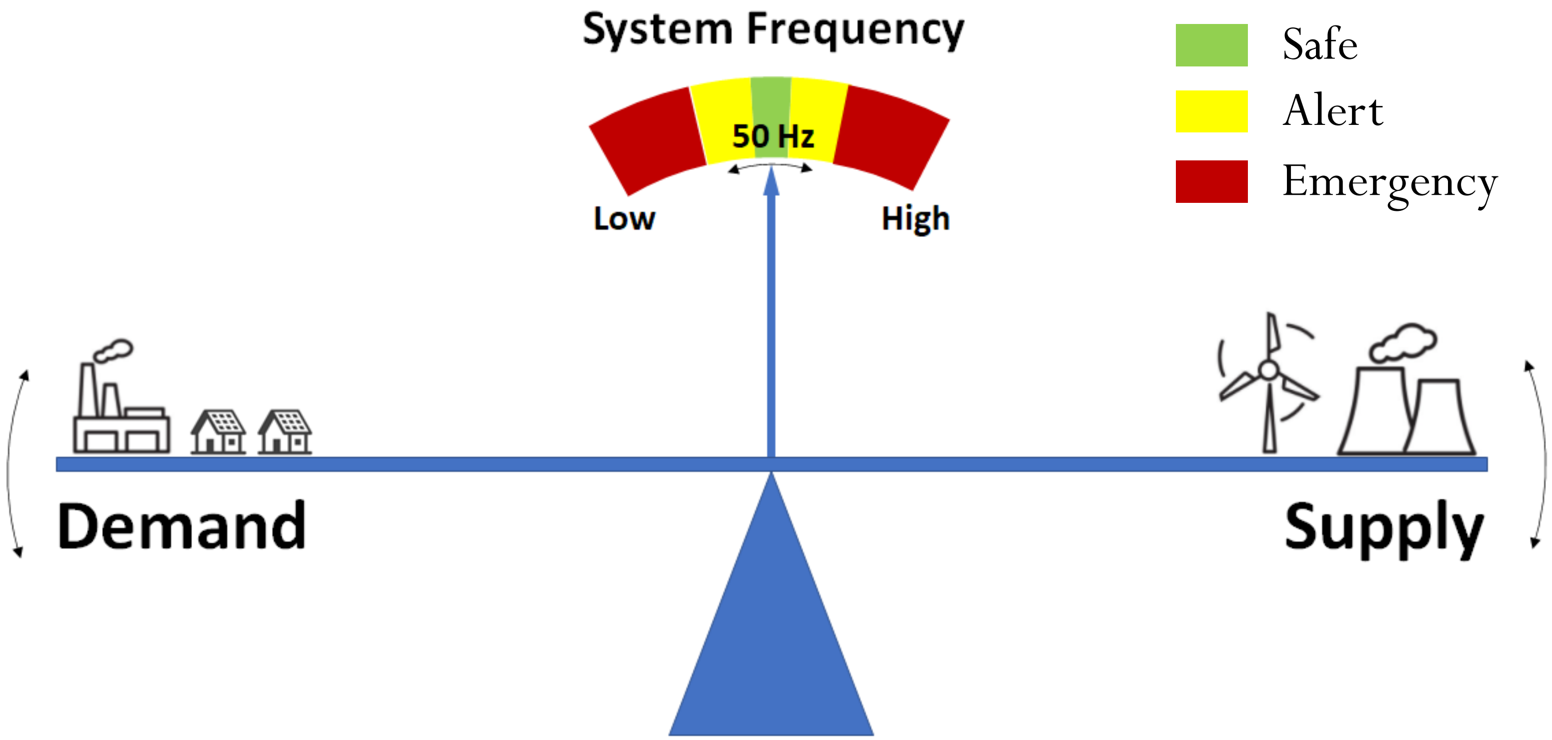
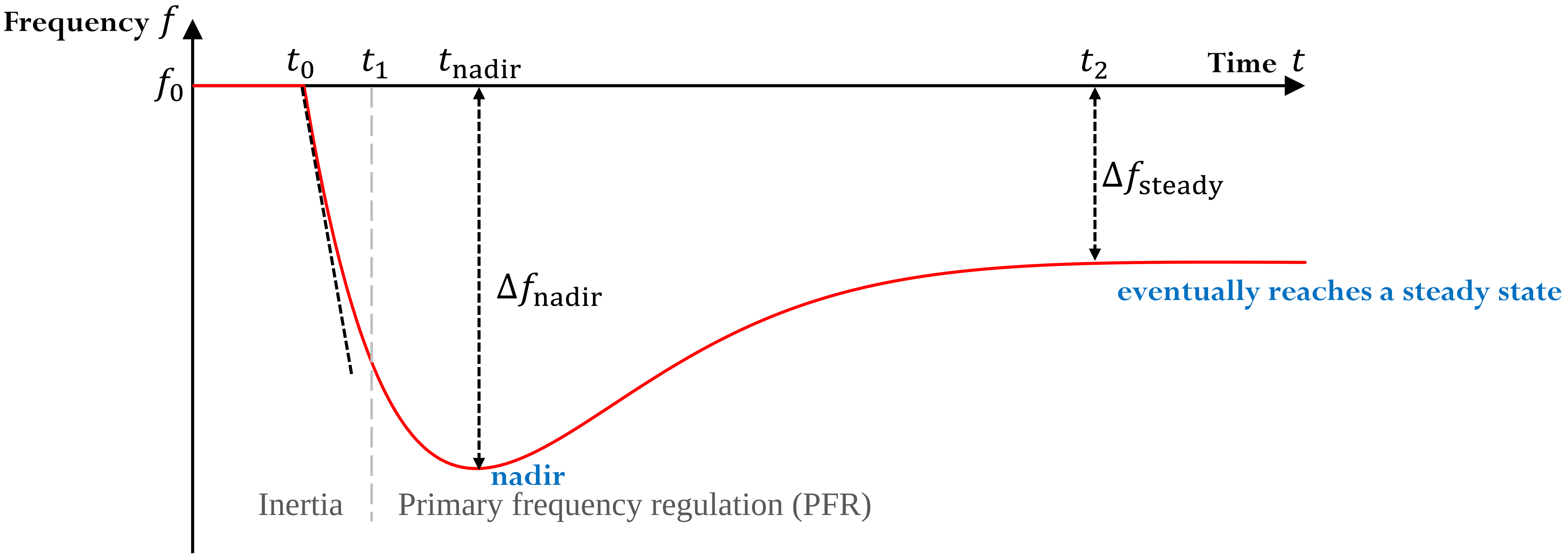
Power System Frequency



The frequency of power systems should be maintained closely around the nominal value

General Frequency Dynamics

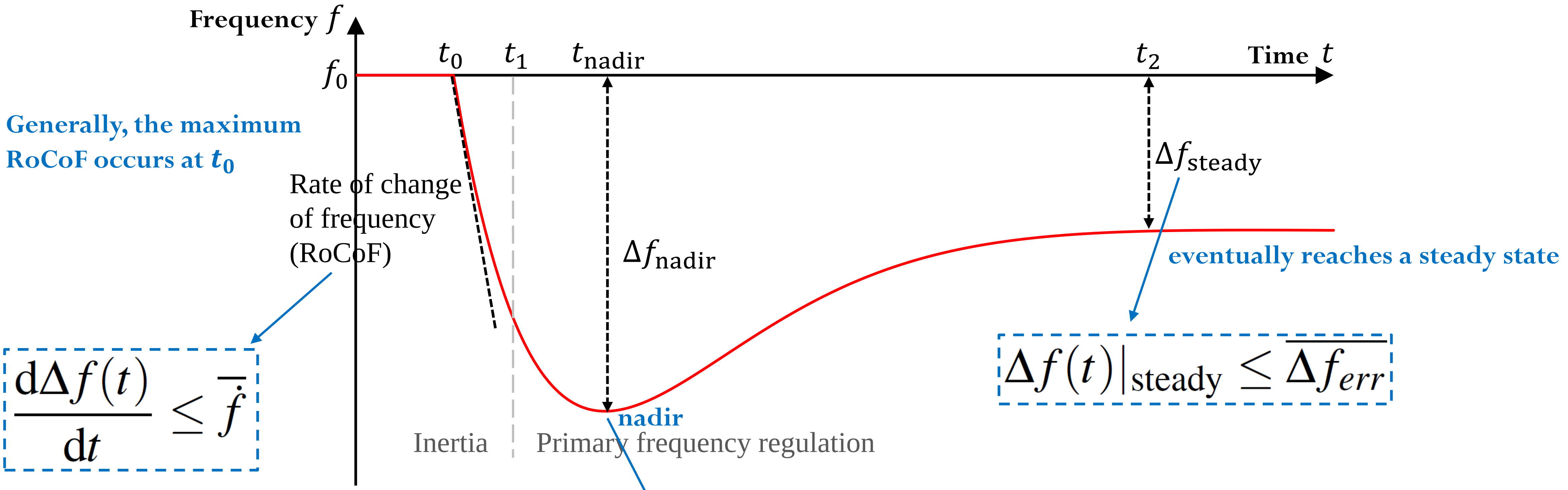
Frequency dynamics during an under-frequency event (a sudden loss of generation at t_0)



- ➢ $t_0 - t_1$: Inertia plays a major role in mitigating frequency drop (PFR does not respond effectively)
- ➢ $t_1 - t_2$: PFR becomes significant, and the frequency eventually reaches a steady state
- ➢ t_{nadir}: Time to reach the nadir during the whole dynamics

Frequency Security Metrics

Frequency dynamics during an under-frequency event (a sudden loss of generation at t_0)



$$\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t} \leq \overline{\dot{f}}$$

$$\Delta f(t)|_{\text{nadir}} \leq \overline{\Delta f}$$

$$\Delta f(t)|_{\text{steady}} \leq \overline{\Delta f_{err}}$$

Governing Equations

Governing equations of system frequency dynamics

$$\underbrace{2H_{sys}\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}}_{\text{inertia}} + \underbrace{k_D P_d \Delta f(t)}_{\text{load damping}} = \underbrace{\Delta P_d}_{\text{imbalance}} - \underbrace{P_{sys}^{PR}(t)}_{\text{PFR}}$$

$$\Delta f(t)|_{t=0} = 0 \quad \textbf{(Initial condition)}$$

Total PFR power

$$P_{sys}^{PR}(t) = \sum_i \left[\underline{P_{g,i}^{PR}(t)} + \underline{P_{w,i}^{PR}(t)}\right]$$

$$T_{g,i}\frac{\mathrm{d}P_{g,i}^{PR}(t)}{\mathrm{d}t} + P_{g,i}^{PR}(t) = G_{g,i} I_{g,i} \Delta f(t)$$

$$P_{g,i}^{PR}(t)|_{t=0} = 0 \quad \textbf{(Initial condition)}$$

$$P_{w,i}^{PR}(t) = G_{w,i}\Delta f(t)$$

PFR power from thermal units

PFR power from wind farms

Notation:
- i: bus index
- k_D: load damping rate
- P_d: total power load
- ΔP_d: power imbalance
- T_g/T_w: response constant
- G_g/G_w: droop factor
- H_{sys}: total inertia
- Δf: frequency deviation
- P_{sys}^{PR}: total PFR power
- P_g^{PR}/P_w^{PR}: PFR power
- I_g: online/offline status

Frequency-Secured Unit Commitment

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_g^{PR}R_{g,i,\tau}^{PR}+c_w^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_s F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to:

- Piecewise linearization of fuel cost calculation
- Logic constraint of unit status
- Minimum online & offline time constraint
- **Frequency security constraint**
- Generation and ramping constraint
- Power balance constraint
- DC power flow constraint

Ref: B. Zhou, J. Fang, X. Ai, et al, "Partial-dimensional correlation-aided convex-hull uncertainty set for robust unit commitment," IEEE Transactions on Power Systems, 38(03), 2434-2446, 2023.

Frequency-Secured Unit Commitm

Notation:
s: scenario index
τ: period index
$c_{(\cdot)}$: cost coefficient
ω_s: probability of scenario s
P_w^A: available wind power
H_g/H_w: inertia constant
P_w: integrated wind power
R_g^{PR}/R_w^{PR}: PFR reserve

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_g^{PR}R^{PR}_{g,i,\tau}+c_w^{PR}R^{PR}_{w,i,\tau}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}$$

Subject to:

- Piecewise linearization of fuel cost calculation
- Logic constraint of unit status
- Minimum online & offline time constraint
- **Frequency security constraint**
- Generation and ramping constraint
- Power balance constraint
- DC power flow constraint

Governing equations

Frequency security metrics

$$H_{sys,\tau}=\sum_i\left(H_{g,i}I_{g,i,\tau}+H_{w,i}\right)$$

$$P^{PR}_{g,i,\tau}(t)\le R^{PR}_{g,i,\tau} \qquad P^{PR}_{w,i,\tau}(t)\le R^{PR}_{w,i,\tau}$$

$$0\le P^{s}_{w,i,\tau}\le P^{A,s}_{w,i,\tau}-R^{PR}_{w,i,\tau}$$

Ref: B. Zhou, J. Fang, X. Ai, et al, "Partial-dimensional correlation-aided convex-hull uncertainty set for robust unit commitment," IEEE Transactions on Power Systems, 38(03), 2434-2446, 2023.

DAE-Constrained Optimization

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_{g}^{PR}R_{g,i,\tau}^{PR}+c_{w}^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_s F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to: Two types of constraints

- **Discrete-time constraints** – mixed-integer linear equations, tractably handled by solvers
- **Continuous-time constraints – differential algebraic equations (DAE)**

$$2H_{sys}\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}+k_D P_d \Delta f(t)=\Delta P_d - P_{sys}^{PR}(t)$$

$$\Delta f(t)|_{t=0}=0$$

$$T_{g,i}\frac{\mathrm{d}P_{g,i}^{PR}(t)}{\mathrm{d}t}+P_{g,i}^{PR}(t)=G_{g,i}I_{g,i}\Delta f(t)$$

$$P_{g,i}^{PR}(t)|_{t=0}=0$$

$$\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}\le\overline{\dot{f}}$$

$$P_{w,i}^{PR}(t)=G_{w,i}\Delta f(t)$$

$$P_{g,i,\tau}^{PR}(t)\le R_{g,i,\tau}^{PR} \qquad P_{w,i,\tau}^{PR}(t)\le R_{w,i,\tau}^{PR}$$

$$\Delta f(t)|_{\text{nadir}}\le\overline{\Delta f} \qquad \Delta f(t)|_{\text{steady}}\le\overline{\Delta f_{err}}$$

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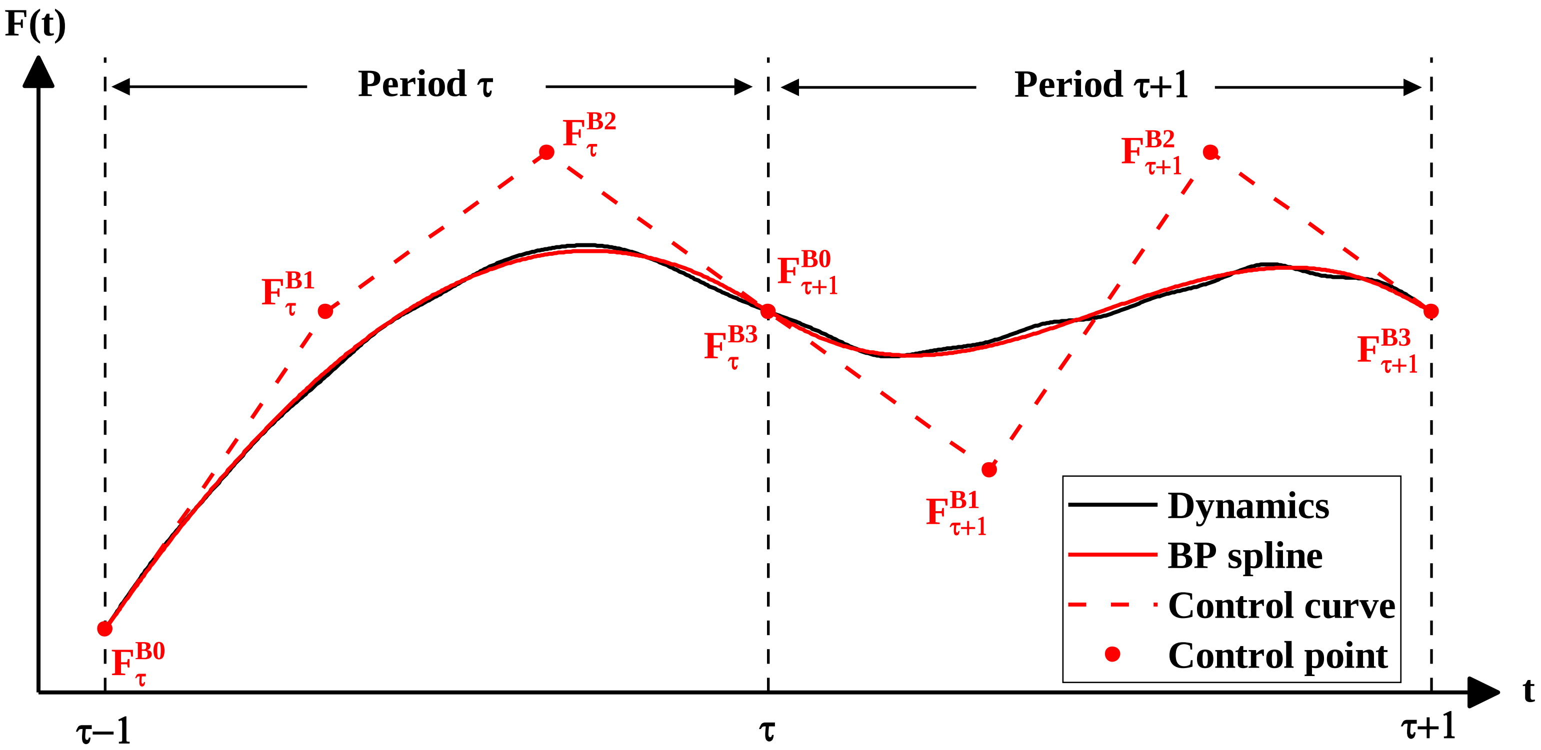
Bernstein Polynomial Approximation

Core idea: Use Bernstein polynomial (BP) spline to approximate dynamics



BP spline $$F(t) = \sum_{k=0}^{3} F^{B,k} B_{3,k}(t) = (F^B)^{\mathrm{T}} B_3(t), t \in [0,1]$$

Cubic BP $$B_{3,k}(t) \coloneqq \binom{3}{k} t^k (1-t)^{3-k}, t \in [0,1]$$

Transformation – Part 1

According to $F(t) = (F^B)^{\mathrm{T}} B_3(t)$, we have

- Integral term

  from 0 to 1

$$\int_0^1 F(t)\mathrm{d}t = (F^B)^{\mathrm{T}} \int_0^1 B_3(t)\,\mathrm{d}t = 1^{\mathrm{T}}\, F^B/4$$

- Derivative term

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = 3[F^{B,1} - F^{B,0}, F^{B,2} - F^{B,1}, F^{B,3} - F^{B,2}]\boldsymbol{B}_2(t) = \left(\boldsymbol{W}_3\boldsymbol{F}^{\boldsymbol{B}}\right)^{\mathrm{T}}\boldsymbol{B}_2(t)$$

quadratic BP

- Equality equation

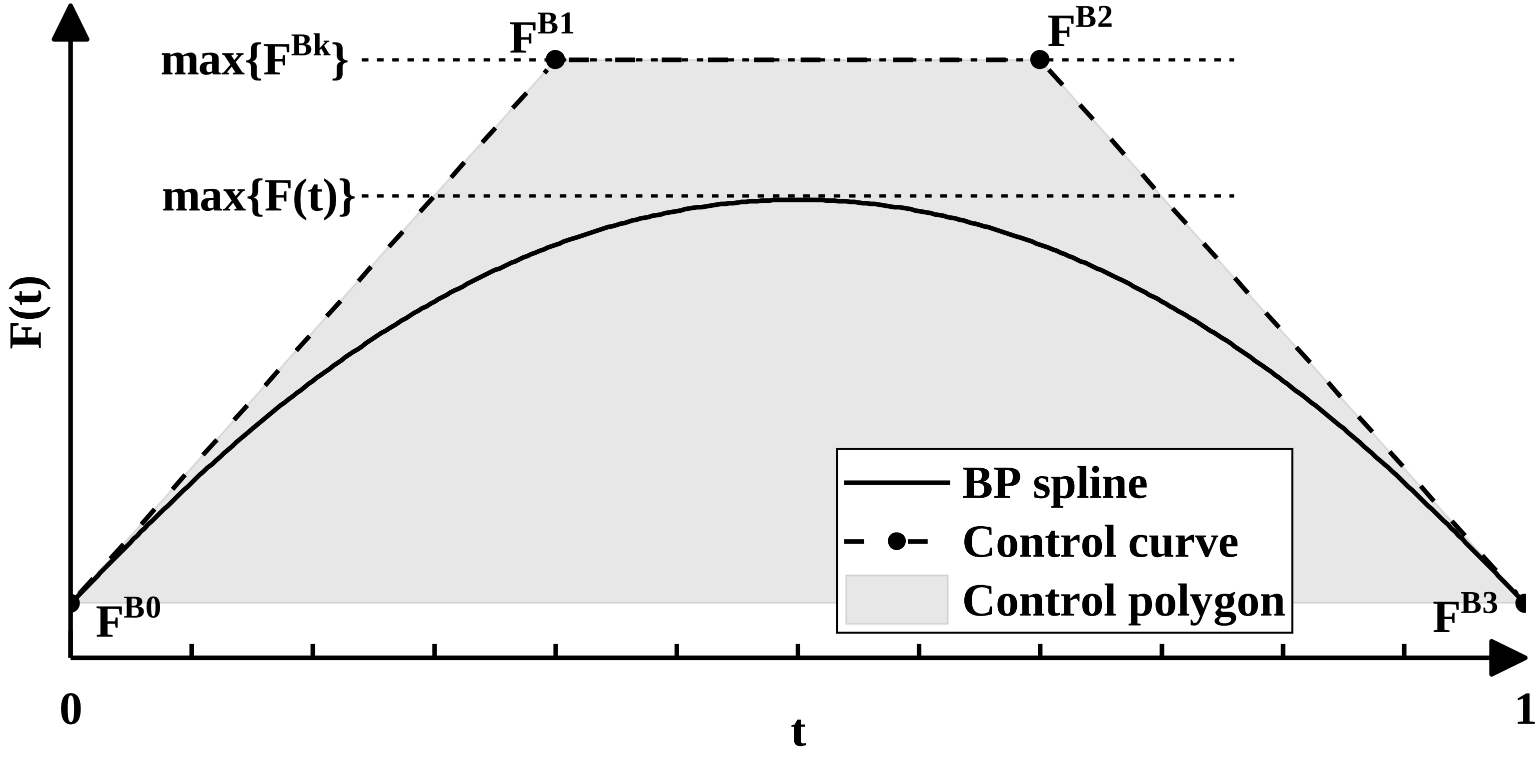
$$F(t) = 0 \Leftrightarrow \left(\boldsymbol{F}^{\boldsymbol{B}}\right)^{\mathrm{T}}\boldsymbol{B}_3(t) = 0 \Leftrightarrow F^{B,k} = 0$$

undetermined coefficient method

How about inequality equations and ODEs?

Convex-hull Property of BP

BP splines must be inside their corresponding control polygons

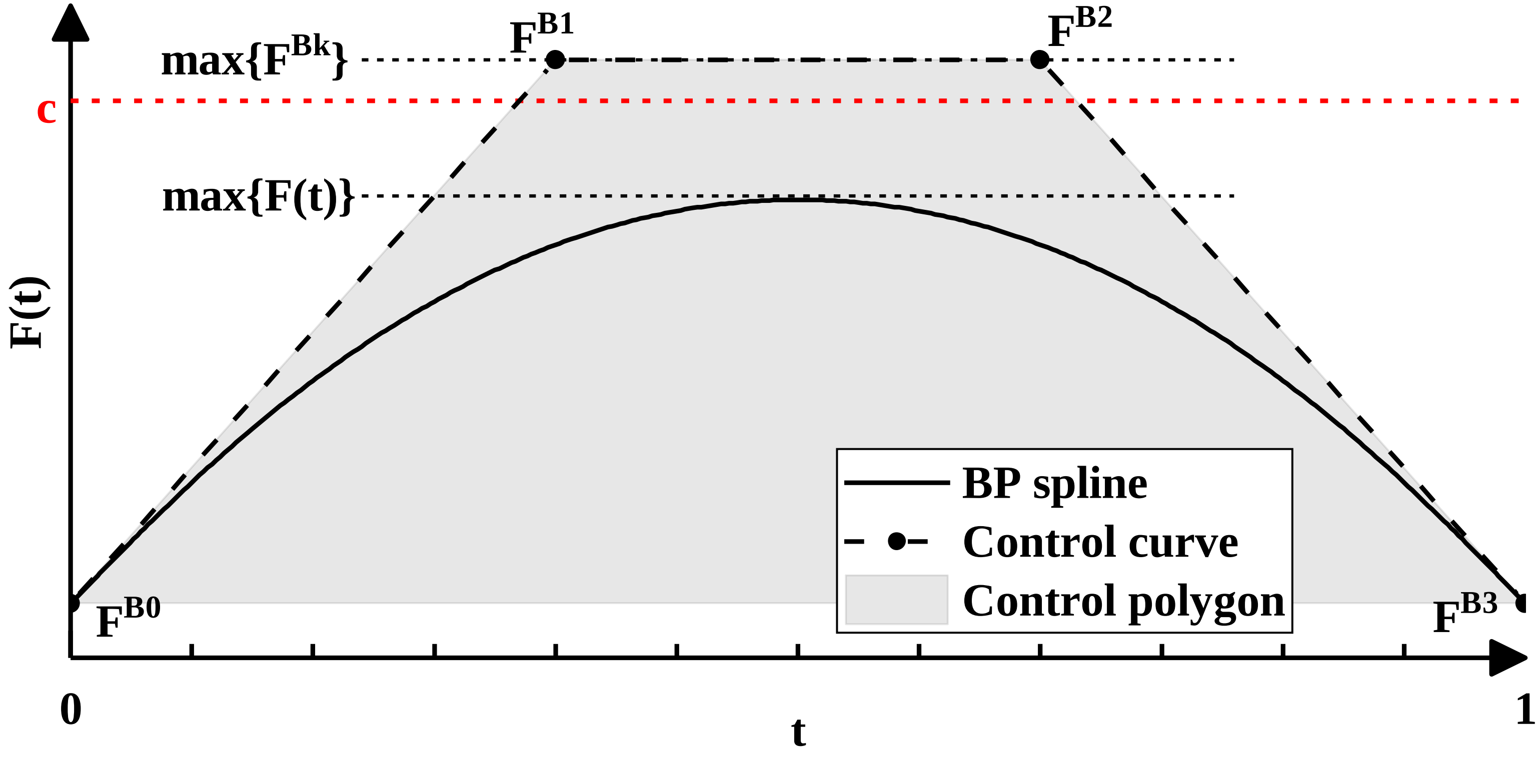


$$\min\{F^{B,k}\} \leq F(t) \leq \max\{F^{B,k}\}$$

- Inequality equation $F(t) \leq c \Leftarrow \max\{F^{B,k}\} \leq c \Leftrightarrow F^{B,k} \leq c$

Convex-hull Property of BP

There exists **gap** between $\max\{F(t)\}$ and $\max\{F^{B,k}\}$

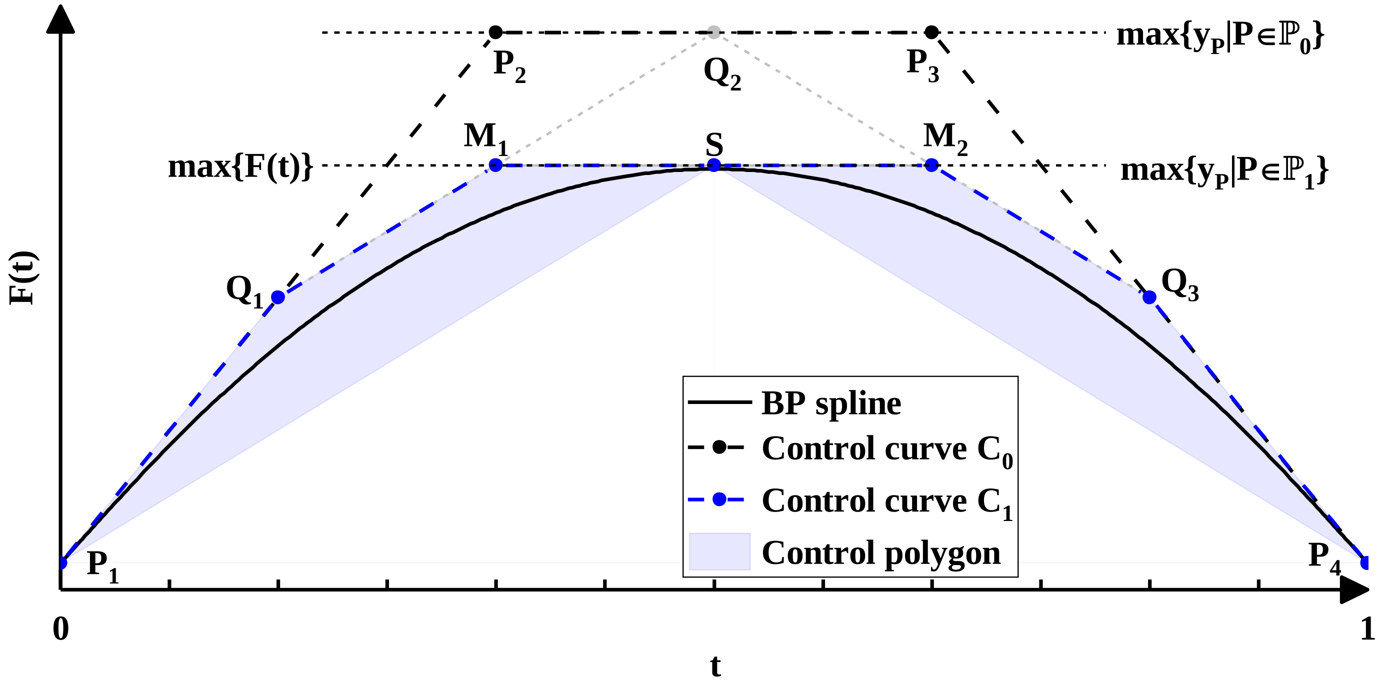


$$\min\{F^{B,k}\} \le F(t) \le \max\{F^{B,k}\}$$

- Inequality equation $\quad F(t) \le c \Leftarrow \max\{F^{B,k}\} \le c \Leftrightarrow F^{B,k} \le c$

Subdivision of BP

Break BP splines into several segments, then each segment is still a BP spline



Notation:

$\mathbb{P}_i$: set of control points after i subdivision

y_P: value of control point P

$Y^{(i)}$: vector of ordinate of control points after i subdivision

de Casteljau's algorithm

$$\begin{cases} \overrightarrow{\mathrm{P_1Q_1}} = t_0\overrightarrow{\mathrm{P_1P_2}}, \overrightarrow{\mathrm{P_2Q_2}} = t_0\overrightarrow{\mathrm{P_2P_3}}, \overrightarrow{\mathrm{P_3Q_3}} = t_0\overrightarrow{\mathrm{P_3P_4}} \\ \overrightarrow{\mathrm{Q_1M_1}} = t_0\overrightarrow{\mathrm{Q_1Q_2}}, \overrightarrow{\mathrm{Q_2M_2}} = t_0\overrightarrow{\mathrm{Q_2Q_3}} \\ \overrightarrow{\mathrm{M_1}S} = t_0\overrightarrow{\mathrm{M_1M_2}} \end{cases}$$

Assume $Y^{(0)} = F^B$, we have $Y^{(1)} = AY^{(0)}$

$$\boldsymbol{A_l} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 - t_0 & t_0 & 0 & 0 \\ (1 - t_0)^2 & 2\tau_0(1 - t_0) & t_0^2 & 0 \\ (1 - t_0)^3 & 3\tau_0(1 - t_0)^2 & 3t_0^2(1 - t_0) & t_0^3 \end{bmatrix}$$

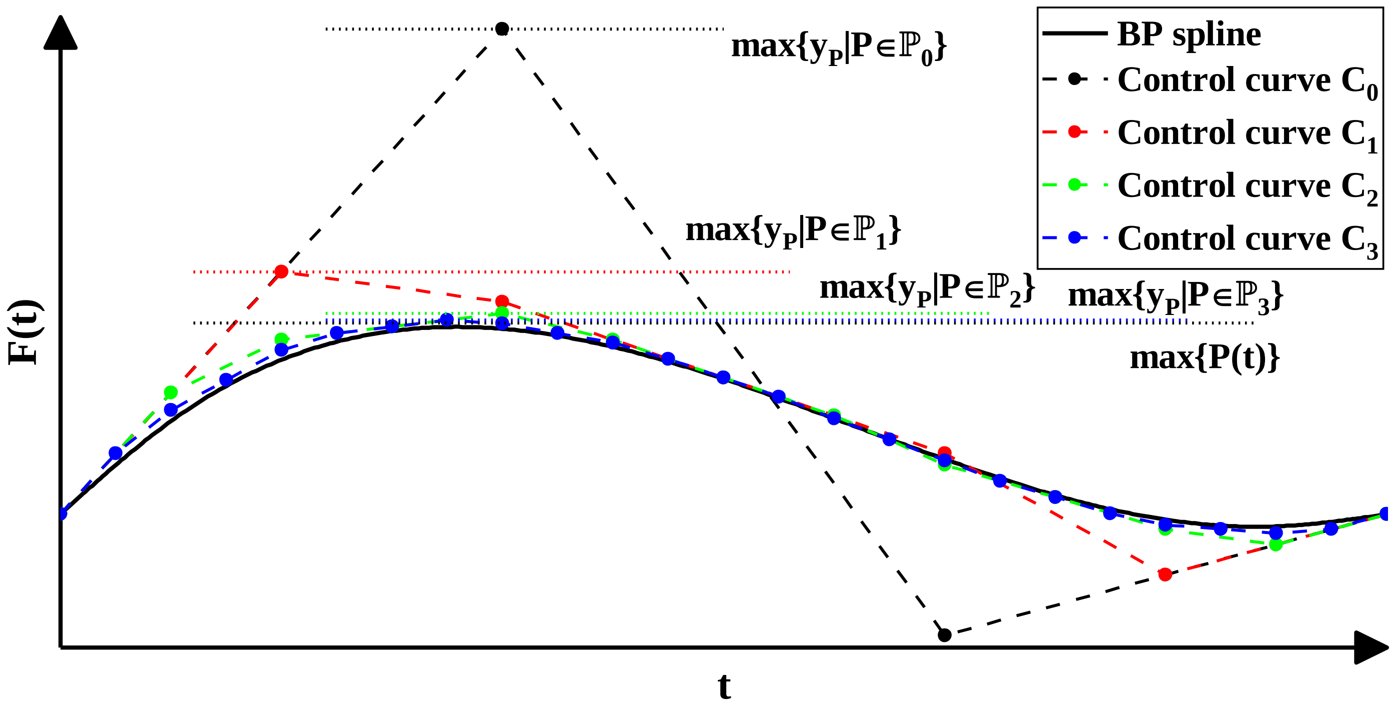
$$\boldsymbol{A_r} = \begin{bmatrix} (1 - t_0)^3 & 3\tau_0(1 - t_0)^2 & 3t_0^2(1 - t_0) & t_0^3 \\ 0 & (1 - t_0)^2 & 2t_0(1 - t_0) & t_0^2 \\ 0 & 0 & 1 - t_0 & t_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A} = \left[\boldsymbol{A}_l^{\mathrm{T}}, \boldsymbol{A}_r^{\mathrm{T}}\right]^{\mathrm{T}}$$

Ref: W. Boehm and A. Mller, "On de Casteljau's algorithm," Computer Aided Geometric Design, vol. 16, no. 7, pp. 587–605, 1999.

Transformation – Part 2

By **repeatedly** implementing the subdivision property, we can narrow the gap between BP splines and control curves



$$Y^{(m)} = D^{(m)}Y^{(m-1)}, m \geq 1$$

$$D^{(m)} = \begin{bmatrix} A & & \\ & \ddots & \\ & & A \end{bmatrix} (2^{m-1}A)$$

$$Y^{(m)} = D^{(m)} \cdots D^{(1)}Y^{(0)} = E^{(m)}Y^{(0)}$$

eliminate repeated control points

$$Y^{(m)} = J^{(m)}Y^{(0)}$$

enhancement matrix

$$\max\{F(t)\} \leq \max\{y_P | P \in \mathbb{P}_m\} \longrightarrow \max\{F(t)\} \leq \max\{J^{(m)}F^B\}$$

➢ **Inequality equation** $F(t) \leq c \Leftarrow J^{(m)}F^B \leq c$

Operation Matrix of BP & Transformation – Part 3

Basic idea: Use $(n-1)$th-order polynomial to approximate nth-order terms

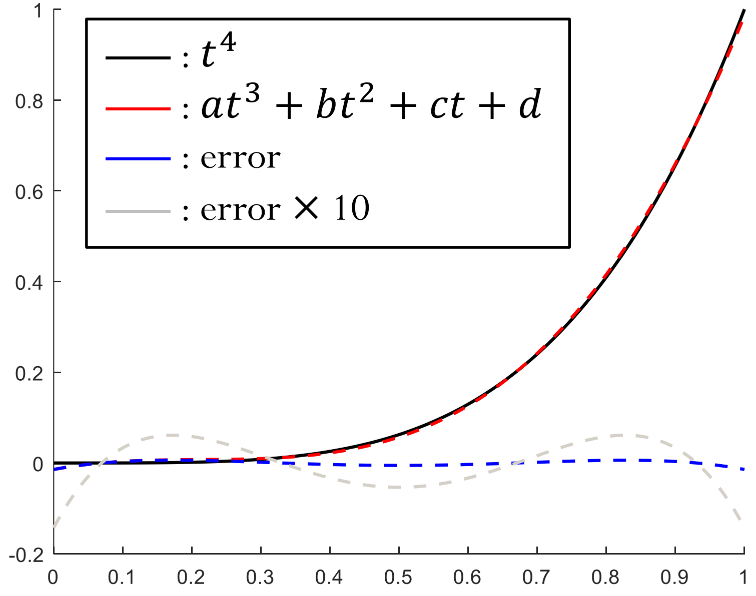
$$t^4 \approx at^3 + bt^2 + ct + d$$

$$\int_0^t B_3(t)\,\mathrm{d}t \approx \boldsymbol{L} B_3(t)$$

operational matrix

- **Integral term** from 0 to t

$$\int_0^t \boldsymbol{F}(\boldsymbol{t})\mathbf{d}\boldsymbol{t} = \left(\boldsymbol{F}^{\boldsymbol{B}}\right)^{\boldsymbol{T}} \int_0^t \boldsymbol{B}_3(\boldsymbol{t})\,\mathbf{d}\boldsymbol{t} \approx \left(\boldsymbol{F}^{\boldsymbol{B}}\right)^{\boldsymbol{T}} \boldsymbol{L} \boldsymbol{B}_3(\boldsymbol{t})$$



Maximum error: 0.006122

Average error: 0.004104

Ref: S. A. Yousefi, M. Behroozifar. Operational matrices of Bernstein polynomials and their applications. International Journal of Systems Science. 2010, 41(6): 709-716

Transformation Rules

According to $F(t) = (F^B)^{\mathrm{T}} B_3(t)$, we have

- Integral term from 0 to 1 $\quad \int_0^1 F(t)\mathrm{d}t = 1^{\mathrm{T}}\, F^B/4$

- Integral term from 0 to t $\quad \int_0^t F(t)\mathrm{d}t \approx (F^B)^{\mathrm{T}} L B_3(t)$

- Derivative term $\quad \frac{\mathrm{d}F(t)}{\mathrm{d}t} = (W_3 F^B)^T B_2(t)$

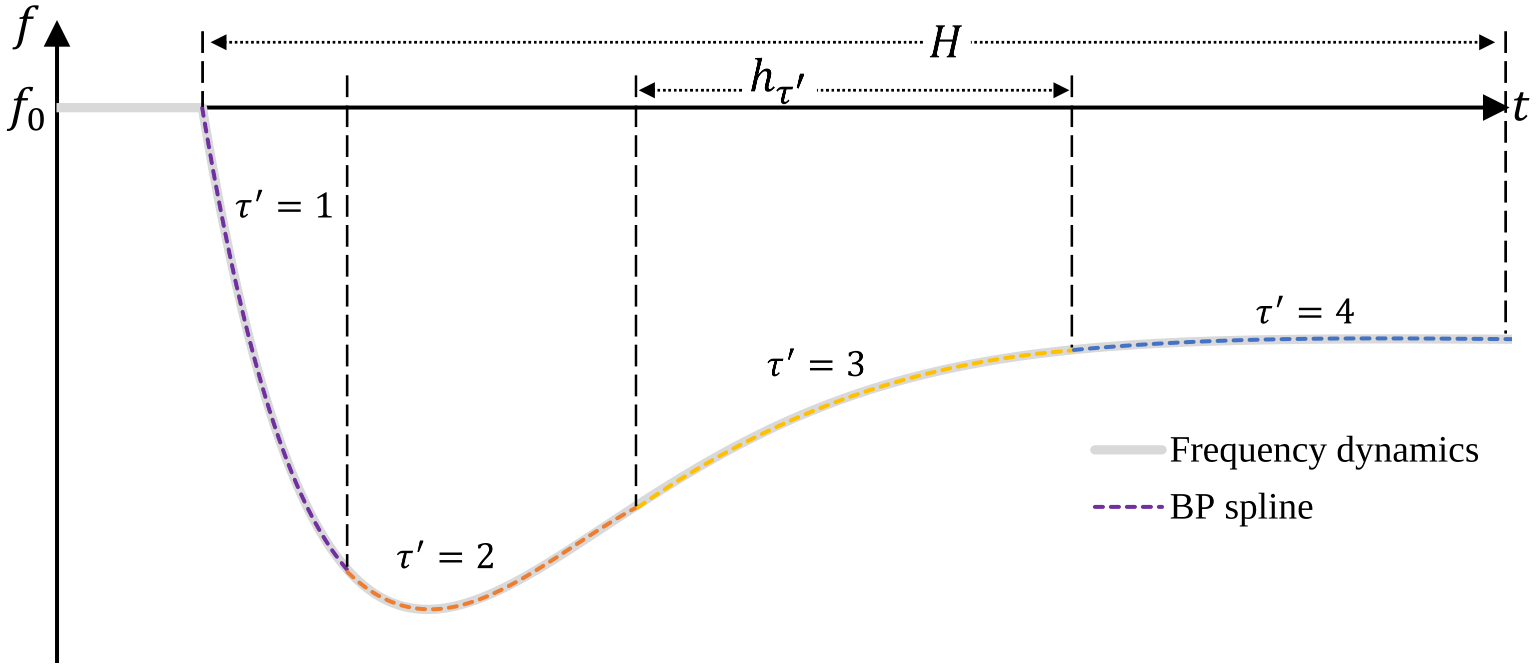
- Equality equation $\quad F(\tau) = 0 \Leftrightarrow F^{B,k} = 0$

- Inequality equation $\quad F(t) \leq c \Leftarrow J^{(m)} F^B \leq c$

Segment-wise Approximation

1-segment BP approximation is not able to fit frequency dynamics for infinitely long

- A multi-segment BP approximation produces better performance
- The number of constraints increases linearly with the number of segments.



We suggest an **uneven division of H**

➢ the early segments should be shorter for a more accurate estimate of nadir

➢ the latter segments should be longer to relieve computational burden

Implementation in Frequency Dynamics

Transformation of governing equations

$$2H_{sys}\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}+k_D P_d \Delta f(t)=\Delta P_d - P_{sys}^{PR}(t)$$
$$\Delta f(t)|_{t=0}=0$$

$$\int_0^t\left[\frac{2H_{sys}}{h_{\tau'}}\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}+k_D P_d \Delta f(t)\right]\mathrm{d}t=\int_0^t\left[\Delta P_d-\Delta P_{sys}^{PR}(t)\right]\mathrm{d}t$$

$$\frac{2H_{sys}}{h_{\tau'}}\left(\Delta f_{\tau'}^B-\Delta f_{\tau',ini}^B\right)+k_D P_d L^{\mathrm{T}}\Delta f_{\tau'}^B=L^{\mathrm{T}}\left(\Delta P_d^B-P_{sys,\tau'}^{PR,B}\right)$$
$$\Delta f_{\tau',ini}^{B,k}|_{\tau'=1}=\Delta f_{DB},\ \Delta f_{\tau',ini}^{B,k}|_{\tau'>1}=\Delta f_{\tau'-1}^{B,3}$$

$$T_{g,i}\frac{\mathrm{d}P_{g,i}^{PR}(t)}{\mathrm{d}t}+P_{g,i}^{PR}(t)=G_{g,i}I_{g,i}\Delta f(t)$$
$$P_{g,i}^{PR}(t)|_{t=0}=0$$

$$\frac{T_{g,i}}{h_{\tau'}}\left(P_{g,i,\tau'}^{PR,B}-P_{g,i,\tau',ini}^{PR,B}\right)+L^{\mathrm{T}}P_{g,i,\tau'}^{PR,B}=G_{g,i}I_{g,i}L^{\mathrm{T}}\left(\Delta f_{\tau'}^B-\Delta f_{DB}^B\right)$$
$$P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'=1}=0,\ P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'>1}=P_{g,i,\tau'-1}^{PR,B,3}$$

$$P_{w,i}^{PR}(t)=G_{w,i}\Delta f(t)$$

$$P_{w,i,\tau'}^{PR,B}=G_{w,i}\left(\Delta f_{\tau'}^B-\Delta f_{DB}^B\right)$$

Implementation in Frequency Dynamics

Frequency security metrics

$$\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t} \leq \overline{\dot{f}} \quad \Rightarrow \quad 2\overline{\dot{f}} H_{sys,\tau} \geq \Delta P_{d,\tau}$$

$$\Delta f(t)|_{\text{nadir}} \leq \overline{\Delta f} \quad \Rightarrow \quad J\Delta f_{\tau'}^{B} \leq \overline{\Delta f}$$

$$\Delta f(t)|_{\text{steady}} \leq \overline{\Delta f_{err}} \quad \Rightarrow \quad k_D P_d \overline{\Delta f_{err}} + G_{sys,\tau}\left(\overline{\Delta f_{err}} - \Delta f_{DB}\right) \geq \Delta P_{d,\tau}$$

PFR reserve

$$P_{w,i,\tau}^{PR}(t) \leq R_{w,i,\tau}^{PR} \quad \Rightarrow \quad JP_{g,i,\tau,\tau'}^{PR,B} \leq R_{g,i,\tau}^{PR}$$

$$P_{g,i,\tau}^{PR}(t) \leq R_{g,i,\tau}^{PR} \quad \Rightarrow \quad JP_{w,i,\tau,\tau'}^{PR,B} \leq R_{w,i,\tau}^{PR}$$

Mixed-integer linear reformulation

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_g^{PR}R_{g,i,\tau}^{PR}+c_w^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_s F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to:

- **Discrete-time constraints** – mixed-integer linear equations, tractably handled by solvers
- **Continuous-time constraints → mixed-integer linear equations**

$$\frac{2H_{sys}}{h_{\tau'}}\left(\Delta f_{\tau'}^{B}-\Delta f_{\tau',ini}^{B}\right)+k_D P_d L^{\mathrm{T}}\Delta f_{\tau'}^{B}=L^{\mathrm{T}}\left(\Delta P_d^{B}-P_{sys,\tau'}^{PR,B}\right)$$

$$\Delta f_{\tau',ini}^{B,k}|_{\tau'=1}=\Delta f_{DB},\ \ \Delta f_{\tau',ini}^{B,k}|_{\tau'>1}=\Delta f_{\tau'-1}^{B,3}$$

$$\frac{T_{g,i}}{h_{\tau'}}\left(P_{g,i,\tau'}^{PR,B}-P_{g,i,\tau',ini}^{PR,B}\right)+L^{\mathrm{T}}P_{g,i,\tau'}^{PR,B}=G_{g,i}I_{g,i}L^{\mathrm{T}}\left(\Delta f_{\tau'}^{B}-\Delta f_{DB}^{B}\right)$$

$$P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'=1}=0,\ \ P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'>1}=P_{g,i,\tau'-1}^{PR,B,3}$$

$$P_{w,i,\tau'}^{PR,B}=G_{w,i}\left(\Delta f_{\tau'}^{B}-\Delta f_{DB}^{B}\right)$$

$$2\bar{\dot{f}}H_{sys,\tau}\geq\Delta P_{d,\tau}\qquad J\Delta f_{\tau'}^{B}\leq\overline{\Delta f}$$

$$k_D P_d\overline{\Delta f_{err}}+G_{sys,\tau}\left(\overline{\Delta f_{err}}-\Delta f_{DB}\right)\geq\Delta P_{d,\tau}$$

$$JP_{g,i,\tau,\tau'}^{PR,B}\leq R_{g,i,\tau}^{PR}\qquad JP_{w,i,\tau,\tau'}^{PR,B}\leq R_{w,i,\tau}^{PR}$$

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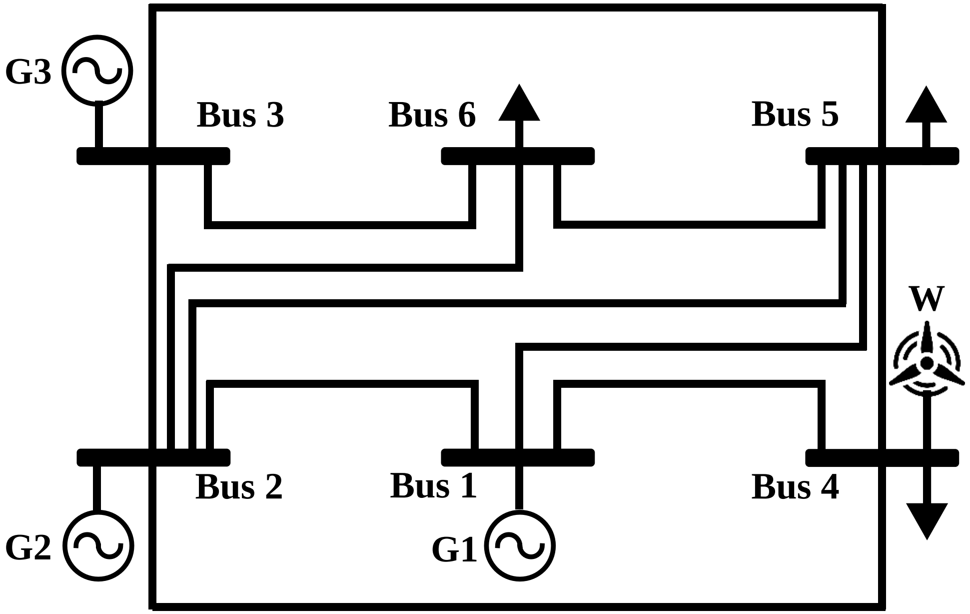
Set up

IEEE 6-bus system



Stochastic UC (power imbalance of 5% load in each period)

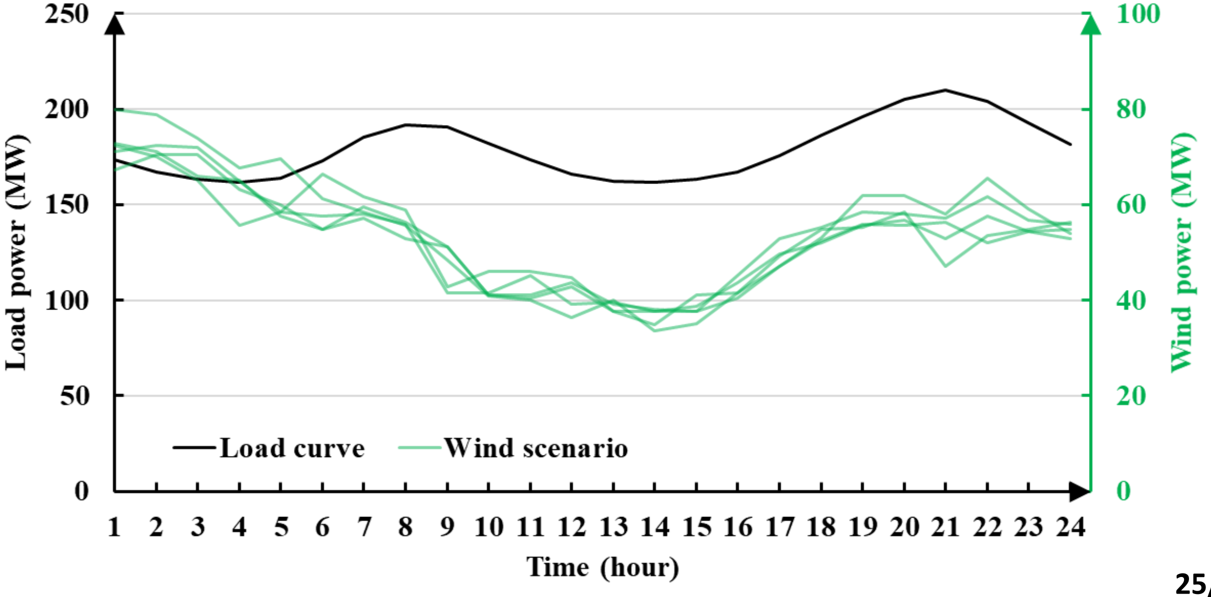
Case 1: without frequency constraints

Case 2: with the proposed DAE frequency constraints

(RoCoF: 0.5 Hz/s, nadir: 0.5 Hz, steady: 0.3 Hz)

MAIN TECHNICAL PARAMETERS

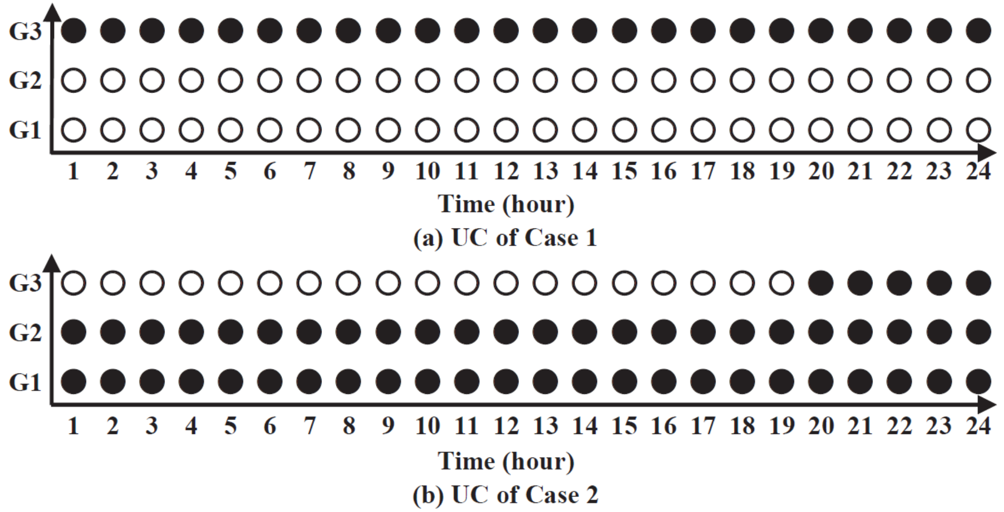
Units	G1	G2	G3	W
Capacity (MW)	200	150	180	80
Minimum generation (MW)	60	45	54	\
Inertia constant (s)	8	5	6	5
Response constant (s)	10	4	6	0
Droop factor (MW/Hz)	20	25	18	20



Stochastic scenarios

Effectiveness

Overall results – UC and costs



Optimal UC in the two cases

- Case 1 violates the frequency security limits
- In Case 2, G1 and G2 are always online and G3 is started up in hour 20

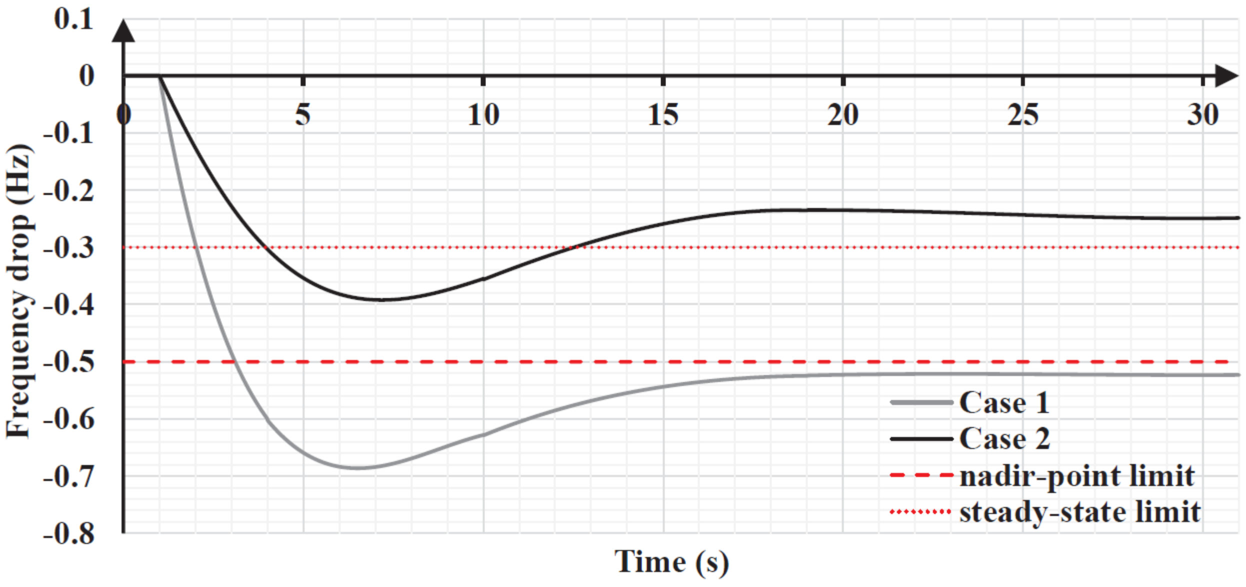
Overall comparison results

Comparison terms	Case 1	Case 2
Startup&shutdown cost ($)	0	5400
PFR reserve cost of units ($)	0	5102.38
Expected fuel cost ($)	41328.08	46944.19
PFR reserve cost of WF ($)	0	947.90
Total cost ($)	41328.08	58394.47
Total PFR reserve of units (MWh)	0	340.16
Total PFR reserve of WF (MWh)	0	189.58
Maximum RoCoF (Hz/s)	0.3547	0.1784
Maximum Δf at the nadir (Hz)	0.6861	0.4474
Maximum Δf at the steady state (Hz)	0.5237	0.2930
	(hour 21)	**(hour 19)**

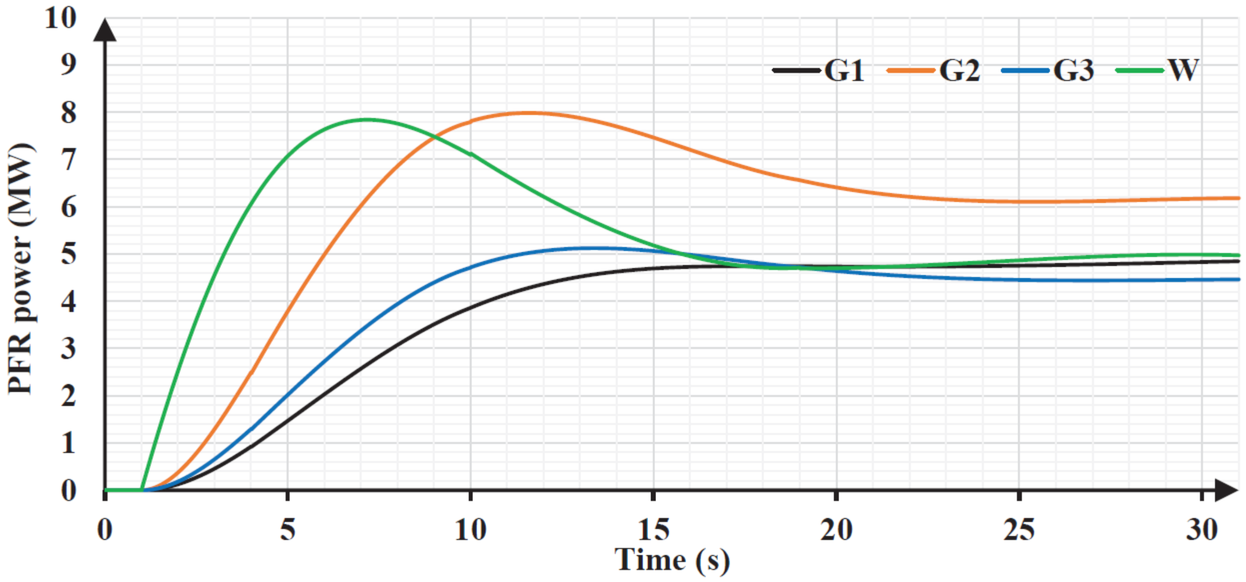
(RoCoF: 0.5 Hz/s, nadir: 0.5 Hz, steady: 0.3 Hz)

Effectiveness

Detailed results – Frequency dynamics in hour 21



Comparison of Frequency Dynamics in Both Cases
(RoCoF: 0.5 Hz/s, nadir: 0.5 Hz, steady: 0.3 Hz)

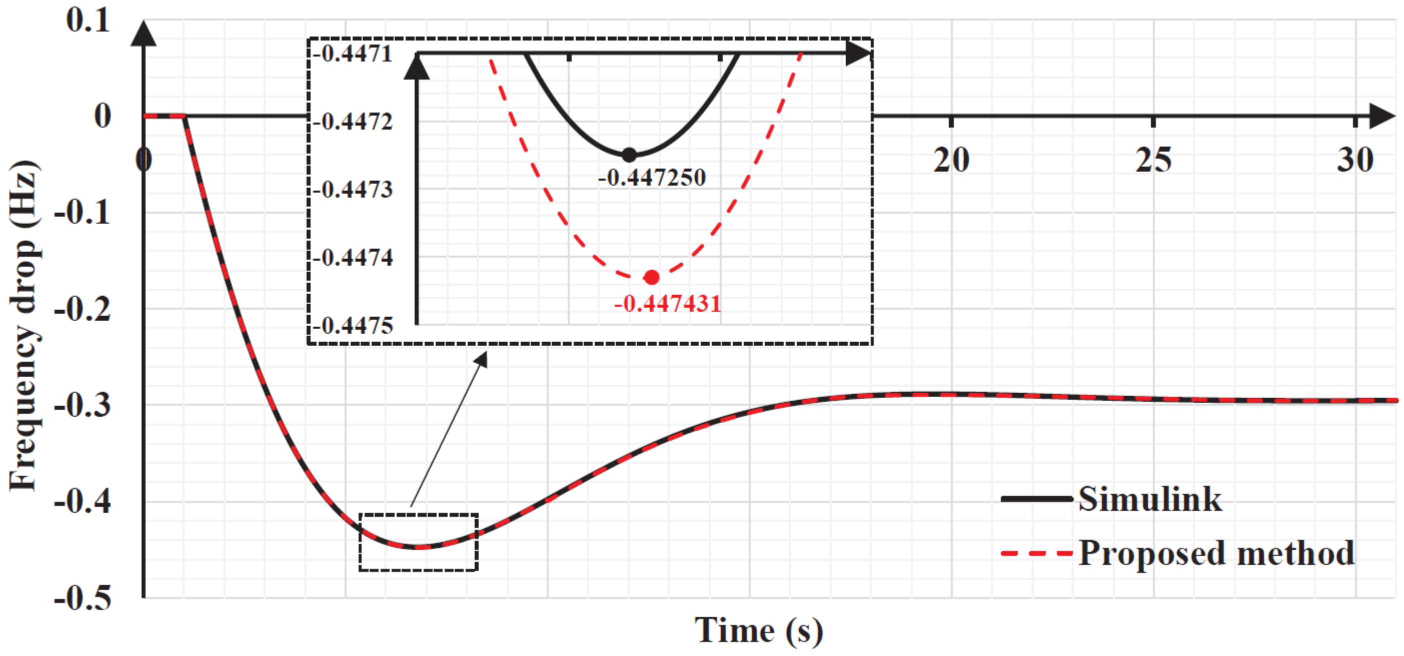


PFR Power Dynamics in Case 2

The proposed method can effectively guarantee frequency security

Accuracy

Comparison with numerical simulation (Simulink) – hour 19 of Case 2



Frequency Dynamics From the Proposed Approximation vs. From Simulink
(RoCoF: 0.5 Hz/s, nadir: 0.5 Hz, steady: 0.3 Hz)

Highly accurate! The relative error in evaluating the nadir-point frequency is about 0.04%

Scalability

Scalability analysis in the IEEE 118-bus system

Overall Comparison in IEEE 118-bus system

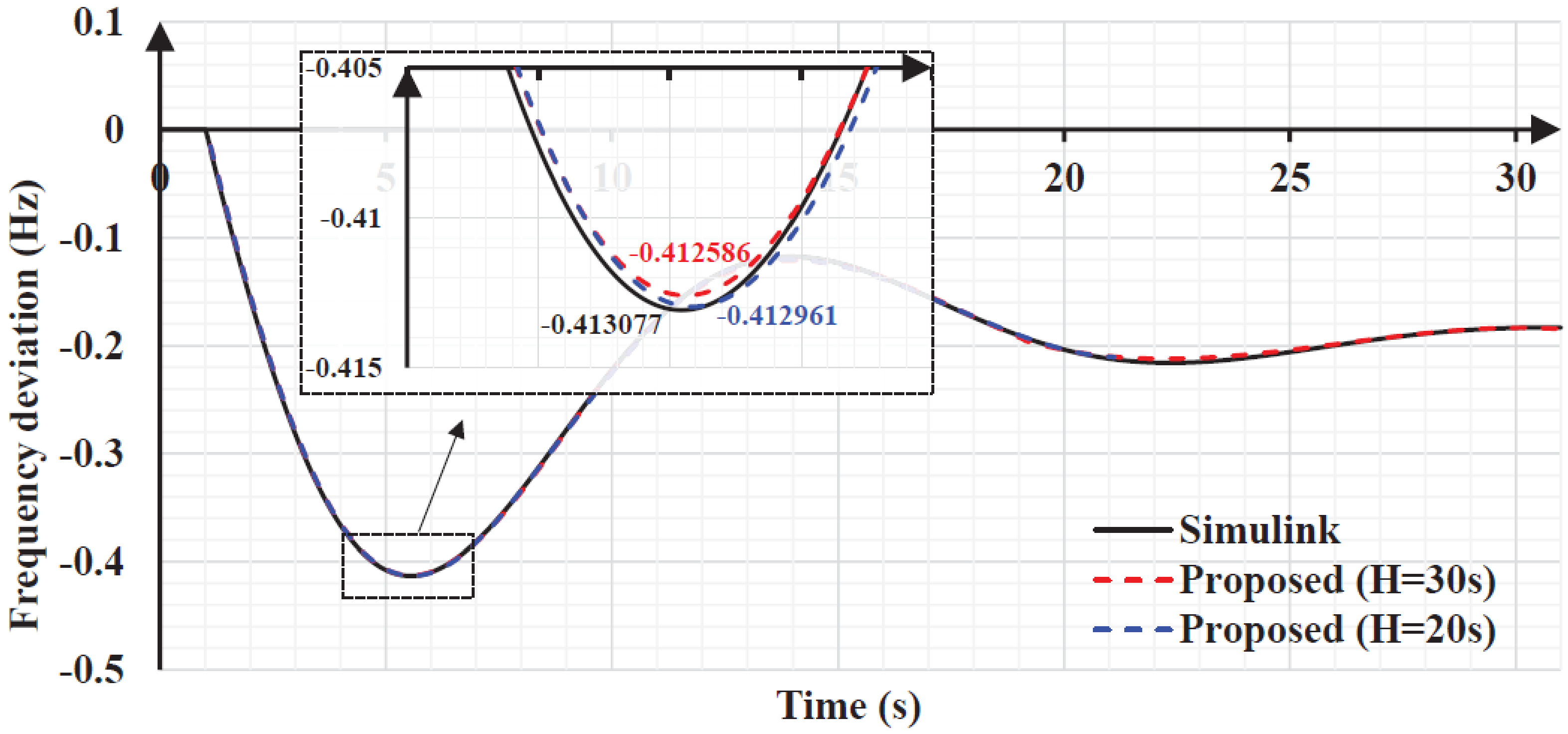
Comparison terms	Case 1	Case 2
Operation cost of thermal units (k$)	2184.18	2394.85
PFR reserve cost of thermal units (k$)	0	155.65
PFR reserve cost of wind farm (k$)	0	9.81
Total cost (k$)	2184.18	2560.32
Maximum RoCoF (Hz/s)	0.2598	0.1696
Maximum Δf at the nadir (Hz)	0.7748	0.4318
Maximum Δf at the quasi-steady state (Hz)	0.4950	0.1935
Computation time (s)	49.98	2472

(RoCoF: 0.5 Hz/s, nadir: 0.5 Hz, steady: 0.3 Hz)

The computation time of the proposed method is about 40 minutes, which is acceptable for day-ahead scheduling

Scalability

Comparison with numerical simulation (Simulink)



Frequency Dynamics From the Proposed Approximation vs. From Simulink

Still highly accurate! Relative error: 0.12% (30s), 0.03% (20s)

Outline

- ❑Introduction and Problem Formulation
- ❑Solution Method
- ❑Case Study
- ❑**Conclusions**

Conclusion

- We incorporated the frequency dynamics using DAEs into the stochastic UC model and validated the effectiveness in deciding UC and PFR reserves for frequency security

- We adopted BP splines to obtain a linear approximation of the DAEs and demonstrated the high accuracy in depicting frequency dynamics

- Future works: transformation of nonlinear constraints

Ref: Bo Zhou, Ruiwei Jiang, Siqian Shen, "Frequency-Secured Unit Commitment:Tight Approximation using Bernstein Polynomials," arXiv preprint, arXiv:2212.12088, 2022.

Thank You for Attention!