Differential-Algebraic Equation-Constrained Frequency-Secured Stochastic Unit Commitment

Bo Zhou, Ruiwei Jiang, Siqian Shen

Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, MI

Introduction

With the increasing share of renewable energy, conventional synchronous generators are being replaced, which threatens the frequency security of power systems and requires a more careful plan and operations of unit commitment and primary frequency response reserve. In this paper, we propose a novel differential-algebraic equations (DAE)-constrained frequency-secured stochastic unit commitment for the first time. A continuous-time function is used to model the frequency variation and DAEs for frequency dynamics are directly incorporated into the formulation of stochastic unit commitment. To solve the established DAE-constrained optimization, we adopt Bernstein polynomial spline-based solution space transformation and operational matrices and reformulate DAEs into general algebraic constraints.

Contribution:

- We directly address the DAEs for frequency dynamics and propose a DAE-constrained frequency-secured stochastic UC.
- We adopt solution space transformation (SST) and operational matrices to reformulate the DAE-constrained formulation for tractable solution.

Problem Formulation

General process of frequency dynamics under a sudden loss of generation:

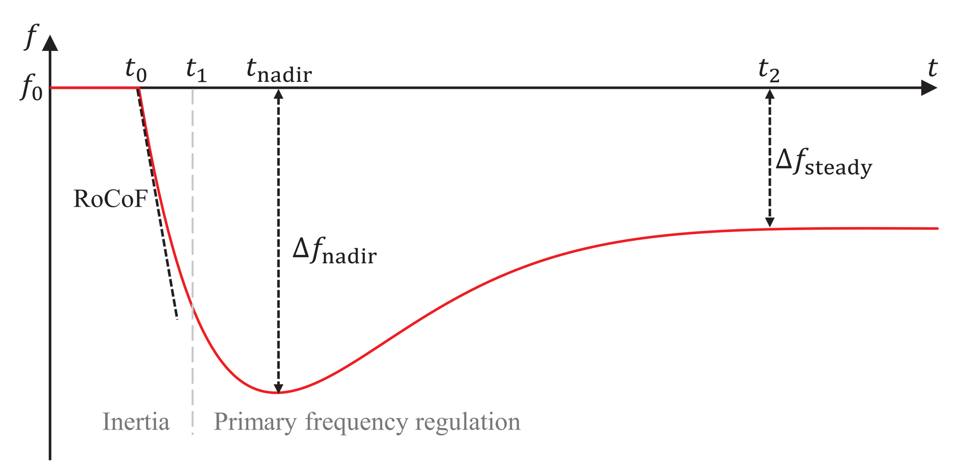

Figure 1. General Process of Frequency Dynamics.

The maximum RoCoF occurs at t_0. During the whole dynamics, frequency first drops and is then lifted, yielding a nadir with deviation $\Delta f_{\mathbf{nadir}}$ at $t_{\mathbf{nadir}}$.

$$2H_{sys}\frac{\mathbf{d}\Delta f(t)}{\mathbf{d}t} + k_D P_d \Delta f(t) = \Delta P_d - P^{PR}_{sys}(t) \tag{1a}$$
$$\Delta f(t)|_{t=0} = 0, \tag{1b}$$

$$P^{PR}_{sys}(t) = \sum_i \left[P^{PR}_{g,i}(t) + P^{PR}_{w,i}(t)\right] \tag{2a}$$
$$T_{g,i}\frac{\mathbf{d}P^{PR}_{g,i}(t)}{\mathbf{d}t} + P^{PR}_{g,i}(t) = G_{g,i} I_{g,i}\Delta f(t) \tag{2b}$$
$$P^{PR}_{g,i}(t)|_{t=0} = 0 \tag{2c}$$
$$P^{PR}_{w,i}(t) = G_{w,i}\Delta f(t), \tag{2d}$$

$$\frac{\mathbf{d}\Delta f(t)}{\mathbf{d}t} \le \bar{\bar{f}} \tag{3a}$$
$$\Delta f(t)|_{\mathbf{nadir}} \le \overline{\Delta f} \tag{3b}$$
$$\Delta f(t)|_{\mathbf{steady}} \le \overline{\Delta f_{err}} \tag{3c}$$

Solution Method

***Continuous-time (CT) optimization*:**

We restrict the functional form of the variables in the DAE constraints using Bernstein polynomial (BP) splines:

$$F(t) = \sum_{k=0}^{3} F^{B,k} B_{3,k}(t) = (F^B)^{\mathrm{T}} B_3(t),\ t \in [0, 1] \tag{4}$$

where $B_{3,k}(t)$ is a cubic BP, and F^B are the spline coefficients.

Using the properties of BP splines, we recast the derivative, integral, and constraints on $F(t)$ as follows:

$$\frac{\mathbf{d}F(t)}{\mathbf{d}t} = \frac{\mathbf{d}(F^B)^{\mathrm{T}}B_3(t)}{\mathbf{d}t} = (WF^B)^{\mathrm{T}}B_2(t) \tag{5a}$$
$$F(t) = 0 \Leftrightarrow (F^B)^{\mathrm{T}}B_3(t) = 0 \Leftrightarrow F^B = 0 \tag{5b}$$
$$F(t) \le 0 \Leftrightarrow (F^B)^{\mathrm{T}}B_3(t) \le 0 \Leftarrow JF^B \le 0 \tag{5c}$$
$$\int_0^t F(t)\mathbf{d}t = \int_0^t (F^B)^{\mathrm{T}}B_3(t)\mathbf{d}t \approx (F^B)^{\mathrm{T}}LB_3(t), \tag{5d}$$

where $B_{2,k}(t)$ represents quadratic BPs, W, J, and L are matrices of known entries.

***DAE Approximation*:**

We apply the CT optimization framework to derive a linear and conservative approximation of the DAE constraints.

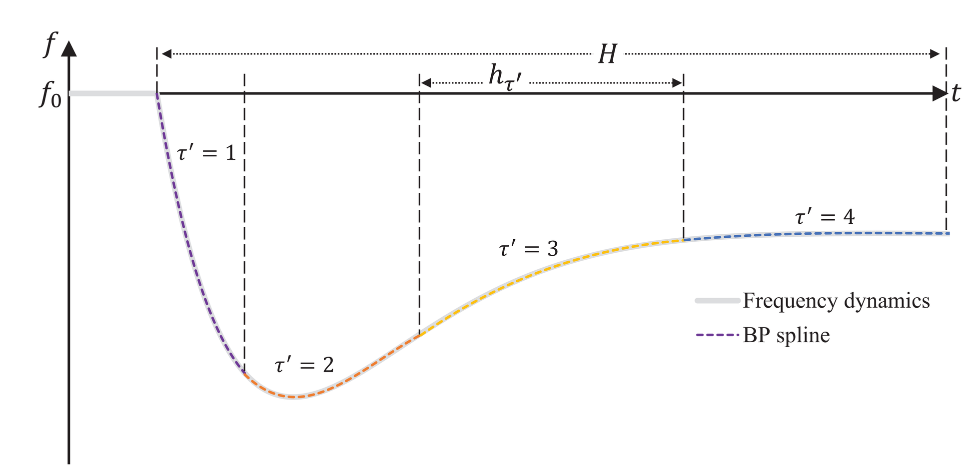

Figure 2. BP Spline-Based Frequency Dynamics

The horizon H is divided into several periods with a time length $h_{\tau'}$, and for each period, we have

$$\frac{2H_{sys}}{h_{\tau'}}\left(\Delta f^B_{\tau'} - \Delta f^B_{\tau',ini}\right) + k_D P_d L^{\mathrm{T}}\Delta f^B_{\tau'} = L^{\mathrm{T}}\left(\Delta P^B_d - P^{PR,B}_{sys,\tau'}\right) \tag{6a}$$
$$\Delta f^{B,k}_{\tau',ini}|_{\tau'=1} = 0,\ \Delta f^{B,k}_{\tau',ini}|_{\tau'>1} = \Delta f^{B,3}_{\tau'-1}, \tag{6b}$$

$$P^{PR,B}_{sys,\tau'} = \sum_i \left(P^{PR,B}_{g,i,\tau'} + P^{PR,B}_{w,i,\tau'}\right) \tag{7a}$$
$$\frac{T_{g,i}}{h_{\tau'}}\left(P^{PR,B}_{g,i,\tau'} - P^{PR,B}_{g,i,\tau',ini}\right) + L^{\mathrm{T}}P^{PR,B}_{g,i,\tau'} = G_{g,i}I_{g,i}L^{\mathrm{T}}\Delta f^B_{\tau'} \tag{7b}$$
$$P^{PR,B,k}_{g,i,\tau',ini}|_{\tau'=1} = 0,\ P^{PR,B,k}_{g,i,\tau',ini}|_{\tau'>1} = P^{PR,B,3}_{g,i,\tau'-1} \tag{7c}$$
$$P^{PR,B}_{w,i,\tau'} = G_{w,i}\Delta f^B_{\tau'}, \tag{7d}$$

$$2\bar{\bar{f}}H_{sys} \ge \Delta P_d \tag{8a}$$
$$J\Delta f^B_{\tau'} \le \overline{\Delta f} \tag{8b}$$
$$k_D P_d \overline{\Delta f_{err}} + \sum_i \left(G_{g,i}I_{g,i,\tau} + G_{w,i}\right)\overline{\Delta f_{err}} \ge \Delta P_{d,\tau} \tag{8c}$$

Case Studies

We use the IEEE 6-bus system for analysis. The maximum admitted RoCoF is set as 0.5 Hz/s and the maximum admitted frequency deviation at the nadir and at the steady state are set as 0.5 Hz and 0.3 Hz, respectively. With the above parameter settings, we compare two cases in stochastic UC:

(Case 1) without frequency constraints; and

(Case 2) with the proposed DAE frequency constraints.

Table 1. Overall Comparison Results

Comparison terms	Case 1	Case 2
Maximum RoCoF (Hz/s)	0.3547	0.1784
Maximum Δf at the nadir (Hz)	0.6861	0.4474
Maximum Δf at the steady state (Hz)	0.5237	0.2930
Total cost (\$)	41328.08	58394.47
Computation time (s)	4.66	6.73

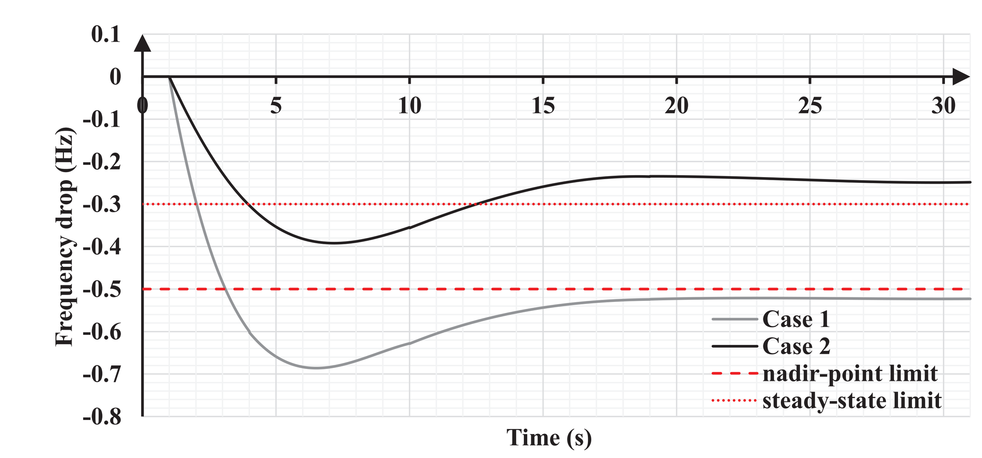

Figure 3. Comparison of Frequency Dynamics in Both Cases

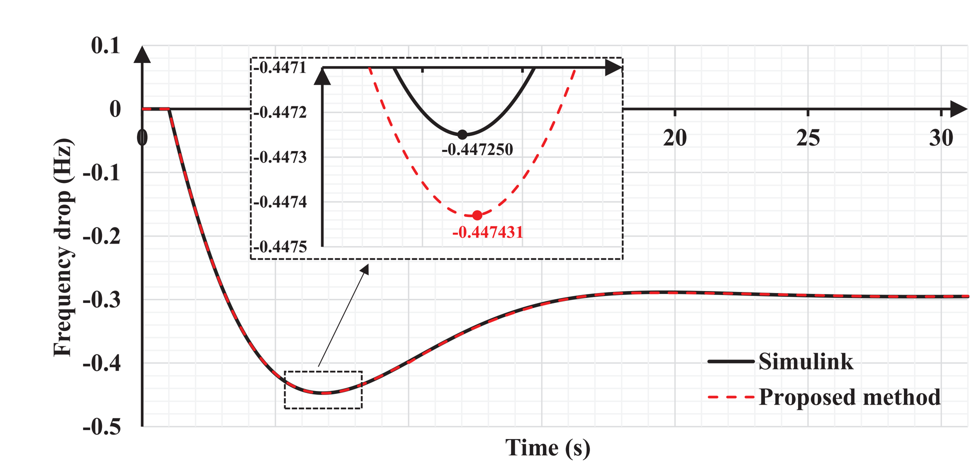

Figure 4. Frequency Dynamics From the Proposed Approximation vs. From Simulink

The above results demonstrate that the proposed model can effectively guarantee frequency security and validate the accuracy of the proposed method.

Conclusions

This paper proposed a DAE-constrained frequency-secured stochastic UC model and studied its solution method. We incorporated the frequency dynamics using DAEs into the stochastic UC model and adopted BP splines to obtain a conservative and linear approximation of the DAEs. Simulation results validated the effectiveness of the proposed model in deciding UC and PFR reserves for frequency security. A comparison with Simulink demonstrated the high accuracy of the proposed approximation in depicting frequency dynamics and evaluating the frequency deviation at the nadir.

Acknowledgements

The work was supported by the U.S. National Science Foundation under Grant ECCS-1845980 and the U.S. Department of Energy, Grant #DE-SC0018018.

bozum@umich.edu