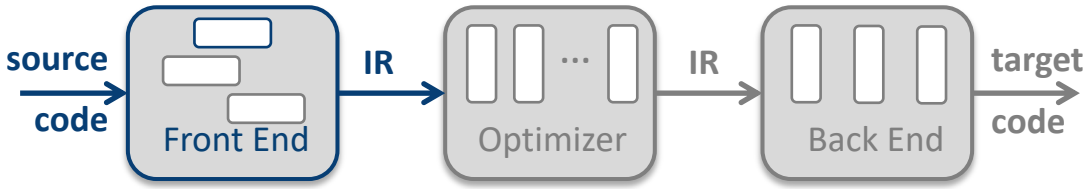
COMP 412
FALL 2018

Syntax Analysis, VII

One more LR(1) example, plus some final thoughts

Comp 412



Chapter 3 in EaC2e

Computing Closures

Review

Closure(*s*) adds all the *possibilities* for the items already in *s*

- Any item $[A \rightarrow \beta \bullet B\delta, \underline{a}]$ where $B \in NT$ implies $[B \rightarrow \bullet \tau, x]$ for each production that has *B* on the *lhs,* and each $x \in \text{FIRST}(\delta\underline{a})$
- Since $\beta B\delta$ is valid, any way to derive $\beta B\delta$ is valid, too

The Algorithm

Closure(*s*)
while (s is still changing)
  $\forall$ *items* $[A \rightarrow \beta \bullet B\delta, \underline{a}] \in s$
    lookahead $\leftarrow$ FIRST($\delta\underline{a}$) // δ might be ε
    $\forall$ *productions* $B \rightarrow \tau \in P$
      $\forall$ $\underline{b} \in$ *lookahead*
        if $[B \rightarrow \bullet \tau, \underline{b}] \notin s$
          then $s \leftarrow s \cup \{ [B \rightarrow \bullet \tau, \underline{b}] \}$

- Classic fixed-point method
- Halts because $s \subset I$, the set of all items (*finite*)
- Worklist version is faster
- ***Closure*** *"fills out"* a state *s*

Generate new lookaheads.
See note on p. 128

Computing Gotos

Review

Goto*(*s*,*x*) computes the state that the parser would reach if it recognized an *x* while in state *s

- ***Goto***({ [$A \rightarrow \beta \bullet X\delta$,a] }, *X*) produces { [$A \rightarrow \beta X \bullet \delta$,a] } ***(obviously)***
- It finds all such items & uses *Closure*() to fill out the state

The Algorithm

```
Goto( s, X )
   new ← Ø
    ∀ items [A→β•Xδ,a] ∈ s
      new ← new ∪ { [A→βX•δ,a] }
    return Closure( new )
```

- ***Goto***( ) models a transition in the automaton
- Straightforward computation
- ***Goto()*** is not a fixed-point method (but it calls ***Closure()***)

***Goto* in this construction is analogous to *Move* in the subset construction.**

Building the Canonical Collection

Review

Start from s_0 = ***Closure***([$S' \rightarrow \bullet\, S$,**EOF**])

Repeatedly construct new states, until all are found

The Algorithm

$s_0 \leftarrow$ ***Closure***({ [$S' \rightarrow \bullet\, S$,**EOF**] })
$S \leftarrow \{ s_0 \}$
$k \leftarrow 1$

while (*S is still changing*)

  $\forall s_j \in S$ *and* $\forall x \in (T \cup NT)$
    $s_k \leftarrow$ ***Goto***(s_j,x)
    record $s_j \rightarrow s_k$ *on* x
  if $s_k \notin S$ *then*
    $S \leftarrow S \cup \{ s_k \}$
    $k \leftarrow k + 1$

- Fixed-point computation
- Loop adds to S (*monotone*)
- $S \subseteq 2^{\text{ITEMS}}$, so S is finite

- *Worklist version is faster because it avoids duplicated effort*

This membership / equality test requires careful and/or clever implementation.

Filling in the ACTION and GOTO Tables

Review

The Table Construction Algorithm

x is the state number

```
∀ set S_x ∈ S
   ∀ item i ∈ S_x
     if  i is [A→β• aδ,b] and goto(S_x,a) = S_k , a ∈ T
        then ACTION[x,a] ← "shift k"
     else if  i is [S'→S •,EOF]
          then ACTION[x ,EOF] ← "accept"
     else if  i is [A→β•,a]
          then ACTION[x,a] ← "reduce A→β"
   ∀ n ∈ NT
    if  goto(S_x ,n) = S_k
       then GOTO[x,n] ← k
```

- before *T* ⇒ *shift*
- have *Goal* ⇒ *accept*
- at end ⇒ *reduce*

Many items generate no table entry

→ Placeholder before a *NT* does not generate an **ACTION** table entry

→ ***Closure***( ) instantiates FIRST(*X*) directly for $[A \rightarrow \beta \bullet X\delta, a]$

Another Example (*grammar & sets*)

Simplified, right recursive expression grammar

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term* - *Expr*
2		|	*Term*
3	*Term*	→	*Factor* * *Term*
4		|	*Factor*
5	*Factor*	→	id

SYMBOL	FIRST
Goal	{ id }
Expr	{ id }
Term	{ id }
Factor	{ id }
–	{ – }
*	{ * }
id	{ id }

Simplified Expression Grammar

Initialization Step

$s_0 \leftarrow$ ***closure***({ [*Goal* $\rightarrow$ •*Expr* , EOF] })

{ [*Goal* $\rightarrow$ • *Expr* , EOF],

[*Expr* $\rightarrow$ • *Term* – *Expr* , EOF], [*Expr* $\rightarrow$ • *Term* , EOF],

[*Term* $\rightarrow$ • *Factor* * Term , EOF], [*Term* $\rightarrow$ • *Factor* * *Term* , –],

[*Term* $\rightarrow$ • *Factor* , EOF], [*Term* $\rightarrow$ • *Factor* , –],

[*Factor* $\rightarrow$ • id , EOF], [*Factor* $\rightarrow$ • id , –], [*Factor* $\rightarrow$ • id , *] }

$S \leftarrow \{s_0\}$

0	*Goal*	$\rightarrow$	*Expr*
1	*Expr*	$\rightarrow$	*Term* - *Expr*
2		|	*Term*
3	*Term*	$\rightarrow$	*Factor* * *Term*
4		|	*Factor*
5	*Factor*	$\rightarrow$	id

Item in *black* is the initial item.
Items in *gray* are added by *closure*().

Simplified Expression Grammar

Iteration 1

$s_1 \leftarrow$ ***goto***$(s_0$, *Expr)*

$s_2 \leftarrow$ ***goto***$(s_0$, *Term)*

$s_3 \leftarrow$ ***goto***$(s_0$, *Factor)*

$s_4 \leftarrow$ ***goto***$(s_0$, id)

Goal, * , & - generate empty sets

Iteration 2

$s_5 \leftarrow$ ***goto***$(s_2$, –)

$s_6 \leftarrow$ ***goto***$(s_3$, *)

Goal, Expr, Term, Factor, & id generate empty sets

Iteration 3

$s_7 \leftarrow$ ***goto***$(s_5$, *Expr*)

$s_8 \leftarrow$ ***goto***$(s_6$, *Term*)

Goal, *, & - generate empty sets. Term, Factor, & id start to re-create existing sets.

Simplified Expression Grammar

The Details

$s_0 \leftarrow$ ***closure***({[*Goal* $\rightarrow$ •*Expr* , EOF]})

{ [*Goal* $\rightarrow$ • *Expr* , EOF],

[*Expr* $\rightarrow$ • *Term* – *Expr* , EOF], [*Expr* $\rightarrow$ • *Term* , EOF],

[*Term* $\rightarrow$ • *Factor* * *Term* , EOF], [*Term* $\rightarrow$ • *Factor* * *Term* , –],

[*Term* $\rightarrow$ • *Factor* , EOF], [*Term* $\rightarrow$ • *Factor* , –],

[*Factor* $\rightarrow$ • id , EOF], [*Factor* $\rightarrow$ • id , –], [*Factor* $\rightarrow$ • id, *] }

$s_1 \leftarrow$ ***goto***(s_0 , *Expr*)

{ [*Goal* $\rightarrow$ *Expr* •, EOF] }

$s_2 \leftarrow$ ***goto***(s_0 , *Term*)

{ [*Expr* $\rightarrow$ *Term* • – *Expr* , EOF], [*Expr* $\rightarrow$ *Term* •, EOF] }

$s_3 \leftarrow$ ***goto***(s_0 , *Factor*)

{ [*Term* $\rightarrow$ *Factor* • * *Term* , EOF],[*Term* $\rightarrow$ *Factor* • * *Term* , –],

[*Term* $\rightarrow$ *Factor* •, EOF], [*Term* $\rightarrow$ *Factor* •, –] }

Items in *black* are core items, generated by moving the placeholder.
Items in *gray* are added by *closure*().

Simplified Expression Grammar

The Details

$s_4 \leftarrow$ ***goto***$(s_0$, id$)$

{ [*Factor* → id •, EOF],[*Factor* → id •, –], [*Factor* → id •, *] }

$s_5 \leftarrow$ ***goto***$(s_2$, –)

{ [*Expr* → *Term* – • *Expr* , EOF], [*Expr* → • *Term* – *Expr* , EOF],
[*Expr* → • *Term* , EOF],
[*Term* → • *Factor* * *Term* , –], [*Term* → • *Factor* , –],
[*Term* → • *Factor* * *Term* , EOF], [*Term* → • *Factor* , EOF],
[*Factor* → • id , *], [*Factor* → • id , –], [*Factor* → • id , EOF] }

$s_6 \leftarrow$ ***goto***$(s_3$, *)

{ [*Term* → *Factor* * • *Term* , EOF], [*Term* → *Factor* * • *Term* , –],
[*Term* → • *Factor* * *Term* , EOF], [*Term* → • *Factor* * *Term* , –],
[*Term* → • *Factor* , EOF], [*Term* → • *Factor* , –],
[*Factor* → • id , EOF], [*Factor* → • id , –], [*Factor* → • id , *] }

Items in *black* are core items, generated by moving the placeholder.
Items in *gray* are added by *closure*().

Simplified Expression Grammar

$s_7 \leftarrow$ ***goto***$(s_5$, *Expr*)

{ [*Expr* $\rightarrow$ *Term* – *Expr* •, EOF] }

goto$(s_5$, *Term*) recreates s_2

goto$(s_5$, *Factor*) recreates s_3

goto$(s_5$, id) recreates s_4

$s_8 \leftarrow$ ***goto***$(s_6$, *Term*)

{ [*Term* $\rightarrow$ *Factor* * *Term* •, EOF], [*Term* $\rightarrow$ *Factor* * *Term* •, –] }

goto$(s_6$, *Term*) recreates s_3

goto$(s_6$, id) recreates s_4

The next iteration creates no new sets.

Items in *black* are core items, generated by moving the placeholder.
Items in *gray* are added by *closure*().

Simplified Expression Grammar

Recorded Transitions

The Goto Relationship

(*recorded during the construction*)

State	—	*	id	Expr	Term	Factor
s_0			4	1	2	3
s_1						
s_2	5					
s_3		6				
s_4						
s_5			4	7	2	3
s_6			4		8	3
s_7						
s_8						

Into shifts in the ACTION Table

Into the GOTO Table

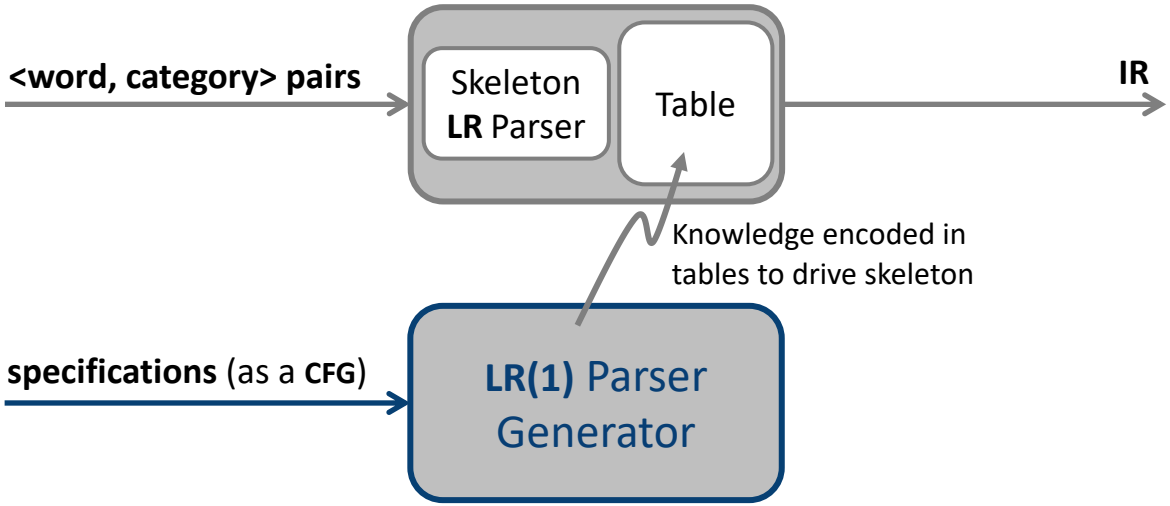
Simplified Expression Grammar

The algorithm produces the following tables

State	ACTION				GOTO		
	id	-	*	EOF	*Expr*	*Term*	*Factor*
s_0	s 4				1	2	3
s_1				acc			
s_2		s 5		r 3			
s_3		r 5	s 6	r 5			
s_4		r 6	r 6	r 6			
s_5	s 4				7	2	3
s_6	s 4					8	3
s_7				r 2			
s_8		r 4		r 4			

Notice that none of these are shift actions, obviously

Brief Commercial: Why Are We Doing This?



The goal of this exercise is to automate construction of parsers

- Compiler writer provides a **CFG** written in modified **BNF**
- Tools provide an efficient and correct parser
  - *One that works well with an automatically generated scanner*
- **LR** parser generators accept the largest class of grammars that are deterministically parsable, and they are highly efficient
  - *Generated parsers are preferable to hand-coded ones for large grammars*

§ 3.6.2 in EaC2e

Shrinking the ACTION and GOTO Tables

Three classic options:

- Combine terminals such as number & identifier, + & -, * & /
  - Directly removes a column, may remove a row
  - For expression grammar, 198 (vs. 384) table entries
- Combine rows or columns
  - Implement identical rows once & remap states
  - Requires extra indirection on each lookup
  - Use separate mappings for ACTION & GOTO
- Use another construction algorithm
  - Both **LALR(1)** and **SLR(1)** produce smaller tables
    - → ***LALR(1)*** *represents each state with its "core" items*
    - → ***SLR(1)*** *uses* ***LR(0)*** *items and the* ***FOLLOW*** *set*
  - Implementations are readily available

Pages 151-153 in EaC2e

Shrinking the Grammar

The Classic Expression Grammar

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	(*Expr*)
8		|	number
9		|	id

Canonical construction produces 32 states

- 32 x (9 + 3) = 384 ACTION/GOTO entries
- Large table, but still just 1.5kb

	Action Table								
State	eof	+	−	×	÷	(	)	num	name
0						s 4		s 5	s 6
1	acc	s 7	s 8						
2	r 4	r 4	r 4	s 9	s 10				
3	r 7	r 7	r 7	r 7	r 7				
4						s 14		s 15	s 16
5	r 9	r 9	r 9	r 9	r 9				
6	r 10	r 10	r 10	r 10	r 10				
7						s 4		s 5	s 6
8						s 4		s 5	s 6
9						s 4		s 5	s 6
10						s 4		s 5	s 6
11		s 21	s 22				s 23		
12		r 4	r 4	s 24	s 25		r 4		
13		r 7	r 7	r 7	r 7		r 7		
14						s 14		s 15	s 16
15		r 9	r 9	r 9	r 9		r 9		
16		r 10	r 10	r 10	r 10		r 10		
17	r 2	r 2	r 2	s 9	s 10				
18	r 3	r 3	r 3	s 9	s 10				
19	r 5	r 5	r 5	r 5	r 5				
20	r 6	r 6	r 6	r 6	r 6				
21						s 14		s 15	s 16
22						s 14		s 15	s 16
23	r 8	r 8	r 8	r 8	r 8				
24						s 14		s 15	s 16
25						s 14		s 15	s 16
26		s 21	s 22				s 31		
27		r 2	r 2	s 24	s 25		r 2		
28		r 3	r 3	s 24	s 25		r 3		
29		r 5	r 5	r 5	r 5		r 5		
30		r 6	r 6	r 6	r 6		r 6		
31		r 8	r 8	r 8	r 8		r 8		

■ **FIGURE 3.31** Action Table for the Classic Expression Grammar.

Pages 151-153 in EaC2e

Shrinking the Grammar

We can combine some of the syntactically equivalent symbols

- Combine **+** and **–** into AddSub
- Combine ***** and **/** into MulDiv
- Combine identifier and number into Val

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* AddSub *Term*
2		|	*Term*
3	*Term*	→	*Term* MulDiv *Factor*
4		|	*Factor*
5	*Factor*	→	(*Expr*)
6		|	Val

The "Reduced" Expression Grammar

This grammar has

- Fewer terminals
- Fewer productions

Which leads to

- Fewer columns in ACTION
- Fewer states, which leads to fewer rows in both tables

Pages 151-153 in EaC2e

Shrinking the Grammar

The Resulting Tables

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* AddSub *Term*
2		|	*Term*
3	*Term*	→	*Term* MulDiv *Factor*
4		|	*Factor*
5	*Factor*	→	(*Expr*)
6		|	Val

	Action Table						*Goto* Table		
	eof	addsub	muldiv	(	)	val	*Expr*	*Term*	*Factor*
0				s 4		s 5	1	2	3
1	acc	s 6							
2	r 3	r 3	s 7						
3	r 5	r 5	r 5						
4				s 11		s 12	8	9	10
5	r 7	r 7	r 7						
6				s 4		s 5		13	3
7				s 4		s 5			14
8		s 15			s 16				
9		r 3	s 17		r 3				
10		r 5	r 5		r 5				
11				s 11		s 12	18	9	10
12		r 7	r 7		r 7				
13	r 2	r 2	s 7						
14	r 4	r 4	r 4						
15				s 11		s 12		19	10
16	r 6	r 6	r 6						
17				s 11		s 12			20
18		s 15			s 21				
19		r 2	s 17		r 2				
20		r 4	r 4		r 4				
21		r 6	r 6		r 6				

(b) *Action* and *Goto* Tables for the Reduced Expression Grammar

■ **FIGURE 3.33** The Reduced Expression Grammar and its Tables.

- 22 states
- 22 * (6 + 3) = 198 ACTION/GOTO entries
- 48.4% reduction (384 - 198) / 384
- Builds (essentially) the same parse tree

§ 3.6.2 in EaC2e

Shrinking the ACTION and GOTO Tables

Three classic options:

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  - For expression grammar, 198 (vs. 384) table entries
- Combine rows or columns
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  - Use separate mappings for ACTION & GOTO
- Use another construction algorithm
  - Both **LALR(1)** and **SLR(1)** produce smaller tables
    - → ***LALR(1)*** *represents each state with its "core" items*
    - → ***SLR(1)*** *uses* ***LR(0)*** *items and the* ***FOLLOW*** *set*
  - Implementations are readily available

Shrinking The Action and Goto Tables

The parser generator can combine identical columns & identical rows

```
for i ← 0 to MaxCol
    MapTo[i] ← i

for i ← 0 to MaxCol
    if MapTo[i] = i then
      for j ← i to MaxCol
          if MapTo[j] = j then
            same ← true
            for k ← 0 to MaxRow
                if Table[i,k] = Table[j,k] then
                   same ← false
                   break
            if same = true then
              MapTo[j] ← i
```

Basic Table Compression Algorithm

- Same algorithm applies to **ACTION** & **GOTO**
  - Also columns in **DFA** transition tables
- Adds a level of indirection, as in a **DFA**'s character classifier
  - More complex table encoding
  - Tradeoff loads vs. table size
- Significant table-size reductions without grammar changes
- Can improve efficiency by using a quick inequality test
  - pop count of non-error states
  - more complex signatures (hash)

§ 3.6.2 in EaC2e

Shrinking the ACTION and GOTO Tables

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- Use another construction algorithm
  - Both **LALR(1)** and **SLR(1)** produce smaller tables
    - → ***LALR(1)*** *represents each state with its "core" items*
    - → ***SLR(1)*** *uses* ***LR(0)*** *items in construction and the* ***FOLLOW*** *set in table-filling algorithm*
  - Implementations are readily available

§ 3.6.2 in EaC2e

Shrinking the ACTION and GOTO Tables

Three classic options:

- Combine terminals such as number & identifier, + & -, * & /
  - Directly removes a column, may remove a row
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  - Implementations are readily available

left-recursive expression grammar with precedence, see § 3.6.2 in EAC

classic space-time tradeoff

Fewer grammars, same languages

Example due to Bill Waite, UC Boulder

LR(1) versus LL(1)

The following **LR(1)** grammar has no **LL(1)** counterpart

- The Canonical Collection has 18 sets of LR(1) Items
  - It is not a simple grammar
  - It is, however, LR(1)

0	*Goal*	→	*S*
1	*S*	→	*A*
2		|	*B*
3	*A*	→	(*A*)
4		|	a
5	*B*	→	(*B*]
6		|	b

- It requires an arbitrary lookahead to choose between *A* & *B*
- An **LR(1)** parser can carry the left context (the '(' s) until it sees a or b
- The table construction will handle it
- In contrast, an **LL(1)** parser cannot decide whether to expand *Goal* by *A* or *B*
  - → *No amount of massaging the grammar and no amount of lookahead will resolve this problem*

More precisely, the language described by this **LR(1)** grammar cannot be described with an **LL(1)** grammar. In fact, the language has no **LL(*k*)** grammar, for finite *k*.

ACTION & GOTO Tables for Waite's Example

	EOF	(	)	a	]
s_0		s 4		s 5	
s_1	acc				
s_2	r 2				
s_3	r 3				
s_4		s 8		s 9	
s_5	r 5				
s_6			s 10		
s_7					s 11
s_8		s 8		s 9	
s_9			r 5		r 7
s_{10}	r 4				
s_{11}	r 6				
s_{12}			s 14		
s_{13}					s 15
s_{14}			r 4		
S_{15}					r 6

	S	*A*	*B*
s_0	1	2	3
s_1			
s_2			
s_3			
s_4		6	7
s_5			
s_6			
s_7			
s_8		12	13
s_9			
s_{10}			
s_{11}			
s_{12}			
s_{13}			
s_{14}			
S_{15}			

0	*Start* → A
1	| B
2	*A* → (*A*)
3	| a
4	*B* → (*B*]
5	| a

LR(*k*) versus LL(*k*)

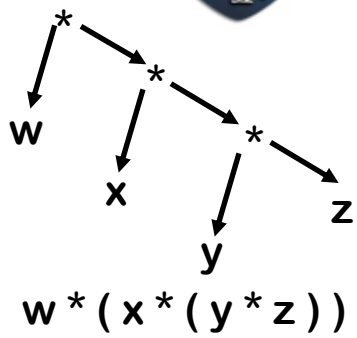
The Bottom Line

*"… in practice, **programming languages** do not actually seem to fall in the gap between LL(1) languages and deterministic languages"*

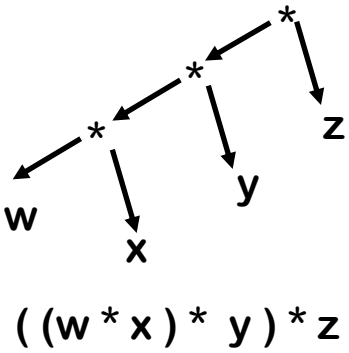
J.J. Horning, "LR Grammars and Analysers", in Compiler Construction, An Advanced Course, Springer-Verlag, 1976

Left Recursion versus Right Recursion

- Right recursion
  - Required for termination in top-down parsers
  - Uses (on average) more stack space
  - Naïve right recursion produces right-associativity



- Left recursion
  - Works fine in bottom-up parsers
  - Limits required stack space
  - Naïve left recursion produces left-associativity



- Rule of thumb
  - Left recursion for bottom-up parsers
  - Right recursion for top-down parsers

Left Recursion versus Right Recursion

A real example, from the front end of the lab 2 ILOC simulator

The simulator was built by two of my successful Ph.D.s

- It is actually a more complex piece of software than you might guess
- The front end is an LR(1) parser, generated by Bison
- The grammar contained the following productions:

```
instruction_list    :   instruction
                    |   label_def instruction
                    |   instruction  instruction_list
                    |   label_def   instruction instruction_list
```

When my colleague first ran the timing blocks through the simulator, it exploded with the error message “memory exhausted”.

⇒ What happened?

Left Recursion versus Right Recursion

A real example, from the lab 1 simulator's front end

instruction_list	:	*instruction*
	|	*label_def instruction*
	|	*instruction* ***instruction_list***
	|	*label_def instruction* ***instruction_list***

right recursion

- The parse stack overflowed as it tried to instantiate the instruction_list

Left Recursion versus Right Recursion

A real example, from the lab 1 simulator's front end

```
instruction_list   :   instruction
                   |   label_def instruction
                   |   instruction  instruction_list
                   |   label_def   instruction instruction_list
```

right recursion

- The parse stack overflowed as it tried to instantiate the instruction_list
- The fix was easy

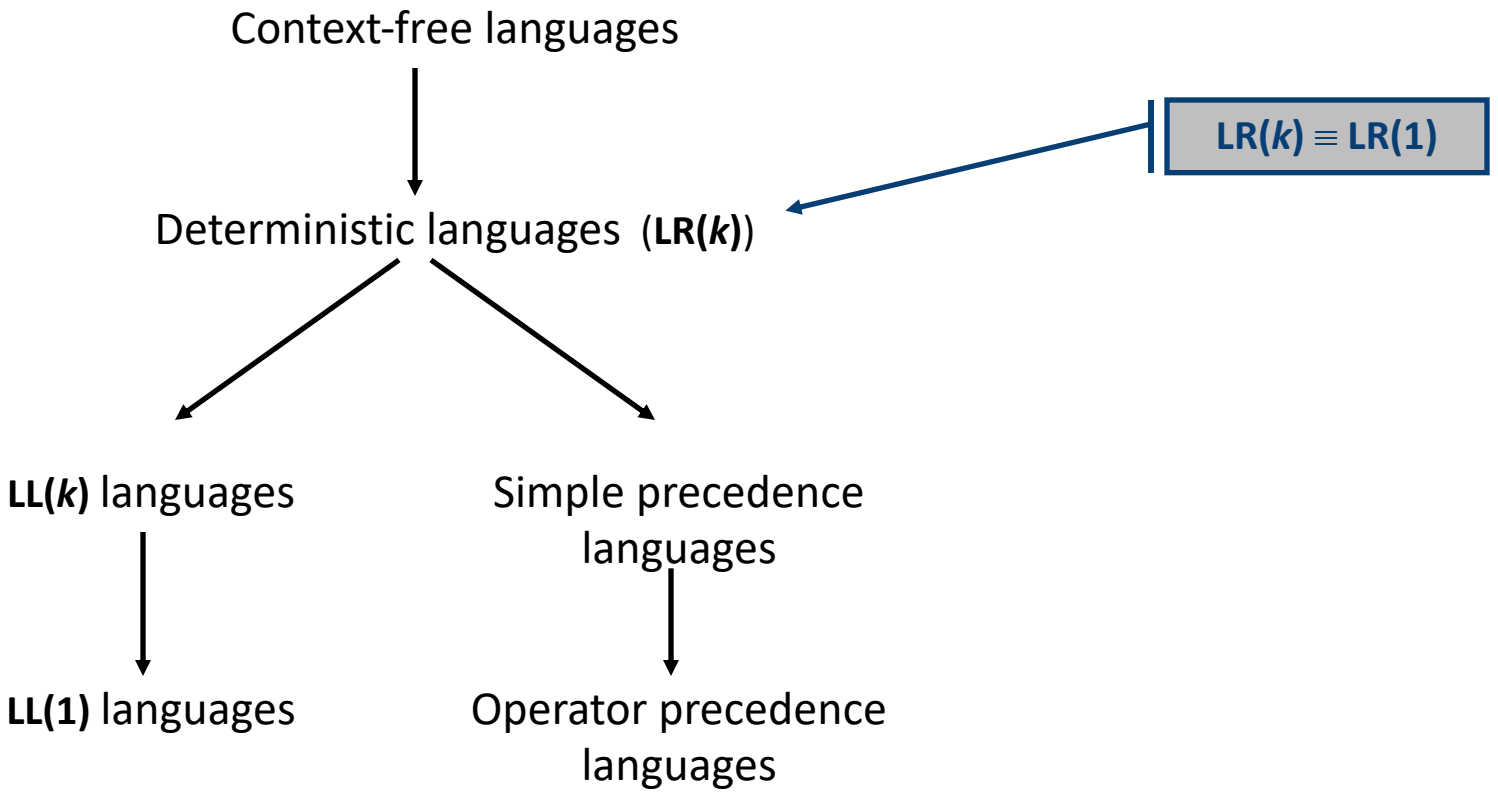
```
instruction_list   :   instruction
                   |   label_def instruction
                   |   instruction_list  instruction
                   |   instruction_list  label_def  instruction
```

left recursion

- This grammar has (small) bounded stack space & (thus) scales well

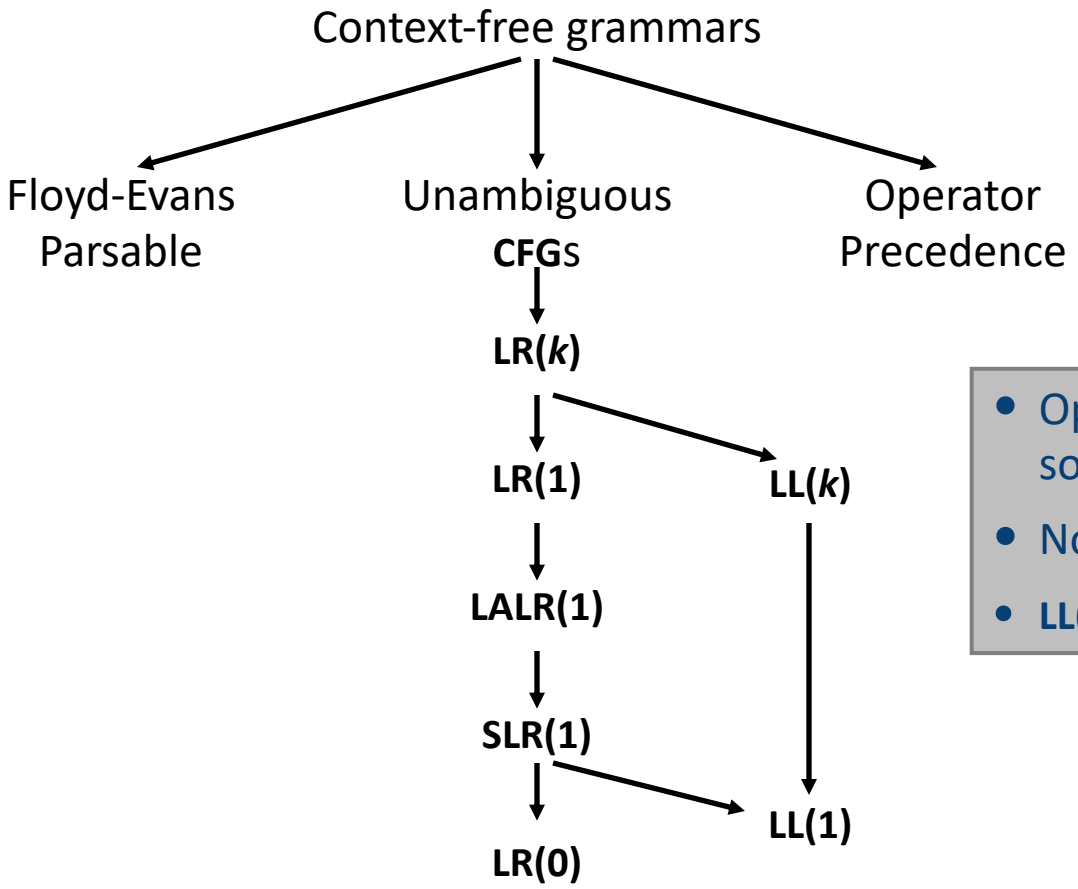
See pp. 145-146 in EaC2e

Hierarchy of Context-Free Languages



The inclusion hierarchy for context-free ***languages***

Hierarchy of Context-Free Grammars



- Operator precedence includes some ambiguous grammars
- Note sub-categories of **LR(1)**
- **LL(1)** is a subset of **SLR(1)**

*The inclusion hierarchy for context-free **grammars***