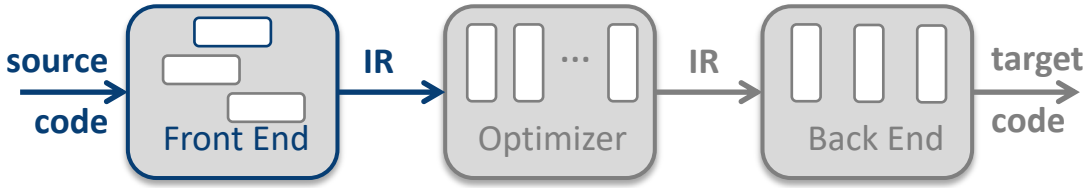
Updated algorithm for removal of indirect left recursion to match EaC3e (3/2018)

COMP 412
FALL 2018

Midterm Exam: Thursday October 18, 7PM Herzstein Amphitheater

Syntax Analysis, III

Comp 412



Chapter 3 in EaC2e

The Classic Expression Grammar

Review

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	(*Expr*)
8		|	number
9		|	id

Classic Expression Grammar, Left-recursive Version

This left-recursive expression grammar encodes standard algebraic precedence and left-associativity (which corresponds to left-to-right evaluation).

We refer to this grammar as either the classic expression grammar or the canonical expression grammar

➡ Both have the same acronym, **CEG**

Top-down Parsing

Review

The Algorithm

- A top-down parser starts with the root of the parse tree
- The root node is labeled with the goal symbol of the grammar

> *Construct the root node of the parse tree*
>
> *Repeat until lower fringe of the parse tree matches the input string*
>
> 1. *At a node labeled with **NT** A, **select a production** with A on its **LHS** and, for each symbol on its **RHS**, construct the appropriate child*
> 2. *When a terminal symbol is added to the fringe and it doesn't match the fringe, backtrack*
> 3. *Find the next node to be expanded* *(label $\in$ NT)*

The key is selecting the right production in step 1

- *That choice should be guided by the input string*

Top-down parsing with the **CEG**

With deterministic choices, the parse can expand indefinitely

Rule	*Sentential Form*	*Input*
—	*Goal*	↑x - 2 * y
0	*Expr*	↑x - 2 * y
1	*Expr +Term*	↑x - 2 * y
1	*Expr + Term +Term*	↑x - 2 * y
1	*Expr + Term +Term + Term*	↑x - 2 * y
1	*… and so on ….*	↑x - 2 * y

Consumes no input!

This expansion doesn't terminate

- Wrong choice of expansion leads to non-termination
- ***Non-termination*** is a bad property for a parser to have
- Parser must make the right choice

Recursion in the **CEG**

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	(*Expr*)
8		|	number
9		|	id

Classic Expression Grammar, Left-recursive Version

The problems with unbounded expansion arise from left-recursion in the **CEG**

- **LHS** symbol cannot derive, in one or more steps, a sentential form that starts with the **LHS**
- Left recursion is incompatible with top-down leftmost derivations †
- A leftmost derivation matches the scanner's left-to-right scan

We need a technique to transform left recursion into right recursion (and, possible, the reverse)

†**Similarly, right recursion is incompatible with rightmost derivaions**

Eliminating Left Recursion

Review

To remove left recursion, we can transform the grammar

Consider a grammar fragment of the form

$$Fee \rightarrow Fee\ \alpha$$
$$\quad\quad | \ \beta$$

where neither α nor β start with *Fee*

Language is β followed by 0 or more α's

We can rewrite this fragment as

$$Fee \rightarrow \beta\ Fie$$
$$Fie \rightarrow \alpha\ Fie$$
$$\quad\quad | \ \varepsilon$$

where *Fie* is a new non-terminal

The new grammar defines the same language as the old grammar, using only right recursion.

Added a reference to the empty string

New Idea: the ε production

Eliminating Left Recursion

Review

The expression grammar contains two cases of left recursion

Expr	→	*Expr + Term*	*Term*	→	*Term * Factor*
	|	*Expr - Term*		|	*Term * Factor*
	|	*Term*		|	*Factor*

Applying the transformation yields

Expr	→	*Term Expr'*	*Term*	→	*Factor Term'*
Expr'	→	*+ Term Expr'*	*Term'*	→	** Factor Term'*
	|	*- Term Expr'*		|	*/ Factor Term'*
	|	ε		|	ε

These fragments use only right recursion

Eliminating Left Recursion

Review

Substituting them back into the grammar yields

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	(*Expr*)
10		|	number
11		|	id

Right-recursive expression grammar

- This grammar is correct, if somewhat counter-intuitive.
- A top-down parser will terminate using it.
- A top-down parser may need to backtrack with it.
- It is left associative, as was the original

Eliminating Left Recursion

*"**LHS** symbol cannot derive, in one or **more** steps, a sentential form that starts with the **LHS**"*

The two-production transformation eliminates immediate left recursion

What about more general, indirect left recursion?

The general algorithm to eliminate left recursion:

arrange the NTs into some order $A_0, A_1, \ldots, A_n$
for $i \leftarrow 1$ *to* n
    for $s \leftarrow 0$ *to* $i - 1$ {
        replace each production $A_i \rightarrow A_s\gamma$ *with* $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$,
            where $A_s \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ *are all the current productions for* A_s
        }
    eliminate any immediate left recursion on A_i *using the direct transformation*

The algorithm assumes that the initial grammar has no cycles ($A_i \Rightarrow^+ A_i$), and no epsilon productions

EaC2e shows the inner loop running from 1 to n, so 1st iteration of inner loop never executes.

Eliminating Left Recursion

How does this algorithm work?

1. Impose arbitrary order on the non-terminals
2. Outer loop cycles through **NT** in order
3. Inner loop ensures that a production expanding A_i cannot directly derive a non-terminal A_s at the start of its **RHS**, for $s < i$
4. Last step in outer loop converts any direct recursion on A_i to right recursion using the transformation showed earlier
5. New non-terminals are added at the end of the order & have no left recursion; they will have right recursion on themselves

At the start of the i^{th} outer loop iteration

For all $k < i$, no production that expands A_k begins with a non-terminal A_s, for $s < k$

Eliminating Indirect Left Recursion

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	A_1 b

Example Grammar

- This grammar generates **a (ba)***
- Subscripts indicate the imposed order
- Indirect left recursion is $A \rightarrow B \rightarrow A$

arrange the NTs into some order A_0, A_1, …, A_n
for i ← 1 to n
  for s ← 0 to i – 1 {
    replace each production $A_i \rightarrow A_s\gamma$ with $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$,
      where $A_s \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ are all the current productions for A_s
    }
  eliminate any immediate left recursion on A_i using the direct transformation

Eliminating Indirect Recursion

General algorithm is a diagonalization, similar to LU decomposition

LHS \ RHS	NT_0	NT_1	NT_2
NT_0	0	**1**	0
NT_1	0	0	**1**
NT_2	0	**1**	0

Original Grammar

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	A_1 b

- If [i, j] element is *k*, then NT_i derives NT_j in a the k^{th} position of its **RHS**
- Diagonal elements represent self-recursion
- Upper triangle represents forward references in the order
- Lower triangle represents backward references in the order
  - *Inner loop systematically zeroes the lower triangle*
  - *Last step of outer loop eliminates ones on the diagonal*

Eliminating Indirect Left Recursion

Applying the Algorithm

	s = 0	*1*	*2*
i = 1	*no action*	*no iteration*	*no iteration*
2	no action	*see below*	*no iteration*

Algorithm iterates through the possible backward references.

- Handles indirect left recursion with forward substitution of **RHS** for **LHS**
- Handles direct left recursion with the transformation

For B_2 (i = 2) and A_1 (s = 1):

First, it rewrites rule 3 with

$B_2 \rightarrow B_2$ a b
| a b

Next, it uses the rule for immediate left recursion to rewrite this pair of rules as:

$B_2 \rightarrow$ a b C_3

$C_3 \rightarrow$ a b C_3
| ε

Eliminating Indirect Left Recursion

Before and After

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	A_1 b

Original Grammar

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	a b C_3
4	C_3	$\rightarrow$	a b C_3
5		|	ε

Transformed Grammar

The transformed grammar (*still*) generates a (ba)*

Eliminating Indirect Recursion

General algorithm is a diagonalization, similar to LU decomposition

	NT_0	NT_1	NT_2
NT_0	0	**1**	0
NT_1	0	0	**1**
NT_2	0	1	0

Original Grammar

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	A_1 b

Original matrix is now upper triangular

	NT_0	NT_1	NT_2	NT_3
NT_0	0	**1**	0	0
NT_1	0	0	**1**	0
NT_2	0	0	0	**3**
NT_3	0	0	0	**3**

Transformed Grammar

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	a b C_3
4	C_3	$\rightarrow$	a b C_3
5		|	ε

Eliminating Indirect Left Recursion

Before and After

0	$Start_0$	$\rightarrow$	A_2
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	A_1 b

Original Grammar

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	a b C_3
4	C_3	$\rightarrow$	a b C_3
5		|	ε

Transformed Grammar

- The transformed grammar generates a (ba)*
- The transformed grammar is still not [illegible]

What does "predictively parsable" even mean?

Predictive Parsing

If a top-down parser picks the "wrong" production, it may need to backtrack. The alternative is to *look ahead in the input & use context to pick the correct production.*

How much lookahead is needed for a context-free grammar?

- In general, an arbitrarily large amount
- Use the Cocke-Younger, Kasami algorithm or Earley's algorithm

Fortunately,

- Large subclasses of CFGs can be parsed with limited lookahead
- Most programming language constructs fall in those subclasses

Among the interesting subclasses are *LL(1)* and *LR(1)* grammars

We will focus, for now, on LL(1) grammars & predictive parsing

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$*, the parser should be able to choose between* α & β

FIRST sets

For some **RHS** $\alpha \in G$, define **FIRST**(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, $\underline{x} \in$ **FIRST**(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

We will defer the problem of how to compute **FIRST** sets for the moment and assume that they are given.

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$, *the parser should be able to choose between* α & β

FIRST sets

For some **RHS** $\alpha \in G$, define **FIRST**(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, $\underline{x} \in$ **FIRST**(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

The Predictive, or LL(1), Property

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ both appear in the grammar, we would like

$$\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \varnothing$$

This would allow the parser to make a correct choice with a lookahead of exactly one symbol!

This rule is intuitive. Unfortunately, it is **not correct**, because it does not handle ε rules. See the next slide

Predictive Parsing

What about ε-productions?

⇒ They complicate the definition of the predictive, or **LL(1)** property

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ and $\varepsilon \in$ FIRST(α), then we need to ensure that FIRST(β) is disjoint from FOLLOW(A), too, where

> **FOLLOW**(A) *is the set of terminal symbols that can appear immediately after A in some sentential form*

Define **FIRST**$^+$($A \rightarrow \alpha$) as

Note that **FIRST**$^+$ is defined over productions, not **NT**s

- **FIRST**(α) $\cup$ **FOLLOW**(A), if $\varepsilon \in$ **FIRST**(α)
- **FIRST**(α), otherwise

Then, a grammar is **LL(1)** *iff* $A \rightarrow \alpha$ and $A \rightarrow \beta$ implies that

FIRST$^+$($A \rightarrow \alpha$) $\cap$ **FIRST**$^+$($A \rightarrow \beta$) = $\varnothing$

The **LL(1)** Property

Our transformed grammar for a (ba)* fails the test

0	$Start_0$	$\rightarrow$	A_1
1	A_1	$\rightarrow$	B_2 a
2		|	a
3	B_2	$\rightarrow$	a b C_3
4	C_3	$\rightarrow$	a b C_3
5		|	ε

Transformed Grammar

Derivations for *A* begin with a:

$A \rightarrow B$ a | a

$B \rightarrow$ a

FIRST$^+(A \rightarrow B$ a) = { a }

FIRST$^+(A \rightarrow$ a) = { a }

These sets are identical, rather than disjoint. So, left recursion elimination did not produce a grammar with the LL(1) condition — a grammar that would be predictively parsable.

The **LL(1)** Property

Can we write an LL(1) grammar for a (ba)* ?

0	$Start_1$	$\rightarrow$	A_2
1	A_2	$\rightarrow$	B_3 a
2		|	a
3	B_3	$\rightarrow$	a b C_3
4	C_3	$\rightarrow$	a b C_3
5		|	ε

Transformed Grammar

0	*Start*	$\rightarrow$	a *B*
1	*B*	$\rightarrow$	b a *B*
2		|	ε

LL(1) Grammar for a (ba)*

What If My Grammar Is Still Not LL(1) ?

Can we transform a non-LL(1) grammar into an LL(1) grammar?

- In general, the answer is no
- In some cases, however, the answer is yes

Assume a grammar *G* with productions $A \rightarrow \alpha\,\beta_1$ and $A \rightarrow \alpha\,\beta_2$

- If α derives anything other than ε, then

$$\text{FIRST}^+(A \rightarrow \alpha\,\beta_1) \cap \text{FIRST}^+(A \rightarrow \alpha\,\beta_2) \neq \varnothing$$

- And the grammar is not LL(1)

If we pull the common prefix, α, into a separate production, we **may** make the grammar LL(1).

$$A \rightarrow \alpha\,A',\ A' \rightarrow \beta_1,\ \text{and}\ A' \rightarrow \beta_2$$

Now, if $\text{FIRST}^+(A' \rightarrow \beta_1) \cap \text{FIRST}^+(A' \rightarrow \beta_2) = \varnothing$, *G* may be LL(1)

What If My Grammar Is Not LL(1) ?

Left Factoring

*For each **NT** A*
  find the longest prefix α common to 2 or more alternatives for A
  if $\alpha \neq \varepsilon$ then
    replace all of the productions
      $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3 \mid \ldots \mid \alpha\beta_n \mid \gamma$
    with
      $A \rightarrow \alpha A' \mid \gamma$
      $A' \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots \mid \beta_n$
*Repeat until no **NT** has alternative rhs' with a common prefix*

This transformation makes some grammars into **LL(1)** grammars

There are languages for which no **LL(1)** grammar exists

Left Factoring Example

Consider a short grammar for subscripted identifiers

0	*Factor*	→	name
1		|	name [*ArgList*]
2		|	name (*ArgList*)
3	*ArgList*	|	*Expr MoreArgs*
4	*MoreArgs*	→	, *Expr MoreArgs*
5		|	ε

Common prefix is name. Introduce a new production

Factor → name *Arguments*

And expand *Arguments* into the three alternatives.

To choose between expanding by 0, 1, or 2, a top-down parser must look beyond the name to see the next word.

$$\text{FIRST}^+(0) = \text{FIRST}^+(1) = \text{FIRST}^+(2) = \{ \text{name} \}$$

Left factoring productions 0, 1, & 2 fixes this problem.

Left factoring cannot fix all non-**LL(1)** grammars.

Left Factoring Example

After left factoring:

0	*Factor*	→	name *Args*
1	*Args*	→	[*ArgList*]
2		|	(*ArgList*)
3		|	ε
4	*ArgList*	|	*Expr MoreArgs*
5	*MoreArgs*	→	, *Expr MoreArgs*
6		|	ε

Left factoring can transform some non-**LL(1)** grammars into **LL(1)** grammars.

There are languages for which no **LL(1)** grammar exists.

See exercise 3.12 in EaC2e.

Clearly, **FIRST**$^+$(1) & **FIRST**$^+$(2) are disjoint.

If neither (or [can follow *Factor*, then **FIRST**$^+$(3) will be, too.

This tiny grammar now has the **LL(1)** property

Predictive Parsing

Given a grammar that has the LL(1) property

- We can write a simple routine to recognize an instance of each **LHS**
- Code is patterned, simple, & fast

Consider $A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3$, with

$\text{FIRST}^+(A \rightarrow \beta_i) \cap \text{FIRST}^+(A \rightarrow \beta_j) = \varnothing$ if $i \neq j$

```
/* find an A */
if (current_word ∈ FIRST+(A→β1))
   find a β1 and return true
else if (current_word ∈ FIRST+(A→β2))
   find a β2 and return true
else if (current_word ∈ FIRST+(A→β3))
   find a β3 and return true
else
   report an error and return false
```

Grammars that have the **LL(1)** property are called **predictive grammars** because the parser can "predict" the correct expansion at each point in the parse.

Parsers that capitalize on the **LL(1)** property are called **predictive parsers**.

One kind of predictive parser is the **recursive descent** parser.

Of course, there is more detail to "find a β_i" — typically a recursive call to another small routine (see pp. 108--111 in EaC2e)

Recursive Descent Parsing

Recall the expression grammar, after transformation

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	(*Expr*)
10		|	number
11		|	id

This grammar leads to a parser that has six **mutually recursive** routines:

1. *Goal*
2. *Expr*
3. *EPrime*
4. *Term*
5. *TPrime*
6. *Factor*

Each routine recognizes an ***rhs*** for that **NT**.

The term **descent** refers to the direction in which the parse tree is built.

This grammar has the LL(1) property

Recursive Descent Parsing (Procedural)

A couple of routines from the expression parser

```
Goal( )
    token ← next_token( );
    if (Expr( ) = true & token = EOF)
       then next compilation step;
       else
          report syntax error;
          return false;

Expr( )
   if (Term( ) = false)
     then return false;
     else return Eprime( );
```

looking for number, identifier, or (, found token instead, or failed to find *Expr* or) after (

```
Factor( )
  if (token = number) then
     token ← next_token( );
     return true;
  else if (token = identifier) then
     token ← next_token( );
     return true;
  else if (token = lparen)
     token ← next_token( );
     if (Expr( ) = true & token = rparen) then
        token ← next_token( );
        return true;
  // fall out of if statement
  report syntax error;
     return false;
```

EPrime, Term, & TPrime follow the same basic lines (Figure 3.10, EaC2e)

Recursive Descent

Page 111 in EaC2e sketches a recursive descent parser for the RR CEG.

- One routine per **NT**
- Check each **RHS** by checking each symbol
- Includes ε-productions

```
Main()
  /* Goal → Expr */
  word ← NextWord();
  if (Expr())
    then if (word = eof)
      then report success;
      else Fail();

Fail()
  report syntax error;
  attempt error recovery or exit;

Expr()
  /* Expr → Term Expr' */
  if (Term())
    then return EPrime();
    else Fail();

EPrime()
  /* Expr' → + Term Expr' */
  /* Expr' → - Term Expr' */
  if (word = + or word = -)
    then begin;
      word ← NextWord();
      if (Term())
        then return EPrime();
        else Fail();
    end;
  else if (word = ) or word = eof)
    /* Expr' → ε */
    then return true;
    else Fail();

Term()
  /* Term → Factor Term' */
  if (Factor())
    then return TPrime();
    else Fail();

TPrime()
  /* Term' → × Factor Term' */
  /* Term' → ÷ Factor Term' */
  if (word = × or word = ÷)
    then begin;
      word ← NextWord();
      if (Factor())
        then return TPrime();
        else Fail();
    end;
  else if (word = + or word = - or
           word = ) or word = eof)
    /* Term' → ε */
    then return true;
    else Fail();

Factor()
  /* Factor → ( Expr ) */
  if (word = ( ) then begin;
    word ← NextWord();
    if (not Expr())
      then Fail();
    if (word ≠ ) )
      then Fail();
    word ← NextWord();
    return true;
  end;
  /* Factor → num */
  /* Factor → name */
  else if (word = num or
            word = name)
    then begin;
      word ← NextWord();
      return true;
    end;
  else Fail();
```

Top-Down Recursive Descent Parser

At this point, you almost have enough information to build a top-down recursive-descent parser

- Need a right-recursive grammar that meets the **LL(1)** condition
  - Can use left-factoring to eliminate common prefixes
  - Can transform direct left recursion into right recursion
  - Need a general algorithm to handle indirect left recursion
- Need algorithms to construct **FIRST** and **FOLLOW**

Next Parsing Lecture

- Algorithms for **FIRST** and **FOLLOW**
- An **LL(1)** skeleton parser, table, and table-construction algorithm

Next Lecture

- Local Register Allocation and Lab 2
- Read materials on local allocation from projects page of web site