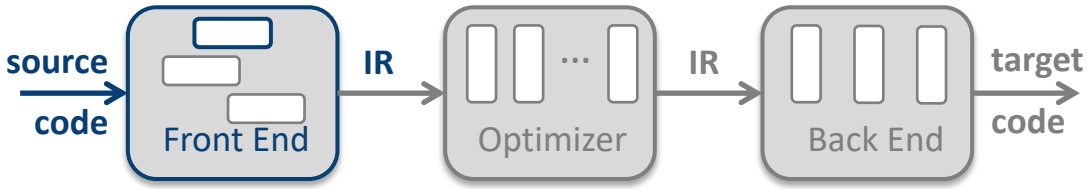
COMP 412
FALL 2018

Syntax Analysis, VII

The Canonical LR(1) Table Construction

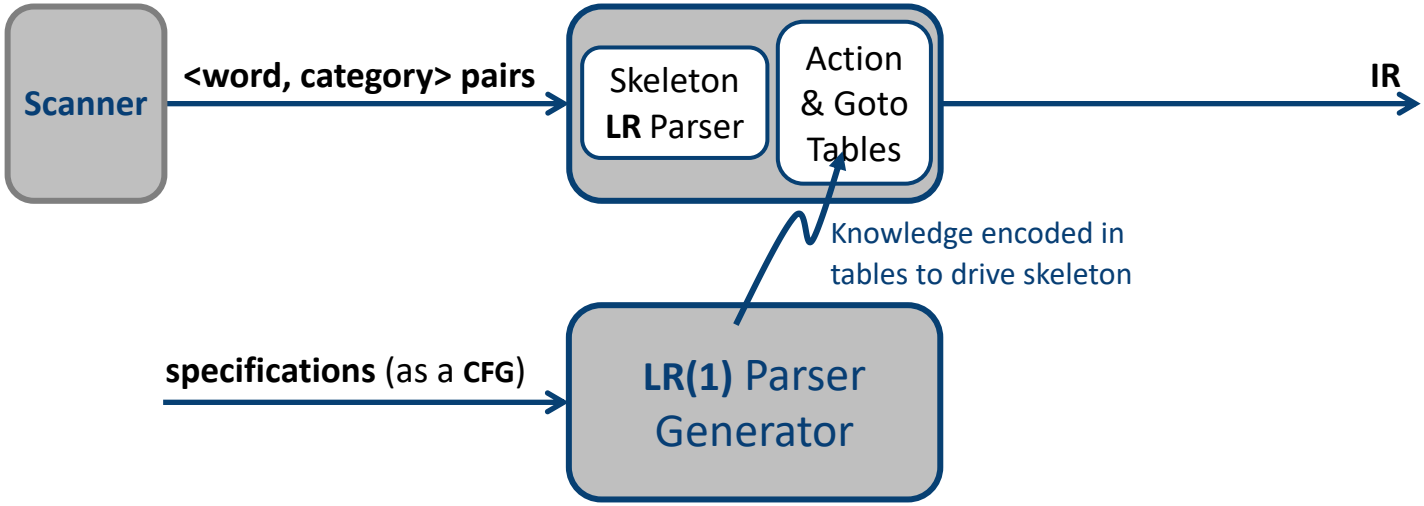
Comp 412



Chapter 3 in EaC2e

Table-Driven LR Parsers

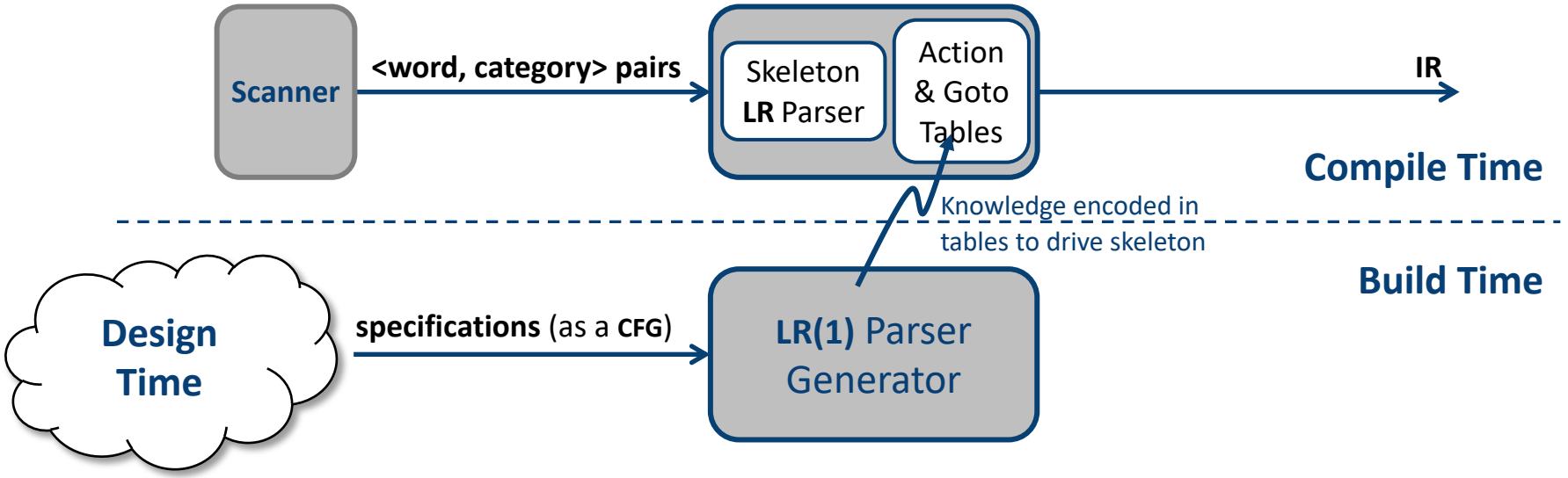
A table-driven LR(1) parser is a bottom-up shift-reduce parser



- Grammatical knowledge is encoded in two tables: *Action* & *Goto*
  - They encode the handle-finding automaton
  - They are constructed by an LR(1) parser generator
- Why two tables?
  - *Reduce* needs more information & more complex information than *shift*
  - *Goto* holds that extra information

Table-Driven LR Parsers

A table-driven LR(1) parser is a bottom-up shift-reduce parser



Back to the Meta Issue

- The compiler writer creates a grammar at **design time**
- The parser generator builds **ACTION** and **GOTO** tables at **build time**
- The compiler uses those tables to parse at **compile time**

Building **LR(1)** Tables

How do we generate the ACTION and GOTO tables?

- Use the grammar to build a model of the Control **DFA**
- Encode actions & transitions in ACTION & GOTO tables
- If construction succeeds, the grammar is **LR(1)**
  - *"Succeeds" means defines each table entry uniquely*

An operational definition

The Big Picture

- Model the state of the parser with **LR(1)** items
- Use two functions *goto(s, X)* and *closure(s)*
  - *goto()* is analogous to *move()* in the subset construction
  - Given a partial state, *closure()* adds all the items implied by the partial state
- Build up the states and transition functions of the **DFA**
- Use this information to fill in the ACTION and GOTO tables

grammar symbol, *T* or *NT*

fixed-point algorithm, similar to the subset construction

LR(1) Table Construction

To understand the algorithms, we need to understand the data structure that they use: LR(1) items

We call • the "placeholder"

An **LR(1)** item is a pair [*P*, δ], where

P is a production $A \rightarrow \beta$ with a • at some position in the **RHS**

δ is a single symbol lookahead (*symbol* ≅ *word* or **EOF**)

The **LR(1)** table construction algorithm models a configuration of the parser, or a state of the parser, as a set of **LR(1)** items

- Each item represents a specific production by which the parser might reuce in the future
- Each item's placeholder represents the progress toward that reduction

The construction ties builds up a set of configurations that model all the possible behaviors implied by the grammar

LR(1) Items

The *intermediate representation* of the **LR(1)** table construction algorithm

An **LR(1)** item is a pair [*P*, δ], where

P is a production $A \rightarrow \beta$ with a • at some position in the **RHS**

δ is a single symbol lookahead (*symbol* ≅ *word* or **EOF**)

The • in an item indicates the position of the top of the stack

[$A \rightarrow \bullet \beta\gamma$,a] means that the input seen so far is consistent with the use of $A \rightarrow \beta\gamma$ immediately after the symbol on top of the stack.
We call an item like this a *possibility*.

[$A \rightarrow \beta \bullet \gamma$,a] means that the input sees so far is consistent with the use of $A \rightarrow \beta\gamma$ at this point in the parse, *and* that the parser has already recognized β (that is, β is on top of the stack).
We call an item like this a *partially complete* item.

[$A \rightarrow \beta\gamma \bullet$,a] means that the parser has seen βγ, *and* that a lookahead symbol of a is consistent with reducing to *A*.
This item is *complete*.

LR(k) parsers rely on items with a lookahead of ≤ *k* symbols. That leads to **LR(k)** items, with correspondingly longer δ.

LR(1) Items

The production $A \rightarrow \beta$, where $\beta = B_1B_2B_3$ with lookahead $\underline{a}$, can give rise to 4 items

$[A \rightarrow \bullet B_1B_2B_3, \underline{a}]$, $[A \rightarrow B_1 \bullet B_2B_3, \underline{a}]$, $[A \rightarrow B_1B_2 \bullet B_3, \underline{a}]$, & $[A \rightarrow B_1B_2B_3 \bullet, \underline{a}]$

The set of LR(1) items for a grammar is ***finite***.

What's the point of all these lookahead symbols?

- Carry them along to help choose the correct reduction
- Lookaheads are bookkeeping, unless item has • at right end
  - Has no direct use in $[A \rightarrow \beta \bullet \gamma, \underline{a}]$
  - In $[A \rightarrow \beta \bullet, \underline{a}]$, a lookahead of $\underline{a}$ implies a reduction by $A \rightarrow \beta$
  - For $\{ [A \rightarrow \beta \bullet, \underline{a}], [B \rightarrow \gamma \bullet \delta, \underline{b}] \}$, $\underline{a} \Rightarrow$ *reduce* to A; **FIRST**$(\delta) \Rightarrow$ *shift*

$\Rightarrow$ Limited right context is enough to pick the actions

$\underline{a} \in$ **FIRST**$(\delta) \Rightarrow$ a conflict, not LR(1)

LR(1) Items

The set of LR(1) items for a grammar is finite

Consider the *SheepNoise* grammar

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

It gives rise to 7 **LR(1)** items

			Prod
[*Goal* → • *SheepNoise*]	[*Goal* → *SheepNoise* •]		0
[*Goal* → • *SheepNoise* baa]	[*Goal* → *SheepNoise* • baa]	[*Goal* → *SheepNoise* baa •]	1
[*Goal* → • baa]	[*Goal* → baa •]		2

7 is finite. Why does this matter?

A **configuration** of the **LR(1)** parser is represented as a set of **LR(1)** items—the collection of productions by which the parser might reduce in the future, given the context seen so far, with the placeholder showing the progress in recognizing each production. The number of such states is, again, finite.

LR(1) Items: Why should you know this stuff?

Debugging a grammar

- When you build an **LR(1)** parser, it is possible (likely) that the initial grammar is not **LR(1)**
- The tools will provide you with debugging output
- To the right is a sample of **bison**'s output for the **if-then-else** grammar

```
goal        :  stmt_list
stmt_list   :  stmt_list  stmt
            |  stmt
stmt        :  IF EXPR THEN stmt
            |  IF EXPR THEN stmt
                        ELSE stmt
            |  OTHER
```

That period is the •

```
state 10

   4  stmt : IF EXPR THEN stmt .
   5       | IF EXPR THEN stmt . ELSE stmt

   ELSE  shift, and go to state 11

   ELSE     [reduce using rule 4 (stmt)]
   $default  reduce using rule 4 (stmt)
```

The state is described by its **LR(1)** items

LR(1) Table Construction

High-level overview

1 Build the Canonical Collection of Sets of **LR(1)** Items, *I*

  a Begin in an appropriate state, s_0

  - [$S' \rightarrow \bullet S$,**EOF**], along with any equivalent items
  - Derive equivalent items as *closure*(s_0)

  b Repeatedly compute, for each s_k, and each *X*, *goto*(s_k,*X*)

  - If the set is not already in the collection, add it
  - Record all the transitions created by *goto*( )

  This eventually reaches a fixed point

S is the start symbol. If needed, we add $S' \rightarrow S$ to create a unique goal production.

goto(s_i, *X*) contains the set of LR(1) items that represent possible parser configurations if the parser recognizes an *X* while in state s_i

2 Fill in the table from the Canonical Collection of Sets of **LR(1)** items

The sets in the canonical collection form the states of the Control **DFA**.

The construction traces the **DFA**'s transitions

LR(1) Table Construction

High-level overview

1 Build the Canonical Collection of Sets of **LR(1)** Items, *I*

- a Begin in an appropriate state, s_0
  - ♦ [*S'* →•*S*,**EOF**], along with any equivalent items
  - ♦ Derive equivalent items as ***closure***(s_0)
- b Repeatedly compute, for each s_k, and each *X*, ***goto***(s_k,*X*)
  - ♦ If the set is not already in the collection, add it
  - ♦ Record all the transitions created by ***goto***( )

  This eventually reaches a fixed point

2 Fill in the table from the Canonical Collection of Sets of **LR(1)** items

Let's build the tables for the left-recursive *SheepNoise* grammar (*S'* is *Goal*)

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

Computing Closures

Closure(*s*) adds all the *possibilities* for the items already in *s*

- Any item $[A \rightarrow \beta \bullet B\delta, \underline{a}]$ where $B \in NT$ implies $[B \rightarrow \bullet \tau, x]$ for each production that has *B* on the *lhs,* and each $x \in \text{FIRST}(\delta\underline{a})$
- Since $\beta B\delta$ is valid, any way to derive $\beta B\delta$ is valid, too

The Algorithm

Closure(*s*)
while (s is still changing)
  $\forall$ *items* $[A \rightarrow \beta \bullet B\delta, \underline{a}] \in s$
    lookahead $\leftarrow$ FIRST($\delta\underline{a}$) // δ might be ε
    $\forall$ *productions* $B \rightarrow \tau \in P$
      $\forall$ $\underline{b} \in$ *lookahead*
        if $[B \rightarrow \bullet \tau, \underline{b}] \notin s$
          then $s \leftarrow s \cup \{ [B \rightarrow \bullet \tau, \underline{b}] \}$

- Classic fixed-point method
- Halts because $s \subset I$, the set of all items (*finite*)
- Worklist version is faster
- ***Closure*** *"fills out"* a state *s*

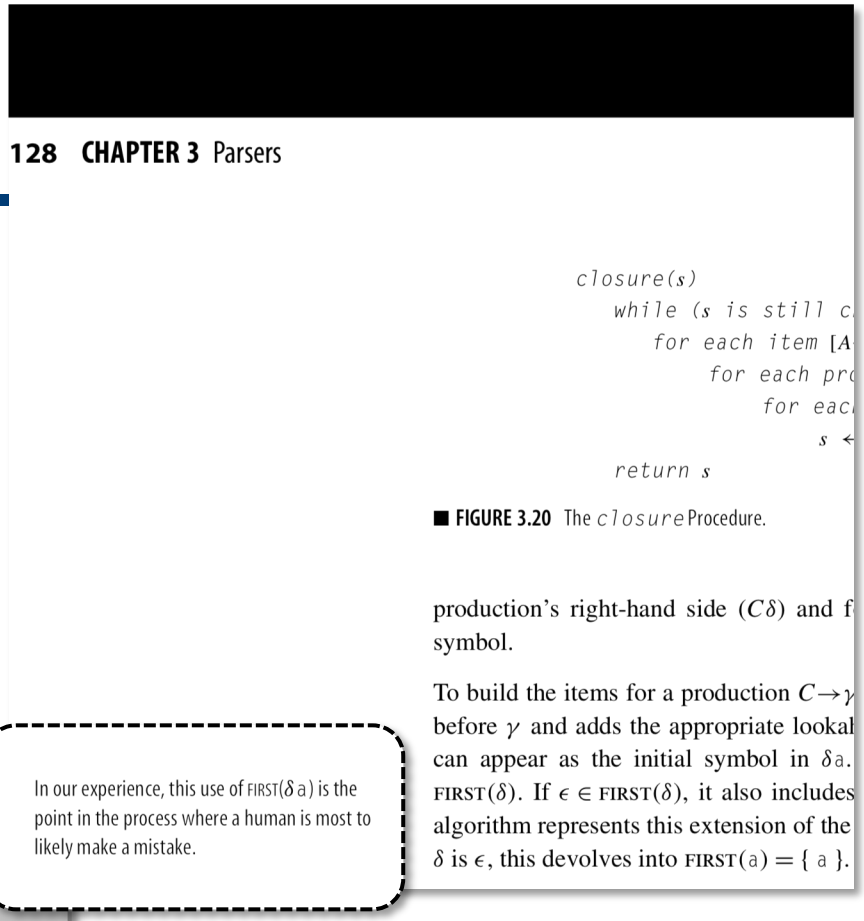
Generate new lookaheads.
See note on p. 128

Computing Closures

Generating Closures is the place where a human is most likely to make a mistake

- With everything going on in the construction, it is easy to lose track of $\delta\underline{a}$ and the fact that it refers to the item, not the current production

Closure *(s)*
while (s is still changing)
$\forall$ *items* $[A \rightarrow \beta \bullet B\delta,\underline{a}] \in s$
$\forall$ *productions* $B \rightarrow \tau \in P$
$\forall$ $\underline{b} \in$ FIRST($\delta\underline{a}$) // δ might be ε
if $[B \rightarrow \bullet\ \tau,\underline{b}] \notin s$
then $s \leftarrow s \cup \{ [B \rightarrow \bullet\ \tau,\underline{b}] \}$



128 CHAPTER 3 Parsers

```
closure(s)
    while (s is still c
        for each item [A
            for each pr
                for eac
                    s ←
    return s
```

■ FIGURE 3.20 The *closure* Procedure.

production's right-hand side ($C\delta$) and f
symbol.

To build the items for a production $C \rightarrow \gamma$
before γ and adds the appropriate lookal
can appear as the initial symbol in δa.
FIRST(δ). If $\epsilon \in$ FIRST(δ), it also includes
algorithm represents this extension of the
δ is ϵ, this devolves into FIRST(a) = { a }.

In our experience, this use of FIRST(δ a) is the point in the process where a human is most to likely make a mistake.

- The lookahead computation is a great example of why these table constructions should be done by computers, not human beings

This is the left-recursive SheepNoise; EaC2e shows the right-recursive version.

Example From SheepNoise

Initial step builds the item [*Goal*→ • *SheepNoise,* EOF] and takes its *Closure*( )

Closure([*Goal*→ • *SheepNoise,* **EOF**])

Item	*Source*
[*Goal* → • *SheepNoise,* **EOF**]	Original item
[*SheepNoise* → • *SheepNoise* baa, **EOF**]	**ITER** 1, **PR** 1, δa is **EOF**
[*SheepNoise* → • baa, **EOF**]	**ITER** 1, **PR** 2, δa is **EOF**
[*SheepNoise* → • *SheepNoise* baa, baa]	**ITER** 2, **PR** 1, δa is baa **EOF**
[*SheepNoise* → • baa, baa]	**ITER** 2, **PR** 2, δa is baa **EOF**

Symbol	*FIRST*
Goal	{ baa }
SheepNoise	{ baa }
baa	{ baa }
EOF	{ **EOF** }

So, S_0 is

{ [*Goal*→ • *SheepNoise*,**EOF**], [*SheepNoise*→ • *SheepNoise* baa,**EOF**],
[*SheepNoise*→• baa,**EOF**], [*SheepNoise*→ • *SheepNoise* baa,baa],
[*SheepNoise*→ • baa,baa] }

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

Computing Gotos

Goto(_s_,_x_) computes the state that the parser would reach if it recognized an _x_ while in state _s_

- ***Goto***({ [$A \rightarrow \beta \bullet X\delta$,a] }, X) produces { [$A \rightarrow \beta X \bullet \delta$,a] } ***(obviously)***
- It finds all such items & uses *Closure*() to fill out the state

The Algorithm

```
Goto( s, X )
   new ← Ø
    ∀ items [A→β•Xδ,a] ∈ s
      new ← new ∪ { [A→βX•δ,a] }
    return Closure( new )
```

- ***Goto***( ) models a transition in the automaton
- Straightforward computation
- ***Goto()*** is not a fixed-point method (but it calls ***Closure()***)

***Goto* in this construction is analogous to *Move* in the subset construction.**

Example from SheepNoise

Assume that S_0 is

{ [*Goal* → • *SheepNoise*,**EOF**], [*SheepNoise* → • *SheepNoise* baa,**EOF**],
[*SheepNoise* → • baa,**EOF**], [*SheepNoise* → • *SheepNoise* baa,baa],
[*SheepNoise* → • baa,baa] }

From earlier slide

Goto(S_0 , baa)

- Loop produces

Item	*Source*
[*SheepNoise* → baa •, **EOF**]	Item 3 in s_0
[*SheepNoise* → baa •, baa]	Item 5 in s_0

- ***Closure*** adds nothing since • is at end of *rhs* in each item

In the construction, this produces s_2

{ [*SheepNoise*→baa •, {**EOF**, baa}] }

New, but *obvious*, notation for two distinct items
[*SheepNoise*→ baa •, **EOF**] & [*SheepNoise*→ baa •, baa]

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

Building the Canonical Collection

Start from s_0 = ***Closure***([$S' \rightarrow \bullet S$,**EOF**])

Repeatedly construct new states, until all are found

The Algorithm

$s_0 \leftarrow$ ***Closure*** ({ [$S' \rightarrow \bullet S$,**EOF**] })
$S \leftarrow \{ s_0 \}$
$k \leftarrow 1$

while (*S is still changing*)

  $\forall s_j \in S$ *and* $\forall x \in (T \cup NT)$
    $s_k \leftarrow$ ***Goto***(s_j,x)
    record $s_j \rightarrow s_k$ *on* x
  if $s_k \notin S$ *then*
    $S \leftarrow S \cup \{ s_k \}$
    $k \leftarrow k + 1$

- Fixed-point computation
- Loop adds to S (*monotone*)
- $S \subseteq 2^{\text{ITEMS}}$, so S is finite

- *Worklist version is faster because it avoids duplicated effort*

This membership / equality test requires careful and/or clever implementation.

Example from SheepNoise

Starts with S_0

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

Iteration 1 computes

S_1 = ***Goto***(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise* → *SheepNoise* • baa, baa] }

S_2 = ***Goto***(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

Nothing more to compute, since • is at the end of every item in S_3 .

Iteration 2 computes

S_3 = ***Goto***(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

Example from SheepNoise

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

S_1 = ***Goto***(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise* → *SheepNoise* • baa, baa] }

S_2 = ***Goto***(S_0 , baa) = { [*SheepNoise* → baa •, EOF], [*SheepNoise* → baa •, baa] }

S_3 = ***Goto***(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

State	*SN*	baa
s_0	s_1	s_2
s_1	—	s_3
s_2	—	—
s_3	—	—

Goto Relationships

Filling in the ACTION and GOTO Tables

The Table Construction Algorithm

x is the state number

$\forall$ *set* $S_x \in S$
  $\forall$ *item* $i \in S_x$
    if i *is* $[A \rightarrow \beta \bullet \underline{a}\delta, \underline{b}]$ *and* $goto(S_x, \underline{a}) = S_k$, $\underline{a} \in T$
      then ACTION$[x, \underline{a}] \leftarrow$ "*shift k*"
    else if i *is* $[S' \rightarrow S \bullet, \underline{EOF}]$
      then ACTION$[x, \underline{EOF}] \leftarrow$ "*accept*"
    else if i *is* $[A \rightarrow \beta \bullet, \underline{a}]$
      then ACTION$[x, \underline{a}] \leftarrow$ "*reduce* $A \rightarrow \beta$"
  $\forall$ $n \in NT$
    if $goto(S_x, n) = S_k$
      then GOTO$[x, n] \leftarrow k$

- before $T \Rightarrow$ *shift*

have *Goal* $\Rightarrow$ *accept*

- at end $\Rightarrow$ *reduce*

Many items generate no table entry

$\rightarrow$ Placeholder before a *NT* does not generate an **ACTION** table entry

$\rightarrow$ ***Closure***( ) instantiates FIRST(*X*) directly for $[A \rightarrow \beta \bullet X\delta, \underline{a}]$

Example from SheepNoise

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

• before $T \Rightarrow$ *shift* k

S_1 = *Goto*(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise*→ *SheepNoise* • baa, baa] }

S_2 = *Goto*(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

so, ACTION[s_0,baa] is "*shift* S_2" (clause 1)
(items define same entry)

S_3 = *Goto*(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

Example from SheepNoise

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

S_1 = *Goto*(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise*→ *SheepNoise* • baa, baa] }

S_2 = *Goto*(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

so, ACTION[S_1,baa] is "*shift* S_3" (clause 1)

S_3 = *Goto*(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

Example from SheepNoise

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

S_1 = *Goto*(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise*→ *SheepNoise* • baa, baa] }

S_2 = *Goto*(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

so, ACTION[S_1,EOF] is "*accept*" (clause 2)

S_3 = *Goto*(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

Example from SheepNoise

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

S_1 = *Goto*(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise*→ *SheepNoise* • baa, baa] }

S_2 = *Goto*(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

so, ACTION[S_2,EOF] is "*reduce 2*"
(clause 3) (baa, too)

S_3 = *Goto*(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

ACTION[S_3,EOF] is "*reduce 1*"
(clause 3) (baa, too)

Building the Goto Table

S_0 : { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

S_1 = *Goto*(S_0 , *SheepNoise*) =
{ [*Goal* → *SheepNoise* •, EOF], [*SheepNoise* → *SheepNoise* • baa, EOF],
[*SheepNoise*→ *SheepNoise* • baa, baa] }

S_2 = *Goto*(S_0 , baa) = { [*SheepNoise* → baa •, EOF],
[*SheepNoise* → baa •, baa] }

S_3 = *Goto*(S_1 , baa) = { [*SheepNoise* → *SheepNoise* baa •, EOF],
[*SheepNoise* → *SheepNoise* baa •, baa] }

The Goto table holds just the entries for nonterminal symbols.

(*the* baa *column went into Action*)

State	*SN*	baa
s_0	s_1	s_2
s_1	—	s_3
s_2	—	—
s_3	—	—

Goto Relationships

ACTION & GOTO Tables

Here are the tables for the left-recursive *SheepNoise* grammar

The tables

ACTION TABLE		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 2*	*reduce 2*
3	*reduce 1*	*reduce 1*

GOTO TABLE	
State	*SheepNoise*
0	1
1	0
2	0
3	0

The grammar

0	*Goal*	→	*SheepNoise*
1	*SheepNoise*	→	*SheepNoise* baa
2		|	baa

Remember, this is the left-recursive SheepNoise; EaC2e shows the right-recursive version.

What can go wrong?

The **if-then-else** grammar is worked as an example in EaC2e

What if a set *s* contains [*A*→β•a̲γ,b̲] and [*B*→β•,a̲] ?

- First item generates "shift", second generates "reduce"
- Both define ACTION[s,a̲] — cannot do both actions
- This is a fundamental ambiguity, called a **shift/reduce error or conflict**
- Modify the grammar to eliminate it *(if-then-else)*
- Shifting will often resolve it correctly

What if a set *s* contains [*A*→γ•, a̲] and [*B*→γ•, a̲] ?

- Each generates "reduce", but with a different production
- Both define ACTION[s,a̲] — cannot do both reductions
- This is a fundamental ambiguity, called a **reduce/reduce error or conflict**
- Modify the grammar to eliminate it *(PL/I's overloading of (...))*

In either case, the grammar is not LR(1)

Implementing the Construction
Building the Canonical Collection

Start from s_0 = ***closure***([$S' \rightarrow \bullet S$,EOF])

Repeatedly construct new states, unti

The algorithm

$s_0 \leftarrow$ ***closure*** ([$S' \rightarrow \bullet S$,EOF])
$S \leftarrow \{ s_0 \}$
$k \leftarrow 1$

while (*S is still changing*)

  $\forall s_j \in S$ *and* $\forall x \in (T \cup NT)$
    $s_k \leftarrow$ ***goto***(s_j,x)
    record $s_j \rightarrow s_k$ *on* x
  if $s_k \notin S$ *then*
    $S \leftarrow S \cup \{ s_k \}$
    $k \leftarrow k + 1$

Remember this comment about implementing the equality test at the bottom of the algorithm to build the Canonical Collection of Sets of LR(1) Items?

- Only need to compare core items — the rest will follow
- Represent items as a triple (R,P,L)
  - R is the rule or production
  - P is the position of the placeholder
  - L is the lookahead symbol
- Order items, then
  1. Compare set cardinalities
  2. Compare (in order) by R, P, L

This membership / equality test requires careful and/or clever implementation.

STOP