

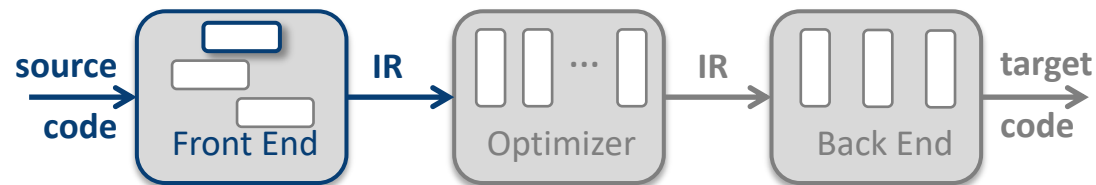


COMP 412  
FALL 2018

Midterm Exam: Thursday  
October 18, 7PM  
Herzstein Amphitheater

## Syntax Analysis, V

*Bottom-up Parsing & The Magic of Handles*  
Comp 412



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Chapter 3 in EaC2e

# Lab 2 Schedule



Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	
9	10	11	12	13	14 Lab 2 Specs available	15	Focus on connecting to Lab 1 IR, and Rename
16	17	18 Tutorial 5 PM McMurtr	19	20	21 Deadline: Code Check 1	22	
23	24 Tutorial 5 PM McMurtry	25 Dan Grove Talk (Dart Group)	26	27	28	29	Allocate correctly
30	1 Deadline: Code Check 2	2 Advice: Pay attention to the timing / scaling tests	3	4	5	6	
7	8 Deadline: Lab 2 Code	9	10	12 Deadline: Lab Report	13	14	Improve performance & allocation

# Parsing Techniques

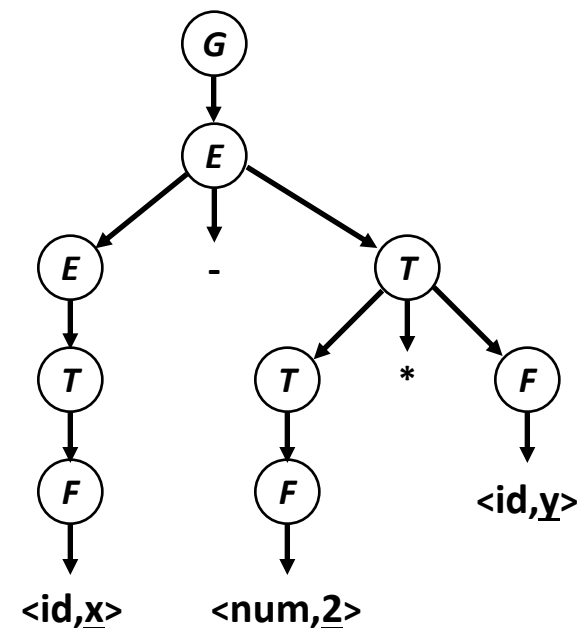


## Top-down parsers (*LL(1), recursive descent*)

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad “pick”  $\Rightarrow$  may need to backtrack
- Some grammars are backtrack-free

## Bottom-up parsers (*LR(1), operator precedence*)

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- We can make the process deterministic



*Parse tree for  $x - 2 * y$*

Bottom-up parsers can recognize a strictly larger class of *grammars* than can top-down parsers. See exercise 3.12 in EaC2e.

# Bottom-up Parsing

(definitions)



**The point of parsing is to construct a *derivation***

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{sentence}$$

- Each  $\gamma_i$  is a sentential form
  - If  $\gamma$  contains only terminal symbols,  $\gamma$  is a **sentence** in  $L(G)$
  - If  $\gamma$  contains 1 or more non-terminals,  $\gamma$  is a **sentential form**
- To get  $\gamma_i$  from  $\gamma_{i-1}$ , expand some NT  $A \in \gamma_{i-1}$  by using  $A \rightarrow \beta$ 
  - Replace the occurrence of  $A \in \gamma_{i-1}$  with  $\beta$  to get  $\gamma_i$
  - In a leftmost derivation, it would be the first NT  $A \in \gamma_{i-1}$

A ***left-sentential form*** occurs in a leftmost derivation

A ***right-sentential form*** occurs in a rightmost derivation

***Bottom-up, LR(1) parsers build a rightmost derivation in reverse***

# Bottom-up Parsing

(definitions)



A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol  $S$

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{sentence}$$

← bottom-up

To **reduce**  $\gamma_i$  to  $\gamma_{i-1}$  match some *rhs*  $\beta$  against  $\gamma_i$  then replace  $\beta$  with its corresponding *lhs*,  $A$  (assuming the reduction is  $A \rightarrow \beta$ )

In terms of the parse tree, it works from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of  $\beta$  with  $A$  shrinks the upper fringe, we call it a **reduction**.
- “Rightmost derivation in reverse” processes words **left to right**

The parse tree need not be built, it can be simulated

$$|\text{parse tree nodes}| = |\text{terminal symbols}| + |\text{reductions}|$$

“Shrinks the upper fringe” implies that the terminals are all instantiated, at least implicitly.

# Finding Reductions



Consider the grammar

0	<i>Goal</i>	$\rightarrow$	<u>a</u> A B <u>e</u>
1	A	$\rightarrow$	A <u>b</u> <u>c</u>
2			<u>b</u>
3	B	$\rightarrow$	<u>d</u>

And the input string abcde

derivation

*Sentential  
Form*

abcde

a A bcde

a A de

a A B e

*Goal*

- The trick is scanning the input and finding the next reduction.
- The mechanism for doing this must be efficient.

\* The reductions are obvious from the derivation. Of course, building the derivation is not a practical way to find it.

**a b<sup>+</sup> (bc)<sup>\*</sup> de**

# Finding Reductions



## Consider the grammar

0	Goal	→	<u>a</u> A B <u>e</u>
1	A	→	A <u>b</u> <u>c</u>
2			<u>b</u>
3	B	→	<u>d</u>

And the input string abcde

parse  
↓

Sentential Form	Next Reduction	
	Prod'n	Pos'n
<u>abcde</u>	2	2
<u>a</u> A <u>bcde</u>	1	4
<u>a</u> A <u>de</u>	3	3
<u>a</u> A B <u>e</u>	0	4
Goal	—	—

- The trick is scanning the input and finding the next reduction
- The mechanism for doing this must be efficient

*“Position” specifies where the right end of  $\beta$  occurs in the current sentential form.*

While the process of finding the next reduction appears to be almost oracular, it can be automated in an efficient way for a large class of grammars.

# Finding Reductions

(Handles)



At each step, the parser needs to find a substring  $\beta$  of the tree's upper frontier that *derives from an expansion by  $A \rightarrow \beta$  in the previous step in the rightmost derivation*

Informally, we call this substring  $\beta$  a **handle**

By convention, we will use  $k$  to mark the **right end** of the handle

Formally,

A **handle** of a right-sentential form  $\gamma$  is a pair  $\langle A \rightarrow \beta, k \rangle$  where  $A \rightarrow \beta \in P$  and  $k$  is the position in  $\gamma$  of  $\beta$ 's rightmost symbol.

If  $\langle A \rightarrow \beta, k \rangle$  is a handle, then replacing  $\beta$  at  $k$  with  $A$  produces the right sentential form from which  $\gamma$  is derived in the rightmost derivation.

Because  $\gamma$  is a right-sentential form, the substring to the right of a handle contains **only terminal symbols**

$\Rightarrow$  the parser doesn't need to scan (*much*) past the handle

*Handles are the **most mystifying** aspect of bottom-up, shift-reduce parsers. It usually takes a couple lectures to comprehend. Assume, **WLOG**, that we can find handles easily ...*



# Using Handles: a Bottom-up Parser



As with top-down parsers, we will use a stack to hold the fringe of the partially completed parse tree. In this case, it is the upper fringe.

## A simple shift-reduce parser:

```
push INVALID
word ← NextWord( )
repeat until (top of stack = Goal and word = EOF)
  if the top of the stack forms a handle  $A \rightarrow \beta$  then
    // reduce  $\beta$  to  $A$ 
    pop  $|\beta|$  symbols off the stack
    push  $A$  onto the stack
  else if (word  $\neq$  EOF) then // shift
    push word
    word ← NextWord( )
  else // need to shift, but out of input
    report an error
report success // accept
```

## What happens on an error?

- Parser fails to find a handle
- Thus, it keeps shifting
- Eventually, it consumes all input

This parser reads all input before reporting an error, not a desirable property.

To fix this issue, the parser must recognize the failure to find a handle earlier.

To make shift-reduce parsers practical, we need good error localization in the handle-finding process.

# Example



0	<i>Goal</i>	$\rightarrow$	<i>Expr</i>
1	<i>Expr</i>	$\rightarrow$	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	$\rightarrow$	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	$\rightarrow$	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

***The Left-recursive, Left-associative  
Classic Expression Grammar***

Bottom-up parsers work with either left-recursive or right-recursive grammars.

The examples will use the obvious left-recursive, left-associative form of the classic expression grammar.

I prefer the obvious left-recursive grammar because its associativity matches the standard rules that we were all taught as children.

# Example



0	<i>Goal</i>	$\rightarrow$	<i>Expr</i>
1	<i>Expr</i>	$\rightarrow$	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	$\rightarrow$	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	$\rightarrow$	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

*The Left-recursive, Left-associative  
Classic Expression Grammar*

derivation

<i>Prod'n</i>	<i>Sentential Form</i>
—	<i>Goal</i>
0	<i>Expr</i>
2	<i>Expr</i> - <i>Term</i>
4	<i>Expr</i> - <i>Term</i> * <i>Factor</i>
9	<i>Expr</i> - <i>Term</i> * <id, <u>y</u> >
6	<i>Expr</i> - <i>Factor</i> * <id, <u>y</u> >
8	<i>Expr</i> - <num, <u>2</u> > * <id, <u>y</u> >
3	<i>Term</i> - <num, <u>2</u> > * <id, <u>y</u> >
6	<i>Factor</i> - <num, <u>2</u> > * <id, <u>y</u> >
9	<id, <u>x</u> > - <num, <u>2</u> > * <id, <u>y</u> >

*Rightmost derivation of x - 2 \* y*

# Example



0	<i>Goal</i>	$\rightarrow$	<i>Expr</i>
1	<i>Expr</i>	$\rightarrow$	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	$\rightarrow$	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	$\rightarrow$	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

*The Left-recursive, Left-associative  
Classic Expression Grammar*

<i>Prod'n</i>	<i>Sentential Form</i>
—	<i>Goal</i>
0	<i>Expr</i>
2	<i>Expr</i> - <i>Term</i>
4	<i>Expr</i> - <i>Term</i> * <i>Factor</i>
9	<i>Expr</i> - <i>Term</i> * <id, <u>y</u> >
6	<i>Expr</i> - <i>Factor</i> * <id, <u>y</u> >
8	<i>Expr</i> - <num, <u>2</u> > * <id, <u>y</u> >
3	<i>Term</i> - <num, <u>2</u> > * <id, <u>y</u> >
6	<i>Factor</i> - <num, <u>2</u> > * <id, <u>y</u> >
9	<id, <u>x</u> > - <num, <u>2</u> > * <id, <u>y</u> >

*parse*

*Rightmost derivation of x - 2 \* y*

# Example



0	Goal	→	Expr
1	Expr	→	Expr + Term
2			Expr - Term
3			Term
4	Term	→	Term * Factor
5			Term / Factor
6			Factor
7	Factor	→	( Expr )
8			<u>number</u>
9			<u>id</u>

*The Left-recursive, Left-associative  
Classic Expression Grammar*

Prod'n	Sentential Form
—	Goal
0	Expr
2	Expr - Term
4	Expr - Term * Factor
9	Expr - Term * <id,y>
6	Expr - Factor * <id,y>
8	Expr - <num,2> * <id,y>
3	Term - <num,2> * <id,y>
6	Factor - <num,2> * <id,y>
9	<id,x> - <num,2> * <id,y>

parse

*Handles for rightmost derivation of  $\underline{x} = \underline{2} * \underline{y}$*

# Back to x - 2 \* y



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	<i>none</i>	<i>shift</i>
\$ <u>id</u>	- <u>num</u> * <u>id</u>		

0	Goal	→	Expr
1	Expr	→	Expr + Term
2			Expr - Term
3			Term
4	Term	→	Term * Factor
5			Term / Factor
6			Factor
7	Factor	→	( Expr )
8			<u>number</u>
9			<u>id</u>

By convention, \$  
represents **INVALID**

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

# Back to $\underline{x} - \underline{2} * \underline{y}$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>		

0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

# Back to $\underline{x} - \underline{2} * \underline{y}$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>		

*Expr* is not a handle at this point because it does not occur at this point in the derivation.

While that statement sounds like **oracular mysticism**, we will see that the decision can be automated efficiently.

0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce



# Back to $\underline{x} - \underline{2} * \underline{y}$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> -	<u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> - <u>num</u>	* <u>id</u>		

0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

1. Shift until the top of the stack is the right end of a handle

2. Find the left end of the handle and reduce

# Back to $x - 2 * y$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> -	<u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> - <u>num</u>	* <u>id</u>	8,3	reduce 8
\$ <i>Expr</i> - <i>Factor</i>	* <u>id</u>	6,3	reduce 6
\$ <i>Expr</i> - <i>Term</i>	* <u>id</u>		

0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

1. Shift until the top of the stack is the right end of a handle

2. Find the left end of the handle and reduce

# Back to $x - 2 * y$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> -	<u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> - <u>num</u>	* <u>id</u>	8,3	reduce 8
\$ <i>Expr</i> - <i>Factor</i>	* <u>id</u>	6,3	reduce 6
\$ <i>Expr</i> - <i>Term</i>	* <u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> *	<u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> * <u>id</u>			

0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

1. Shift until the top of the stack is the right end of a handle

2. Find the left end of the handle and reduce

# Back to $x - 2 * y$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce 3
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> -	<u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> - <u>num</u>	* <u>id</u>	8,3	reduce 8
\$ <i>Expr</i> - <i>Factor</i>	* <u>id</u>	6,3	reduce 6
\$ <i>Expr</i> - <i>Term</i>	* <u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> *	<u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> * <u>id</u>		9,5	reduce 9
\$ <i>Expr</i> - <i>Term</i> * <i>Factor</i>		4,5	reduce 4
\$ <i>Expr</i> - <i>Term</i>		2,3	reduce 2
\$ <i>Expr</i>		0,1	reduce 0
\$ <i>Goal</i>		none	accept

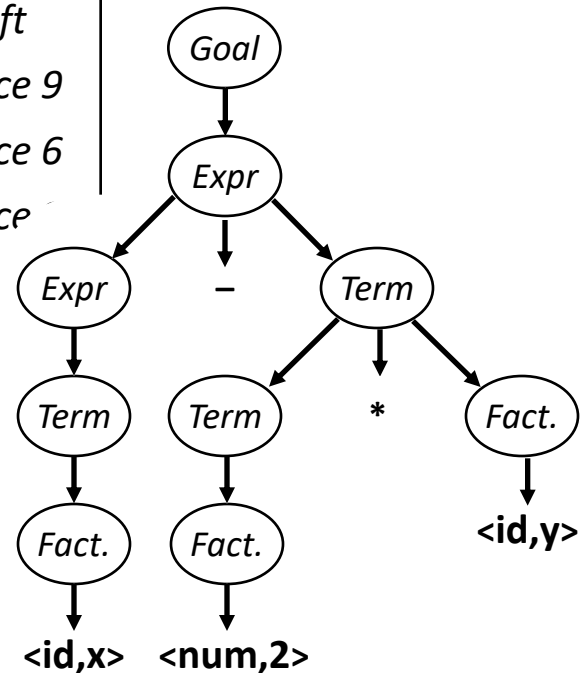
0	Goal	→	<i>Expr</i>
1	<i>Expr</i>	→	<i>Expr</i> + <i>Term</i>
2			<i>Expr</i> - <i>Term</i>
3			<i>Term</i>
4	<i>Term</i>	→	<i>Term</i> * <i>Factor</i>
5			<i>Term</i> / <i>Factor</i>
6			<i>Factor</i>
7	<i>Factor</i>	→	( <i>Expr</i> )
8			<u>number</u>
9			<u>id</u>

5 shifts +  
9 reduces +  
1 accept

# Back to $x - 2 * y$



Stack	Input	Handle	Action
\$	<u>id</u> - <u>num</u> * <u>id</u>	none	shift
\$ <u>id</u>	- <u>num</u> * <u>id</u>	9,1	reduce 9
\$ <i>Factor</i>	- <u>num</u> * <u>id</u>	6,1	reduce 6
\$ <i>Term</i>	- <u>num</u> * <u>id</u>	3,1	reduce
\$ <i>Expr</i>	- <u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> -	<u>num</u> * <u>id</u>	none	shift
\$ <i>Expr</i> - <u>num</u>	* <u>id</u>	8,3	reduce
\$ <i>Expr</i> - <i>Factor</i>	* <u>id</u>	6,3	reduce
\$ <i>Expr</i> - <i>Term</i>	* <u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> *	<u>id</u>	none	shift
\$ <i>Expr</i> - <i>Term</i> * <u>id</u>		9,5	reduce
\$ <i>Expr</i> - <i>Term</i> * <i>Factor</i>		4,5	reduce
\$ <i>Expr</i> - <i>Term</i>		2,3	reduce
\$ <i>Expr</i>		0,1	reduce 0
\$ <i>Goal</i>		none	accept



Corresponding Parse Tree

|Parse tree nodes| = |shifts| + |reduces|

Took |Parse tree nodes| + 1 steps

# More on Handles



## Shift reduce parsers find a rightmost derivation in reverse order

- Rightmost derivation  $\Rightarrow$  rightmost NT expanded at each step in the derivation
- Processed in reverse  $\Rightarrow$  parser proceeds left to right

These two statements are somewhat counter-intuitive

# More on Handles



## Shift-reduce parsers find a reverse rightmost derivation

- Process input left to right
  - Upper fringe of partially completed parse tree is  $(NT \mid T)^* T^*$
  - The handle always appears with its right end at the junction between  $(NT \mid T)^*$  and  $T^*$  *(the hot spot for LR parsing)*
  - We can keep the prefix of the upper fringe of the partially completed parse tree on a stack — that is,  $(NT \mid T)^*$
- Handles appear at the top of the stack
  - Right end of handle is *always* at the top of the stack
  - The stack makes the position information irrelevant
- All the information for the decision is at the hot spot
  - The next word in the input stream
  - The rightmost **NT** on the fringe & its immediate left neighbors
  - An **LR** parser keeps additional information on the stack
    - *the “state” of a handle-recognizing automaton*

# Handles



Consider  $x - 2 * y$  with the expression grammar

Sentential Form			
Goal			
Expr			
Expr	—	Term	
Expr	—	Term	* Factor
Expr	—	Term	* <id,y>
Expr	—	Factor	* <id,y>
Expr	—	<num,2>	* <id,y>
Term	—	<num,2>	* <id,y>
Factor	—	<num,2>	* <id,y>
<id,x>	—	<num,2>	* <id,y>

derivation

parse

**Unambiguous grammar implies unique rightmost derivation**

- At each step, we have one step that leads to  $x - 2 * y$
- Any other choice leads to another distinct expression

**A bottom-up parse reverses the rightmost derivation**

- Each step has a unique reduction
- The key is finding the reduction at each step that leads to the derivation



# Handles



Consider  $x - 2 * y$  with the expression grammar

Sentential Form			
Goal			
Expr			
Expr	—	Term	
Expr	—	Term	* Factor <sup>↑</sup>
Expr	—	Term	* <id,y>
Expr	—	Factor	* <id,y>
Expr	—	<num,2>	* <id,y>
Term	—	<num,2>	* <id,y>
Factor	—	<num,2>	* <id,y>
<id,x>	—	<num,2>	* <id,y>

Now, look at the sentential forms in the example

- They have a specific form
- $NT^* (NT \mid T)^* T^*$
- Handles are found in the  $(NT \mid T)^*$  portion
  - Track right end of region
  - Search left from there
  - Finite set of rhs strings

We know that each step has a unique reduction

That reduction is the **handle**

# Handles



Consider  $x - 2 * y$  with the expression grammar

<i>Sentential Form</i>				<i>Reduction</i>
<i>Goal</i>				
<i>Expr</i>				$Goal \rightarrow Expr$
<i>Expr</i>	—	<i>Term</i>		$Expr \rightarrow Expr - Term$
<i>Expr</i>	—	<i>Term</i>	*	$Term \rightarrow Term * Factor$
<i>Expr</i>	—	<i>Term</i>	*	$Factor \rightarrow \underline{id}$
<i>Expr</i>	—	<i>Factor</i>	*	$Term \rightarrow Factor$
<i>Expr</i>	—	<num,2>	*	$Factor \rightarrow \underline{num}$
<i>Term</i>	—	<num,2>	*	$Expr \rightarrow Term$
<i>Factor</i>	—	<num,2>	*	$Term \rightarrow Factor$
<id,x>	—	<num,2>	*	$Factor \rightarrow \underline{id}$

# Finding Reductions

We saw this slide earlier.  
It deserves a second take.



At each step, the parser needs to find a substring  $\beta$  of the tree's upper frontier that *derives from an expansion by  $A \rightarrow \beta$  in the previous step in the rightmost derivation*

Informally, we call this substring  $\beta$  a **handle**

By convention, we will use  $k$  to mark the **right end** of the handle

Formally,

A **handle** of a right-sentential form  $\gamma$  is a pair  $\langle A \rightarrow \beta, k \rangle$  where  $A \rightarrow \beta \in P$  and  $k$  is the position in  $\gamma$  of  $\beta$ 's rightmost symbol.

If  $\langle A \rightarrow \beta, k \rangle$  is a handle, then replacing  $\beta$  at  $k$  with  $A$  produces the right sentential form from which  $\gamma$  is derived in the rightmost derivation.

Because  $\gamma$  is a right-sentential form, the substring to the right of a handle contains **only terminal symbols**

$\Rightarrow$  the parser doesn't need to scan (*much*) past the handle

Handles are the **most mystifying** aspect of bottom-up, shift-reduce parsers. It usually takes a couple lectures to comprehend.  
Assume, **WLOG**, that we can find handles easily ...

# Handles Are Unique



## Theorem:

*If  $G$  is unambiguous, then every right-sentential form has a **unique** handle.*

*Recall: Right sentential form is a string that appears as one step in a rightmost derivation.*

## Sketch of Proof:

- 1  $G$  is unambiguous  $\Rightarrow$  rightmost derivation is unique
- 2  $\Rightarrow$  a unique production  $A \rightarrow \beta$  applied to derive  $\gamma_i$  from  $\gamma_{i-1}$
- 3  $\Rightarrow$  a unique position  $k$  at which  $A \rightarrow \beta$  is applied
- 4  $\Rightarrow$  a unique handle  $\langle A \rightarrow \beta, k \rangle$

This all follows from the definitions

If we can find the handles, we can build a derivation!

*The handle always appears with its right end at the stack top.*

*$\Rightarrow$  We can make the handles relative to the stack top, which makes the “position” implicit. Now the handle is just the “right” RHS at stack top.*

# From Handles to Parsers



**The sentential forms in the derivation have the form:**

$$NT^* (NT \mid T)^* T^*$$

- They form the upper fringe of partially completed parse tree
- The suffix consisting of  $T^*$  is, at each step, the unread input
  - *The first word in the trailing string of terminals is the current word or token*

**The shift-reduce parser operates by:**

- Keeping the portion  $NT^* (NT \mid T)^*$  on a stack
  - Leftmost symbol at bottom of stack, rightmost at stack top
- Handles always appear with right end at the top of stack
  - *The border between  $NT^* (NT \mid T)^*$  and  $T^*$  is the critical spot*
- Searching for handles from stack top to stack bottom
- If search fails, shift another terminal onto stack

All the info that the parser needs  
to decide is at TOS

# The Critical Lesson about Handles

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Simply looking for right hand sides that match is not good enough

**Critical Question:** How can we know when we have found a handle without generating lots of different derivations?

- **Answer:** We use left context, encoded in the sentential form, left context encoded in a “parser state”, and a lookahead — the next word in the input. (Formally, 1 word beyond the handle.)
- Parser states are derived by reachability analysis on grammar that resembles the subset construction from Chapter 2.
- The set of handles is *finite*  $\Rightarrow$  we can recognize handles with a **DFA**
  - Invoke **DFA** recursively to find terminal symbols as “sub goals”
  - Store **DFA** states (at recursive invocation) on the stack as “parser state”

The additional left context is precisely the reason that LR(1) grammars express a superset of the languages that can be expressed as LL(1) grammars

# The Critical Lesson about Handles

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