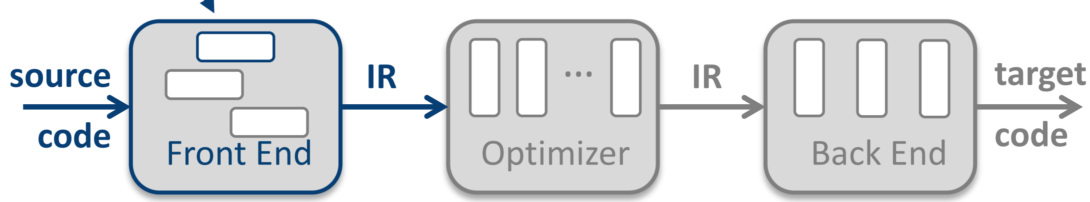
COMP 412
FALL 2018

Introduction to Parsing

Context-free grammars

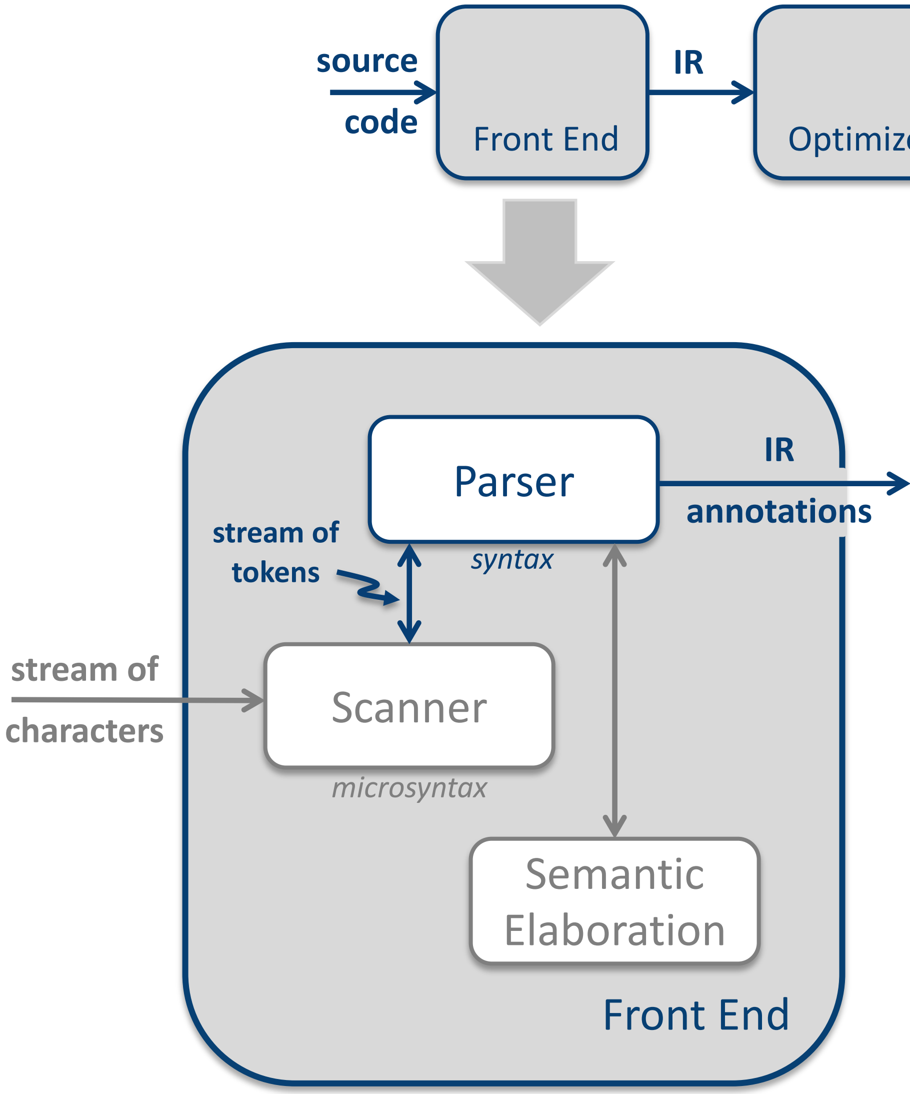
Comp 412

Finally, we changed boxes …



Chapter 3 in EaC2e

The Front End



Scanner looks at every character

- Converts stream of chars to stream of classified words:
  - <word, part of speech>
  - Sometimes call this pair a “token”
- Efficiency & scalability matter

Parser looks at every token

- Determines if the stream of tokens forms a sentence in the source language
- Fits tokens to some syntactic model, or grammar, for the source language

The Study of Parsing

Parsing is the process of discovering a *derivation* for some sentence

- Need mathematical model of syntax — a grammar G
- Need an algorithm to test membership in $L(G)$
- Need to remember that our goal is to build parsers, not to study the interesting if arcane mathematics of arbitrary languages

Roadmap for our study of parsing

1. Context-free grammars & derivations
2. Top-down parsing
   - Top-down parsers explore the possibilities of syntax in a systematic way
   - A file of code has a limited number of words that can occur at its start
3. Bottom-up parsing
   - Bottom-up parsers build on the detailed structure of the input stream
   - Each classified word can affect the interpretation of past & future words

Review: See Lecture 3

Specifying Syntax: Context-Free Grammars

Context-free syntax is specified with a *context-free grammar* (CFG)

This **CFG** defines the set of noises that sheep normally make

0	***SheepNoise***	→	***SheepNoise*** **baa**
1		|	**baa**

See the digression about BNF on p. 87 of EaC2e

It is written in a variant of Backus-Naur form (**BNF**)

Formally, a **CFG** is a four tuple, $G = (S, N, T, P)$

- S is the *start symbol* of the grammar
  - *L(G) is the set of sentences that can be derived from* S
- N is a set of *nonterminal symbols* or syntactic variables — *SheepNoise*
- T is a set of *terminal symbols* or words — baa
- P is a set of *productions* or *rewrite rules* — $P: N \rightarrow (N \cup T)^+$

We will defer the definition of "context free" for a few slides.

Review: See Lecture 3

Deriving Sentences with a CFG

We can use the *SheepNoise* grammar to derive sentences

– *use the productions as rewrite rules*

Rule	Sentential Form
—	*SheepNoise*
1	baa

Rule	Sentential Form
—	*SheepNoise*
0	*SheepNoise* baa
1	baa baa

Rule	Sentential Form
—	*SheepNoise*
0	*SheepNoise* baa
0	*SheepNoise* baa baa
1	baa baa baa

And, so on …

While this example is cute, it becomes trite pretty quickly …

A ***sentential form*** is a string of terminal & nonterminal symbols that is a valid step in some derivation.

Context-Free Grammars

What makes a grammar "context free" ?

Productions in the *SheepNoise* grammar have a specific form:

0	***SheepNoise***	→	***SheepNoise*** **baa**
1		|	**baa**

Each production has *a single nonterminal symbol on its left hand side*, which makes it impossible to encode either left or right context.

⇒ The grammar is *context free*

A context-sensitive grammar can have ≥ 1 symbol on its lhs.

- **CSG**'s have not found widespread application in compilers

lhs ≅ *left-hand side*
rhs ≅ *right hand side*

Notice that *L*(*SheepNoise*) is actually a regular language: baa baa* **RLs ⊂ CFLs**

Digression: The Chomsky Hierarchy

Noam Chomsky proposed a hierarchy of languages & grammars

PL microsyntax

Type 3 grammars are regular grammars (equivalent to **RE**s)

- Single ***NT*** on *lhs*; *rhs* has one ***T*** & (optionally) one ***NT***
- Corresponds to a **DFA**

PL syntax

Type 2 grammars are context-free grammars

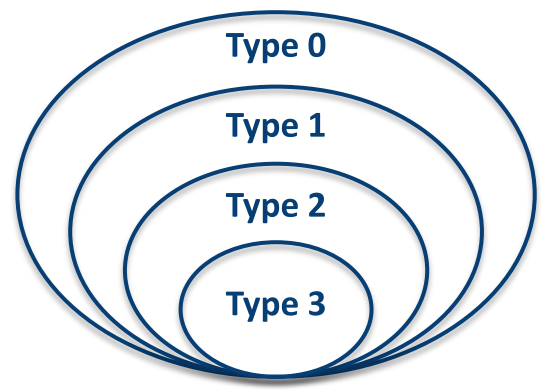
- Single ***NT*** on *lhs*; *rhs* has string of grammar symbols
- Corresponds to a **push-down automaton** (**PDA**)

Type 1 grammars are *context-sensitive grammars*

- Productions of form $\alpha A \beta \rightarrow \gamma$, where α, β, and γ are strings in (***T*** ∪ ***NT***)
- Corresponds to a **linear bounded automaton**

Type 0 grammars are unrestricted grammars

- Includes all formal grammars
- Corresponds to a **Turing machine**



The Chomsky Hierarchy of Grammars

Relating COMP 412 to your friends who major in linguistics.

Limits of Regular Languages

RL's $\subset$ CFL's

Does it matter that RL's $\subset$ CFL's ?

You cannot construct **DFA**'s to recognize these languages

- $L = \{ p^k q^k \}$ *(parentheses, brackets)*
- $L = \{ wcw^r \mid w \in \Sigma^* \}$

Neither of these is a regular language *(nor an **RE**)*

Constructs like these are important to programming languages

But, this is a little subtle. You can construct **DFA**'s for

- Strings with alternating 0's and 1's

  $(\varepsilon \mid 1) (01)^* (\varepsilon \mid 0)$

- Strings with and even number of 0's and 1's

RE's can count bounded sets and bounded differences

Terminology for Derivations

The point of parsing is to discover a derivation

A derivation consists of a series of rewrite steps

$$\textit{Start} \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a **sentence** in *L(G)*
  - If γ contains 1 or more non-terminals, γ is a **sentential form**
- To get γ_i from γ_{i-1}, expand some **NT** $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - Replacing the leftmost **NT** at each step, creates a **leftmost** derivation
  - Replacing the rightmost **NT** at each step, creates a **rightmost** derivation
    - → *We are only interested in systematic derivations*

A **left-sentential form** occurs in a *leftmost* derivation

A **right-sentential form** occurs in a rightmost derivation

NT ≅ *nonterminal symbol*

Terminology for Derivations

The point of parsing is to discover a derivation

Rule	Sentential Form
—	*SheepNoise*
0	*SheepNoise* baa
0	*SheepNoise* baa baa
1	baa baa baa

Top down

Bottom up

Three-word SheepNoise

- A top-down parse begins with the grammar's start symbol and works toward the sentence
- A bottom-up parse starts with the words in the sentence and works towards the start symbol

In the general case[1], discovering a derivation looks expensive

- Many alternatives & combinations, possible backtracking
- Derivation must be guided by the actual words in the sentence
- Fortunately, programming languages tend to have simple syntax
- Understanding parsing will help you see why PLs look as they do!

[1] e.g., Chomsky 0 or 1 grammars

A Better Example

Not a regular language

***SheepNoise* is quite limited. Let's consider a more interesting example.**

0	*Start*	→	*Brackets*
1	*Brackets*	→	**(** *Brackets* **)**
2		|	**[** *Brackets* **]**
3		|	**( )**
4		|	**[ ]**

Two flavors of nested brackets

Rule	***Sentential Form***
—	*Start*
0	*Brackets*
1	**(** *Brackets* **)**
2	**([** *Brackets* **])**
3	**([( )])**

Derivation of "([( )])"

- A sequence of rewrites that produces a sentence is a *derivation*
- Process of discovering a derivation is called *parsing*

We denote this derivation: *Start* $\Rightarrow^*$ ([( )])

We had a question about the goal symbol in lecture 3. This grammar has a specific symbol, *Start*, that serves as the goal symbol of the grammar.

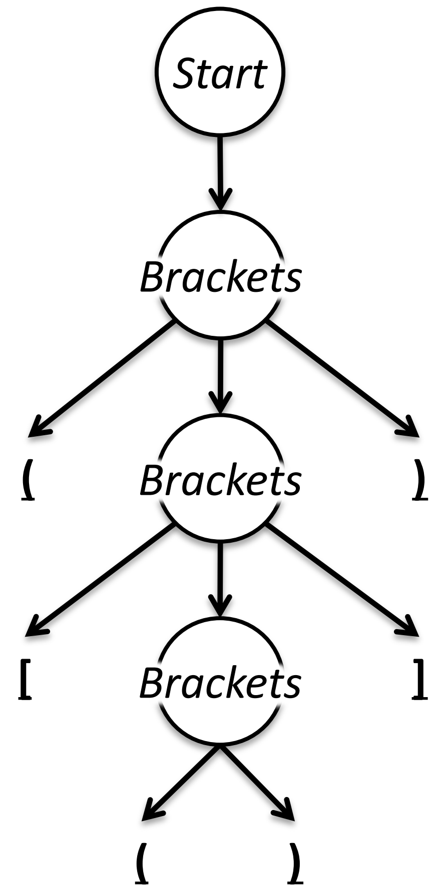
Brackets

A derivation corresponds to a *derivation tree* or *parse tree*

Rule	*Sentential Form*
—	*Start*
0	*Brackets*
1	(*Brackets*)
2	([*Brackets*])
3	([( )])

$Start \Rightarrow^{*}$ ([( )])

The derivation gives us the grammatical structure of the input sentence, which was completely missing in **RE / DFA** recognizers.



Derivation Tree or Parse Tree for this derivation

A Simple Expression Grammar

CFGs are used to define many programming language constructs

0	*Expr*	→	*Expr Op Expr*
1		|	number
2		|	identifier
3	*Op*	→	plus
4		|	minus
5		|	times
6		|	divide

Expressions over +, -, *, /
numbers, & identifiers

Rule	***Sentential Form***
—	*Expr*
0	*Expr Op Expr*
2	<id,x> *Op Expr*
4	<id,x> – *Expr*
0	<id,x> – *Expr Op Expr*
1	<id,x> – <num,2> *Op Expr*
5	<id,x> – <num,2> * *Expr*
2	<id,x> – <num,2> * <id,y>

Derivation of x – 2 * y

When a syntactic category has just one lexeme, as with plus and minus, we will often write it as just the lexeme.

And, if you skipped class & are reading the slides, you should know that this grammar is a very bad way to define expressions

A Simple Expression Grammar

Constructing a derivation

- At each step, we select an **NT** in the current string to replace
- Different choices can lead to different derivations

Two derivations are of interest

- ***Leftmost derivation*** — replace, at each step, the leftmost **NT**
- ***Rightmost derivation*** — replace, at each step, the rightmost **NT**

These are the two systematic derivations (*We don't care about random orders*)

The example on the preceding slide was a *leftmost* derivation

- Of course, there is also a *rightmost* derivation
- In this example, the rightmost derivation is different

Leftmost and Rightmost Derivations

Rule	Sentential Form
—	*Expr*
0	*Expr Op Expr*
2	<id,x> *Op Expr*
4	<id,x> – *Expr*
0	<id,x> – *Expr Op Expr*
1	<id,x> – <num,2> *Op Expr*
5	<id,x> – <num,2> * *Expr*
2	<id,x> – <num,2> * <id,y>

Leftmost Derivation of x – 2 * y

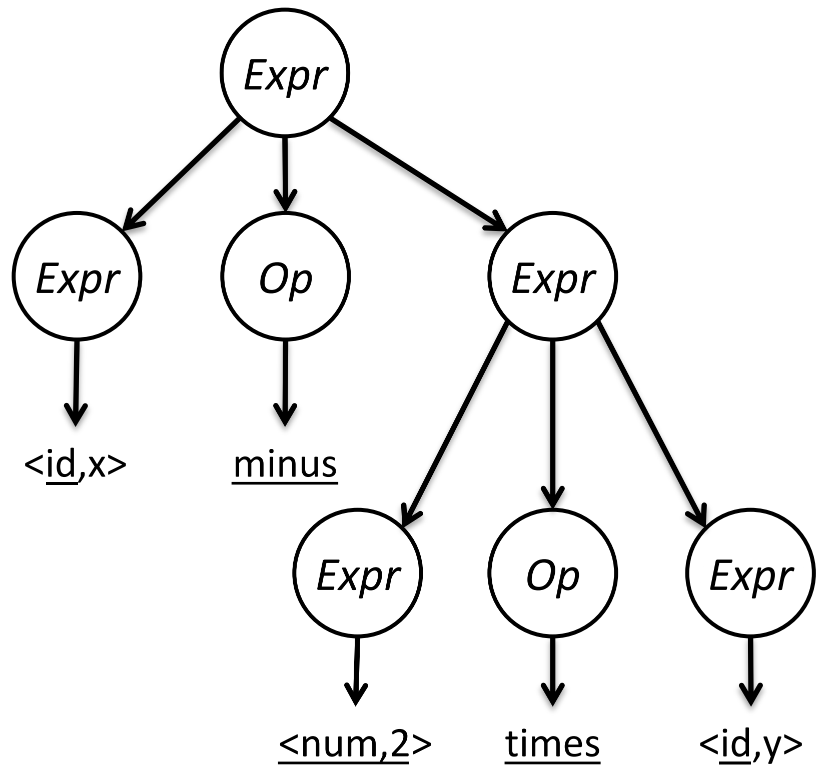
Rule	Sentential Form
—	*Expr*
0	*Expr Op Expr*
2	*Expr Op* <id,y>
5	*Expr* * <id,y>
0	*Expr Op Expr* * <id,y>
1	*Expr Op* <num,2> * <id,y>
4	*Expr* – <num,2> * <id,y>
2	<id,x> – <num,2> * <id,y>

Rightmost Derivation of x – 2 * y

- In both cases, *Expr* $\Rightarrow^*$ identifier – number + identifier
- The two derivations produce different parse trees & evaluation orders

Leftmost Derivation

Rule	Sentential Form
—	*Expr*
0	*Expr Op Expr*
2	<id,x> *Op Expr*
4	<id,x> – *Expr*
0	<id,x> – *Expr Op Expr*
1	<id,x> – <num,2> *Op Expr*
5	<id,x> – <num,2> * *Expr*
2	<id,x> – <num,2> * <id,y>

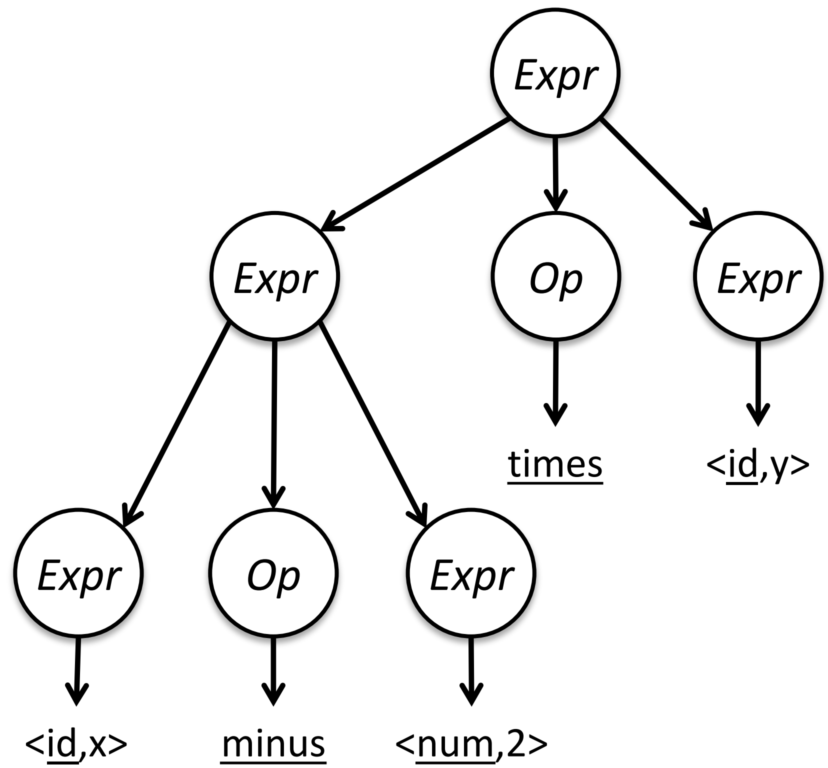


In a postorder treewalk, this parse tree evaluates as **x – (2 * y)**

Parse tree for the leftmost derivation

Rightmost Derivation

Rule	Sentential Form
—	*Expr*
0	*Expr Op Expr*
2	*Expr Op* <id,y>
5	*Expr* * <id,y>
0	*Expr Op Expr* * <id,y>
1	*Expr Op* <num,2> * <id,y>
4	*Expr* – <num,2> * <id,y>
2	<id,x> – <num,2> * <id,y>



In a postorder treewalk, this parse tree evaluates as **(x – 2) * y**

Parse tree for the rightmost derivation

Evaluation Order: Why Do We Care?

The leftmost & rightmost derivations for x – 2 * y produced different evaluation orders.

- These two orders may produce different results, even with integers
  - x − (2 * y) is different than (x − 2) * y, for most values of x & y
  - In floating point, the problem can arise with a string of the same operator
- Standard algebra specifies both an evaluation order (*left to right*) and a precedence (*parentheses; multiply and divide; add and subtract*)

The compiler must pay attention to the intended order of evaluation

Numbers on a computer are not real numbers

- Finite magnitude (e.g., -2^{31} to $2^{31}-1$) introduces overflow & underflow
- Floating-point arithmetic causes unexpected losses of precision
  - There exist x, y, & z such that $x + y > 0$, $(x + y) + z > z$, but $x + (y + z) = z$

Reminder: Why Do We Care About Ambiguity?

It is easy to get lost in language theory

The point of this course is language translation

- Build an executable image that implements the source program
- *Implements* implies that the source program has well-defined meaning

Ambiguity is the opposite of “well-defined”

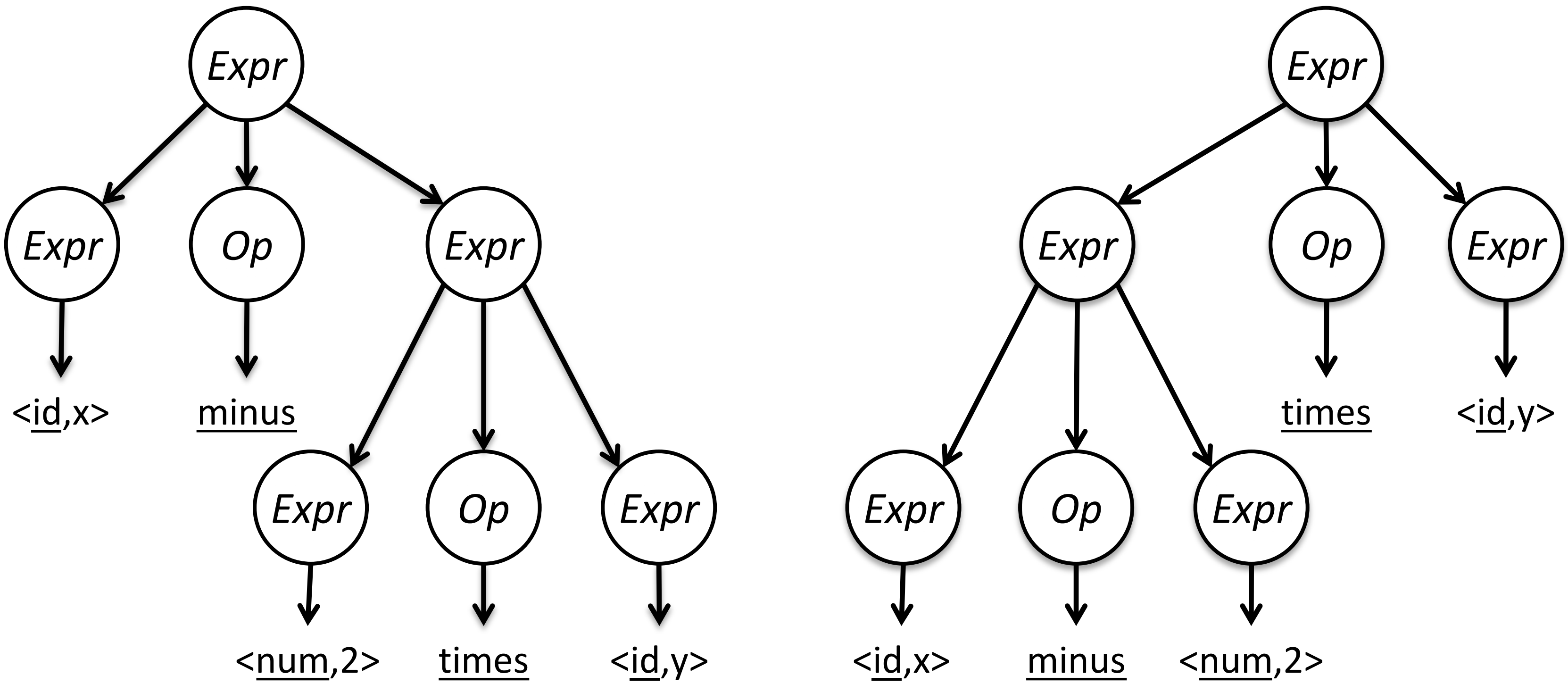
- Ambiguous constructs have multiple meanings
- A program with multiple meanings is not, in general, a good thing

Programming languages (& their designers) should abhor ambiguity

$\Rightarrow$ *except when it is planned and useful, as with operator overloading*

Ambiguity

A grammar that can produce two different parse trees from one input string is, by definition, an ambiguous grammar.



Not a good thing

Ambiguity

A grammar that can produce two leftmost (or two rightmost) derivations from one input string is, by definition, ambiguous.

Rule	*Sentential Form*
—	*Expr*
0	*Expr Op Expr*
2	<id,x> *Op Expr*
4	<id,x> – *Expr*
0	<id,x> – *Expr Op Expr*
1	<id,x> – <num,2> *Op Expr*
5	<id,x> – <num,2> * *Expr*
2	<id,x> – <num,2> * <id,y>

Rule	*Sentential Form*
—	*Expr*
0	*Expr Op Expr*
0	*Expr Op Expr Op Expr*
2	<id,x> *Op Expr Op Expr*
4	<id,x> – *Expr Op Expr*
1	<id,x> – <num,2> *Op Expr*
5	<id,x> – <num,2> * *Expr*
1	<id,x> – <num,2> * <id,y>

Not a good thing

Ambiguity

What should you do with an ambiguous grammar?

You rewrite it to remove the ambiguity.

0	*Expr*	→	*Expr Op Expr*
1		|	number
2		|	identifier
3	*Op*	→	plus
4		|	minus
5		|	times
6		|	divide

Ambiguous Grammar

0	*Expr*	→	*Expr Op Value*
1		|	*Value*
2	*Value*	→	number
3		|	identifier
4	*Op*	→	plus
5		|	minus
6		|	times
7		|	divide

Rewritten Grammar †

In this case, the ambiguity that we see arises from the fact that rule 0 generates *Expr*, its *lhs*, at both the right & left ends of its *rhs*.

† **This rewritten grammar has its own set of problems.**

Ambiguity

Leftmost derivation of x – 2 * y with the rewritten grammar

0	*Expr*	→	*Expr Op Value*
1		|	*Value*
2	*Value*	→	<u>number</u>
3		|	<u>identifier</u>
4	*Op*	→	<u>plus</u>
5		|	<u>minus</u>
6		|	<u>times</u>
7		|	<u>divide</u>

Rule	***Sentential Form***
—	*Expr*
0	*Expr Op Value*
0	*Expr Op Value Op Value*
1	*Value Op Value Op Value*
3	<id,x> *Op Value Op Value*
5	<id,x> – *Value Op Value*
2	<id,x> – <num,2> *Op Value*
6	<id,x> – <num,2> * Value
3	<id,x> – <num,2> * <id,y>

The unambiguous grammar requires one more step in this derivation: the rewrite through *Value* for x.

Seems like a small price to pay. (**TANSTAAFL**)

Ambiguity

Definitions

- A context-free grammar *G* is **ambiguous** if there exists has more than one leftmost derivation for some sentence in *L*(*G*)
- A context-free grammar *G* is **ambiguous** if there exists has more than one rightmost derivation for some sentence in *L*(*G*)
- A context-free grammar *G* is **ambiguous** if the rightmost and leftmost derivations produce different parse trees
  - *However, the rightmost and leftmost derivations may differ*

The classic example — the if-then-else problem

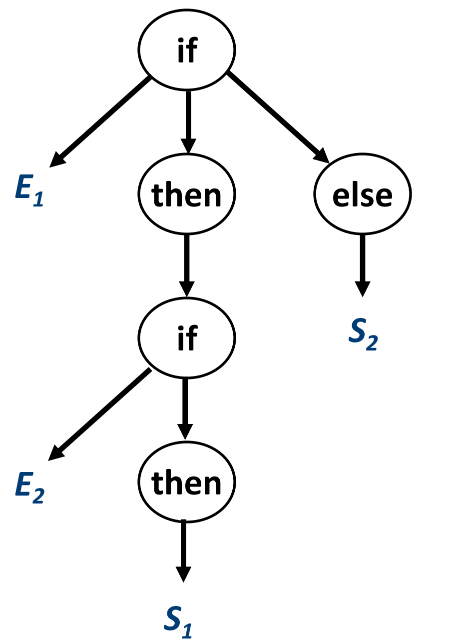
0	*Stmt*	→	if *Expr* then *Stmt*
1		|	if *Expr* then *Stmt* else *Stmt*
2		|	… *other statements* …

Ambiguity

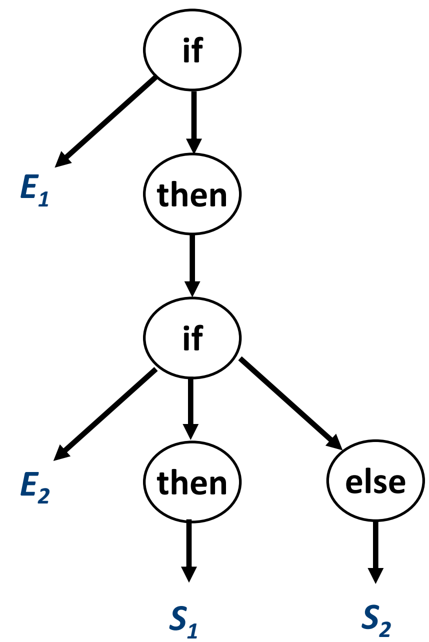
The straightforward if-then-else grammar is ambiguous

Consider the sentential form:

<u>if</u> *Expr*$_1$ <u>then</u> <u>if</u> *Expr*$_2$ <u>then</u> *Stmt*$_1$ <u>else</u> *Stmt*$_2$



production 2, then production 1



production 1, then production 2

Two parse trees, two meanings

Ambiguity

The new grammar forces the structure to match the desired meaning.

Rewriting the grammar to remove the ambiguity

- Must rewrite the grammar to avoid generating the problem
- Match each else to innermost unmatched if (*common sense rule*)

0	*Stmt*	→	if *Expr* then *Stmt*
1		|	if *Expr* then *WithElse* else *Stmt*
2		|	… *other statements* …
3	*WithElse*	→	if *Expr* then *WithElse* else *WithElse*
4		|	… *other statements* …

The critical point: the **if-then** case is not in … *other statements* …

With this grammar, the example has only one rightmost derivation

Intuition: once inside a *WithElse*, derivation cannot generate an unmatched else

… a final if without an else can only come through rule 2 …

This solution is not particularly obvious. Memorize it.

Ambiguity

Derivation for the example sentence

if $Expr_1$ then if $Expr_2$ then $Stmt_1$ else $Stmt_2$

Rule	*Sentential Form*
—	*Stmt*
0	if *Expr* then *Stmt*
1	if *Expr* then if *Expr* then *WithElse* else *Stmt*
2	if *Expr* then if *Expr* then *WithElse* else S_2
4	if *Expr* then if *Expr* then S_1 else S_2
?	if *Expr* then if E_2 then S_1 else S_2
?	if E_1 then if E_2 then S_1 else S_2

Other productions to derive *Expr*s

The new grammar has only one **rightmost** derivation for the example

A Final Word on **IF-THEN-ELSE**

The IF-THEN-ELSE ambiguity is a bit more subtle than it looks

Stmts	→	*Stmts Stmt*
	|	*Stmt*
Stmt	→	*Reference* <u>=</u> *Expr*
	|	<u>IF</u> <u>(</u> *Expr* <u>)</u> <u>THEN</u> *Stmt*
	|	<u>IF</u> <u>(</u> *Expr* <u>)</u> <u>THEN</u> *Stmt* <u>ELSE</u> *Stmt*

… where *Reference* and *Expr* are **NT**s defined elsewhere

We know how to fix this ambiguity using the "withelse" rewrite

What happens if we add a *Stmt* that contains *Stmt*?

Stmt	→	<u>WHILE</u> <u>(</u> *Expr* <u>)</u> *Stmts*

Stmt can derive **IF-THEN-ELSE**, which creates an ambiguity when a **WHILE** is inside an **IF-THEN** or an **IF-THEN-ELSE**

→ *Either disallow **IF-THEN** inside while or require brackets around Stmts list*

Deeper Ambiguity

Ambiguity usually refers to confusion in the CFG

Overloading can create deeper confusions about meaning

a = f(17)

In many Algol-like languages, f can be either a function or a subscripted variable

Disambiguating this confusion requires context

- Need values of declarations
- Really an issue of *type*, not context-free syntax
- Requires an extra-grammatical solution (not in the **CFG**)
- Must handle these with a different mechanism
  - Step outside grammar rather than use a more complex grammar

The alternative: *change the syntax*
C introduced square brackets for subscripts
BCPL used !, the indirection operator

Ambiguity - the Final Word

Ambiguity arises from two distinct sources

- Confusion in the context-free syntax (*if*-*then*-*else*)
- Confusion that requires context to resolve (*overloading*)

Resolving ambiguity

- To remove context-free ambiguity, rewrite the grammar
- To handle context-sensitive ambiguity takes cooperation
  - Knowledge of declarations, types, …
  - Accept a superset of *L(G)* & check it by other means†
  - This is a language design problem

Sometimes, the compiler writer accepts an ambiguous grammar

- Parsing techniques that "do the right thing"
  - → *i.e.,* always select the same derivation
- Occasional language features that put ambiguity to good use

†See Chapter 4