

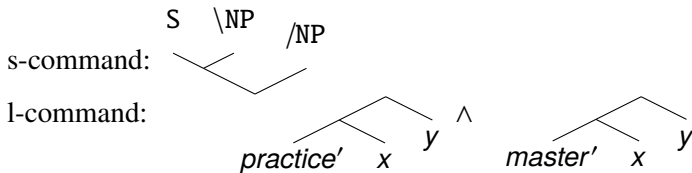
Models of Grammar: Training

Cem Bozşahin

Boğaziçi Linguistics, Ling488

- Suppose that we have developed a grammar which is well-formed according to a theory. Say it studies an idea about some NL phenomenon.
- ‘Well-formed’ means the models of the grammar would be ready for **model validation**. The stage of **model verification** has been reached.
- In our particular case, Bozsahin (2025), we do this by studying linguistic analysis using **linguistic categories alone**.
- These are categories of two command relations: syntactic command (s-command) and semantic command (l-command).

(1) a. **played** :: (S\NP)/NP : $\lambda x \lambda y . practice' xy \wedge master' xy$



b. **sleep** :: S\NP : $\lambda x.torpid'x$



Examples of grammar in TheBench notation:

(2)

```
likes | v :: (s\^np[agr=3s])/^np : \x\y.like x y <120, 1.0>
#np-raise np[agr=?x] : lf --> s/(s\np[agr=?x]) : \lf\p. p lf <34, 1.0>
runs | tense :: s[t=pres,agr=3s]\np:\x.pres run x <2, 1.0>
ran | tense :: s[t=past]\np:\x.past run x <76, 1.0>
```

<key,parameter> : element's unique key and its parameter's value
(added by the system.)

- We now want to put the grammar-idea to experiment. That is, we turn the grammar into a **model**.
- What is the experiment for? Depends on what you wanted to capture with the grammar. (word order, lang. acq., case, grammatical relations etc.)

- We first initialize the grammar so that all and only the elements (data points) get a parameter (aka. **data parameters**). There are no intermediaries.
- We obtain **form:meaning pairs** that we think are correct pairings about the phenomenon we are studying.
- Training will bias the grammar toward certain elements, depending on the experiment.
- Bias and variance control in an experiment is like granma's recipe for cooking: not too much, not too little.
- **Bias** means different things in modeling and statistics. In modeling, it is essentially a **consequent (if not deliberate) error**: what assumptions are made in the model to simplify learning. High bias: strong assumptions. Low bias: flexible.

- Because our grammar is well-formed wrt. a theory, its initial model has in fact passed **model verification**.
- In training, we do **model validation**, that is, check how well the grammar model fits the world (rep. by training pairs).
- We do this by **model training, parameter re-estimation, and model selection**.

- Last thing first: Once parameters are re-estimated and a model is chosen, we can assess the quality of the chosen model using a **parse-ranking algorithm**.
- The one we use is summarized from Zettlemoyer and Collins (2005). This is what the `r`-command of TheBench does (Bozsahin, 2024).

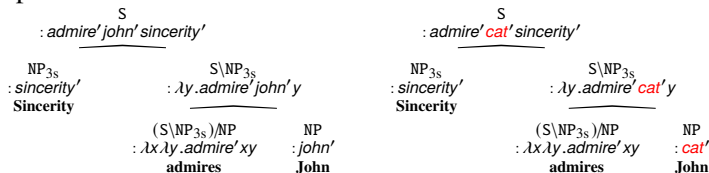
$$\arg \max_L P(L \mid S; \bar{\theta}) = \arg \max_L \sum_D P(L, D \mid S; \bar{\theta}) \quad (1)$$

S is the expression to be parsed, L is the l-command for it,

D is a sequence of derivations for the (S, L) pair,

$\bar{\theta}$ is the n -dimensional **parameter vector** for a grammar of size n (the total number of elements).

Example: Suppose we have two alternative analyses (D s) for the same expression:



$$\arg \max_L P(L \mid \text{sincerity admires John}; \bar{\theta}) = \arg \max_L \sum_D P(L, D \mid \text{sincerity admires john}; \bar{\theta}) \quad (2)$$

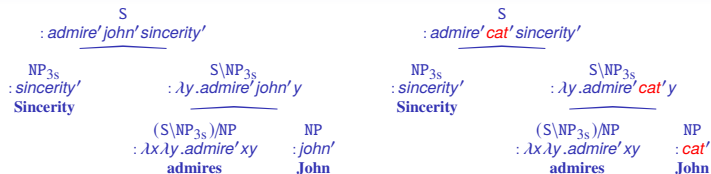
There are two L s. Each has one D . It is a simple choice for the analysis that maximizes the l-command prob. of the expression.

How do we measure $P(L, D \mid \text{sincerity admires john}; \bar{\theta})$?

- It is induced from the following relation of probabilities and parameters.

$$P(L, D \mid S; \bar{\theta}) = \frac{e^{\bar{f}(L, D, S) \cdot \bar{\theta}}}{\sum_L \sum_D e^{\bar{f}(L, D, S) \cdot \bar{\theta}}} \quad (3)$$

- \bar{f} is a vector of 3-argument functions $\langle f_1(L, D, S), \dots, f_n(L, D, S) \rangle$.
- The functions of \bar{f} count local substructure in D . By default, f_i is the number of times the lexical element i (item or rule) is used in D , sometimes called the **feature** i .



- Parameters can be re-estimated from training data of (L_i, S_i) pairs where L_i is the meaning associated with sentence S_i .
- This is what the `t`-command of TheBench does.
- sample training data in TheBench format:

Mary persuaded Harry to study : persuade (study harry) harry mary
Mary promised Harry to study : promise (study mary) harry mary
Mary expected Harry to study : expect (study harry) mary

- The **log-likelihood** of the training data is:

$$O(\bar{\theta}) = \sum_{i=1}^n \log P(L_i \mid S_i; \bar{\theta}) = \sum_{i=1}^n \left(\sum_D P(L_i, D \mid S_i; \bar{\theta}) \right) \quad (5)$$

(To see how likely our training data is according to our grammar, analyze S_i and add up all analyses (D) that led to L_i).

$$O(\bar{\theta}) = \sum_{i=1}^n \log P(L_i | S_i; \bar{\theta}) = \sum_{i=1}^n \left(\sum_D P(L_i, D | S_i; \bar{\theta}) \right) \quad (6)$$

- You can see how syntax is marginalized by summing over all derivations D of (L_i, S_i) . This needs a PARSER. (next lecture)
- For individual parameters we look at the partial derivative of (6) with respect to parameter θ_j .
- The **local gradient** of θ_j with feature f_j for the training pair (L_i, S_i) is the difference between two expected values:
 $E_{\text{true}, f_j}^i = E(L_i, D, S_i)$ $E_{\text{false}, f_j}^i = E(L_k, D, S_i), k \neq i$

$$\frac{\partial O_i}{\partial \theta_j} = \text{if } E_{\text{false}, f_j}^i \approx 0 \text{ then } 0 \text{ else } (E_{\text{true}, f_j}^i - E_{\text{false}, f_j}^i) \quad (7)$$

$$\frac{\partial O_i}{\partial \theta_j} = \begin{cases} \text{if } E_{\text{false}, f_j}^i \approx 0 \text{ then } 0 \text{ else } (E_{\text{true}, f_j}^i - E_{\text{false}, f_j}^i) \end{cases}$$

- The gradient will be negative if feature f_j contributes more to incorrect parses than it does to the correct parses of (L_i, S_i) .
- It will be zero if all parses are correct,
- and positive otherwise.

- Expected values of f_j are therefore calculated under the distributions $P(D \mid S_i, L_i; \bar{\theta})$ and $P(L_k, D \mid S_i; \bar{\theta}), k \neq i$.
- For the overall training set, using sums, the partial derivative is:

$$\frac{\partial O}{\partial \theta_j} = \text{if } E_{\text{false}, f_j} \approx 0 \text{ then } 0 \text{ else } (E_{\text{true}, f_j} - E_{\text{false}, f_j}) \quad (8)$$

$$E_{\text{true}, f_j} = \sum_{i=1}^n \sum_D f_j(L_i, D, S_i) P(D \mid S_i, L_i; \bar{\theta}) \quad (9)$$

$$E_{\text{false}, f_j} = \sum_{i=1}^n \sum_D f_j(L_k, D, S_i) P(D \mid S_i, L_k; \bar{\theta}), k \neq i \quad (10)$$

- We can think of this gradient search as a way to investigate the **Continuity Hypothesis** of Crain and Thornton (1998) in linguistics.

- Every candidate model of grammar would be a possible grammar if the model follows from a theory of NL grammar.
- Once we have the derivative, we use **Stochastic Gradient Ascent** to re-estimate the parameters:

Initialize $\bar{\theta}$ to some value. (11)

‘ for $k = 0 \dots N - 1$

 for $i = 1 \dots n$

$$\bar{\theta} = \bar{\theta} + \frac{\alpha_0}{1 + c(i + kn)} \frac{\partial \log P(L_i | S_i; \bar{\theta})}{\partial \bar{\theta}}$$

- N is the number of passes over the training set,
- n is the training set size,
- α_0 and c are learning-rate parameters.¹

¹Specified in **experiment files**; see TheBench Guide, §7.5, §7.6.

- This is gradient *ascent*, so initialize $\bar{\theta}$ accordingly. Default is 1.0. (Note: This is not a probability).
- **Stochastic** gradient search? Are our grammars stochastic?
 - No. Every grammar is a proxy for **categorical** understanding of the form-meaning relation. **Linguistic grammars** are symbolic empirical species. **Formal grammars** are, ehm, formal species.
 - What is stochastic is the **space** of all (and hopefully only) human grammars.

- After model training and development, we can do **model selection**.
- During training, we tend to generate many models, depending on training parameters (data and hyperparameters).
- This is what the experiment facility of TheBench's `t`-command is designed for. There are as many experiments as the number of lines in an experiment file. See TheBench Guide §7.5, §7.6.
- Unlike LLMs, scientific models do not tweak their response so that model choice can be **independently replicable**.

- Model selection can be
 - performance-based (e.g. accuracy, precision, recall, log-likelihood)
 - cross validation (e.g. split the data into N subsets, train on N-1 subsets and test on 1)
 - generalized testing (check with really unseen data, cf. cross-validation)
 - Bias check (e.g. **overfitting**: high variance, low bias, too little complexity in data for finding patterns, **poor generalization to unseen patterns**)
(**underfitting**: high bias, low variance, too much complexity already in data to allow discovery)
- Model selection has not been streamlined in TheBench. We leave it to the experimenter (for now).

Bozsahin, Cem. 2024. *THEBENCH Guide*.

<https://github.com/bozsahin/thebench>.

Bozsahin, Cem. 2025. *Connecting Social Semiotics, Grammaticality and Meaningfulness: The Verb*. Newcastle upon Tyne: Cambridge Scholars.

Crain, Stephen, and Rosalind Thornton. 1998. *Investigations in Universal Grammar*. Cambridge MA: MIT Press.

Zettlemoyer, Luke, and Michael Collins. 2005. Learning to map sentences to logical form: Structured classification with probabilistic categorial grammars. In *Proc. of the 21st Conf. on Uncertainty in Artificial Intelligence*. Edinburgh.